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# Output Control of Linear Time-invariant Systems Under Input and Output Disturbances ⋆

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**Abstract:** The novel control algorithm for linear time-invariant systems under disturbances and measurement noises is proposed. The designed control law, based on the noise and disturbance estimation, ensures the accuracy in steady state depending on the disturbance, only one component of noise vector and its first derivatives. Sufficient conditions in terms of linear matrix inequality (LMI) providing stability of the closed-loop system are obtained. The simulations show efficiency of the proposed method compared with existing ones.

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*Keywords:* Linear system, disturbance, measurement noise, compensation, LMI.

## 1. INTRODUCTION

The algorithms from Ahrens and Khalil (2009); Anderson and Moore (2005); Astolfy and Marconi (2015); Haykin (1991); Paarmann (2001); Khlebnikov (2017); Prasov and Khalil (2013); Sanfelice and Praly (2011); Wang et al. (2015) are based on disturbance and noise attenuation approaches, using filters and high-gain feedback. The accuracy in steady state, given by these algorithms, sufficiently depends on upper bounds of disturbances, noises and its derivatives. In contrast to Ahrens and Khalil (2009); Astolfy and Marconi (2015); Khlebnikov (2017); Prasov and Khalil (2013); Sanfelice and Praly (2011); Wang et al. (2015) there are disturbance compensation methods allowing to effectively reject disturbances without using filters or high-gain feedback and increase the accuracy in steady state. The idea of disturbance compensation method consists in on-line disturbance estimation and using this estimate to design the control law. As a result the value of the control signal is opposite to the value of the disturbance estimate. The classical results of disturbance decoupling problem (DDP) are considered in Isidori (1995); Wonham (1979). Currently, there are many solutions of DDP proposed for various kinds of plant (see e.g. Conte et al. (2015), Wen et al. (2016)). In Chen et al. (2015) the DDP is solved by the linear quadratic Gaussian optimization and $H^\infty$-optimization assuming that disturbances are random or belong to the class of $L_2$. The DDP with optimal compensation of the arbitrary bounded disturbances is formulated in Yakubovich (1975) and for some special cases this problem is solved in Vidyasagar (1986). The full solution of the DDP with optimal disturbance compensation is obtained in Dahleh and Pearson (1987) by using $l_1$-optimization methods. However, the algorithms based on $l_1$-optimization have high dynamical order and weak convergence to an equilibrium point. In contrast to Dahleh and Pearson (1987); Vidyasagar (1986); Yakubovich (1975) the quality of transients can be improved if disturbances are described by the sum of sinusoidal signals, see for instance Fedele and Ferrise (2013); Marino and Tomei (2002); Pigg and Bodson (2010); Xia (2002). However, the complexity of calculation and implementation of algorithms Fedele and Ferrise (2013); Marino and Tomei (2002); Pigg and Bodson (2010); Xia (2002) significantly depends on number of sinusoidal signals.

Thus, disturbance compensation algorithms Chen et al. (2015); Conte et al. (2015); Dahleh and Pearson (1987); Fedele and Ferrise (2013); Isidori (1995); Marino and Tomei (2002); Vidyasagar (1986); Wen et al. (2016); Wonham (1979); Xia (2002); Yakubovich (1975) provide better accuracy in steady state than disturbance attenuation algorithms Ahrens and Khalil (2009); Anderson and Moore (2005); Astolfy and Marconi (2015); Haykin (1991); Paarmann (2001); Khlebnikov (2017); Prasov and Khalil (2013); Sanfelice and Praly (2011); Wang et al. (2015) without noises. However, these disturbance compensation algorithms cannot be directly applied to systems in presence of noises.

Differently from Chen et al. (2015); Conte et al. (2015); Dahleh and Pearson (1987); Fedele and Ferrise (2013);

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Isidori (1995); Marino and Tomei (2002); Vidyasagar (1986); Wen et al. (2016); Wonham (1979); Xia (2002); Yakubovich (1975), the disturbance and noise compensation method is obtained in Furtat (2017). In contrast to Khlebnikov (2017) and Stoorvogel (1992) in Furtat (2017) the disturbances and noises are different and matrices of the plant model have more general structure. Also, differently from Fedele and Ferrise (2013); Marino and Tomei (2002); Pigg and Bodson (2010); Xia (2002), in Furtat (2017) disturbances and noises may be non-sinusoidal and unlike Astolfy and Marconi (2015); Wang et al. (2015) in Furtat (2017) the noises can have an arbitrary frequency spectrum. Also, in contrast to Ahrens and Khalil (2009); Anderson and Moore (2005); Astolfy and Marconi (2015); Haykin (1991); Paarmann (2001); Khlebnikov (2017); Prasov and Khalil (2013); Sanfelice and Praly (2011); Wang et al. (2015) in Furtat (2017) the accuracy in steady state depends on the first derivative of disturbances and only one component of vector of noises. Thus, algorithm Furtat (2017) is effective under disturbances and noises with large magnitudes.

Despite the abovementioned advantages of algorithm Furtat (2017), it is available only for measured state vector and scalar control signal. Thus, the present paper is dedicated to development of approach Furtat (2017) for solution of these problems.

The rest of the paper is outlined as follows. The problem formulation is presented in Section 2. In Sections 3, 4 the noise estimation algorithm and the control law are designed. In Section 4 the equation of the closed-loop system and main results are given. The simulations illustrate an efficiency of the proposed scheme and confirm analytical results in Section 5. Section 6 collects some conclusions. Finally, the proof of theorems are considered in Appendixes.

The following *notations* are used in the paper: $\mathbb{R}$ is the set of real numbers; $I$ is the identity matrix; $A^+$ is the left pseudo-inverse matrix to $A$; $E_j = [0, ..., 0, 1, 0, ..., 0]^{\mathrm{T}}$ is the vector of the corresponding dimension where the $j$th component is equal to 1 and other components are equal to zero; $\tilde{E} = [E_1, ..., E_{r-1}, E_{r+1}, ..., E_m]$; $|\cdot|$ and $\|\cdot\|$ denotes Euclidean norm of a vector and matrix respectively; $p = \frac{d}{dt}$.

## 2. PROBLEM STATEMENT

Consider a plant model in the form

$$\dot{x} = Ax + Bu + Bf(t), \quad y = Lx, \tag{1}$$
$$z = y + \xi, \tag{2}$$

where $x = x(t) \in \mathbb{R}^n$ is the state vector, $u = u(t) \in \mathbb{R}^l$ is the control signal, $y = y(t) \in \mathbb{R}^m$ is the unmeasured output signal ($m \geq 2$), the signal $z = z(t) \in \mathbb{R}^m$ is available for measurement, $\xi = \xi(t) = [\xi_1(t), ..., \xi_m(t)]^{\mathrm{T}}$ is the bounded noise with bounded component $\xi_r$, $r \in \{1, 2, ..., m\}$. Denote $\chi_1 = \lim\limits_{t\to\infty} \sup\limits_{t\geq 0} |\xi_r(t)|$ and $\chi_2 = \lim\limits_{t\to\infty} \sup\limits_{t\geq 0} |\dot{\xi}_r(t)|$. The unknown function $f = f(t) \in \mathbb{R}^l$ satisfies the conditions $\chi_3 = \lim\limits_{t\to\infty} \sup\limits_{t\geq 0} |f(t)|$ and $\chi_4 = \lim\limits_{t\to\infty} \sup\limits_{t\geq 0} |\dot{f}(t)|$. The matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, $D \in \mathbb{R}^{n\times l}$, $L \in \mathbb{R}^{m\times n}$ are known and $A$ is Hurwitz.

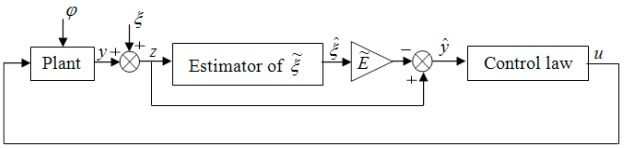



Fig. 1. Structure of the control scheme.

It is necessary to design the algorithm that provides the input-to-state stability of (1) leading to ultimate bound

$$\lim_{t\to\infty} \sup_{t\geq 0} |y(t)| \leq \delta, \tag{3}$$

where $\delta = \Theta(\sum_{i=1}^{4} \chi_i)$ is obtained in Theorem 1 below, $\Theta(\chi)$ for $\chi \in \mathbb{R}$ means that $\lim\limits_{\chi\to 0} \frac{\Theta(\chi)}{\chi} = C$, $C$ is a constant. Thus, the accuracy in steady state depends only on disturbance $\phi$, $r$th component of the noise vector and its first derivatives. In Ahrens and Khalil (2009); Anderson and Moore (2005); Astolfy and Marconi (2015); Furtat et al. (2015); Haykin (1991); Paarmann (2001); Khlebnikov (2017); Prasov and Khalil (2013); Sanfelice and Praly (2011); Wang et al. (2015) the accuracy $\delta$ depends on disturbance $\phi$, all component of noise vector and its derivatives.

Let us briefly consider the main steps of the control scheme synthesis. Plant equations (1), (2) contain two independent unknown signals $f$ and $\xi$. Therefore, at least two independent measurements of the plant output are required for simultaneous compensation of $f$ and $\xi$, consequently, $m \geq 2$. Thus, the algorithm synthesis consists of two steps. At the first step the new noise vector without $r$th component $\tilde{\xi} = [\xi_1, ..., \xi_{r-1}, \xi_{r+1}, ..., \xi_m]^{\mathrm{T}}$ is estimated in Section 3. To this end, the estimation algorithm of $\tilde{\xi}$ is designed (Estimator $\tilde{\xi}$ in Fig. 1). As a result, only the component $\xi_r$ influences on the accuracy of the noise estimation, but not the whole vector $\xi$. At the second step the estimate $\hat{y}$ (see Fig. 1) of the plant output $y$ is constructed in Section 4 by using information about $\hat{\xi}$. Further, the unknown function $f$ is estimated and compensated by using the control law ("Control law" in Fig. 1).

## 3. NOISE ESTIMATION ALGORITHM

In the present section an algorithm for estimation of the vector $\tilde{\xi} = [\xi_1, ..., \xi_{r-1}, \xi_{r+1}, ..., \xi_m]^{\mathrm{T}}$ is constructed. First we obtain equation in variable $\tilde{\xi}$ and then design the estimate of $\tilde{\xi}$.

Eliminate the $r$th equation in (2) and rewrite the result w.r.t. $\tilde{\xi}$. To this end, pre-multiplying (2) by $\tilde{E}^{\mathrm{T}}$, we have

$$\tilde{\xi} = \tilde{E}^{\mathrm{T}} z - \tilde{E}^{\mathrm{T}} L x. \tag{4}$$

It follows from (2) that the variables $x$ and $\xi$ are related, and $x$ is contained in (4). However, the variable $x$ cannot be expressed through $\xi$ from (2). Therefore, according to the structures of (1), (2) and taking into account $\xi = \tilde{E}\tilde{\xi} +$

$E_r\xi_r$, introduce the new variable $\tilde{x} = L^+[z - \tilde{E}\tilde{\xi} - E_r\xi_r]$. Considering (2) and (4), we have $x-\tilde{x} = \tilde{E}^{\mathrm{T}}LA(I-L^+L)x$. Integrating (1) w.r.t. $t$ and substituting results in (4), we get

$$\begin{aligned}\tilde{\xi} &= \int_0^t \Big[\tilde{A}\tilde{\xi}(s) - \tilde{A}_1 z(s)\Big]\,ds + \tilde{E}^{\mathrm{T}}z - \tilde{E}^{\mathrm{T}}Lx(0)\\ &- \int_0^t \Big[\tilde{B}u(s) + \tilde{B}f(s) - \tilde{A}_2\xi_r(s) + \tilde{A}_3 x(s)\Big]ds,\end{aligned} \tag{5}$$

where

$\tilde{A} = \tilde{E}^{\mathrm{T}}LAL^+\tilde{E}$, $\tilde{A}_1 = \tilde{E}^{\mathrm{T}}LAL^+$, $\tilde{A}_2 = \tilde{E}^{\mathrm{T}}LAL^+E_r$, $\tilde{A}_3 = \tilde{E}^{\mathrm{T}}LA(I - L^+L)$, $\tilde{B} = \tilde{E}^{\mathrm{T}}LB$.

Obviously, expression (5) cannot be used to obtain the information about the signal $\tilde{\xi}$, because (5) contains unmeasured signals. However, following the structure of (5), introduce the algorithm for estimation of the vector $\tilde{\xi}$ (Estimator of $\tilde{\xi}$ in Fig. 1) in the form

$$\hat{\xi} = \int_0^t \Big[\tilde{A}\hat{\xi}(s) - \tilde{A}_1 z(s)\Big]\,ds + \tilde{E}^{\mathrm{T}}z, \tag{6}$$

where $\hat{\xi}$ is the estimate of $\tilde{\xi}$. To evaluate the performance of noise estimator (6), consider the following error

$$e = \tilde{\xi} - \hat{\xi}. \tag{7}$$

Differentiate (7) along the trajectories of (5), (6) and rewrite the result in the form

$$\dot{e} = \tilde{A}e - \tilde{B}u - \tilde{B}f + \tilde{A}_2\xi_r - \tilde{A}_3 x. \tag{8}$$

The ultimate bound of $|e|$ depends on the values of $u$ and $f$. Thus, appropriate choice of the control $u$ allows to reduce the influence of $f$ on the value of $|e|$. Therefore, the next section is devoted to design the control law $u$ compensated the influence of $f$.

## 4. CONTROL LAW DESIGN AND MAIN RESULT

Now we obtain information about unknown function $f$ and use this information for design the control law $u$. First clarify information about the output signal $y$ by using the signal $\hat{\xi}$ given by (6). Let $\hat{y}$ be the estimate of $y$ which introduced as (see Fig. 1)

$$\hat{y} = z - \tilde{E}\hat{\xi}. \tag{9}$$

The following assumption is needed for derivation the control law.

Corresponding to (2), (7) and $\xi = \tilde{E}\tilde{\xi} + E_r\xi_r$, rewrite (9) in the form $\hat{y} = Lx + \tilde{E}e + E_r\xi_r$. Taking into account (1), differentiate $\hat{y}$ w.r.t. $t$ and rewrite the result as follows

$$LBf = \dot{\hat{y}} - LAx - LBu - \tilde{E}\dot{e} - E_r\dot{\xi}_r. \tag{10}$$

Consider the method Furtat (2017) for design the control law. It follows from (1) that unknown function $f$ can be compensated if the control law is chosen such that $u = -f$. However, the signal $f$ cannot be used from (10), because it depends on unmeasured signals $x$, $\dot{e}$ and $\dot{\xi}_r$. Therefore, define the control law $u$ as follows

$$u = -\hat{f}, \tag{11}$$

where $\hat{f}$ is the estimate of $f$. According to the structure of (10) and the second equation of (1), define the signal $\hat{f}$ in the form

$$\hat{f} = \dot{\hat{y}} - LAL^+\hat{y} - \alpha(p)LBu. \tag{12}$$

Here $\alpha(p)$ is a scalar differential operator and $p = d/dt$. Let us explain the choice of the operator $\alpha(p)$. Substituting (12) into (11), we have

$$[1 - \alpha(p)]u = -\dot{\hat{y}} + LAL^+\hat{y}. \tag{13}$$

Considering $\alpha(p) = 1 - \mu p$ and taking into account (13), introduce the control law in the form

$$u = -\frac{1}{\mu}\big[\hat{y} - LAL^+\int_0^t \hat{y}(s)ds\big]. \tag{14}$$

Before formulation the main result, consider the following notations

$$\begin{aligned}
&A_{21} = \frac{1}{\mu}[c_0A - BLA(I - L^+L)],\\
&A_{22} = \frac{1}{\mu}[-c_0I + \mu A], \quad A_{23} = \frac{1}{\mu}BLAL^+\tilde{E},\\
&A_{24} = -\frac{1}{\mu}B\tilde{E}, \quad B_{21} = -\frac{1}{\mu}B,\\
&B_{23} = \frac{1}{\mu}BLAL^+E_r, \quad B_{24} = -\frac{1}{\mu}BE_r.\\
&A_{41} = \frac{1}{\mu}(\tilde{B} - c_0\tilde{E}^{\mathrm{T}})LA(I - L^+L),\\
&A_{43} = \frac{1}{\mu}[c_0\tilde{A} - \tilde{B}LAL^+\tilde{E}],\\
&A_{44} = \frac{1}{\mu}[-c_0I + \mu\tilde{A} + \tilde{B}\tilde{E}],\\
&B_{41} = \frac{1}{\mu}\tilde{B},\ B_{43} = \frac{1}{\mu}[c_0\tilde{A}_2 - \tilde{B}LAL^+E_r],\\
&B_{44} = \frac{1}{\mu}[\tilde{B}E_r + \mu\tilde{A}_2].\\
&A_e = \begin{bmatrix} 0 & I & 0 & 0\\ A_{21} & A_{22} & A_{23} & A_{24}\\ 0 & 0 & 0 & I\\ A_{41} & -\tilde{A}_3 & A_{43} & A_{44}\end{bmatrix},\\
&B_e = \begin{bmatrix} 0 & 0 & 0 & 0\\ B_{21} & D & B_{23} & B_{24}\\ 0 & 0 & 0 & 0\\ B_{41} & -\tilde{D} & B_{43} & B_{44}\end{bmatrix},\\
&C_1 = [\,I\ 0\ 0\ 0\,], \quad C_2 = [\,0\ I\ 0\ 0\,],\\
&\tilde{\Psi}_1 = \alpha_1^2C_1^TC_1,\\
&\tilde{\Psi}_2 = 2\alpha_2^2C_2^{\mathrm{T}}C_2 + 2\alpha_3^2C_1^{\mathrm{T}}C_1,\\
&\Psi_{11} = A_e^{\mathrm{T}}P + PA_e + 2\beta P + \tau_1\tilde{\Psi}_1 + \tau_2\tilde{\Psi}_2.
\end{aligned}$$

Setting $x_e = col\{x, \dot{x}, e, \dot{e}\}$ and $d = col\{\varphi, \dot{\varphi}, \xi_r, \dot{\xi}_r\}$, rewrite the closed-loop systems in the form

$$\dot{x}_e = A_ex_e + B_ed. \tag{15}$$

The following result is in order.

*Theorem 1.* Let Assumption 1 and 2 hold. Consider the control system consisting of plant (1), (2), noise estimator (6) and control law (14), (9). Given coefficients $\beta > 0$ and $\mu > 0$, let there exist $\rho > 0$ and the matrix $P > 0$ such that the following LMI holds

$$\Psi = \begin{bmatrix} \Psi_{11} & PB_e \\ * & -\rho I \end{bmatrix} < 0. \tag{16}$$

Here "*" denotes a symmetrical block of a symmetric matrix. Then algorithm (6), (9), (14) ensures goal (3).

Theroem 1 is proved in Appendix A. In Theorem 1 LMI (16) depends on unknown parameter $c_0$ which belongs to the known set $[c_{\min}, c_{\max}]$. Therefore, we formulate the following result to verify (16).

*Theorem 2.* LMI (16) is feasible if the following two LMIs are feasible

$$\Psi^- < 0 \quad \text{and} \quad \Psi^+ < 0, \tag{17}$$

where $\Psi^- = \Psi\big|_{A_e=A_e^-,\ B_e=B_e^-}$, $\Psi^+ = \Psi\big|_{A_e=A_e^+,\ B_e=B_e^+}$, $A_e^- = A_e\big|_{c_0=c_{\min}}$, $A_e^+ = A_e\big|_{c_0=c_{\max}}$, $B_e^- = B_e\big|_{c_0=c_{\min}}$, $B_e^+ = B_e\big|_{c_0=c_{\max}}$.

**Remark.** It follows from (8) that the error $e$ depends only on one component $\xi_r$ of noise vector $\xi$. Assume *a priori* we know that the vector $\xi$ has two components $\xi_i$ and $\xi_j$ such that $\lim\limits_{t\to\infty}\sup\limits_{t\geq 0}|\xi_i(t)| < \lim\limits_{t\to\infty}\sup\limits_{t\geq 0}|\xi_j(t)|$. Then, choosing $r = i$, the value of $\lim\limits_{t\to\infty}\sup\limits_{t\geq 0}|e(t)|$ can be reduced. Also, if we know the smallest component of the vector $\xi$ then it is recommended to choose this one as $r$th component.

## 5. EXAMPLES

Consider plant (1), (2). The following parameters are known: $A = \begin{bmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 3 & 2 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Other unknown parameters and functions in (1), (2) are defined below.

Choose parameters in algorithm (6), (9), (14). Employing $r = 2$ and $L^+ = L^{\mathrm{T}}$, we get $\tilde{A} = -1$, $\tilde{A}_1 = [-1\ \ 1]$, $LAL^+ = \begin{bmatrix} -1 & 1 \\ -3 & 0 \end{bmatrix}$.

Verify the conditions of Theorem 2 using Yalmip package. LMIs given by (17) are feasible for $\mu \in [10^{-6},\ 0.09]$ in (14). Simulations in Matlab Simulink show that solutions of (1) are bounded for $\mu \in (0,\ 0.3]$.

Compare the proposed results with algorithms from Furtat et al. (2015) and Khlebnikov (2017). Let $\mu = 0.01$ in (14). The robust algorithm Furtat et al. (2015) is presented by

$$u = -100v,\ \ v = \begin{bmatrix} v_1^1 + 2v_1^2 + v_1^3 \\ v_2^1 + 2v_2^2 + v_2^3 \end{bmatrix},$$
$$v_i^1 = z_i,\ \ v_i^2 = \tfrac{z_i - z_i(t-0.01)}{0.01}, v_i^3 = \tfrac{v_i^2 - v_i^2(t-0.01)}{0.01},\quad i = 1, 2.$$

The algorithm Khlebnikov (2017) are defined as

$$u = -J\hat{z},\quad \dot{\hat{z}} = A\hat{z} + Bu + S(\hat{z} - z),$$

where the matrices $J = \begin{bmatrix} 0.09 & 0.62 & 1.13 \\ 1.12 & 0.11 & 1.26 \end{bmatrix}$ and $S = -\begin{bmatrix} 0.27 & 0.66 & 0.28 \\ 0.69 & 2.53 & 2.99 \\ 0.24 & 2.95 & 6.27 \end{bmatrix}$ are calculated such that the ellipsoid $x^{\mathrm{T}}Px = 1$, $P > 0$ has the smallest semiaxes.

Consider plant (1), (2) with $x(0) = [1\ \ 1\ \ 1]^{\mathrm{T}}$, $c_0 = 0$, $\xi_1 = 1 + 10\sin(3t)$, $\xi_2 = 0.01\sin(0.8t)$, $\psi$ and $\varphi$ are given by (**??**). Fig. 2 and Fig. 3 show the transients of $y = [y_1\ \ y_2]^{\mathrm{T}}$ obtained by algorithms Furtat et al. (2015), Khlebnikov (2017) and the proposed one.

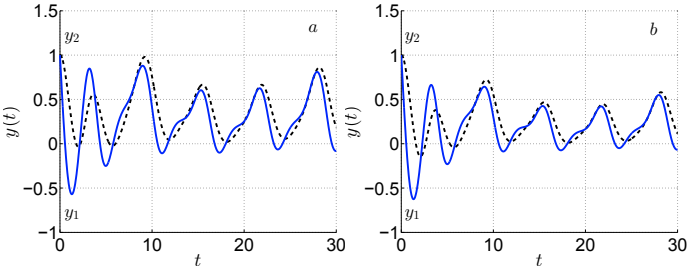



Fig. 2. The transients of $y(t)$ for the algorithms from Furtat et al. (2015) ($a$) and Khlebnikov (2017) ($b$).

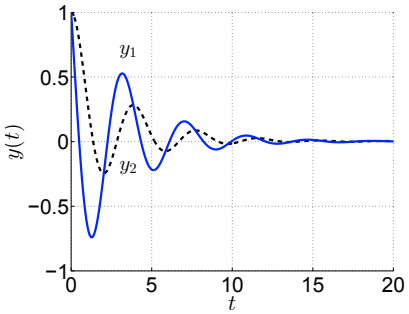



Fig. 3. The transients of $y(t)$ for the proposed algorithm.

The transients obtained by algorithms Furtat et al. (2015) and Khlebnikov (2017) depend on signals $\xi_1$, $\xi_2$, while the transients obtained by the proposed algorithm depend on signal $\xi_2$ only. Also, the simulations have been demonstrated that upper bound of $\mu$ obtained by (17) has a permissible deviation from the upper bound of $\mu$ obtained by Matlab Simulink.

## 6. CONCLUSIONS

The output feedback control algorithm with compensation of disturbances and measurement noises is designed for nonlinear multi-input multi-output systems. In contrast to Ahrens and Khalil (2009); Astolfy and Marconi (2015); Pigg and Bodson (2010); Khlebnikov (2017); Prasov and Khalil (2013); Sanfelice and Praly (2011); Wang et al. (2015) in the presented paper the dimension of noise can be equal to the dimension of plant output, disturbances and noises are independent, and the control law has only one design parameter $\mu$. Sufficient conditions in terms of linear matrix inequalities provide the stability of the closed-loop system. The accuracy in the steady state depends on the disturbance, one component of noise and its first

derivatives. Numerical examples illustrate the efficiency of the proposed method under smooth and random noises and disturbances.

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## Appendix A. PROOF OF THEOREM 1

For the input-to-state stability analysis of (15) consider the following Lyapunov function

$$V = x_e^{\mathrm{T}} P x_e. \tag{A.1}$$

Find the condition such that the following inequality holds

$$\dot{V} + 2\beta V - \rho d^{\mathrm{T}} d \le 0. \tag{A.2}$$

To this end, employing (15) and (A.1), rewrite (A.2) as follows

$$\begin{aligned} &x_e^{\mathrm{T}} \left(A_e^{\mathrm{T}} P + P A_e\right) x_e \\ &+2x_e^{\mathrm{T}} P B_e d + 2\beta x_e^{\mathrm{T}} P x_e - \rho d^{\mathrm{T}} d \le 0. \end{aligned} \tag{A.3}$$

Denoting $z = col\{x_e, d\}$ the closed-loop system is stable if LMI (16) holds.

Now let us proof the boundedness of all signals in the closed-loop system. Since $x_e$ is ultimate bounded, the signals $x$, $\dot{x}$, $e$ and $\dot{e}$ are ultimate bounded. The ultimate boundedness of $z$ follows from (2). The signal $\hat{\xi}$ is bounded from (7). The signal $\hat{y}$ is bounded from (9). The boundedness of $\int_0^t \hat{y}(s)ds$ follows from (14). Thus, the function $\int_0^t \left[\tilde{A}\hat{\xi}(s) - \tilde{A}_1 z(s)\right] ds$ is bounded from (6). Consequently, all signals are bounded in the closed-loop system.

## Appendix B. PROOF OF THEOREM 2

The matrix $\Psi$ is affine w.r.t. the matrices of the system (15). It follows from (4) that the matrices $A_e$ and $B_e$ are linearly dependent on the parameter $c_0$. Thus, according to Remark 2 in Fridman (2010), LMI (16) is feasible if two LMIs (17) are feasible.