Row Echelon Form (REF)

A matrix is in **row echelon form** if:

- All rows having only zero entries are at the bottom.
- If a row has a leading entry, every row below it has its leading entry in a column to the right.
- If a row has a leading entry, the column entries below it are all 0.

Determine which of the following matrices are in REF.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Reduced Row Echelon Form (RREF)

A matrix is in **reduced row echelon form** if:

- The matrix is in row echelon form.
- The leading entry in each nonzero row is 1.
- If a row has a leading entry, the column entries above and below it are all 0.

Of the matrices above that are in REF, which are also in RREF? For any matrices that are not in RREF, what row operations could you do to get them into RREF?

1. Apply row operations to get the matrix into RREF.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 3 & 3 & 2 \\ 4 & 4 & 4 & 3 \\ 5 & 5 & 5 & 4 \end{bmatrix}$$

2. Apply row operations to get the augmented matrix into RREF. What can you say about the solutions of the corresponding linear system?

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 2 & 6 & -1 \end{array}\right]$$

3. The following augmented matrices are in REF. What can you say about the number of solutions of the corresponding linear systems?

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & -5 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$