

SIMULATED ANNEALING: USING RANDOMNESS TO SOLVE THE TRAVELING SALESMAN PROBLEM

Tutor: Guillaume CHENNETIER, guillaume.chennetier@polytechnique.edu

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Mathematical context. In black box global optimization, neither the analytical form nor the gradient of the objective function to be minimized are known and no assumption of regularity or convexity can be made. Introducing randomness in the minimization method can be an efficient way to explore the space of possible solutions. The term stochastic optimization is used to designate optimization methods that generate and rely on random variables.

In this work, we will focus in particular on the so-called "simulated annealing" method. At each iteration, a new point (potential solution) close to the previous one is proposed. If this one is better, it is systematically accepted, and if this one is worse, it is accepted only with low probability. Not systematically discarding the "bad" candidates allows to explore more the space and to eventually find a much better solution. In other words, it avoids getting stuck in a local minimum. A well-calibrated method should obviously explore more in the first iterations and less as good candidates become scarce. The method is parameterized by a temperature T which is decreased with each iteration. The higher the temperature, the more easily a bad candidate will be accepted.

The suggestion of new candidates and the acceptance policy for these candidates is based on a Metropolis-Hastings algorithm which is a Monte-Carlo Markov Chain (MCMC) sampling method. This work will therefore require to study some fundamental properties of Markov chains (in particular ergodicity).

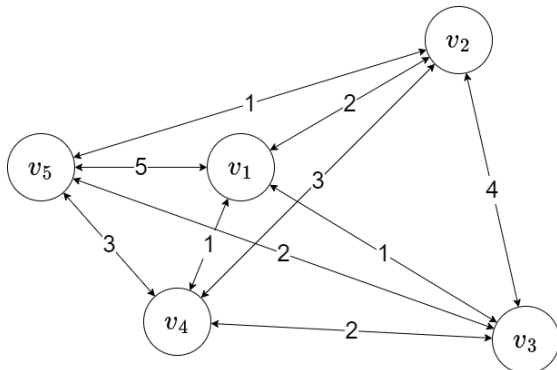


Figure 1: TSP representation with five cities $(v_i)_{i=1}^5$ and distances between these cities.

Application. A classical (NP-hard) combinatorial optimization problem is the traveling salesman problem (TSP): "given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?".

This is a problem that lends itself very well to the implementation of simulated annealing. The student should be able to implement the method in Python and test different parameterizations to get the best possible performance.

Key points:

- Understand some fundamental results about Markov chains (invariant measures, ergodicity, Doeblin condition).
- Learn about random sampling with Markov chain Monte Carlo (MCMC) methods, in particular Metropolis-Hastings.
- Implement a simulated annealing method that performs well in practice on the TSP.

Bonus objectives:

- Theoretical aspects: give conditions of convergence of the algorithm.
- Implement simulated annealing on other optimization problems.
- Compare simulated annealing to other optimization methods.

References

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