

I) QUANTUM MECHANICS Lec 8 Spg 15
RECIPE PLUS
I) ENSEMBLE APPROACH (READING ①
ch 3, 4, 2)

1) EMPHASIZES USE OF (SIMPLE
IT TURNS OUT) PROBABILITIES,
P_r ≡ prob. of observing

system in (eigen) energy
state r

2) THIS PROBABILISTIC APPROACH,
IT TURNS OUT, IS MUCH
EASIER TO WORK WITH --

~~by~~ ^Savoiding state-counting
(eg., determination of
 $\Omega(E, V, N)$).

3) PICTURE / CONCEPT =

a) IMAGINE ^A LARGE
NUMBER, $\tilde{N}$, OF REPLICAS

OF YOUR SYSTEM OF
INTEREST, $\Rightarrow$ THE SET = ENSEMBLE

b) AT ANY GIVEN INSTANT, t,
 n_i OF THE SYSTEMS (w/in

THE SET OF N REPLICAS)
are in (eigen) energy state

1, n_2 are in energy state

2, etc. etc. (NONE OF THE REPLICAS ARE IN

c) THE set of (energy state) ^A MIXED state)
occupation numbers,

$$\mathcal{E} \equiv \{n_1, n_2, n_3, \dots, n_M\},$$

TYPICALLY VARIES (VERY

RAPIDLY $\rightarrow$ ON ATOMIC TIME

SCALES. THUS, $n_r(t_1) \neq$

$n_r(t_2)$ (where $t_1 \neq t_2$),

and where $r = 1, 2, 3, \dots, M$

d) THE NUMBER OF energy
states excited at any

given instant, M , ~~also~~

also VARIES w/ time, i.e.

$$M = M(t)$$

e) The ^{set of} N (eigen) energy states associated w/ any given system, are in principle, gotten via a Quantum Mechanics calcn.

ASIDE:

Quantum Mechanics Recipe
For computing E's!

- 1) GROSS A CLASSICAL microscale HAMILTONIAN, H , FOR YOUR (microscale) system,
- 2) CONVERT H TO A QM OPERATOR $\hat{H}$

$$\text{eg. i) } H = \sum_i \frac{\vec{P}_i \cdot \vec{P}_i}{2m} + \sum_{i,j} V(\vec{r}_i - \vec{r}_j)$$

$$\text{ii) } \Rightarrow \vec{P}_i \Rightarrow \hat{\vec{P}}_i = -i\hbar \nabla$$

$$V \Rightarrow \hat{V} = \sum_{i,j} V(\vec{r}_i - \vec{r}_j)$$

$$\therefore \hat{H} = \sum_i \frac{\vec{P}_i \cdot \vec{P}_i}{2m} + \hat{V}$$

$$= -\frac{\hbar^2 \nabla^2}{2m} + \hat{V}$$

iii)

④

3) use $\hat{H}$ in Schrodinger's eqn:

$$\boxed{i\hbar \frac{d}{dt} |\psi\rangle_t = \hat{H} |\psi\rangle_t} \quad (4.0)$$

i) $|\psi\rangle =$ "ket" = a vector
that "rotates" (super-fast)
w/in some state space, $\in$
state-space

ii) $\mathcal{H}$ is defined / spanned
by a complete set
of basis vectors,
 $\{|\phi_1\rangle, |\phi_2\rangle, \dots, |\phi_n\rangle\}$,
where $|\phi_k\rangle$ is the k th
eigen vector obtained
by soln of the
stationary S.E.

$$(4.1) \quad \boxed{\hat{H} |\phi_r\rangle = E_r |\phi_r\rangle}$$

$\oplus E_r = \hbar \omega_r = r$ th eigenvalue of $\hat{H}$

$\oplus$ (4.1) follows from (4.0)

By assuming

$$\text{or } \boxed{\left[-\frac{\hbar^2}{2m} \nabla^2 + V \right] |\phi_r\rangle = E_r |\phi_r\rangle} \quad (4.1.a)$$

(5)

$$|\psi\rangle = \sum_n e^{i\omega_n t} |\phi_n\rangle$$

i.) constructing $|\psi\rangle$ via

superposn of a set of plane waves

ii) Thus, the energy eigenvalues, $E_1, E_2, \dots, E_n, \dots$ are gotten by solving (4.1) or equiv (4.1a)

iii) Two prominent / useful solns of SB, that will use a lot for modeling (and 1-ptcl) N-ptcl. quantum systems are the solns for the non-interacting (i.e. $V=0$) single ptcl in a box: ch 1 path 1a:

eqn. 1.4.5	$n_x^2 + n_y^2 + n_z^2 = \frac{8m E_{n_x, n_y, n_z}}{\hbar^2} V^{3/2}$
------------	--

QM Recipe contra less saying is
 and the ^{LD} harmonic oscillator (6)

$$E_n = \hbar \omega_n \left(n + \frac{1}{2} \right)$$

$$\textcircled{+} m \omega^2 = K$$

$\textcircled{+} m = \text{ptcl mass (rest mass?)}$

$\textcircled{+} K = \text{effective spring constant}$

END QM Recipe

IMPORTANT

ASIDE 2 When DO we use
 a QUANTUM MICROSCOPE
 MODEL AND WHEN IS
 it CLASSICAL MODEL:

ANSWER: IT TURNS OUT
 WE MUST USE
 QM models when the
 characteristic thermal
 wavelength

QM $\left[\lambda \approx \hbar / \sqrt{mKT} \approx \text{Order } 1 \right]$

lec 8 Sep 15

Classical
microstate
model

$$\lambda \approx \frac{h}{\sqrt{m k T}} \ll 1$$

From these it is soon that
QUANTUM EFFECTS ONLY
become dominant when:

i) $T \rightarrow$ small (order
1-10 K or less)

and/or ii) $m \rightarrow$ small