

Maclaurin SeriesRadius of Convergence

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$$R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$R = \infty$$

$$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$R = 1$$

1. Using the table above, match the following functions with their corresponding Maclaurin series.

(a) $f(x) = e^{x^2/2}$

V

I. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

similar to $\ln(1+x)$

II. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4^n (2n)!}$

similar to $\cos x$

(b) $g(x) = \cos\left(\frac{x}{2}\right)$

II

III. $\sum_{n=1}^{\infty} \frac{-x^n}{n}$

$\ln(1+x)$

(c) $h(x) = \ln(1-x)$

III

IV. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+1}$

$\arctan x$

V. $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$

e^x

(d) $k(x) = x^3 \arctan x$

IV

VI. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+3}}{2n+1}$

$\arctan x$

2. Use the table of known Maclaurin series to find the sum of each series.

Hint: **Do the left column first.** Then use your answer from the left column to find the sum of the corresponding series in the right column. Part (h) is a challenge problem :)

$$(a) \sum_{n=0}^{\infty} (-1)^n \frac{(1/2)^{2n+1}}{(2n+1)} = \arctan\left(\frac{1}{2}\right)$$

$$(e) \sum_{n=1}^{\infty} (-1)^n \frac{(1/2)^{2n+1}}{(2n+1)} = \arctan\left(\frac{1}{2}\right) - \frac{1}{2}$$

↑ subtract the first term from your answer from (a)

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (1/3)^n}{n}$$

$$= \ln\left(1 + \frac{1}{3}\right)$$

$$= \ln(4/3)$$

doesn't change the value

$$(f) \sum_{n=1}^{\infty} \frac{(-1)^n}{n 3^n} = (-1)^2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n 3^n}$$

$$= (-1) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n 3^n}$$

$$= -\ln(4/3)$$

$$(c) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n+1} (2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n (\pi/3)^{2n+1}}{(2n+1)!}$$

$$= \sin(\pi/3)$$

$$= \sqrt{3}/2$$

$$(g) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n} (2n+1)!} = \frac{3}{3} \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n} (2n+1)!}$$

$$= 3 \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n+1} (2n+1)!}$$

$$= 3 \left(\sqrt{3}/2 \right) = \frac{3\sqrt{3}}{2}$$

$$(d) \sum_{n=0}^{\infty} \frac{1}{n!} = e$$

$$(h) \sum_{n=0}^{\infty} \frac{1}{(2n)!} = \frac{e + e^{-1}}{2}$$

Notice that: $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$

$$+ e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$$

$$e + e^{-1} = \frac{2}{0!} + \frac{2}{2!} + \frac{2}{4!} + \dots$$