

8.6- Power Series Examples

examples: Find the radius and IOC

$$\textcircled{1} \sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$$

← base point is $a=0$
(because " x^n ")

Ratio Test:

remember absolute value!

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{10^{n+1} x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{10^n x^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{\cancel{10^n} \cdot 10 \cdot \cancel{x^n} \cdot x \cdot n^3}{(n+1)^3 \cdot \cancel{10^n} \cdot \cancel{x^n}} \right| \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \left| \frac{10 x n^3}{(n+1)^3} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{10 |x| n^3}{(n+1)^3}$$

$$= 10|x| \lim_{n \rightarrow \infty} \frac{n^3}{(n+1)^3}$$

$$= 10|x| < 1$$

By Ratio Test, this series:

converges if

$$10|x| < 1$$

$$|x| < \frac{1}{10}$$

$$-\frac{1}{10} < x < \frac{1}{10}$$

diverges if

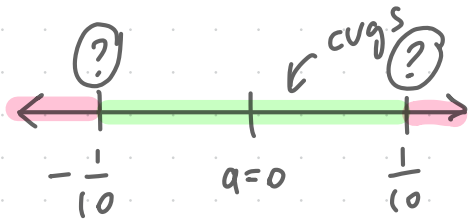
$$10|x| > 1$$

??? if

$$10|x| = 1$$

$$|x| = \frac{1}{10}$$

$$x = \pm \frac{1}{10}$$



$$R = \frac{1}{10}$$

Check endpoints:

$$x = \frac{1}{10} : \sum_{n=1}^{\infty} \frac{10^n \left(\frac{1}{10}\right)^n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3} \quad \begin{array}{l} p=3 > 1 \\ \text{so} \\ \text{converges} \end{array}$$

Means $x = \frac{1}{10}$ is included

$$X = -\frac{1}{10} : \sum_{n=1}^{\infty} \frac{10^n \left(-\frac{1}{10}\right)^n}{n^3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$$

absolute value
converges so
this series is absolutely
convergent hence
convergent
(or use AST)

$$\text{IOC is } \left[-\frac{1}{10}, \frac{1}{10}\right]$$

↑ ↑
both
endpoints included

②

$$\sum_{n=0}^{\infty} \frac{(x+1)^n}{n+1}$$

$$\leftarrow (x+1)^n = (x - (-1))^n$$

$$a = -1$$

Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x+1)^{n+1}}{n+2} \cdot \frac{n+1}{(x+1)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{\cancel{(x+1)^n} (x+1) (n+1)}{(n+2) \cancel{(x+1)^n}} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(x+1) (n+1)}{(n+2)} \right|$$

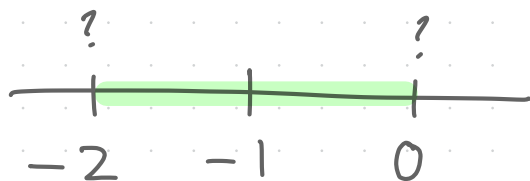
$$= \lim_{n \rightarrow \infty} \frac{|x+1| (n+1)}{(n+2)}$$

$$= |x+1| \lim_{n \rightarrow \infty} \frac{(n+1)}{(n+2)}$$

$$= |x+1| < 1$$

$$\boxed{R=1}$$

cvgs if $|x - (-1)| < 1$ by Ratio Test



~~$(-2, 0)$~~

$[-2, 0)$

~~$(-2, 0]$~~

~~$[-2, 0]$~~

check endpoints:

$$x = -2: \sum_{n=0}^{\infty} \frac{(-2+1)^n}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$$

plug in
to the
original
series

AST, cvgs
(fill in details)

$$b_n = \frac{1}{n+1}$$

$$x = 0: \sum_{n=0}^{\infty} \frac{(0+1)^n}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}$$

harmonic, diverges

-2 is included and 0 is not so
IOC is $[-2, 0)$.

The radius of convergence of a
power series $\sum c_n (x-a)^n$ is $R \geq 0$
such that:

if $|x-a| < R$, then the P.S. converges
and if $|x-a| > R$, the P.S. diverges.

Ratio Test: if $L < 1$ then converges
if $L > 1$ then diverges

1st ex: $L = 10|x-0| < 1$
 $|x-0| < \frac{1}{10}$

for $\sum_{n=0}^{\infty} c_n (x-a)^n$ ← generic P.S.

ex: Suppose $\lim_{n \rightarrow \infty} \left| \frac{c_{n+1}}{c_n} \frac{(x-a)^{n+1}}{(x-a)^n} \right| = L$

What is the radius of convergence?

(a) $L = \frac{|x-a|}{3} < 1$

$$|x-a| < 3$$

$$\boxed{R=3}$$

(b) $L = 5|x-a| < 1$

$$|x-a| < \frac{1}{5}$$

$$\boxed{R=\frac{1}{5}}$$

(c) $L = \infty > 1$

always, P.S. diverges

for all x
(except the base pt)

$$\boxed{R=0}$$

and IOC is $\{a\}$

(d) $L = 0 < 1$ always

P.S. converges for all x

$$\Rightarrow \boxed{R = \infty}$$

and I.O.C. is $(-\infty, \infty)$

ex: $\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n 3^n}$

base point: $\left(3 \cdot \left(x - \frac{2}{3}\right)\right)^n$
 $= 3^n \left(x - \frac{2}{3}\right)^n$

$$\boxed{a = \frac{2}{3}}$$

Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{(3x-2)^{n+1}}{(n+1) 3^{n+1}} \cdot \frac{n 3^n}{(3x-2)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{\cancel{(3x-2)^n} (3x-2) \cdot \cancel{n 3^n}}{(n+1) \cdot \cancel{3^n} \cdot 3 \cdot \cancel{(3x-2)^n}} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(3x-2)n}{3(n+1)} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{|3x-2| \cdot n}{3(n+1)}$$

$$= \frac{|3x-2|}{3} \lim_{n \rightarrow \infty} \frac{n}{n+1}$$

$$= \frac{|3x-2|}{3} < 1$$

$$a = \frac{2}{3}$$

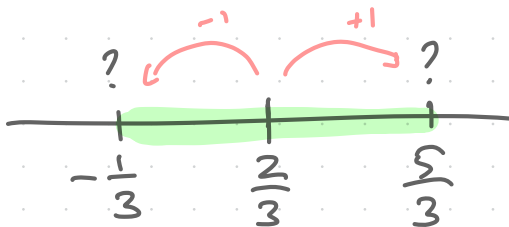
$$|3x-2| < 3$$

$$\boxed{R=3} \quad ?$$

$$3 \left| x - \frac{2}{3} \right| < 3$$

$$\left| x - \frac{2}{3} \right| < 1$$

$$\leftarrow \boxed{R=1}$$



OR

$$|3x-2| < 3$$

$$-3 < 3x-2 < 3$$

$$-1 < 3x < 5$$

$$-\frac{1}{3} < x < \frac{5}{3}$$

check endpoints: $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n 3^n}$

$$x = \frac{5}{3} : \sum_{n=1}^{\infty} \frac{(3 \cdot \frac{5}{3} - 2)^n}{n 3^n} = \sum_{n=1}^{\infty} \frac{3^n}{n 3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

diverges
(harmonic)

$$x = -\frac{1}{3} : \sum_{n=1}^{\infty} \frac{(3 \cdot (-\frac{1}{3}) - 2)^n}{n 3^n} = \sum_{n=1}^{\infty} \frac{(-3)^n}{n 3^n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{3^n}}{n \cancel{3^n}}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

cvgs by AST
(include details)

Ioc is $[-\frac{1}{3}, \frac{5}{3})$

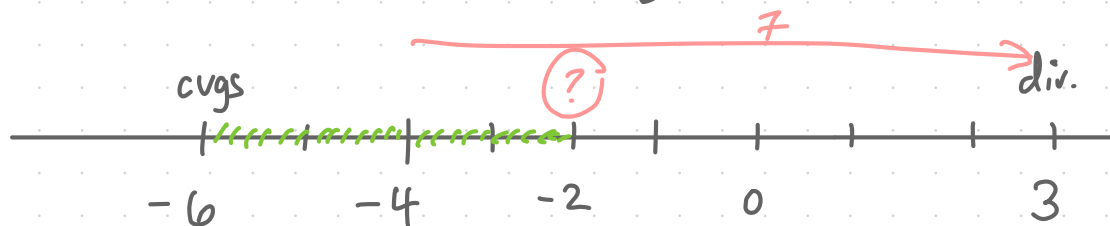
example: (similar to a WebAssign problem)

Suppose $\sum_{n=0}^{\infty} C_n (x+4)^n$ is a P.S. that

↖ base pt $a = -4$

converges at $x = -6$ and diverges at $x = 3$.

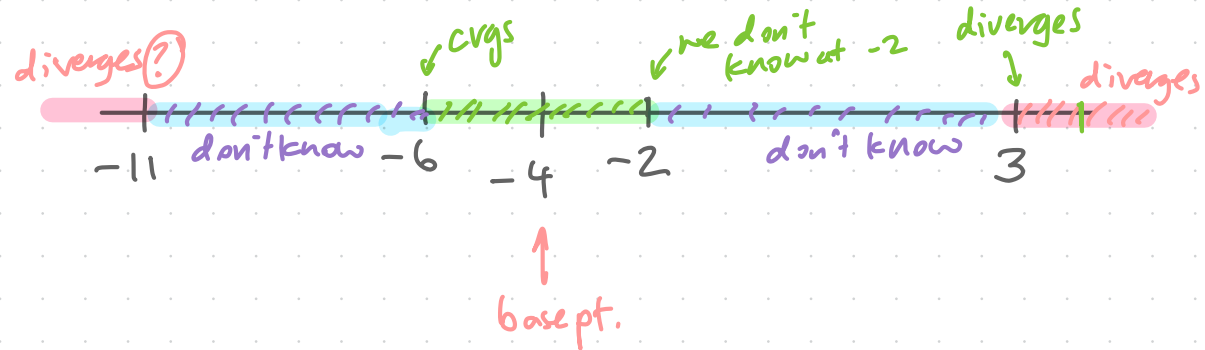
(a) What can you say about the radius of convergence?



I.o.c could be $[-6, -2)$ or $[-6, -2]$

$R \geq 2$ because cvgs at $x = -6$ which is 2 away from base point

$R \leq 7$ because div. at $x = 3$ which is 7 away from base point



(b) Determine if each of the following converges, diverges or you can't tell.

P.S. given is $\sum_{n=0}^{\infty} C_n (x+4)^n$

same

$$\sum_{n=0}^{\infty} C_n 8^n$$

← plug in $x=4$, so this series diverges

(because the P.S. diverges at 3 and 4 is farther from the base point)

$$\sum_{n=0}^{\infty} C_n (-1)^n \leftarrow \text{plug in } x=-5 \text{ so converges}$$

$$\sum_{n=0}^{\infty} C_n 1^n \leftarrow \text{plug in } x=-3 \text{ so converges}$$

$$\sum_{n=0}^{\infty}$$

$$c_n \cdot 2^n$$

← plug in $x = -2$,
we don't know