Arc Length Formula
If a smooth curve with parametric equations $x = f(t)$, $y = g(t)$, $a \leq t \leq b$, is traversed exactly once as t increases from a to b, then its length is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

1. Follow the steps below to use the arc length formula to compute the circumference of a circle of radius r.

   (a) Give a parametrization of the circle of radius r centered at the origin.

   (b) Check that your parametrization traverses the circle exactly once. What is the starting point of your parametrization? Which direction does your parametrization go, clockwise or counter clockwise?

   (c) Compute the length of the curve, i.e. the circumference of the circle. Is your answer what you expected?

2. (a) Graph the curve $y = \ln x$ where $1 \leq x \leq 3$. Find a parametrization of the curve.

(b) Does your parametrization traverse this curve exactly once? How do you know?

(c) Set up an integral that represents the arc length of this curve. You do not have to evaluate the integral.

3. (a) Graph the curve $x = y^2$ where $-1 \leq y \leq 1$. Find a parametrization of the curve.

(b) Does your parametrization traverse this curve exactly once? How do you know?

(c) Set up an integral that represents the arc length of this curve. You do not have to evaluate the integral.

4. Think about how you were able to find a parametrization for the curve in problem 2. Can you use that process to find a parametrization for any curve given as $y = f(x)$ where $a \leq x \leq b$? What would the arc length formula be in that case?