

1. For each integral, determine the appropriate trig sub ($x = \dots$) and draw the corresponding triangle.

(a) $\int \frac{x^2}{\sqrt{x^2 + 4}} dx$

(b) $\int \frac{1}{\sqrt{6 - x^2}} dx$

(c) $\int \frac{x^2}{(x^2 - 9)^{3/2}} dx$

(d) $\int x^3 \sqrt{1 - 9x^2} dx$

2. What is the form of the partial fraction decomposition for the following rational functions?

(a) $f(x) = \frac{x + 1}{x^3 - x}$

(b) $g(x) = \frac{x^3 - 1}{(x + 2)^2(x^2 + 9)}$

3. Write the improper integral as a limit (or sum of limits).

(a) $\int_0^e \frac{\ln(x)}{x} dx$

(b) $\int_1^5 \frac{x}{3 - x} dx$

(c) $\int_{-2}^{\infty} \frac{1}{x^3} + 1 dx$

4. Evaluate the integrals.

(a) $\int \frac{x}{x^2 - 4} dx$

(b) $\int \frac{1}{x^2 - 4} dx$

(c) $\int \frac{1}{x^2 + 4} dx$

(d) $\int \frac{x^3}{x^2 - 4} dx$

(e) $\int \frac{1}{(x^2 - 1)^{3/2}} dx$

(f) $\int \frac{x^3 + 2x + x}{x^2 + 1} dx$

(g) $\int_0^{\pi/2} \cos^5 x dx$

(h) $\int x \arctan x dx$

(i) $\int \frac{e^{1/x}}{x^2} dx$

(j) $\int \sec^3 x \tan x dx$

(k) $\int e^{2x} \sin x dx$

(l) $\int_1^e \frac{\ln x}{x^2} dx$

A u -sub trick: “Back Substitution”

Example: $\int x\sqrt{4-x} \, dx$

We can compute this using a u -sub with

$$\begin{aligned} u &= 4 - x \\ du &= -dx \end{aligned}$$

But we have a hanging x when we do the substitution: $\int \textcolor{red}{x} \sqrt{u} (-1) \, du$.

To fix this we can use the fact that $u = 4 - x$ so $x = 4 - u$.

The full solution looks like:

$$\begin{aligned} \int x\sqrt{4-x} \, dx &= \int (4-u)\sqrt{u} (-1) \, du & \begin{aligned} u &= 4 - x \implies x = 4 - u \\ du &= -dx \end{aligned} \\ &= \int -4\sqrt{u} + u\sqrt{u} \, du \\ &= \int -4u^{1/2} + u^{3/2} \, du \\ &= -4 \left(\frac{2}{3} u^{3/2} \right) + \frac{2}{5} u^{5/2} + C \\ &= -\frac{8}{3} (4-x)^{3/2} + \frac{2}{5} (4-x)^{5/2} + C \end{aligned}$$

5. Use back substitution to compute the integrals.

(a) $\int \frac{x^3}{\sqrt{x^2-4}} \, dx$

(b) $\int e^{2x} \ln(1+e^x) \, dx$

6. Use the Comparison Theorem to determine if the integral converges or diverges.

(a) $\int_2^\infty \frac{x}{\sqrt{x^4-x}} \, dx$

(b) $\int_1^\infty \frac{\sin^2(x)}{\sqrt{x^3}} \, dx$