Goal: To evaluate integrals of the form $\int \sin^m x \cos^n x\, dx$.

1. $\int \sin x \cos^4 x\, dx$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\, dx \end{array}} \Rightarrow (-1)du = \sin x\, dx$

$= \int u^4 (-1)\, du$

$= -\frac{1}{5}u^5 + C = -\frac{1}{5}\cos^5 x + C$

2. $\int \sin^3 x\, dx$ (*Hint: Use the identity* $\sin^2 x + \cos^2 x = 1$, *then do a u-substitution.*)

$\hookrightarrow \sin^2 x = 1 - \cos^2 x$

$= \int \sin^2 x \cdot \sin x\, dx$

$= \int (1-\cos^2 x) \cdot \sin x\, dx$ $\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\, dx \end{array}} \Rightarrow (-1)du = \sin x\, dx$

$= \int (1-u^2)(-1)\, du$

$= \int u^2 - 1\, du = \frac{1}{3}u^3 - u + C = \frac{1}{3}\cos^3 x - \cos x + C$

3. $\int \sin^5 x \cos^2 x\, dx$ (*Hint: write* $\sin^5 x = (\sin^2 x)^2 \sin x$.)

$= \int (\sin^2 x)^2 \cdot \sin x \cdot \cos^2 x\, dx$

$= \int (1-\cos^2 x)^2 \cdot \sin x \cdot \cos^2 x\, dx$ $\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\, dx \end{array}}$

$= \int (1-u^2)^2 \cdot u^2 \cdot (-1)\, du$

$= -\int (1-2u^2+u^4)u^2\, du = -\int u^2 - 2u^4 + u^6\, du$

$= -\left(\frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7\right) + C = -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$

4. Use the same strategy as the previous problem. (The algebra gets hairy, so stop once you do the substitution.)

$$\int \sin^7 x \cos^4 x \, dx = \int (\sin^2 x)^3 \cdot \sin x \cdot \cos^4 x \, dx$$
$$= \int (1-\cos^2 x)^3 \cdot \sin x \cdot \cos^4 x \, dx$$
$$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array}}$$
$$= \int \underbrace{(1-u^2)^3}_{\text{to finish, expand this out}} \cdot u^4 \cdot (-1)\, du$$

5. Describe your strategy to evaluate any integral of the form $\int \sin^m x \cos^n x \, dx$ where m is odd.

Save one copy of $\sin x$ (for du) and convert the remaining even power of sines to cosines. Use $u = \cos x$ (so $(-1)du = \sin x \, dx$.)

6. The same type of trick works if the power on $\cos x$ is odd. What trig identity and u-sub would you use to evaluate the following integral?

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cdot \cos^2 x \cdot \cos x \, dx$$
$$= \int \sin^2 x \cdot (1-\sin^2 x) \cdot \cos x \, dx$$
$$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$$
$$= \int u^2 (1-u^2)\, du$$

Use $\cos^2 x = 1 - \sin^2 x$ and $u = \sin x$.

7. Describe your strategy to evaluate any integral of the form $\int \sin^m x \cos^n x \, dx$ where n is odd.

Save one copy of $\cos x$ (for du) and convert the remaining even power of cosines into sines. Use $u = \sin x$ (so $du = \cos x \, dx$)