

Choose a strategy to complete each line integral.

- Compute directly (by parameterizing C)
- Use the Fundamental Theorem of Line Integrals (FTLI) $\leftarrow$ need conservative VF
- Use Green's Theorem $\leftarrow$ need closed C in $\mathbb{R}^2$
- Use Stokes' Theorem $\leftarrow$ need closed C in $\mathbb{R}^3$
- Bonus trick: complete the curve and use Green's Theorem or Stokes' Theorem

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle \cos(x^2) - y^3, x^3 + \sqrt{y^4 + 1} \rangle$ and C is $x^2 + y^2 = 1$ oriented clockwise.

$\leftarrow C$ is closed
 $\mathbf{F}$ looks complicated.

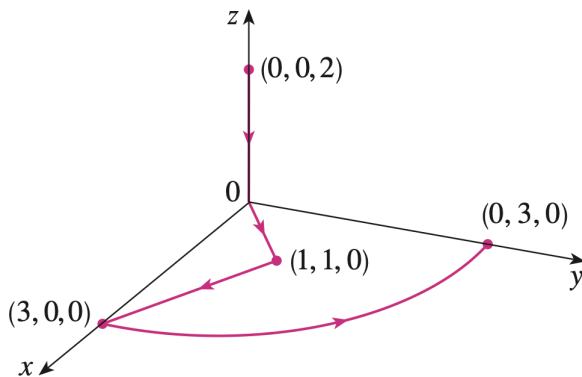
$$Q_x - P_y = 3x^2 + 3y^2 \text{ is much nicer!}$$

Use Green's Theorem.

$\leftarrow$ fixes orientation

$$\left(\text{Note: } \int_C \mathbf{F} \cdot d\mathbf{r} = - \iint_D (Q_x - P_y) dA \right)$$

2. Let $\mathbf{F} = (3x^2yz - 3y)\mathbf{i} + (x^3z - 3x)\mathbf{j} + (x^3y + 2z)\mathbf{k}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve below.



Complicated curve, would NOT want to parameterize!

Also C is not closed and even if we close it (to use Stokes') there's no obvious surface with this boundary

$$\text{curl } \mathbf{F} = \langle x^3 - x^3, -(3x^2y - 3x^2y), (3x^2z - 3) - (3x^2z - 3) \rangle = \mathbf{0}$$

$\mathbf{F}$ is conservative

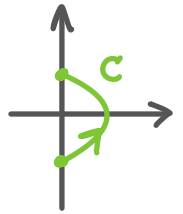
(Note: Need to find potential function to use FTLI)

Use FTLI!

3. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle ye^{xy}, xe^{xy} \rangle$ and C is the arc of the parabola $x = 1 - y^2$ from $(0, -1)$ to $(0, 1)$.

Notice $f(x,y) = e^{xy}$
 is a potential function
 for this VF $\Rightarrow \mathbf{F}$ is conservative
 Use FTLI

$\hookrightarrow C$ not closed



4. Compute $\int_C y^3 dx + x^2 dy$ where C is the arc of the parabola $x = 1 - y^2$ from $(0, -1)$ to $(0, 1)$.

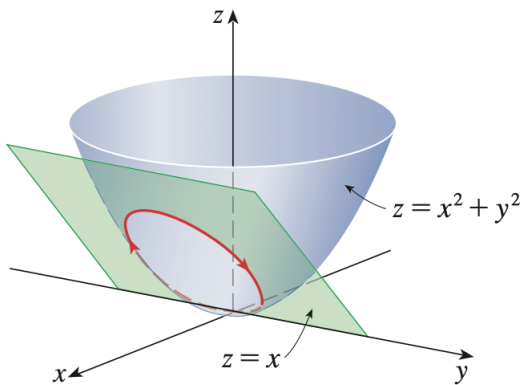
$$P = y^3, \quad Q = x^2$$

$$Q_x - P_y = 2x - 3y^2 \neq 0 \text{ so not conservative}$$

$\hookrightarrow$ same curve as #3,
not closed

Compute directly (or use Green's Thm Bonus Trick)

5. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (x^3 - z)\mathbf{i} + xy\mathbf{j} + (y + z^2)\mathbf{k}$ and C is the intersection of the paraboloid $z = x^2 + y^2$ and the plane $z = x$ with a counterclockwise orientation relative to the positive z -axis.



C is a closed curve in $\mathbb{R}^3$
 $\Rightarrow$ use Stokes' Theorem

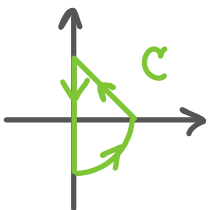
(Note: use $z = x$ as the surface
 since it's simpler than
 $z = x^2 + y^2$)

6. Find the work done by the force field $\mathbf{F}(x, y) = \langle \tan^2(\frac{x}{2}) + xy^2, x^2y + x \rangle$ on a particle that moves from $(0, 2)$ to $(0, -2)$ along the y -axis, then counterclockwise along $x^2 + y^2 = 4$ to $(2, 0)$ and then along $x + y = 2$ back to $(0, 2)$.

C is closed, oriented counterclockwise

Also C has a lot of pieces so would be
 annoying to parameterize.

$$Q_x - P_y = 2xy + 1 - 2xy = 1 \leftarrow \text{Much nicer!}$$



Use Green's Theorem