

For each of the following, sketch or describe the region of integration and change the coordinate system in which the iterated integral is written to one of the remaining two.

1. $\int_0^{\pi/2} \int_0^{\pi} \int_0^1 \rho^3 \sin \phi \, d\rho \, d\phi \, d\theta$

← from the bounds

$$0 \leq \rho \leq 1$$

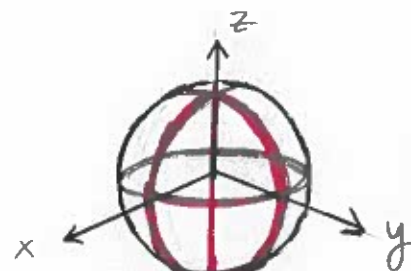
$$0 \leq \phi \leq \pi$$

$$0 \leq \theta \leq \pi/2$$

convert to rectangular (from spherical)

$$\begin{aligned} \rho^3 \sin \phi \, d\rho \, d\phi \, d\theta \\ = \rho \cdot \underbrace{\rho^2 \sin \phi \, d\rho \, d\phi \, d\theta}_{dx \, dy \, dz} \end{aligned}$$

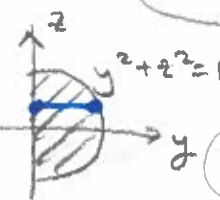
$$= \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz$$



new bounds

$$0 \leq x \leq \sqrt{1 - y^2 - z^2}$$

$$\begin{aligned} 0 \leq y \leq \sqrt{1 - z^2} \\ -1 \leq z \leq 1 \end{aligned}$$



$$\int_{-1}^1 \int_0^{\sqrt{1-z^2}} \int_0^{\sqrt{1-y^2-z^2}} \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz$$

2. $\int_0^{2\pi} \int_0^1 \int_r^1 r^2 \cos \theta \, dz \, dr \, d\theta$

← from the bounds

$$r \leq z \leq 1$$

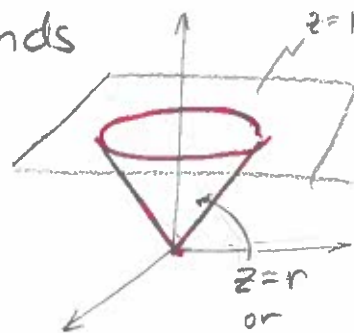
$$0 \leq r \leq 1$$

$$0 \leq \theta \leq 2\pi$$

convert to rectangular
(from cylindrical)

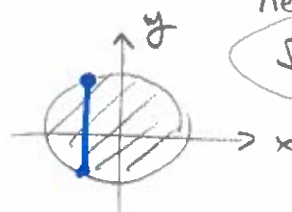
$$r^2 \cos \theta \, dz \, dr \, d\theta = \underbrace{r \cos \theta}_x \cdot \underbrace{r \, dz \, dr \, d\theta}_{dz \, dy \, dx}$$

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 x \, dz \, dy \, dx$$



new bounds

$$\sqrt{x^2 + y^2} \leq z \leq 1$$



$$\begin{aligned} -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} \\ -1 \leq x \leq 1 \end{aligned}$$

