

Parametric Curves

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

$a \leq t \leq b$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

slope of tangent line
at (c, d)

solve for t that
gives you (c, d)

both true $\rightarrow$ $f(t) = c$
and
 $g(t) = d$

Arc length:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\downarrow$$
$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

OR if $y = f(x)$, $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

OR if $x = g(y)$, $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$

Polar Curves : $r = f(\theta)$

Can write a polar curve parametrically
(treat θ as a parameter)

$$\begin{cases} x = r \cos \theta = f(\theta) \cos \theta \\ y = r \sin \theta = f(\theta) \sin \theta \end{cases}$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

And for arc length of polar curve

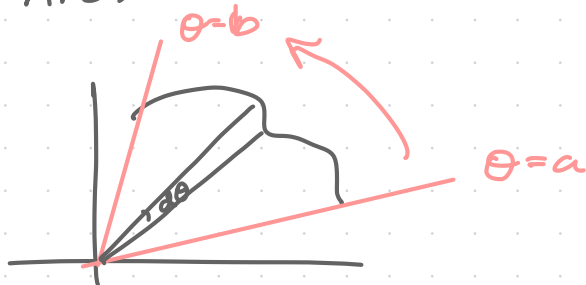
$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

$\uparrow$ $\uparrow$
 $f(\theta)$ $f'(\theta)$

For arc length : $L = \int_a^b ds$

- perfect square inside $\sqrt{\quad}$
- u-sub
- trig sub

Areas Between Polar ($r=f(\theta)$)



$$\text{Area of a sector} = (\pi r^2) \cdot \frac{\theta}{2\pi} = \frac{1}{2} r^2 \theta$$

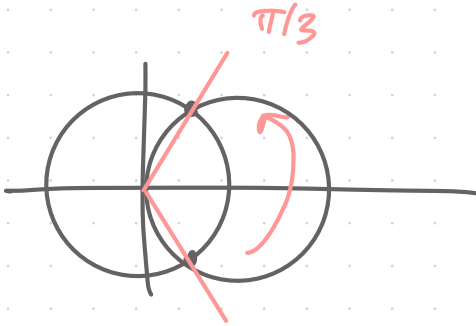


$$dA = \frac{1}{2} r^2 d\theta$$

$$A = \int_a^b dA$$

$$= \int_a^b \frac{1}{2} r^2 d\theta$$

inside $r = 4 \cos \theta$, outside $r = 2$



$$4 \cos \theta = 2$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

$-\pi/3$ NOT $\frac{7\pi}{3}$

$$\int_{-\pi/3}^{\pi/3} \frac{1}{2} \left(\underset{\substack{\uparrow \\ (\text{outer} \\ \text{curve})^2}}{(4 \cos \theta)^2} - \underset{\substack{\uparrow \\ (\text{inner} \\ \text{curve})^2}}{(2)^2} \right) d\theta$$

$$dA = \frac{1}{2} r^2 d\theta$$