

Matrix Operations

Consider the matrices below.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

1. If possible, perform each of the following computations.

(a) AB

(g) $D + F$

(b) BA

(h) $A + B$

(c) CE

(i) $C - 2E$

(d) BC

(j) $2(D + F)$

(e) D^2

(k) $(2 + 3)F$

(f) B^2

2. Some of the following properties are true and some are false. Put a check next to the ones you think are true. Assume the matrices A , B and C are the appropriate size so that all operations are defined. Here r and s are real numbers.

☐ $A + B = B + A$

☐ $(A + B) + C = A + (B + C)$

☐ $AB = BA$

☐ $r(A + B) = rA + rB$

☐ $(r + s)A = rA + sA$

☐ $A(B + C) = AB + AC$

☐ $(B + C)A = AB + AC$

3. What is the *additive identity*? I.e. if A is an $m \times n$ matrix, what matrix B has the property that $A + B = A$?

4. What is the *multiplicative identity*? I.e. if A is an $m \times n$ matrix, what matrix B has the property that $AB = A$?

5. Consider the product AB where

$$A = \begin{bmatrix} 9 & 13 & 5 & 6 & -1 \\ 14 & 15 & -7 & -6 & 4 \\ -8 & -9 & 12 & -5 & -9 \\ -2 & 5 & 0 & 6 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 7 & 5 & 4 & -9 & 5 & 6 \\ 10 & 6 & 2 & 16 & -4 & 7 \\ 12 & 8 & 12 & 7 & 5 & -3 \\ -8 & -6 & -2 & 5 & 4 & -9 \\ 2 & 3 & -5 & 4 & 2 & -6 \end{bmatrix}$$

(a) What size matrix is the result AB ?

(b) Without computing the entire product, find the entry in the 4th row and 3rd column of AB .

6. The transpose of a matrix A is formed by interchanging the rows and columns of A . The transpose of A is denoted A^T . (See <https://en.wikipedia.org/wiki/Transpose> for a very nice visual.)

$$\text{For example, if } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \text{then} \quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{where } a, b, c, d \in \mathbb{R}.$$

(a) Compute AB and $(AB)^T$.

(b) Compute $A^T B^T$ and $B^T A^T$.

(c) Which is true? $(AB)^T = A^T B^T$ or $(AB)^T = B^T A^T$

7. Let $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

(a) Compute the products $\mathbf{u}^T \mathbf{v}$ and $\mathbf{v}^T \mathbf{u}$. (Do you recognize this operation?)

(b) Compute the products $\mathbf{v} \mathbf{u}^T$ and $\mathbf{u} \mathbf{v}^T$.