

Written HW - IBP on #5-7, 12, 13

↑
some

Integration by Parts

ex: $\int x^3 \ln x \, dx$

~~$u = x^3$ $dv = \ln x \, dx$
 $du = 3x^2 \, dx$ $v = \underline{\quad ?? \quad}$~~

$u = \ln x$	$dv = x^3 \, dx$
$du = \frac{1}{x} \, dx$	$v = \frac{1}{4} x^4$

$$= uv - \int v \, du$$

$$= \ln x \cdot \left(\frac{1}{4} x^4\right) - \int \frac{1}{4} x^4 \cdot \frac{1}{x} \, dx$$

$$= \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^3 \, dx$$

$$= \frac{1}{4} x^4 \ln x - \underbrace{\frac{1}{4} \cdot \frac{1}{4} x^4}_{\frac{1}{16}} + C$$

example : $\int \underbrace{\ln x}_u \underbrace{1 \cdot dx}_{dv}$ (trick: $dv = dx$)

$$\left[\begin{array}{ll} u = \ln x & dv = dx \\ du = \frac{1}{x} dx & v = x \end{array} \right]$$

$$\begin{aligned} \int \ln x \, dx &= uv - \int v \, du \\ &= \ln x \cdot x - \int x \cdot \frac{1}{x} \, dx \\ &= x \ln x - \int \underline{1} \, dx \\ &= x \ln x - x + C \end{aligned}$$

IBP for Definite Integrals:

$$\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$$

example: $\int_0^1 \arctan x \, dx$ ↙

$$\begin{array}{ll} u = \arctan x & dv = dx \\ du = \frac{1}{1+x^2} dx & v = x \end{array} \quad \} \quad \swarrow$$

$$= uv \Big|_0^1 - \int_0^1 v du$$

$$= (\arctan x) x \Big|_0^1 - \int_0^1 x \cdot \frac{1}{1+x^2} dx$$

$$= \arctan(1) - \cancel{\arctan(0) \cdot 0} - \int_{x=0}^{x=1} \frac{x}{1+x^2} dx$$

$$u = 1+x^2$$

$$du = 2x dx$$

$$\hookrightarrow \frac{1}{2} du = x dx$$

$$\text{bounds: } x=0 \Rightarrow u=1$$

$$x=1 \Rightarrow u=2$$

$$= \arctan(1) - \int_{u=1}^{u=2} \frac{1}{u} \cdot \frac{1}{2} du$$

$$= \arctan(1) - \frac{1}{2} \ln|u| \Big|_1^2$$

$$= \arctan(1) - \frac{1}{2} (\ln 2 - \ln 1)$$

$$= \pi/4 - \frac{1}{2} \ln 2$$



$$\theta = \pi/4$$

example: $\int e^{2x} \cos x \, dx$ Boomerang trick

$$\int e^{2x} \cos x \, dx$$

$$\begin{aligned} u &= e^{2x} & dv &= \cos x \, dx \\ du &= 2e^{2x} \, dx & v &= \sin x \end{aligned}$$

$$\begin{aligned} \int e^{2x} \cos x \, dx &= uv - \int v \, du \\ &= e^{2x} \sin x - \int \sin x \cdot 2e^{2x} \, dx \\ &= e^{2x} \sin x - 2 \int e^{2x} \sin x \, dx \end{aligned}$$

$$\begin{aligned} u &= e^{2x} & dv &= \sin x dx \\ du &= 2e^{2x} & v &= -\cos x \end{aligned}$$

★ stay consistent with choice of u & dv

$$= e^{2x} \sin x - 2 \left(-e^{2x} \cos x - \int -\cos x \cdot 2e^{2x} dx \right)$$

$$= e^{2x} \sin x - 2 \left(-e^{2x} \cos x + 2 \int e^{2x} \cos x dx \right)$$

$$= e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x dx$$

Let $I = \int e^{2x} \cos x \, dx$.

$$I = e^{2x} \sin x + 2e^{2x} \cos x - 4I$$

+4 I

$+4I$

$$5I = e^{2x} \sin x + 2e^{2x} \cos x$$

$$I = \frac{1}{5} e^{2x} \sin x + \frac{2}{5} e^{2x} \cos x + C$$

Picking u & dv . Consider the examples we've done.

examples: $\int x \sin x dx$ $\int x^2 \cos(2x) dx$

$\int x^3 \ln x dx$ $\int \ln x dx$
 $dv = dx$

$\int_0^1 \arctan x dx$ $\int e^{2x} \cos x dx$
 $dv = dx$

Types of functions: trig functions
exponentials
power functions
logs
inverse trig

Q: What happens to each type when we differentiate?

- ④ { trig functions ($\sin x / \cos x$) — same
exponentials ($e^x, e^{2x}, 2^x$) — same
- ③ power functions (x^n) — get a little nicer

(degree decreases
by 1)

① logs ($\ln x \rightarrow \frac{1}{x}$) - gets a lot nicer!

② $\begin{cases} \arctan x \rightarrow \frac{1}{1+x^2} & \text{a lot nicer!} \\ \arcsin x \rightarrow \frac{1}{\sqrt{1-x^2}} & \text{nicer} \end{cases}$

Guideline for choosing u (from best to worst choice)

LIPET

L - logs

I - inverse trig

P - powers of x

E - exponentials

T - trig functions

exercise: Pick u & dv for the following.

① $\int x^2 (\ln x)^2 dx$

$$u = (\ln x)^2 \quad dv = x^2 dx$$

$$du = 2 \ln x \cdot \frac{1}{x} dx$$

$$(2) \int \frac{1}{x^2} \arctan x \, dx$$

$$u = \arctan x \quad dv = \frac{1}{x^2} dx$$

$$du = \frac{1}{1+x^2} dx$$

$$(3) \int x \sec^2 x \, dx$$

$$u = x \quad dv = \sec^2 x \, dx$$

$$du = dx \quad v = \tan x$$

$$(4) \int \frac{x}{e^{2x}} \, dx = \int \overset{\substack{\uparrow \\ \text{power} \\ \text{of } x}}{x} \cdot \underbrace{\frac{1}{e^{2x}}}_{\text{exponential}} \, dx$$

$$u = x \quad dv = e^{-2x} \, dx$$

$$du = dx \quad v = -\frac{1}{2} e^{-2x}$$

$$= x \left(-\frac{1}{2} e^{-2x} \right) - \int -\frac{1}{2} e^{-2x} \, dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} \, dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \left(-\frac{1}{2} e^{-2x} \right) + C$$

ex: $\int (x^2+1) \cdot e^x dx$

$\underbrace{\hspace{10em}}$ "power of x"
 $\uparrow$ exp.

$\underbrace{\hspace{15em}}$

$$uv - \int v du = \int u dv = \int f(x) \cdot g'(x) dx$$

$$\int \underbrace{(x^2+1)}_u \cdot \underbrace{e^x}_{dv} dx$$