Choose a strategy to complete each problem.

- Compute directly
- Use Stokes' Theorem
- Use Divergence Theorem
- Use a bonus trick (close S and use Divergence Theorem OR switch to a better surface)

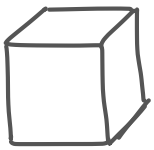
1. Suppose $\mathbf{F}(x, y, z) = (x^3 - e^y)\mathbf{i} + (y^3 + \sin z)\mathbf{j} + (z^3 - xy)\mathbf{k}$ and S is the surface of the solid bounded above by $z = \sqrt{4 - x^2 - y^2}$ and below by the xy-plane, oriented outward. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$.



includes the base

↳ S is closed and $\operatorname{div} \vec{F} = 3x^2 + 3y^2 + 3z^2$ is nicer! ⇒ use Divergence Theorem

2. Compute the flux of $\mathbf{F}(x, y, z) = y\mathbf{j} - z\mathbf{k}$ through S, where S is the surface of the cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 3$, $z = 0$ and $z = 4$, oriented outward.



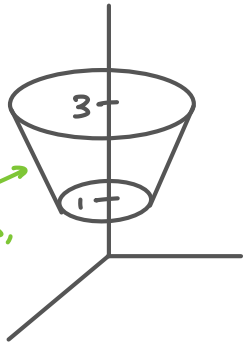
S is closed (and would be a pain to parameterize!)

Also $\operatorname{div} \vec{F} = 1 - 1 = 0$ ⇒ use Divergence Theorem

$$\left(\text{flux thru } S = \iint_S \vec{F} \cdot d\vec{S} = \iiint_E 0 \, dV = 0\right)$$

3. Let S be the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$. Compute $\iint_S x^2 z^2 \, dS$.

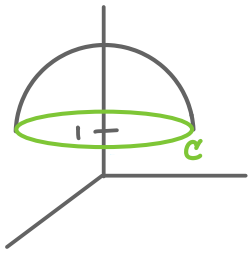
↖ this is a surface integral of a scalar field so we MUST compute directly (no theorems apply)



just the sides, no top or bottom

Recall: $dS = \|\vec{r}_u \times \vec{r}_v\| \, dA$ where $\vec{r}(u, v)$ is a parametrization of S

4. Compute $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z)=\langle x^2yz,\ yz^2,\ z^3e^{xy}\rangle$ and S is the part of the sphere $x^2+y^2+z^2=5$ that lies above the plane $z=1$, oriented upward.

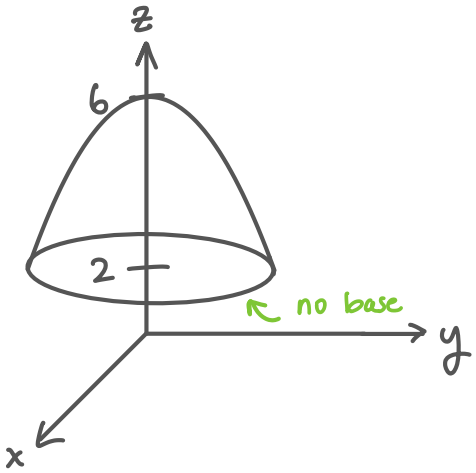


S is not closed

integrating curl $\vec{F}$ $\Rightarrow$ use Stokes'

(Notice if we try to compute directly we'd have to compute curl $\vec{F}$ since we're only given $\vec{F}$. For this VF, it's pretty involved.)

5. Let $\mathbf{F}(x,y,z)=y\mathbf{i}+z\mathbf{j}+x\mathbf{k}$ and let S be the surface $z=6-x^2-y^2$ where $z\geq 2$, oriented upwards. Compute $\iint_S \mathbf{F}\cdot d\mathbf{S}$.



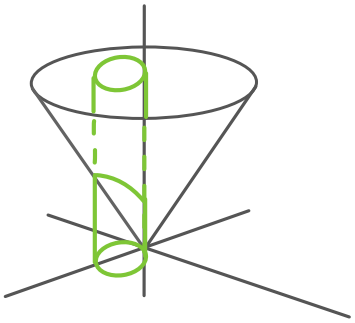
S is not closed

option 1: compute directly

option 2: close S and use Div. Thm. (Div. Thm. trick)

6. Find the surface area of the portion of the cone $z=\sqrt{x^2+y^2}$ that lies inside the cylinder $x^2+y^2=2x$.

↳ $\iint_S 1\, dS$ where $dS = \|\vec{r}_u \times \vec{r}_v\|\, dA$



Another integral of a scalar function so we MUST compute directly