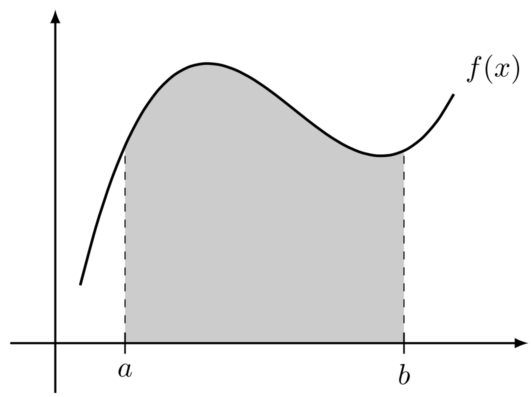
Moments and Center of Mass Formulas

Consider a lamina (flat plate) in the shape of region in the plane bounded by a function $f(x)$ and the x-axis on an interval $[a, b]$. Assume the lamina has uniform density ρ.



Total Mass: $\boxed{m = \int_a^b \rho f(x)\,dx}$

Moments: These measure the tendency of the lamina to rotate about the x or y-axis

$$\boxed{M_y = \int_a^b \rho x f(x)\,dx} \quad \text{and} \quad \boxed{M_x = \int_a^b \frac{1}{2}\rho f(x)^2\,dx}$$

(y-moment) (x-moment)

Center of Mass: The center of mass is $(\overline{x}, \overline{y})$ where $\overline{x}$ and $\overline{y}$ are given by the formulas below.

$$\boxed{\overline{x} = \frac{M_y}{m} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx} = \frac{1}{A}\int_a^b x f(x)\,dx} \quad \text{and} \quad \boxed{\overline{y} = \frac{M_x}{m} = \frac{\int_a^b \frac{1}{2} f(x)^2\,dx.}{\int_a^b f(x)\,dx} = \frac{1}{A}\int_a^b \frac{1}{2} f(x)^2\,dx}$$

(x-coordinate) (y-coordinate)