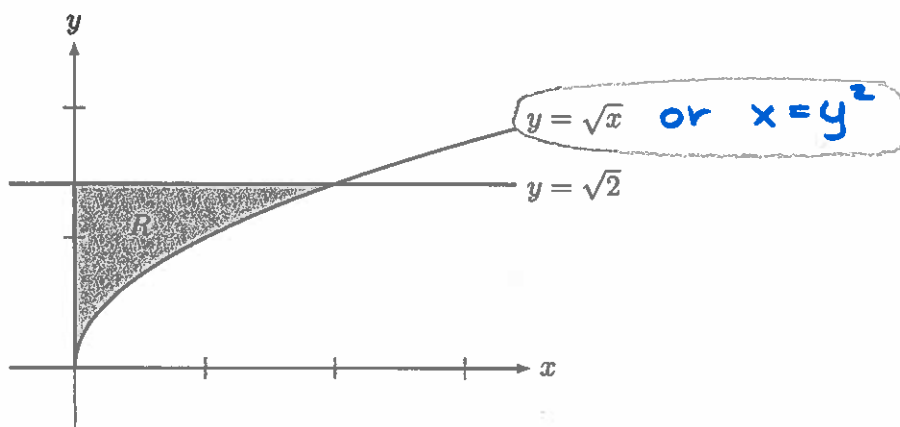


1. For each region R , write $\iint_R f(x,y) dA$ as an iterated integral using $dA = dx dy$ and $dA = dy dx$.

(a)



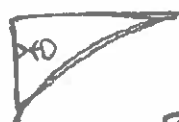
$$dA = dy dx$$



ceiling: $y = \sqrt{2}$
floor: $y = \sqrt{x}$

$$\int_0^2 \int_{\sqrt{x}}^{\sqrt{2}} f(x,y) dy dx$$

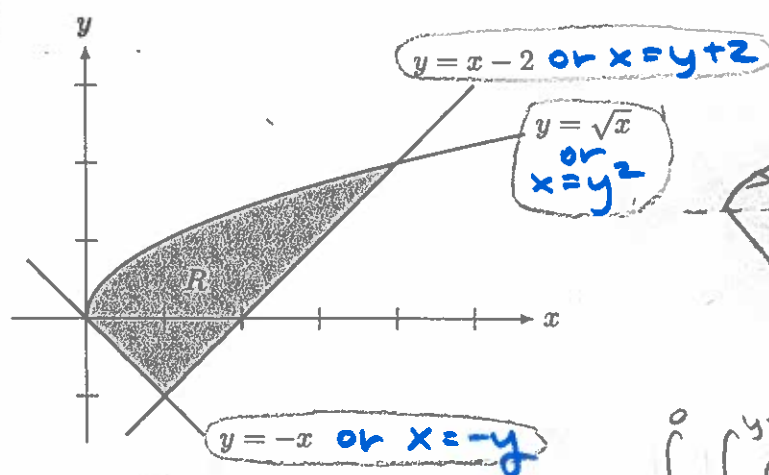
$$dA = dx dy$$



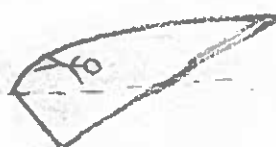
ceiling: $x = y^2$
floor: $x = 0$

$$\int_0^{\sqrt{2}} \int_0^{y^2} f(x,y) dx dy$$

(b)



$$dA = dx dy$$



When $0 \leq y \leq 2$
ceiling: $x = y + 2$
floor: $x = y^2$

when $-1 \leq y \leq 0$,
ceiling: $x = y + 2$
floor: $x = -y$

$$\int_{-1}^0 \int_{-y}^{y+2} f(x,y) dx dy + \int_0^2 \int_{y^2}^{y+2} f(x,y) dx dy$$

$$dA = dy dx$$



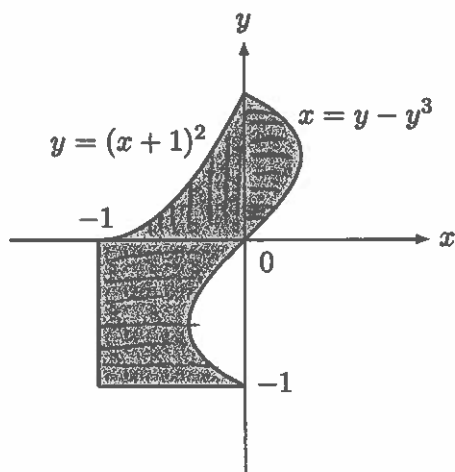
when $0 \leq x \leq 1$, ceiling: $y = \sqrt{x}$
floor: $y = -x$

when $1 \leq x \leq 4$, ceiling: $y = \sqrt{x}$
floor: $y = x - 2$

$$\int_0^1 \int_{-x}^{\sqrt{x}} f(x,y) dy dx + \int_1^4 \int_{x-2}^{\sqrt{x}} f(x,y) dy dx$$

* floor changes
at $x = 1$

2. Compute $\iint_R y \, dA$ where R is the shaded region.



QI: $x=0 \rightarrow x=y-y^3, 0 \leq y \leq 1$

QII: $y=0 \rightarrow y=(x+1)^2, -1 \leq x \leq 0$

QIII: $x=-1 \rightarrow x=y-y^3, -1 \leq y \leq 0$

Answer is
 $\left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{10}\right) + \left(\frac{1}{3} - \frac{1}{5} - \frac{1}{2}\right)$
 $= \frac{20 - 12 + 3 - 15}{30} = -\frac{4}{30}$

$$\int_0^1 \int_0^{y-y^3} y \, dx \, dy = \int_0^1 [xy]_{x=0}^{x=y-y^3} dy$$

$$= \int_0^1 y(y-y^3) - 0 \, dy$$

$$= \int_0^1 y^2 - y^4 \, dy = \left[\frac{1}{3}y^3 - \frac{1}{5}y^5\right]_0^1 = \frac{1}{3} - \frac{1}{5}$$

$$\int_{-1}^0 \int_0^{y=(1+x)^2} y \, dy \, dx = \int_{-1}^0 \left[\frac{1}{2}y^2\right]_{y=0}^{y=(1+x)^2} dx$$

$$= \int_{-1}^0 \frac{1}{2}(1+x)^4 - 0 \, dx$$

$$= \left[\frac{1}{2} \cdot \frac{1}{5}(1+x)^5\right]_{-1}^0 = \frac{1}{10} - 0 = \frac{1}{10}$$

$$\int_{-1}^0 \int_{x=-1}^{x=y-y^3} y \, dx \, dy = \int_{-1}^0 [xy]_{x=-1}^{x=y-y^3} dy = \int_{-1}^0 y^2 - y^4 - (-y) \, dy$$

$$= \left[\frac{1}{3}y^3 - \frac{1}{5}y^5 + \frac{1}{2}y^2\right]_{-1}^0 = 0 - \left(-\frac{1}{3} + \frac{1}{5} + \frac{1}{2}\right) = \frac{1}{3} - \frac{1}{5} - \frac{1}{2}$$