Aggregating distributed energy resources for grid flexibility services: a distributed game theoretic approach

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Overview

1. Aggregative Game Formulation
2. Nash Equilibrium Seeking Algorithm
3. Numerical Study
4. Conclusion

Background

- Increasing penetration of uncertain and volatile renewable energy generation into power system
- Increasing need for flexibility
- Demand side management for Distributed Energy Resources (DERs)

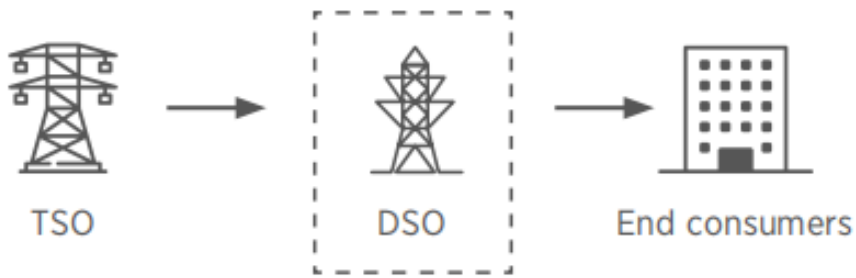


Figure: Traditional power system structure

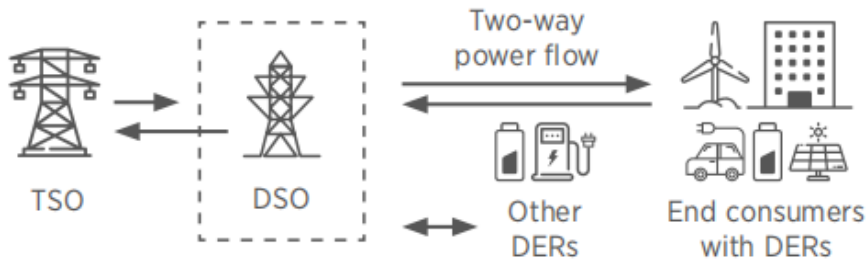


Figure: Power system structure with DERs deployment

Problem setting

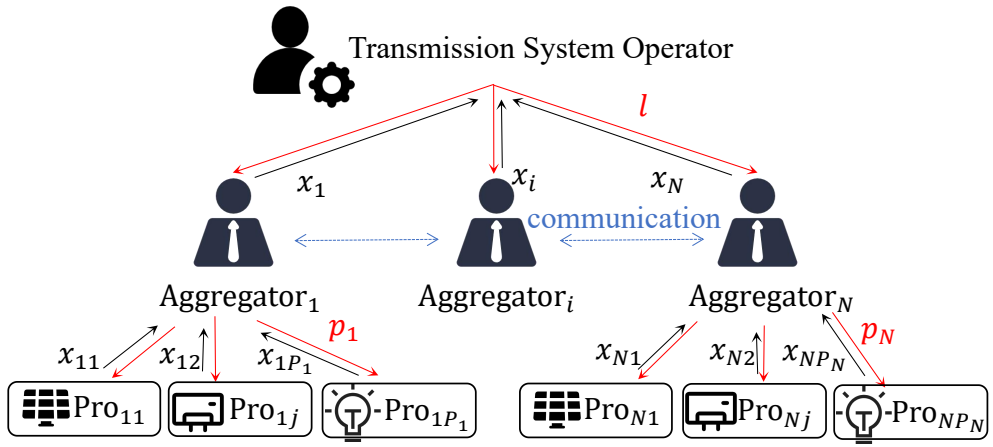
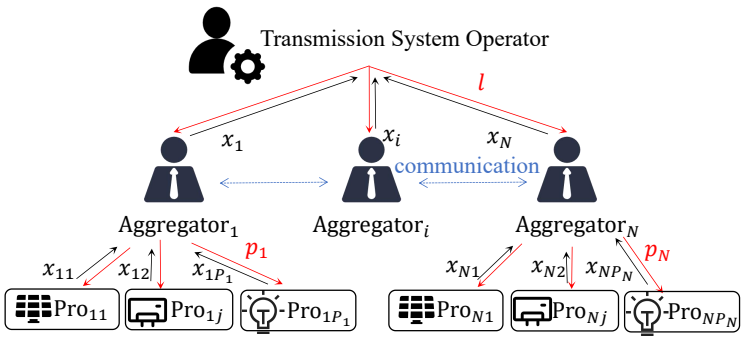


Figure: Hierarchical energy management scheme

- **TSO:** procuring energy flexibility services from DERs of prosumers.
- **Aggregators:** buying flexibility from prosumers and selling it to TSO.
- **Prosumers:** providing flexibility services with DERs.

Mathematical model: Prosumer

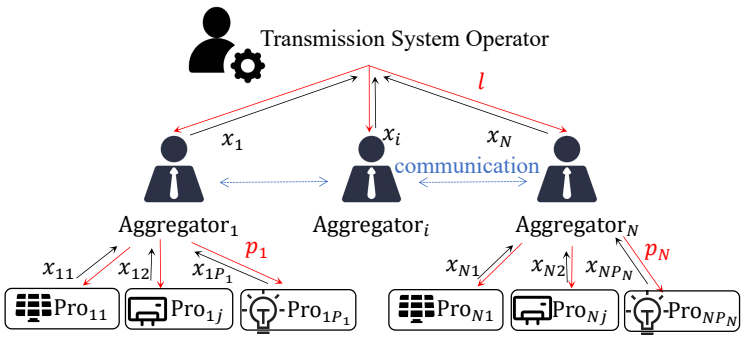


Prosumer ij's goal: maximizing the net profit based on the incentive price p_i proposed by aggregator i.

$$\max_{x_{ij} \geq 0} U_{ij}(x_{ij}), \quad U_{ij}(x_{ij}) := x_{ij} p_i - g_{ij}(x_{ij}),$$

g_{ij}: the cost/inconvenience of the prosumer for providing the flexibility x_{ij}.

Mathematical model: Prosumer



The aggregated flexibility from all prosumers managed by aggregator i:

$$x_i = \sum_{j \in \mathcal{P}_i} x_{ij}$$

Lemma 1

$$p_i(x_i) = \beta_i + \alpha_i x_i, \quad \forall x_i \geq \underline{x}_i,$$

where $\alpha_i = 1/\sum_{j \in \mathcal{P}_i} \frac{1}{a_{ij}}$, $\beta_i = \sum_{j \in \mathcal{P}_i} \frac{b_{ij}}{a_{ij}} / \sum_{j \in \mathcal{P}_i} \frac{1}{a_{ij}}$ and $\underline{x}_i = (\max_{j \in \mathcal{P}_i} b_{ij} - \beta_i)/\alpha_i$.

Mathematical model: Aggregator

Aggregator i's goal: maximizing its net profit,

$$\min_{x_i \in \Omega_i} J_i(x_i, \mathbf{x}), \quad J_i(x_i, \mathbf{x}) := x_i p_i(x_i) - l(\mathbf{x}) x_i.$$

The flexibility price $l(\cdot) : \mathbb{R}^N \to \mathbb{R}$ linearly depends on the aggregated flexibility $s(\mathbf{x}) = \frac{1}{N} \mathbb{1}_N^\top \mathbf{x}$

$$l(\mathbf{x}) = l_0 - \lambda s(\mathbf{x}),$$

The set of feasible strategy set of the aggregator i

$$\Omega_i = \{x_i \in \mathbb{R} \mid \underline{x}_i \leq x_i \leq \bar{x}_i\},$$

$\bar{x}_i$: the maximum available flexibility
$\underline{x}_i$: the minimum available flexibility

Aggregative game formulation

The aggregative game among aggregators:

$$\mathcal{G} = \{\mathcal{I}, \Omega, \text{col}(J_i(x_i, s(\mathbf{x})))_{i\in\mathcal{I}}\}$$

The Nash Equilibrium (NE) of the game:

$$x_i^* \in \arg\min_{\Omega_i} J_i(x_i, \mathbf{x}_{-i}^*)$$

Lemma 2

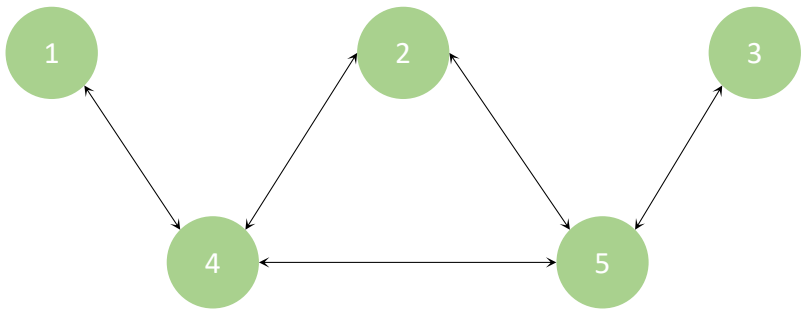
The game $\mathcal{G}$ has a unique NE. Moreover, the NE is the unique solution of the variational inequality $\text{VI}(\Omega, \hat{F})$.

where $\hat{F}(\mathbf{x})$ is the pseudo-gradient mapping of the game $\mathcal{G}$.

Challenges

Gradient-based algorithm:

$$x_i^{k+1} = x_i^k - \tau f_i(x_i^k, s(x^k))$$



1. **Full Information setting**: knowing all x_{-i} and sharing x_i with all aggregators
2. **Partial Information setting**: estimating all x_{-i} and sharing x_i with neighbouring aggregators

Challenge: What if each aggregator does not know x_{-i} or is not willing to share x_i with neighbouring aggregators?

Proposed Algorithm

Shared Information:

x_i: the quantity of flexibility. $\times$

σ_i: a local estimate of the aggregate quantity $s(\mathbf{x})$. ✓

ψ_i: an additional auxiliary variable aiming to reach a consensus on the local estimate σ_i. ✓

- **Initialization:** for all $i \in \mathcal{I}$, set $x_i(0) \in \Omega_i$, $\sigma_i(0) \in \mathbb{R}$, $\psi_i(0) \in \mathbb{R}$
- **Communication:** for all $i \in \mathcal{I}$, aggregator i exchanges $\sigma_i(k)$, $\psi_i(k)$ with its neighbors $\mathcal{N}_i$.
- **Iterations:** for all $i \in \mathcal{I}$,

$$x_i(k+1) = \mathrm{Proj}_{\Omega_i}\Big(x_i(k) - \tau_i k_i f_i\big(x_i(k), \sigma_i(k)\big)\Big),$$

$$\psi_i(k+1) = \psi_i(k) + v_i \sum_{j \in \mathcal{N}_i} w_{ij}\big(\sigma_i(k) - \sigma_j(k)\big),$$

$$\sigma_i(k+1) = \sigma_i(k) + \rho_i\big(x_i(k) - \sigma_i(k)\big) - \rho_i \sum_{j \in \mathcal{N}_i} w_{ij}\Big(2\big(\psi_i(k+1) - \psi_j(k+1)\big) + \big(\psi_i(k) - \psi_j(k)\big)\Big).$$

Proposed Algorithm

The algorithm in a compact form:

$$\mathbf{x}(k+1) = \mathrm{Proj}_{\Omega}\Big(\mathbf{x}(k) - \tau K F\big(\mathbf{x}(k), \boldsymbol{\sigma}(k)\big)\Big),$$
$$\boldsymbol{\psi}(k+1) = \boldsymbol{\psi}(k) + \mathbf{v} L \boldsymbol{\sigma}(k),$$
$$\boldsymbol{\sigma}(k+1) = \boldsymbol{\sigma}(k) + \boldsymbol{\rho}\Big(\mathbf{x}(k) - \boldsymbol{\sigma}(k) - L\big(2\boldsymbol{\psi}(k+1) - \boldsymbol{\psi}(k)\big)\Big),$$

Question 1: Is the fix point $\mathbf{x}$ of this algorithm is the NE of our game?
Question 2: Could this algorithm converge to the fix point?

Proposed Algorithm

Question 1: Is the fix point $\mathbf{x}$ of this algorithm is the NE of our game?

Proposition 1

If the step sizes are chosen properly, the algorithm is equivalent to the forward-backward iteration,

$$\begin{aligned}\boldsymbol{\omega}(k+1) &= (\mathrm{Id} + \Phi^{-1}\mathcal{B})^{-1} \circ (\mathrm{Id} - \Phi^{-1}\mathcal{A})(\boldsymbol{\omega}(k)) \\ &= \mathcal{V}_\Phi \circ \mathcal{U}_\Phi(\boldsymbol{\omega}(k)),\end{aligned}$$

Moreover, any fixed point of this iteration is given by $\overline{\boldsymbol{\omega}} = \mathrm{col}(\overline{\mathbf{x}}, \overline{\boldsymbol{\psi}}, \overline{\boldsymbol{\sigma}}) = \mathrm{col}(\mathbf{x}^*, \overline{\boldsymbol{\psi}}, s(\mathbf{x}^*)\mathbb{1}_N)$, where $\mathbf{x}^*$ is the NE.

$\boldsymbol{\omega} := \mathrm{col}(\mathbf{x}, \boldsymbol{\psi}, \boldsymbol{\sigma})$, $\mathcal{U}_\Phi := \mathrm{Id} - \Phi^{-1}\mathcal{A}$, $\mathcal{V}_\Phi := (\mathrm{Id} + \Phi^{-1}\mathcal{B})^{-1}$, and

$$\mathcal{A}: \begin{bmatrix}\mathbf{x}\\ \boldsymbol{\psi}\\ \boldsymbol{\sigma}\end{bmatrix} \mapsto \begin{bmatrix}KF(\mathbf{x},\boldsymbol{\sigma})\\ 0\\ -\mathbf{x}+\boldsymbol{\sigma}\end{bmatrix}, \mathcal{B}: \begin{bmatrix}\mathbf{x}\\ \boldsymbol{\psi}\\ \boldsymbol{\sigma}\end{bmatrix} \mapsto \begin{bmatrix}N_\Omega(\mathbf{x})\\ -L\boldsymbol{\sigma}\\ L\boldsymbol{\psi}\end{bmatrix}, \Phi = \begin{bmatrix}\boldsymbol{\tau}^{-1} & 0 & 0\\ 0 & \mathbf{v}^{-1} & L\\ 0 & L & \boldsymbol{\rho}^{-1}\end{bmatrix}.$$

Proposed Algorithm

Question 2: Could this algorithm converge to the fix point?

Theorem 1

If the step sizes and parameter k_i are chosen properly, any sequence generated by the algorithm converges to the fixed point $\overline{\boldsymbol{\omega}} = \text{col}(\overline{\mathbf{x}}, \overline{\boldsymbol{\psi}}, \overline{\boldsymbol{\sigma}}) = \text{col}(\mathbf{x}^*, \overline{\boldsymbol{\psi}}, s(\mathbf{x}^*)\mathbb{1}_N)$.

$$k_i \in \left(\frac{(\sqrt{\mu_i + \lambda} - \sqrt{\mu_i})^2}{\lambda^2}, \frac{(\sqrt{\mu_i + \lambda} + \sqrt{\mu_i})^2}{\lambda^2} \right)$$

$$\tau_i < 2\epsilon, \quad \nu_i < 2\epsilon, \quad \rho_i^{-1} > \lambda_{\max}^2(L)\left(\frac{1}{\max_{i\in\mathcal{I}} \nu_i} - \frac{1}{2\epsilon}\right)^{-1} + \frac{1}{2\epsilon},$$

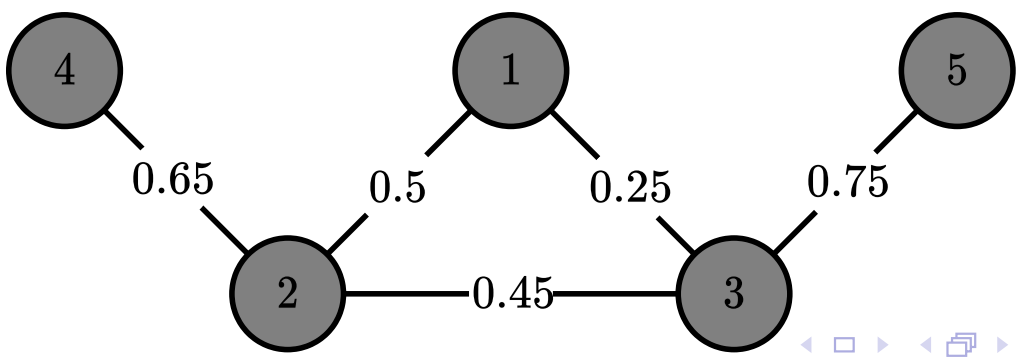
with $\mu_i = 2\alpha_i + \frac{1}{N}\lambda$, $\epsilon := \frac{\lambda_{\min}\left(R+R^\top\right)}{2\lambda_{\max}(R^\top R)}$,

$R := \text{blockdiag}(R_i)_{i\in\mathcal{I}}$, $R_i := \begin{bmatrix} k_i\mu_i & k_i\lambda \\ -1 & 1 \end{bmatrix}$.

Numerical Study

We consider $N = 5$ aggregators providing flexibility services with $\lambda = 1$ and $l_0 = 30\big(\$/(\text{kWh})\big)$.

Aggregator	α_i	β_i	$\overline{x}_i$	$\underline{x}_i$
1	0.1403	3.1408	36.4000	9.6792
2	0.1582	3.0646	38.7000	8.7302
3	0.1215	2.9669	37.0000	9.4634
4	0.1315	3.0169	40.4000	11.0666
5	0.1829	3.0023	34.8000	7.5663



Numerical Study

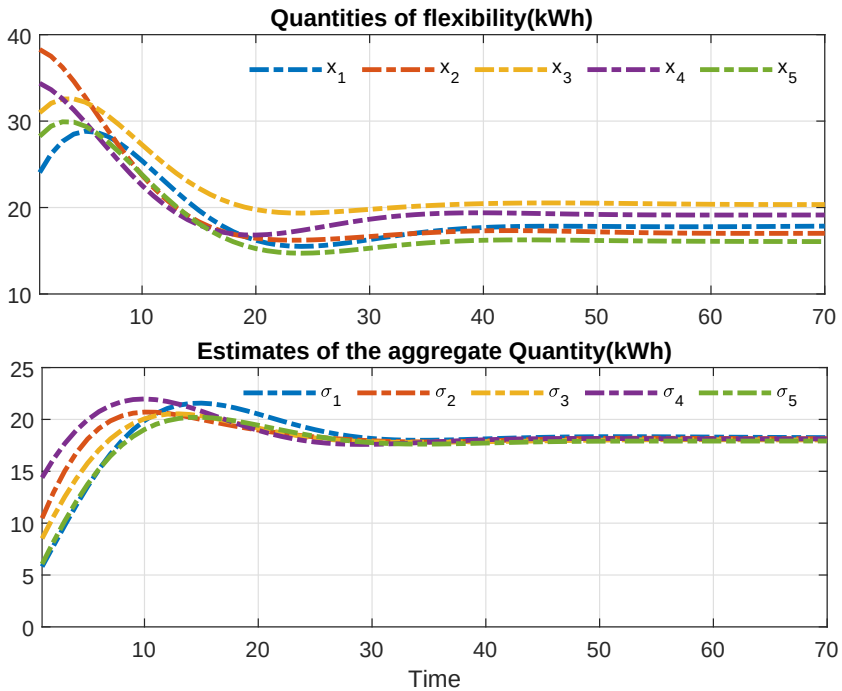


Figure: The evolution of actions

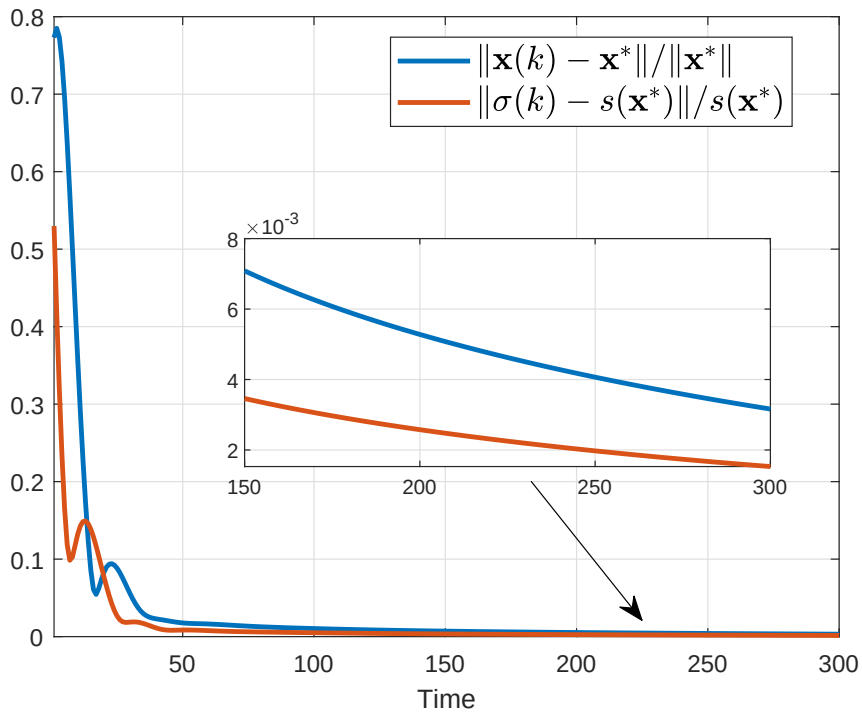


Figure: The normalized errors

Conclusion

Conclusion

- We propose a hierarchical energy management scheme for aggregating distributed energy resources for grid flexibility services.
- A fully distributed algorithm is designed, where only a local estimate of the aggregated flexibility and an auxiliary variable are shared with neighboring aggregators.

Future work

- Including coupling constraints among different aggregators, e.g. due to line capacity constraints, which leads to a generalized Nash equilibrium problem.

Thank You