

MATH 2130: Intro to Linear Algebra – Exam 3 Practice Problems

1. Find the determinant each matrix. Be as efficient as possible with your computations.

$$(a) \quad A = \begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 1 & 0 & 3 & 4 & 7 & 1 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(b) \quad B = \begin{bmatrix} 0 & 0 & 7 & 0 & 0 \\ 0 & 7 & 7 & 7 & 0 \\ 7 & 7 & 7 & 7 & 7 \\ 0 & 7 & 7 & 7 & 0 \\ 0 & 0 & 7 & 0 & 0 \end{bmatrix}$$

2. Suppose A and B are $n \times n$ matrices where $\det(A) = 2$ and $\det(B) = 3$. Evaluate each of the following.

- (a) $\det(A^3)$
- (b) $\det(A^T B)$
- (c) $\det(2B)$

3. Suppose A is a 3×3 matrix with one eigenvalue $\lambda = 4$ with algebraic multiplicity 3.

- (a) Is A invertible?
- (b) Explain why A may or may not be diagonalizable. What must be true for A to be diagonalizable?
- (c) Show that if A is diagonalizable, then $A = 4I$.

4. Suppose A is a matrix with characteristic polynomial $p(\lambda) = \lambda(1 - \lambda)^3(4 - \lambda)^2$.

- (a) What are the eigenvalues of A ?
- (b) List all possible dimensions of each eigenspace.

5. Let $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be bases for a vector space V . Suppose that $\mathbf{a}_1 = -\mathbf{b}_1 + 4\mathbf{b}_2$ and $\mathbf{a}_2 = 5\mathbf{b}_1 - 3\mathbf{b}_2$.

- (a) Find $[\mathbf{x}]_{\mathcal{B}}$ given $[\mathbf{x}]_{\mathcal{A}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.
- (b) Find $[\mathbf{x}]_{\mathcal{B}}$ for an arbitrary vector $[\mathbf{x}]_{\mathcal{A}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

6. (a) Find two vectors in $\mathbb{R}^2$ that are orthogonal to $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$. Can you find two that are not parallel?

- (b) Find two vectors in $\mathbb{R}^3$ that are orthogonal to $\mathbf{w} = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$. Can you find two that are not parallel?

7. Determine if each of the following statements is **True (T)** or **False (F)**. Justify your answers.
- (T / F) If A is a 5×5 matrix with five distinct eigenvalues, then A is diagonalizable.
- (T / F) If a 5×5 matrix B has fewer than 5 distinct eigenvalues, B is not diagonalizable.
- (T / F) If A and B have the same characteristic polynomial then $A = B$.
- (T / F) If A is diagonalizable and the only eigenvalue of A is 0, then A is the zero matrix.
- (T / F) If the only eigenvalue of A is 0, then A is the zero matrix.
8. **Give an example of each.** Justify your answers.
- (a) A nonzero matrix with $\det(A) = 0$
- (b) A matrix with eigenvalues $\lambda = 0, 1, 2, 3$
- (c) A 3×3 matrix with exactly one real eigenvalue.
- (d) A matrix that is not diagonalizable
- (e) A matrix that is diagonalizable but not invertible