

Why is it called Dirac point exactly?

In high-energy physics, in electronics, in photonics, in phononics, using particles or quasi-particles, a plethora of areas in Physics are talking about Dirac points. The recent development of graphene and topology made it even more apparent. For instance, if we plot the dispersion of graphene (plotting the energy versus the wavevector in the reciprocal space) it appears 6 Dirac points (see fig. 1), which are degeneracy points where the lower band (in red) and the upper band (in blue) are touching.

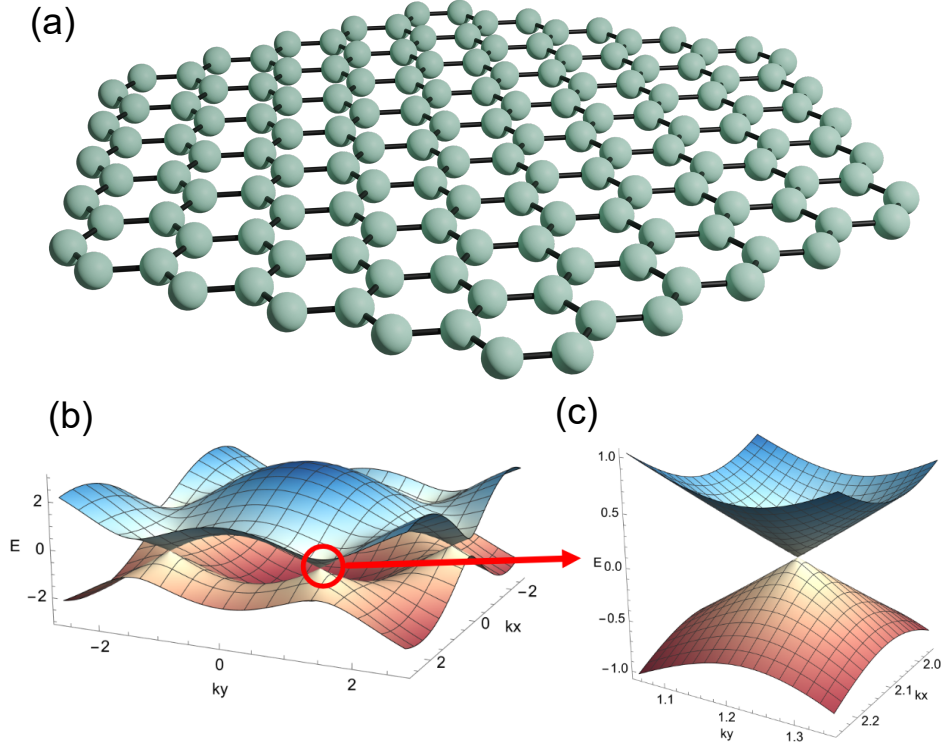


Figure 1: (a) Graphene sheet in real space. (b) Dispersion of graphene. (c) Zoom on the Dirac point (at the K point in the reciprocal space)!

But why is it called Dirac points? To answer these questions, we need to take a look at the Dirac equation introduced by the famous physicist Paul Dirac in 1928. This equation was derived for a relativistic quantum mechanical wavefunctions, and it describes particles with spin 1/2

$$H = c\mathbf{p} \cdot \boldsymbol{\alpha} + mc^2\beta \quad (1)$$

where m is the rest mass of a particle and c the speed of light. α_i and β are the Dirac matrices which must satisfy

$$\begin{aligned} \alpha_i^2 &= \beta^2 = 1 \\ \alpha_i\alpha_j + \alpha_j\alpha_i &= 0 \\ \alpha_i\beta + \beta\alpha_i &= 0 \end{aligned} \quad (2)$$

In 3D, these matrices can be expressed in term of Pauli matrices σ_i ($i = x, y, z$) such as

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} \quad (3)$$

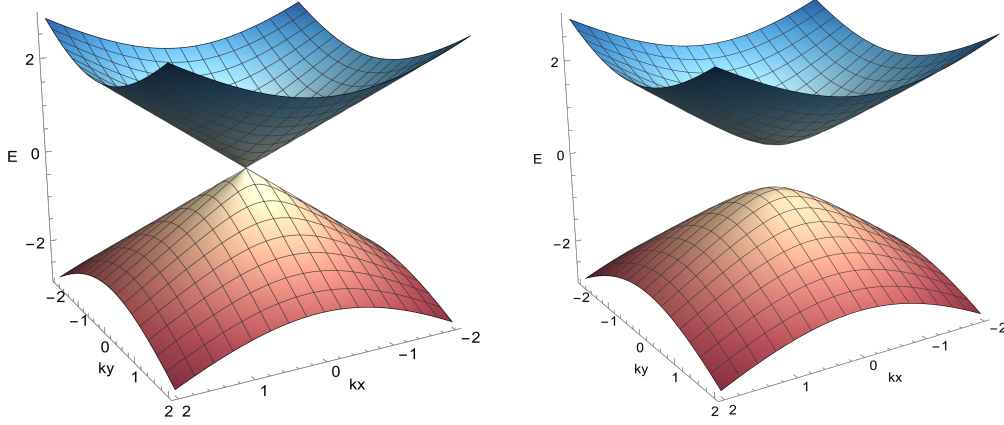


Figure 2: Dispersion of the 2D Dirac Hamiltonian (a) without mass $m = 0$ (b) with a non-zero mass $m \neq 0$. The gap is proportional to the mass.

with σ_0 the identity matrix. The eigenvalues of eq. 1 are $E^2 = m^2c^4 + p^2c^2$, which ultimately gives the stunning result that exists antiparticles (particles with negative mass and energy), and predicts the existence of positrons, the electron antiparticle.

Let us consider a 2D systems, like graphene (see fig. 1) or cavities, which confine the particles in one direction (generally z). In this case, the matrices take a simpler form

$$\alpha_x = \sigma_x, \quad \alpha_y = \sigma_y, \quad \beta = \sigma_z \quad (4)$$

so eq. 1 transforms into

$$H = \hbar c \mathbf{k} \cdot \boldsymbol{\sigma} + mc^2 \sigma_z \quad (5)$$

where $\mathbf{k} = (k_x, k_y, 0)$ and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector of Pauli matrices. The eigenvalues of this Hamiltonian are plotted in fig. 2. We observe that without a mass (left panel, $m = 0$), the dispersion is similar to the dispersion of graphene in the K point plotted in fig. 1: it is why this point is called a Dirac point! We can also get the dispersion of a massive Dirac Hamiltonian (right panel, $m \neq 0$). In this case, the bands are not crossing and we observe the emergence of a gap proportional to the mass.

Answer: The dispersion of the Dirac point (in graphene or other systems) is similar to the massless dispersion of a particle described by the Dirac equation!