

dV in Spherical Coordinates

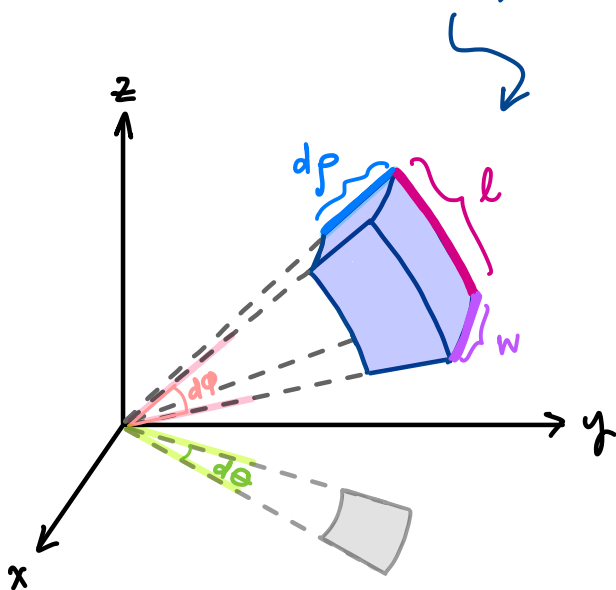
When converting a triple integral from rectangular to spherical coordinates, we change the " dV " that we are integrating with respect to.

$$\iiint_E f(x,y,z) dV_{xyz} = \iiint_E f(\rho, \phi, \theta) dV_{\rho\phi\theta}$$

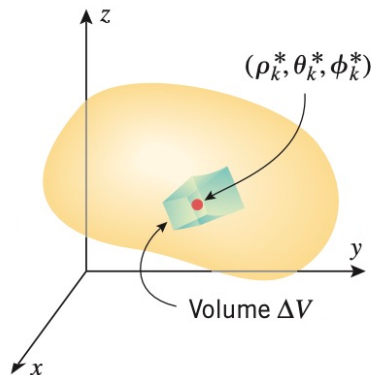
↑ rectangular dV
 $dV = dx dy dz = dz dy dx = \dots$

↑ spherical dV

For the spherical dV , the idea is to divide up E into chunks like this, instead of boxes. →



Fill in E with chunks like this:



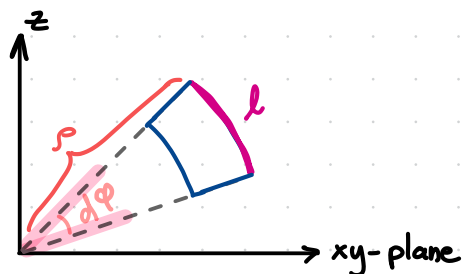
$$\text{spherical } dV = \text{volume of chunk} \approx (l)(w)(d\rho)$$

We need to express l and w in terms of ρ, φ and θ .

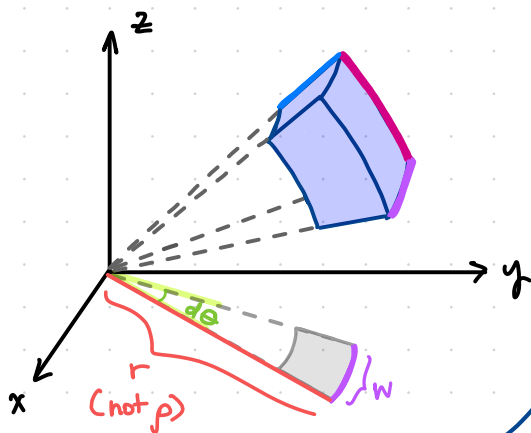
Think of ρ, φ and θ as the spherical coordinates of a test point and $d\rho, d\varphi$ and $d\theta$ are small increments in each of those variables.

l is a sector of a circle of radius ρ with angle $d\varphi$

$$\Rightarrow l = \rho d\varphi$$

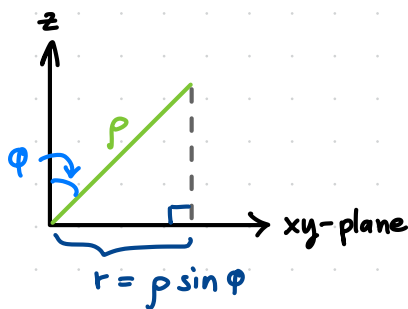


w is a sector of a circle of a different radius (call it r) because the circle we use is in the xy -plane



$$w = r d\theta$$

But what is r ?



$$\Rightarrow w = \rho \sin \varphi d\theta$$

$$\begin{aligned} \text{Therefore, } dV &= (\rho d\varphi)(\rho \sin \varphi d\theta) d\rho \\ &= \rho^2 \sin \varphi d\rho d\varphi d\theta \end{aligned}$$