

Final Exam Topics

The final exam is cumulative. We have covered the following sections (not in order): 1.7, 5.5–5.7, 5.9–5.10, 6.1–6.6, 8.1–8.8 and Appendix H1 & H2. Important topics from these sections are listed below.

New topics that have not been tested on a previous midterm are **bolded**.

Important Topics:

- Integration techniques
 - u -substitution
 - Integration by parts
 - Partial fraction decomposition
 - Trig identities and u -sub (for integrals involving trig functions)
 - Trig substitution
- Evaluating improper integrals
- Calculating volumes using...
 - Method of Disks/Washers
 - Method of Shells
 - Known cross sections
- Applications
 - Average rate of change
 - Work problems (ropes, springs, water pumping problems)
 - Center of mass
- Parametric equations
 - Finding the slope & equation of a tangent line to a parametric curve
 - Finding the arc length of a parametric curve
- Polar coordinates
 - Sketching a polar curve/recognizing standard polar curves
 - Finding the slope & equation of a tangent line to a polar curve
 - Finding the arc length of a polar curve
 - Finding the area enclosed by a polar curve or area between two polar curves
- Sequences
- Series
 - Definition using partial sums
 - Telescoping series
- Series tests (for convergence or divergence)
 - Divergence Test
 - Geometric Series Test
 - Integral Test

- p -series Test
- Comparison and Limit Comparison Tests
- Alternating Series Test
- Ratio Test
- Absolute vs conditional convergence
- Power series
 - Determining the radius and interval of convergence for a power series
 - Using $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to find a power series representation for a similar function (section 8.6)
- **Taylor/Maclaurin series** & Taylor polynomials
 - Finding the n th degree Taylor polynomial for a function
 - **Finding a Taylor series representation of a function and its radius of convergence**
 - **Using known Maclaurin series to find a power series representation of a function**
- Series remainder estimates
 - Alternating Series Remainder Estimate
 - Integral Test Remainder Estimate
 - **Taylor's Inequality**

The following formulas WILL BE INCLUDED on the exam. You do not need to memorize them.

Arc Length:

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$L = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Area in Polar Coordinates:

$$A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$$

Remainder Estimate for the Alternating Series Test:

Suppose $S = \sum a_k$ is a convergent alternating series and $a_k = (-1)^k b_k$ or $a_k = (-1)^{k+1} b_k$ where $b_k \geq 0$ for each n . Then the remainder $R_n = |S - S_n|$ satisfies

$$|R_n| = |S - S_n| \leq b_{n+1}.$$

Remainder Estimate for the Integral Test:

Suppose $f(k) = a_k$ where $f(x)$ is a continuous, positive, decreasing function and $\sum a_k$ is convergent. Then the remainder $R_n = |S - S_n|$ satisfies

$$\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx.$$

Maclaurin Series:

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
- $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
- $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$
- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

Taylor's Inequality:

If $|f^{(n+1)}(x)| \leq M$ for all x satisfying $|x-a| \leq d$, then

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1} \quad \text{for } |x-a| \leq d.$$

OR

If $|f^{(n+1)}(x)| \leq M$ for all x between a and x_0 , then

$$|R_n(x_0)| \leq \frac{M}{(n+1)!} |x_0-a|^{n+1}.$$