

For each of the positive-term series $\sum a_n$ below, we can determine convergence by comparing to another series $\sum b_n$. Choose the sequence b_n to compare to, say whether $\sum b_n$ converges or diverges, and indicate whether you would use Term-size Comparison test or the Limit Comparison Test to carry out the justification. Also give the inequality you will use for Term-size Comparison, or the value of the limit, for the Limit Comparison Test. For this assignment, the goal is to get the big picture, so just for today we'll leave out the detailed justification.

1. $\sum_{n=1}^{\infty} \frac{n-1}{n^3+n}$

$b_n = \frac{1}{n^2}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{n-1}{n^3+n} < \frac{1}{n^2}$

2. $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$

$b_n = \frac{1}{n}$, $\sum b_n$ diverges

Use Limit comparison

Note: $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 2$

3. $\sum_{n=1}^{\infty} \frac{n^3+2n-1}{n^5-2n+3}$

$b_n = \frac{1}{n^2}$, $\sum b_n$ converges

Use Limit comparison

Note: $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1$

4. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+3n+1}{n^2+\sqrt{n}+1}$

$b_n = \frac{1}{n}$, $\sum b_n$ diverges

Use Limit comparison

Note: $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 3$

5. $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^7+2n+1}}$

$b_n = \frac{1}{n^{3/2}}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{n^2}{\sqrt{n^7+2n+1}} < \frac{1}{n^{3/2}}$

6. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

$b_n = \frac{1}{n}$, $\sum b_n$ diverges

Use Term-size comparison

Note: $\frac{\ln n}{n} > \frac{1}{n}$ (for $n > e$)

7. $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$

$b_n = \frac{1}{n^2}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{1}{n^2 \ln n} < \frac{1}{n^2}$ (for $n > e$)

8. $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

$b_n = \frac{1}{n^2}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{\ln n}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$

9. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2 + 1}$

$b_n = \frac{2}{n^2}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{\arctan n}{n^2 + 1} < \frac{2}{n^2}$

13. $\sum_{n=2}^{\infty} \frac{5 + 3 \sin n}{n - 1}$

$b_n = \frac{2}{n}$, $\sum b_n$ diverges

Use Term-size comparison

Note: $\frac{5 + 3 \sin n}{n - 1} > \frac{2}{n}$

10. $\sum_{n=2}^{\infty} \frac{\arctan n}{\sqrt{n^3 - 1}}$

$b_n = \frac{1}{n^{3/2}}$, $\sum b_n$ converges

Use Limit comparison

Note: $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \pi/2$

14. $\sum_{n=1}^{\infty} \frac{5}{n2^n}$

$b_n = \frac{5}{2^n}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{5}{n2^n} < \frac{5}{2^n}$

11. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{4n^5 + n}$

$b_n = \frac{1}{n^5}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{\sin^2 n}{4n^5 + n} < \frac{1}{n^5}$

15. $\sum_{n=1}^{\infty} \frac{3}{2^n - n}$

$b_n = \frac{1}{2^n}$, $\sum b_n$ converges

Use Limit comparison

Note: $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 3$

12. $\sum_{n=1}^{\infty} \frac{5 + 3 \sin n}{n^3 + 1}$

$b_n = \frac{8}{n^3}$, $\sum b_n$ converges

Use Term-size comparison

Note: $\frac{5 + 3 \sin n}{n^3 + 1} < \frac{8}{n^3}$

16. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln n}$

$b_n = \frac{1}{n}$, $\sum b_n$ diverges

Use Term-size comparison

Note: $\frac{1}{\sqrt{n} \ln n} > \frac{1}{\sqrt{n} \sqrt{n}} = \frac{1}{n}$