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Brigadeiro state: relaxation to a delicious condensed matter phase characterized by the entanglement between cocoa powder and condensed milk

Krissia Zawadzki[1] and The QuCoA Collaboration[1]

[1] *QuCoA – Quantum Correlations and Applications Group, Instituto de Física de São Carlos, Universidade de São Paulo*

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We introduce a minimal nonequilibrium model for the preparation of brigadeiro, one of the most delicious Brazilian strongly correlated dessert states that emerges from the driven interaction between cocoa powder and condensed milk. By formulating the preparation protocol within the framework of quantum thermodynamics, we investigate whether the texture of the final confectionery phase exhibits genuine many-body correlations, anomalous relaxation, and signatures of entanglement.

I. INTRODUCTION

Originally rooted in the social and political history of Brazil [1, 2], brigadeiro has evolved into one of the most emblematic dessert states of Brazilian cuisine. The traditional recipe consists of a mixture of condensed milk and cocoa powder, cooked to perfection to become an indulgent chocolate truffle. A representative realization of an ensemble of brigadeiros is shown in Fig. 1. More than a national dessert capable of satisfying sugar cravings at any time, brigadeiro has an important role in promoting social connections and fostering scientific collaboration. It has been demonstrated as the main catalyst for interactions between Brazilian researchers and various groups abroad.

From the perspective of condensed matter and many-body physics, brigadeiro may be viewed as an emergent strongly correlated state resulting from the nonequilibrium coupling between condensed milk and cocoa powder. Such a phase emerges as the stationary solution of the driven dissipative dynamics. Differences in the ability of individuals preparing the delicacy, as well as in the cooking accessories employed (pan, spatula, etc.), could not be explained classically. Moreover, the remarkable reproducibility of the final phase suggests the stabilization of a strongly correlated phase between cocoa and condensed milk, with possible quantum correlations.



FIG. 1. Experimental realization of a finite strongly correlated brigadeiro ensemble prepared under the clean reference protocol.

This perspective naturally raises a fundamental question: does the brigadeiro preparation protocol generate genuine many-body correlations, and perhaps even entanglement, between its microscopic constituents? This is the question we intend to address in this paper. To this aim, we introduce a minimal microscopic model, including the preparation protocol, as well as propose the use of quantum state texture [3] as the entanglement witness.

This paper is organized as follows. In Sec. II, we introduce the minimal microscopic model and the preparation protocol. Section III presents the open-quantum system description and the corresponding master equation governing the nonequilibrium dynamics. The experimental protocol is detailed in Sec. IV, including both instructions for the cooking and classical rolling. In Sec. V, we discuss how texture can be used as a witness of the brigadeiro phase and present the phase diagram. Finally, Sec. VI is devoted to the discussion of anomalous relaxation pathways, including Mpemba and inverse-Mpemba effects, as well as future perspectives.

II. BRIGADEIRO'S HAMILTONIAN

We consider the effective Hamiltonian

$$H_{\text{brigadeiro}}(t) = H_{\text{cond-milk}} + H_{\text{cocoa}} + H_{\text{heat-bath}} + H_{\text{stirring}}(t), \tag{1}$$

where the first two terms contain all the degrees of freedom required to describe the system (sugar + milk and pure cocoa powder) before and after turning into brigadeiro. The third describes the heat bath which provides the energy necessary for condensation, while the last is the external time-dependent drive associated with stirring, which both couples milk and cocoa and assists the transfer of heat.

The stirring protocol is modeled as

$$H_{\text{stirring}}(t) = g(t) \sum_i \left(a_i^\dagger c_i + c_i^\dagger a_i \right), \tag{2}$$

with $g(t) \propto \Omega$ proportional to the stirring rate.

This term acts both as an interaction channel and as a time-dependent external drive that promotes homog-

enization. Such driven preparation protocols are conceptually related to control strategies used in nonequilibrium quantum thermodynamics and anomalous relaxation studies [4].

III. OPEN-SYSTEM DYNAMICS AND MASTER EQUATION

The initial state of the system – condensed milk+cocoa powder+bath – is known. We therefore have all the ingredients and instructions to solve this problem. It is reasonable to assume that the state of the heat bath remains unchanged during the process. Of course, a more accurate description should include additional effects of the oven, the pan, related environmental effects (etc, etc, etc). For the purposes of this initial model, this assumption is reasonable.

One can then derive a master equation for the mixture (cocoa + condensed milk). Suppose that the effective dynamics is described by the Liouvillian master equation. Since the preparation of brigadeiro involves both external heating and active stirring, the relevant dynamics is intrinsically nonequilibrium and should be described at the level of an open quantum system. This description closely follows standard open-system approaches used in the study of dissipative many-body dynamics and anomalous relaxation [4].

We therefore consider the reduced density matrix $\rho(t)$ associated with the effective cocoa–condensed-milk degrees of freedom, obtained after tracing out the heat bath. Its evolution is modeled by the master equation

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -i[H_{\mathrm{eff}}(t), \hat{\rho}] + \mathcal{D}_{\mathrm{heat}}[\hat{\rho}] + \mathcal{D}_{\mathrm{stir}}[\hat{\rho}], \tag{3}$$

where

$$\hat{H}_{\mathrm{eff}}(t) = \hat{H}_{\text{cond-milk}} + \hat{H}_{\mathrm{cocoa}} + \hat{H}_{\mathrm{stirring}}(t). \tag{4}$$

The dissipator $\mathcal{D}_{\mathrm{heat}}$ accounts for irreversible energy exchange with the thermal environment and drives the system toward the desired condensed phase. A minimal choice is a Lindblad form,

$$\mathcal{D}_{\mathrm{heat}}[\hat{\rho}] = \sum_{\mu} \gamma_{\mu} \left(L_{\mu} \hat{\rho} L_{\mu}^{\dagger} - \frac{1}{2} \left\{ L_{\mu}^{\dagger} L_{\mu}, \hat{\rho} \right\} \right), \tag{5}$$

where the jump operators L_{μ} encode local condensation, energy redistribution, and viscosity-renormalization processes.

In principle, a Davies map could be used to model the heating dissipator, as shown in a recent paper exploring the genuine Mpemba effect [4].

To capture the homogenizing role of stirring, we further introduce an effective mixing dissipator,

$$\mathcal{D}_{\mathrm{stir}}[\hat{\rho}] = \kappa(\Omega) \sum_{i} \left(S_i \hat{\rho} S_i^{\dagger} - \frac{1}{2} \left\{ S_i^{\dagger} S_i, \hat{\rho} \right\} \right), \tag{6}$$

where $\kappa(\Omega)$ is an increasing function of the stirring rate and the operators S_i promote local cocoa–milk homogenization.

The brigadeiro phase emerges as a nonequilibrium steady state,

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = 0, \tag{7}$$

stabilized by the competition between coherent mixing, thermal relaxation, and external driving.

An important observable is the relaxation time τ_B required for the system to approach this steady state within a prescribed tolerance. In practice, this time scale may be estimated from the approach of the spoon adhesion order parameter $M(t)$ or the texture observable $\mathcal{R}(t)$ to their stationary values. The use of texture-inspired observables is motivated by recent developments in quantum state texture and rugosity as probes of many-body correlations [3].

IV. EXPERIMENTAL PROTOCOL

The protocol to prepare brigadeiro is straightforward, but special attention must be paid in choosing the ingredients and implementing the stirring drive.

We suggest brands of condensed milk produced in the global south and pure cocoa powders. This choice suppresses disorder-induced granulation associated with excess sugar and provides a cleaner route toward the stabilization of the correlated brigadeiro phase. This choice is further supported by the distinct compositional and nutritional properties of cocoa powder when compared with commercial chocolate beverage formulations [5, 6].

The preparation consists of an initial driven dissipative cooking stage involving cocoa powder and condensed milk. A single tablespoon of butter is added only at the late stages of the dynamics in order to regularize the effective viscosity and facilitate detachment from the pan. This guarantees that the texture of the final brigadeiro state is as smooth as possible.

Illustrative images of the step-by-step preparation guide are shown in Fig. 2.

A. Classical rotational shaping protocol

After cooling, the final spherical states are generated through a purely classical rotational protocol.

The shaping dynamics is governed by

$$I \frac{\mathrm{d}\omega}{\mathrm{d}t} = \tau_{\mathrm{ext}}, \tag{8}$$

while the effective centrifuge force reads

$$F_c = m\omega^2 R. \tag{9}$$

This stage is entirely classical and should therefore be understood as a post-quantum rotational stabilization procedure.



FIG. 2. Step-by-step preparation of the brigadeiro phase. (a) 2 tablespoons of cocoa powder are sieved before being mixed to the condensed milk. (b) the mixture is combined until it acquires a uniform texture (c)-(d). The mixture is cooked over medium heat, stirring constantly until thickened (e). When thickened, lumps will form. Turn off the heat and add a tablespoon of unsalted butter (f). Stir the mixture until it gets silky and homogeneous (g). Pour over a plate and let it cool.



FIG. 3. Classical rotational shaping protocol using water-assisted low-friction boundary conditions.

V. TEXTURE AND PHASE DIAGRAM

We define the spoon adhesion order parameter

$$M = \frac{m_{\text{retained}}}{m_{\text{total}}}, \tag{10}$$

and the texture observable

$$\mathcal{R} = -\log \mathcal{T}, \tag{11}$$

where $\mathcal{T} = \langle f_1|\hat{\rho}|f_1\rangle$ is the overlap with the textureless state $|f_1\rangle = \frac{1}{D}\sum_j |j\rangle$, defined in a given basis $\{|j\rangle\}$ of dimension D.

Our reasoning for choosing the texture as an order parameter is motivated by its potential use to probe quantum phase transitions [3]. Here, we seek a silky, smooth, and lump-free brigadeiro state. Therefore, the overlap of the prepared state with the reference textureless perfect brigadeiro state should be maximized.

The reference basis to define the textureless brigadeiro state is that of a brigadeiro sample prepared by someone who mastered the perfect texture after many experimental realizations. It should be noted that a crucial step in the preparation involves sieving the cocoa powder before mixing it with the condensed milk. If this step is forgotten, lumps will occur regardless of the efficiency of the stirring protocol. This should be signalled by the texture measurement, which might show a reduced overlap with the textureless reference state.

The final state is controlled by the stirring rate Ω and bath temperature T.

VI. DISCUSSION AND OUTLOOK

In order to rigorously establish that the texture serves as an order parameter and entanglement witness for the brigadeiro phase, we suggest the reader improve the mi-

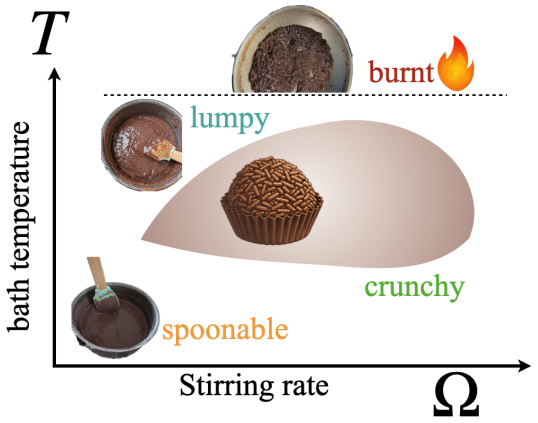


FIG. 4. Nonequilibrium phase diagram of brigadeiro preparation. Intermediate temperatures and moderate stirring stabilize the strongly correlated phase.

croscopic model with relevant degrees of freedom and, more importantly, try the brigadeiro recipe at home.

A remaining question concerns the hierarchy of preparation time scales. In particular, we define the characteristic brigadeiro formation time τ_B and the characteristic burn time $\tau_{\rm burn}$. The useful preparation window is then determined by the regime

$$\tau_B < \tau_{\rm burn}. \tag{12}$$

A natural continuation of the present work is inspired by recent investigations of the genuine Mpemba effect in quantum systems [4]. An inverse Mpemba effect could arise when refrigerated condensed milk assists the acceleration of the preparation protocol, leading to

$$\tau_B(T_{\rm cold}) < \tau_B(T_{\rm hot}), \qquad T_{\rm cold} < T_{\rm hot}. \tag{13}$$

Another important direction concerns the design of shortcut-to-adiabaticity protocols based on engineered stirring profiles $\Omega(t)$ and bath temperature ramps $T(t)$. Such protocols may minimize τ_B while suppressing leakage into burnt and glassy excited states.

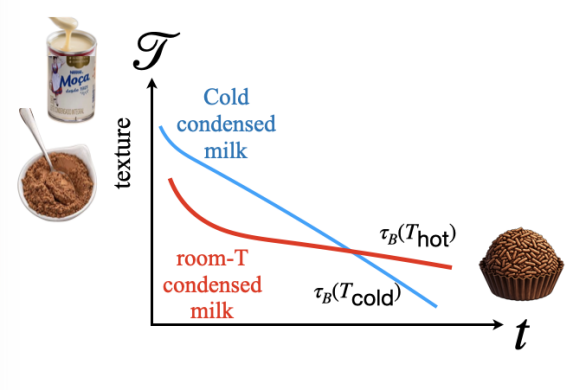


FIG. 5. Texture as a witness of the inverse Mpemba effect. Within a suitable temperature window, a colder initial mixture (e.g., condensed milk left in the fridge before use) can reach the correlated brigadeiro phase faster than a warmer one, i.e., $\tau_B(T_{\rm cold}) < \tau_B(T_{\rm warm})$ for $T_{\rm cold} < T_{\rm warm}$.

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[1] P. v. M. Meirelles, Aedos (2019), historical review of brigadeiro.
[2] B. Sheen, *Foods of Brazil*, A Taste of Culture (Greenhaven Publishing LLC, 2007).
[3] F. Parisio, Phys. Rev. Lett. **133**, 260801 (2024).
[4] M. Moroder, O. Culhane, K. Zawadzki, and J. Goold, Phys. Rev. Lett. **133**, 140404 (2024).
[5] M. d. C. Razola-Díaz, M. J. Aznar-Ramos, V. Verardo, S. Melgar-Locatelli, E. Castilla-Ortega, and C. Rodríguez-Pérez, Antioxidants **12**, 10.3390/antiox12030716 (2023).
[6] V. Barišić, N. C. Icyer, S. Akyil, O. S. Toker, I. Flanjak, and Đurđica Ačkar, Trends in Food Science & Technology **132**, 65 (2023).