

using the Midpoint Rule with 12 subintervals and $\Delta t = 2$:

$$\begin{aligned}\int_0^{24} P(t) dt &\approx [P(1) + P(3) + P(5) + \cdots + P(21) + P(23)] \Delta t \\ &\approx (440 + 400 + 420 + 620 + 790 + 840 + 850 \\ &\quad + 840 + 810 + 690 + 670 + 550)(2) \\ &= 15,840\end{aligned}$$

The energy used was approximately 15,840 megawatt-hours. ■

A note on units How did we know what units to use for energy in Example 7? The integral $\int_0^{24} P(t) dt$ is defined as the limit of sums of terms of the form $P(t_i^*) \Delta t$. Now $P(t_i^*)$ is measured in megawatts and Δt is measured in hours, so their product is measured in megawatt-hours. The same is true of the limit. In general, the unit of measurement for $\int_a^b f(x) dx$ is the product of the unit for $f(x)$ and the unit for x .

5.4 Exercises

1–4 Verify by differentiation that the formula is correct.

1. $\int \ln x dx = x \ln x - x + C$

2. $\int \tan^2 x dx = \tan x - x + C$

3. $\int \frac{1}{x^2 \sqrt{1+x^2}} dx = -\frac{\sqrt{1+x^2}}{x} + C$

4. $\int x \sqrt{a+bx} dx = \frac{2}{15b^2} (3bx - 2a)(a+bx)^{3/2} + C$

5–24 Find the general indefinite integral.

5. $\int (3x^2 + 4x + 1) dx$

6. $\int (5 + 2\sqrt{x}) dx$

7. $\int (x + \cos x) dx$

8. $\int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \right) dx$

9. $\int (x^{1.3} + 7x^{2.5}) dx$

10. $\int \sqrt[4]{x^5} dx$

11. $\int \left(5 + \frac{2}{3}x^2 + \frac{3}{4}x^3 \right) dx$

12. $\int (u^6 - 2u^5 - u^3 + \frac{2}{7}) du$

13. $\int (u + 4)(2u + 1) du$

14. $\int \sqrt{t}(t^2 + 3t + 2) dt$

15. $\int \frac{1 + \sqrt{x} + x}{x} dx$

16. $\int \left(x^2 + 1 + \frac{1}{x^2 + 1} \right) dx$

17. $\int \left(e^x + \frac{1}{x} \right) dx$

18. $\int (2 + 3^x) dx$

19. $\int (\sin x + \sinh x) dx$

20. $\int \left(\frac{1+r}{r} \right)^2 dr$

21. $\int (2 + \tan^2 \theta) d\theta$

22. $\int \sec t (\sec t + \tan t) dt$

23. $\int 3 \csc^2 t dt$

24. $\int \frac{\sin 2x}{\sin x} dx$

 **25–26** Find the general indefinite integral. Illustrate by graphing several members of the family on the same screen.

25. $\int (\cos x + \frac{1}{2}x) dx$

26. $\int (e^x - 2x^2) dx$

27–54 Evaluate the definite integral.

27. $\int_{-2}^3 (x^2 - 3) dx$

28. $\int_1^2 (4x^3 - 3x^2 + 2x) dx$

29. $\int_1^4 (8t^3 - 6t^{-2}) dt$

30. $\int_0^8 \left(\frac{1}{8} + \frac{1}{2}w + \frac{1}{3}w^{1/3} \right) dw$

31. $\int_0^2 (2x - 3)(4x^2 + 1) dx$

32. $\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3} \right) dx$

33. $\int_1^3 \left(\frac{3x^2 + 4x + 1}{x} \right) dx$

34. $\int_{-1}^1 t(1 - t)^2 dt$

35. $\int_1^4 \left(\frac{4 + 6u}{\sqrt{u}} \right) du$

36. $\int_0^1 \frac{4}{1 + p^2} dp$

37. $\int_{\pi/6}^{\pi/3} (4 \sec^2 y) dy$

38. $\int_0^{\pi/2} (\sqrt{t} - 3 \cos t) dt$