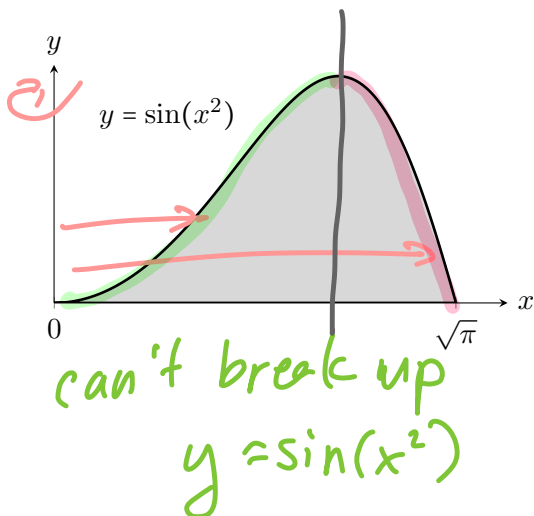


Turn in the following problems:

1. Let S be the solid obtained by rotating the region shown in the figure about the y -axis.



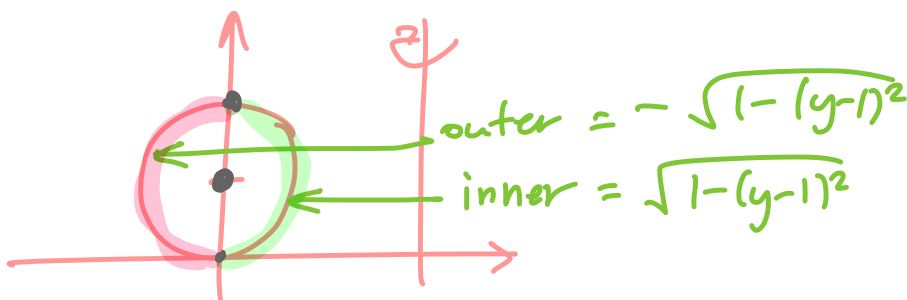
- (a) If you were to find the volume of S , would you use the washer method or the method of cylindrical shells? Explain.
- (b) Draw a typical cross-section and label any relevant dimensions.
- (c) What is the volume of S ?

2. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $x = 3 + 2y - y^2$ and $x + y = 3$ about the x -axis. Sketch the region and a typical shell.

3. Find the volume obtained by rotating the disk bounded by the graph of the circle

$$x^2 + (y-1)^2 = 1 \quad \leftarrow x = \pm \sqrt{1 - (y-1)^2}$$

about the line $x = 2$ in two ways: (a) by the washer method; (b) by the shell method. Are your answers the same?



4. THEOREM of PAPPUS: Let $\mathcal{R}$ be a plane region that lies entirely on one side of a line l in the plane. If $\mathcal{R}$ is rotated about l , then the volume of the resulting solid is the product of the area of $\mathcal{R}$ and the distance d traveled by the centroid of $\mathcal{R}$.

↪ center

Use the Theorem of Pappus to find the volume of the solid obtained by rotating

$$x^2 + (y-1)^2 = 1 \text{ about the line } x = 2.$$

↪ centroid/center is $(0, 1)$

Theorem says

$$\text{vol} = (\text{area of region}) \cdot \left(\text{distance traveled by the center as it goes around the axis of revolution} \right)$$

5. The circumference of a tree at different heights above the ground is given in the table below:

Height (in)	0	20	40	60	80	100	120
Circumference (in)	31	28	21	17	12	8	2

⇒ method of disks

Assuming all of the tree's horizontal cross sections are circular, estimate the volume of the tree.

$$V = \int \pi r^2 dy$$

← r is function of y

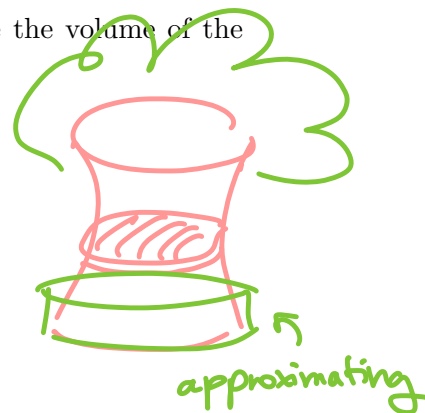
approx.
with a sum

$$\sum_{n=1}^?$$

$$\pi r^2 \Delta y$$

or Δh

figure out from the measurements



approximating

6. If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring?

Set up your coordinate system so that 0 is at one end:



x_0 is equilibrium
⇒ natural length

Using the info, get a system of equations involving k and x_0 .
Solve for x_0 .

$$\text{Hooke's Law} \\ F(x) = k(x - x_0)$$

7. A skyrocket firework burns fuel as it climbs. Its mass, $m(h)$ kg, at height h m is given by

$$m(h) = 0.96 + \frac{0.065}{1+h}$$

- What is the mass of the rocket at $h = 0$ just before launch?
- Show the mass of the rocket decreases as the rocket climbs.
- How much work is required to lift the rocket 10 m if the rocket's mass did not decrease?
- Approximate the work done as the rocket goes from height h to $h + \Delta h$.
- The rocket explodes at a height $h = 250$ m. Find the total work done by the rocket from its launch to its explosion.

8. Jamie and Mike are working to understand the following problem:

A chain lying on the ground is 10 m long, and its mass is 80 kg. How much work is required to raise one end of the chain to a height of 6 m?

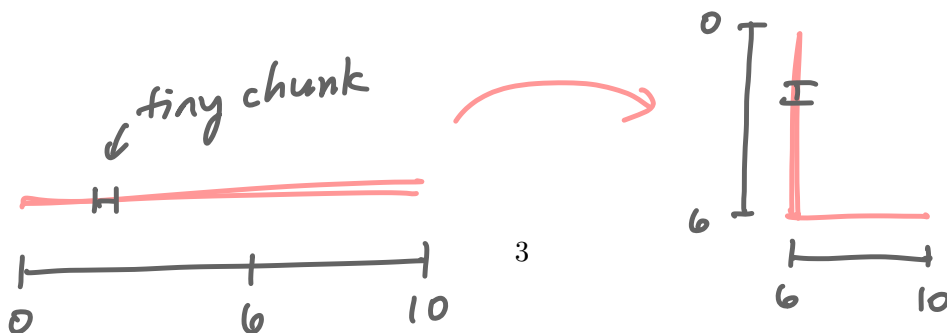
(Assume no friction)

They went to the MARC for help, and found the solution could be found by evaluating the following integral:

$$W = 78.4 \int_0^6 (6 - x) dx$$

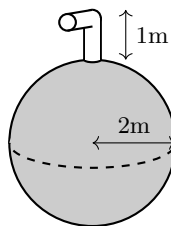
When they got home, they found they could not remember how they arrived at this solution. Help Jamie and Mike by explaining what each part of the integral represents in the context of the problem.

- Where did the 78.4 come from?
- What do the bounds represent?
- Where did the $(6 - x)$ come from?
- What does the dx represent?
- Why does the integral give Jamie and Mike the work required to raise one end of the chain to a height of 6 m?

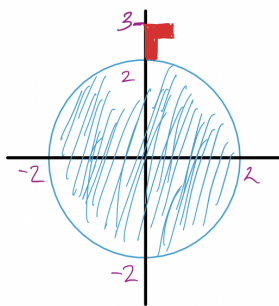


9. Moira and Alexis are working on the following problem:

Given spherical tank with a radius of 2 m is full of water, with density 1000 kg/m^3 . Assume the water is pumped from the surface and there is no water in the spout at the beginning. Find the work required to pump the water out of the spout.

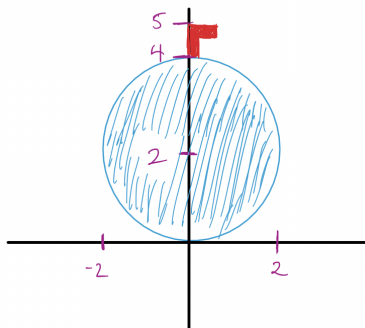


(a) Moira created the following picture for the problem:



Explain if Moira's picture is valid for approaching and solving the problem

(b) Alexis came up with the diagram for the problem:



Explain if Alexis's diagram is valid for approaching and solving the problem.

(c) Find the work required to pump the water out of the spout.