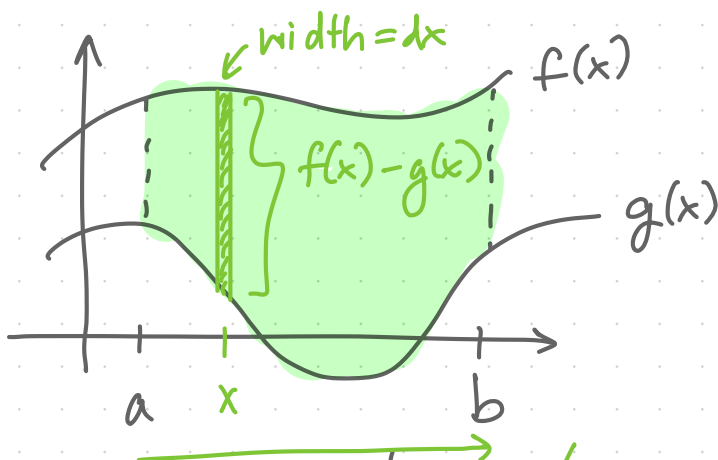


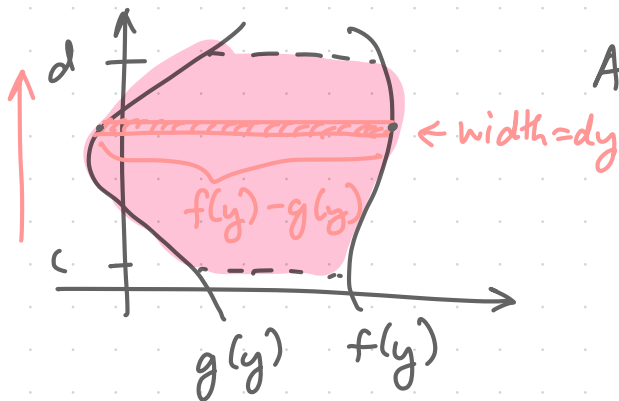
6.1 - Areas Between Curves



$$\text{Area} = \int_a^b \text{top fn} - \text{bottom fn} \, dx$$

$$= \int_a^b \underbrace{f(x) - g(x)}_{\text{height}} \underbrace{dx}_{\text{width}}$$

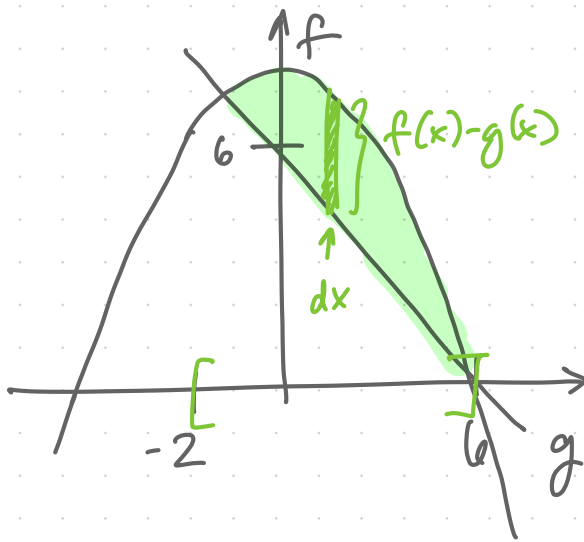
add them up



$$\text{Area} = \int_c^d \text{right fn} - \text{left fn} \, dy$$

$$= \int_c^d f(y) - g(y) \, dy$$

ex: Find the area between $f(x) = 9 - \left(\frac{x}{2}\right)^2$ and $g(x) = 6 - x$.



intersection pts

$$9 - \left(\frac{x}{2}\right)^2 = 6 - x$$

$$9 - \frac{x^2}{4} - 6 + x = 0$$

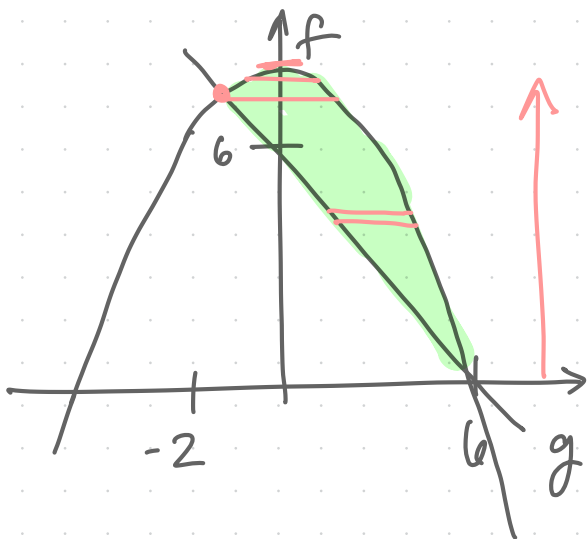
$$-4 \left(-\frac{x^2}{4} + x + 3 \right) = (-4)(4)$$

$$x^2 - 4x - 12 = 0$$

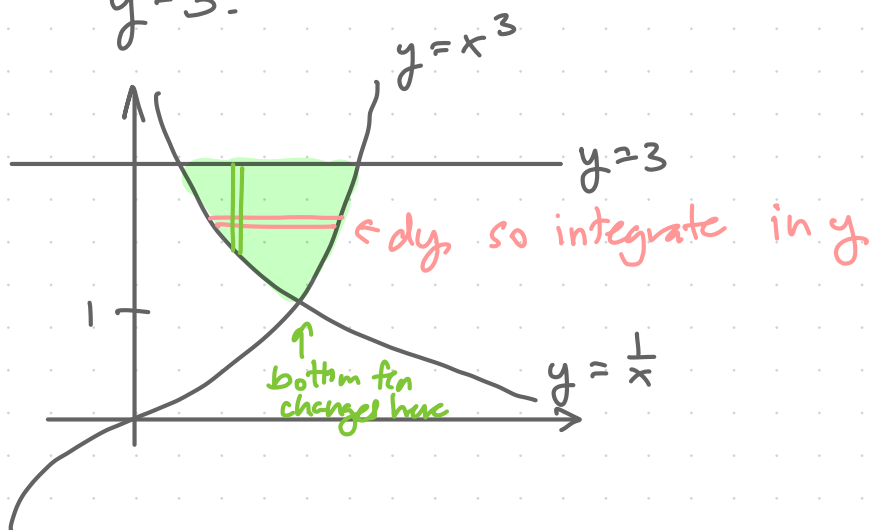
$$(x - 6)(x + 2) = 0$$

$$x = 6, -2$$

$$\int_{-2}^6 f(x) - g(x) dx = \int_{-2}^6 \left(9 - \left(\frac{x}{2}\right)^2 \right) - (6 - x) dx$$
$$= \frac{64}{3}$$



ex: Find the area between $y=x^3$, $y=\frac{1}{x}$ and $y=3$.



y -value of the intersection of $y=x^3$, $y=\frac{1}{x}$
 $x^3 = \frac{1}{x} \Rightarrow x^4 = 1 \Rightarrow x = \pm 1$
 so $x=1$

if $x=1$, $y=(1)^3$
 $y=1$

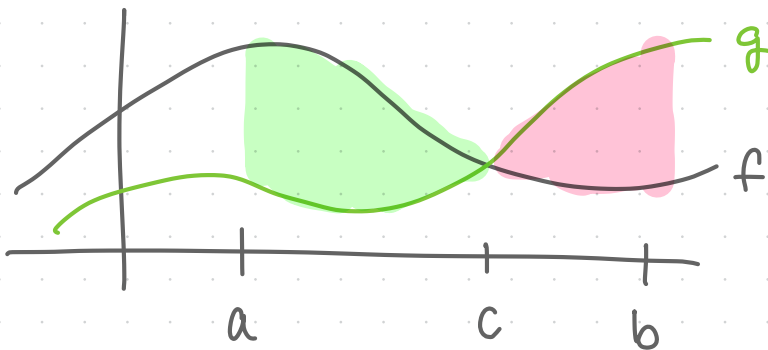
$\Rightarrow$ Bounds are 1 to 3

Now, need to convert our fens to fens of y .

right function: $y = x^3 \Rightarrow x = \sqrt[3]{y}$

left function: $y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$

$$\text{Area} = \int_1^3 \sqrt[3]{y} - \frac{1}{y} dy = \text{some number.}$$

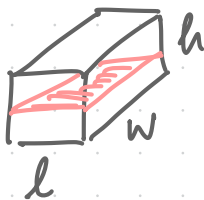


$$\text{Area} = \int_a^c f(x) - g(x) dx + \int_c^b g(x) - f(x) dx$$

6.2 - Volumes

We know how to compute the volume of

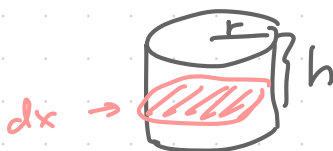
a cube



$$V = lwh$$

$$= \underbrace{(lw)}_{\text{area of cross section}} \cdot \overset{\uparrow}{h} \text{ height}$$

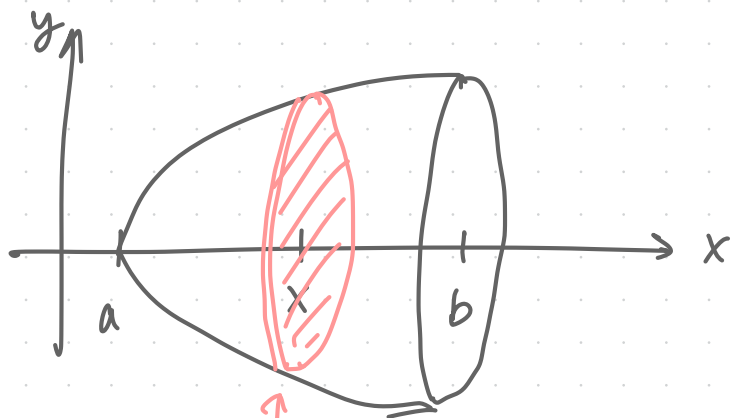
a cylinder



$$V = \underbrace{(\pi r^2)}_{\text{cross sectional area}} \cdot \overset{\uparrow}{h} \text{ height}$$

$$\text{Volume} = \left(\begin{array}{c} \text{cross sectional} \\ \text{area} \end{array} \right) \times \left(\begin{array}{c} \text{height} \end{array} \right)$$

What about a solid like this?



cross
sectional
area
depends on x

3d-slice
with width dx

dV = volume of a slice (tiny bit of volume)

$A(x)$ = cross sectional area at x

Volume : $V = \int_a^b dV = \int_a^b A(x) dx$

$\underbrace{\int_a^b}_{\text{add them up}}$ $\underbrace{dV}_{\text{bits of volume}}$

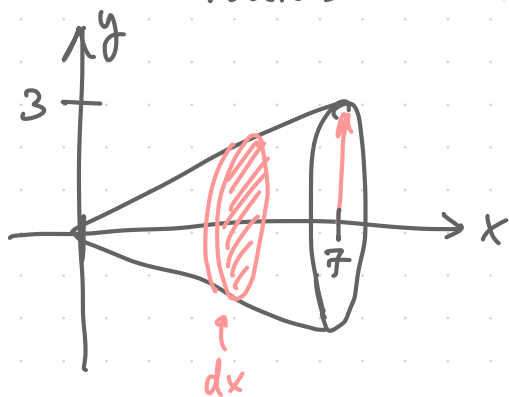
Summarize: $dV = A(x)dx$

and $V = \int_a^b A(x) dx$

Big Idea:

Volume = integral of cross sectional area

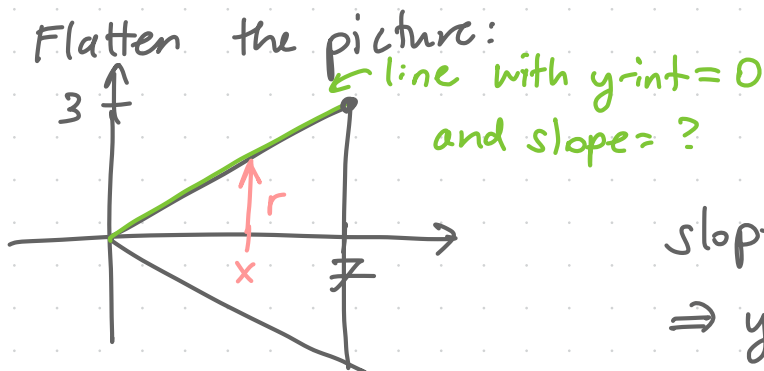
example: volume of a cone with base radius 3 and height 7



cross sectional area = πr^2

where r is a function of x

Flatten the picture:



$$\text{slope} = \frac{3}{7}$$

$$\Rightarrow y = \frac{3}{7}x + 0$$

$$\text{So } r = \frac{3}{7}x$$

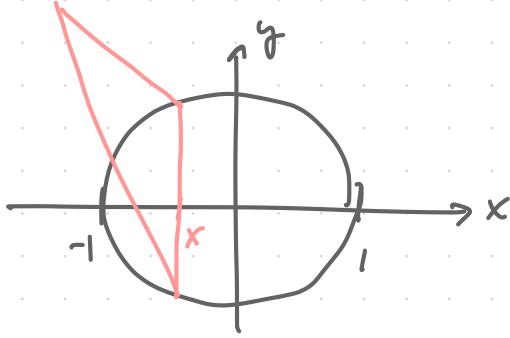
$$\Rightarrow A(x) = \pi \left(\frac{3}{7}x \right)^2$$

$$\begin{aligned} V &= \int_0^7 \pi \left(\frac{3}{7}x \right)^2 dx = \frac{9\pi}{49} \int_0^7 x^2 dx \\ &= \frac{9\pi}{49} \left(\frac{1}{3}x^3 \right) \Big|_0^7 \\ &= 21\pi \end{aligned}$$

ex: "volumes of solids with known cross sections"

A solid object has $x^2 + y^2 = 1$ as its base and each cross section perpendicular to the x -axis is a triangle whose height equals the length of the base. Find the volume.

★ see visualization: <https://www.desmos.com/3d/uvv23qujst>

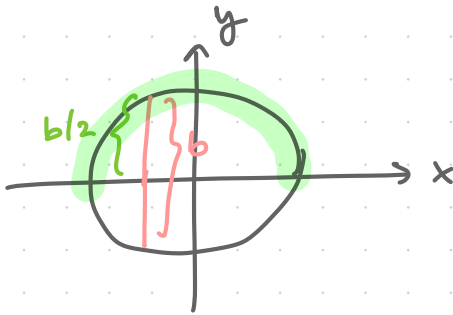


cross sections are triangles and $b=h$

$$A(x) = \frac{1}{2} b h$$

$$= \frac{1}{2} b^2$$

Goal: write b as a function of x



$$y = \pm \sqrt{1-x^2}$$

$$\text{top half: } y = \sqrt{1-x^2}$$

$$\frac{b}{2} = \sqrt{1-x^2}$$

$$\Rightarrow b = 2\sqrt{1-x^2}$$

to be continued ...

