

8.5 Power Series

Up to now, we've been studying the convergence or divergence of series like

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

With power series, we introduce a variable x .

Definition: A **power series** is a series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$$

Here $\{c_n\}$ is the sequence of coefficients and a is the **center** or **base point** of the power series.

When we plug a value for x into a power series, we get a “normal” series.

Example: $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

Definition: The **interval of convergence** (IOC) of a power series is the set of all points, x , for which the power series converges.

Example: $\sum_{n=0}^{\infty} x^n$

Facts about the Interval of Convergence

- If a power series converges at two separate points, it also converges at all points between them. In other words, the interval of convergence will always be an interval (unless it's a single point).
- The base point is always in the interval of convergence. Another way of saying this is that a power series always converges at its base point.
- The interval of convergence is always symmetric about the base point.

Definition: The **radius of convergence** of a power series $\sum c_n(x-a)^n$ is a number $R \geq 0$ such that

- (i) if $|x-a| < R$, the series converges at x
- (ii) if $|x-a| > R$, the series diverges at x

Facts about the Radius of Convergence

- The radius of convergence does not tell you what happens when $|x-a| = R$.
- We allow $R = \infty$, which means the power series converges at every real number.
- $R = 0$ means the power series converges ONLY at the base point a .
- We use the Ratio Test to find the radius of convergence.