

Virasoro Blocks and the Black Hole Information Paradox

A Thesis Defence Presentation

Aaditya Datar

Indian Institute of Science Education and Research, Pune
and
Center for High Energy Physics, Indian Institute of Science, Bengaluru



Supervisor: Prof. Chethan Krishnan

The AdS/CFT correspondence

The AdS/CFT correspondence is a conjectured relationship between theories of quantum gravity (usually some variety of string theory) in anti-de Sitter (AdS) space and a quantum field theory with conformal symmetries operating in Minkowski spacetime (CFTs).

AdS (Bulk Theory)	CFT (Boundary Theory)
3-dimensional AdS space	2-dimensional conformal field theory
Gravitational constant G_N	Central Charge $c = \frac{2l}{3G_N}$
Bulk scalar field $\phi(\rho, \tau, \theta)$	Scalar operator $\mathcal{O}_h(z)$
Mass of bulk field $m_{\text{AdS}}^2 = \Delta(\Delta - 2)$	Scaling dimension Δ $\Delta = 2h$

Euclidean BTZ black hole

It has horizon(s), a Hawking temperature $T_H = 2\pi/r_+$ and other interesting thermodynamic properties which makes it a simplified setting for understanding quantum black holes.

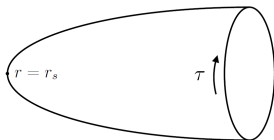


Figure: Cigar representation

$$ds^2 = \left(\frac{r^2 - r_+^2}{l^2} \right) d\tau^2 + \left(\frac{r^2 - r_+^2}{l^2} \right)^{-1} dr^2 + r^2 d\phi^2$$

CFT₂ correlators and Virasoro blocks

Four-point functions of local scalar operators $\mathcal{O}_{h_i}(z_i)$ are interesting to calculate.

$$\langle \mathcal{O}_1(\infty) \mathcal{O}_2(1) \mathcal{O}_3(z, \bar{z}) \mathcal{O}_4(0) \rangle = \sum_{h, \bar{h}} P_{h, \bar{h}} \mathcal{V}_h(c, h_i, h; z) \mathcal{V}_{\bar{h}}(c, h_i, \bar{h}; \bar{z})$$

A block decomposition expression can be made for this because the basis of the Hilbert space consists of primary operators $|\mathcal{O}_h\rangle$ and its descendants which are given by the action of the Virasoro generators L_n that satisfy the algebra

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

Features of black hole information loss in correlators

According to Kaplan et al. [1] the periodic singularities in the (semi-classical $G_N \rightarrow 0$) thermal AdS correlators and thus, their presence in the dual ($c \rightarrow \infty$) CFT correlators is a manifestation of the information paradox

$$\langle \mathcal{O}_L(t) \mathcal{O}_L \rangle_{\text{black hole}} \sim \langle \mathcal{O}_H(\infty) \mathcal{O}_L(1) \mathcal{O}_L(z, \bar{z}) \mathcal{O}_H(0) \rangle$$

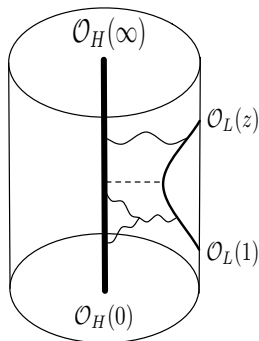
This is because periodic singularities are forbidden in pure state CFT correlators. Essentially, the AdS side is allowed to “win” when we allow a semi-classical approximation, and the problem statement lies in identifying and understanding the departures between the approximate large c and exact finite- c calculations.

Why we like the heavy-light limit to study Virasoro blocks

It turns out that these periodic singularities are presented by the Virasoro blocks individually, and this can be computed analytically using a certain “semi-classical heavy-light” limit wherein $\frac{h_H}{c}$, h_L , h are fixed for $c \rightarrow \infty$.

$$\mathcal{V}_h(c, h_i, h; z) \sim \left[\sin \left(\frac{r_+ \tau}{2} \right) \right]^{h-2h_L} {}_2F_1(h, h; 2h; 1 - e^{ir_+ \tau})$$

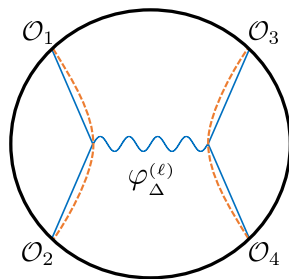
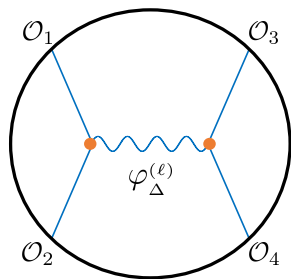
Furthermore, this limit also has a very appealing AdS interpretation.



What we wish to learn

- Is there a way to understand this from the bulk? Specifically, is there some prescription that can calculate this block in the bulk?
- Periodic singularities as a signature of the information paradox is not a very strong statement, especially given that these exist in other thermal systems as well. Can we get a stronger statement about the information paradox, preferably from the bulk?

Witten Diagrams to Geodesic Witten Diagrams



We go from

$$\int_y \int_{y'} G_{b\partial}(y, x_1) G_{b\partial}(y, x_2) G_{bb}(y, y'; \Delta, l) G_{b\partial}(y', x_3) G_{b\partial}(y', x_4)$$

to

$$\int d\lambda G_{b\partial}(x_1, y(\lambda)) G_{b\partial}(x_2, y(\lambda)) \int d\lambda' G_{b\partial}(x_3, y(\lambda')) G_{b\partial}(x_4, y(\lambda')) \\ G_{bb}(y(\lambda), y(\lambda'); 2h)$$

Holographic dual of the scalar conformal block

Such a calculation has been undertaken before [2], but it resorted to using the conic defect metric

$$ds^2 = \alpha^2 \sec^2 \rho \left(\frac{d\rho^2}{\alpha^2} + d\tau^2 + \sin^2 \rho d\phi^2 \right)$$

and then analytically continued the parameter $\alpha \rightarrow ir_+$. This means that while the calculation gets a consistent result, any bulk understanding that the construction provides is lost.

We wanted to investigate what this bulk construction has to say about the information loss problem, and thus, we did the calculation in a BTZ black hole set-up.

Setting up the calculation

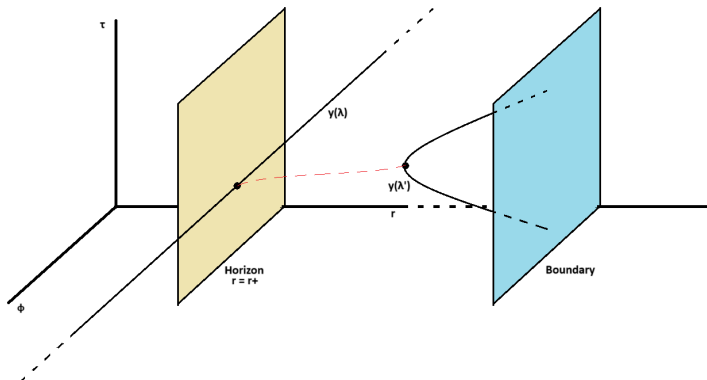


Figure: A bulk prescription to find the conformal block using a geodesic Witten diagram in BTZ geometry. The “heavy” geodesic resides at the source: the horizon, while the “light” geodesic comes from the asymptotic boundary.

- Metric:

$$ds^2 = \frac{dr^2}{r^2 - r_+^2} + (r^2 - r_+^2)d\tau^2 + r^2 d\phi^2$$

- Bulk-bulk propagator:

$$G_{bb}(y, y'; 2h) = \xi^{2h} {}_2F_1\left(h, h + \frac{1}{2}; 2h; \xi^2\right)$$

with

$$\xi = \frac{r_+^2}{rr' \cosh(r_+ \Delta\phi) - \sqrt{r^2 - r_+^2} \sqrt{r'^2 - r_+^2} \cos(r_+ \Delta\tau)}$$

- Bulk-boundary propagator:

$$G_{b\partial}(x', y) = \left(\frac{r_+^2}{r \cosh(r_+ \Delta\phi) - \sqrt{r^2 - r_+^2} \cos(r_+ \Delta\tau)} \right)^{2h}$$

Results

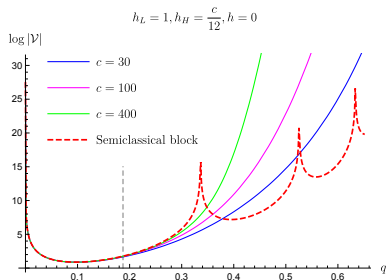
- We end up getting

$$W_{\Delta,l} \sim \mathcal{V}_h(c, h_i, h; z) \mathcal{V}_{\bar{h}}(c, h_i, \bar{h}; \bar{z})$$

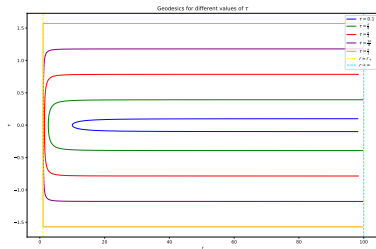
which verifies that the bulk prescription works.

- We observe that the departures of the exact Virasoro blocks and semi-classical approximation begins at $\tau = \beta/2$, which is precisely when the light geodesic starts straddling the horizon.

Essentially, this bulk prescription gives a bulk geometric perspective on the origin of where the (presumably non-perturbative) corrections that ameliorate the periodic singularity pathology kick in: as soon as one gets near the horizon.



(a) Comparison between analytic semi-classical limit and numerical calculations.



(b) Geodesics in the BTZ geodesic Witten diagram prescription

This result clarifies the role of the horizon in the bulk interpretation of Euclidean periodic singularities, which is otherwise obscure. Having a horizon statement for a Euclidean black hole is also a stronger statement given the horizon's heavy involvement in the information paradox.

Furthermore, it also opens up certain natural directions to explore:

- Are there any finite- c modifications to the propagators that might restore unitarity? *$O(1/c)$ master equation*
- Now that the role of the horizon is known, can this bulk prescription be made to include the physics of firewall naturally, that is, without imposing an ad hoc Dirichlet condition at the stretched horizon?
Mapping of BPZ equation to the radial BTZ equation

References

- [1] A. Liam Fitzpatrick et al. “On information loss in AdS₃/CFT₂”. In: *Journal of High Energy Physics* 2016.5 (May 2016), p. 109. ISSN: 1029-8479. DOI: 10.1007/JHEP05(2016)109. URL: [https://doi.org/10.1007/JHEP05\(2016\)109](https://doi.org/10.1007/JHEP05(2016)109).
- [2] Eliot Hijano et al. “Semiclassical Virasoro blocks from AdS₃ gravity”. In: *Journal of High Energy Physics* 2015.12 (Dec. 2015), pp. 1–31. ISSN: 1029-8479. DOI: 10.1007/JHEP12(2015)077. URL: [https://doi.org/10.1007/JHEP12\(2015\)077](https://doi.org/10.1007/JHEP12(2015)077).