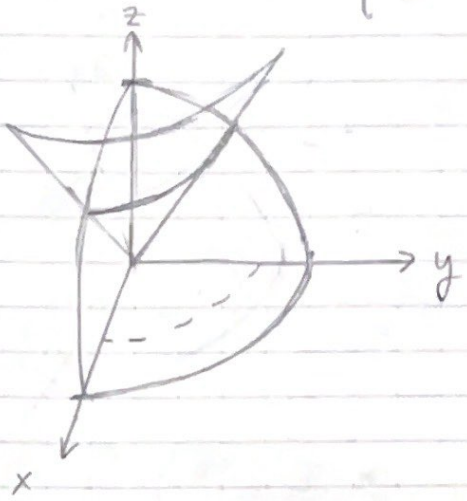
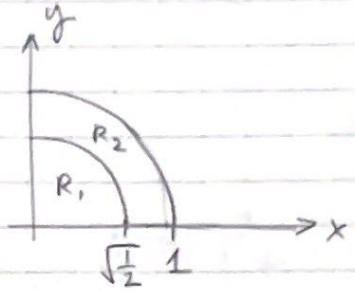
ex: Find the volume that lies under the (half) cone $z = \sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=1$ in the first quadrant





Desmos: tinyurl.com/4m95mz3b

intersects when $(z^2) + z^2 = 1$

$2z^2 = 1$

$z = \pm\sqrt{\frac{1}{2}}$

$\Rightarrow \sqrt{x^2+y^2} = \sqrt{\frac{1}{2}}$

$x^2+y^2 = \frac{1}{2}$

Let $f(x,y) = \sqrt{x^2+y^2}$ and $g(x,y) = \sqrt{1-x^2-y^2}$

$$\iint_{R_1} f(x,y)\,dA + \iint_{R_2} g(x,y)\,dA$$

$$= \iint_{R_1} \sqrt{x^2+y^2}\,dA + \iint_{R_2} \sqrt{1-x^2-y^2}\,dA \quad \text{convert to polar}$$

$$= \int_0^{\pi/2}\int_0^{\sqrt{1/2}} \sqrt{r^2}\cdot r\,dr\,d\theta + \int_0^{\pi/2}\int_{\sqrt{1/2}}^{1} \sqrt{1-r^2}\cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/2} 1\, d\theta \cdot \int_0^{\sqrt{1/2}} r^2\, dr + \int_0^{\pi/2} 1\, d\theta \cdot \int_{\sqrt{1/2}}^{1} r\sqrt{1-r^2}\, dr$$

$u = 1 - r^2$
$du = -2r\,dr$

$r = \sqrt{\frac{1}{2}} \Rightarrow u = \frac{1}{2}$
$r = 1 \Rightarrow u = 0$

$$= [\theta]_0^{\pi/2} \cdot \left[\frac{1}{3}r^3\right]_0^{\sqrt{1/2}} + [\theta]_0^{\pi/2} \cdot \int_{1/2}^{0} \sqrt{u} \cdot \left(-\frac{1}{2}\right) du$$

$$= \frac{\pi}{2} \cdot \frac{1}{3}\left(\frac{1}{2}\right)^{3/2} + \frac{\pi}{2} \cdot \left(-\frac{1}{2} \cdot \frac{2}{3} u^{3/2}\right)\Big|_{1/2}^{0}$$

$$= \frac{\pi}{6}\left(\frac{1}{2}\right)^{3/2} - \frac{\pi}{6}\left(0 - \left(\frac{1}{2}\right)^{3/2}\right)$$

$$= 2\left(\frac{\pi}{6}\left(\frac{1}{2}\right)^{3/2}\right)$$

$$= \cancel{2} \cdot \frac{\pi}{6} \cdot \frac{1}{\cancel{2}} \cdot \sqrt{\frac{1}{2}}$$

$$= \frac{\pi}{6}\sqrt{\frac{1}{2}}$$