

OBT. I - Repeat Pathria's argument leading to ~~the~~ ω_0

where $\frac{1}{\omega_0} = \frac{\text{No. of Quantum States}}{\text{Unit Vol. of (classical) phase space}}$

NOTE - MOST OF THOSE NOTES USE AN ALTERNATIVE DERIVATION OF THIS

AND

①

$$\omega_0 = h^{\tilde{N}}$$

$\tilde{N} = \text{D.O.F. FREEDOM IN SYSTEM OF PARTICLES}$

Estimate

Repeat ~~derivation~~ of ~~the~~ ω_0

$$\Sigma(N, V, E) \equiv \left(\frac{V}{h^3}\right)^N \frac{(2\pi m E)^{3N/2}}{(3N/2)!} \quad \text{EQ. (1.4.17)}$$

FOR NO. OF QUANTUM ENERGY STATES TAKEN UP BY N INDEPENDENT, IDENTICAL FREE PARTICLES (IDEAL GAS)

OBT. II Repeat calculation of

$$\omega \approx \frac{\Delta}{E} V^N \frac{(2\pi m E)^{3N/2}}{(3N/2 - 1)!} \quad \text{EQ. (2.4.2)}$$

which is the "volume" of phase space occupied by the same N particles of an ideal gas

④ WE HAVE TO DO (II) & (II)
FIRST IN ORDER TO SHOW ①.

II) DEMONSTRATION OF (1.4.17):

1) FROM CH. I, the energy levels of

an single free free particle
(no internal d.o.f.), in a ~~box~~ box
of volume $V = L^3$ is

$$n_x^2 + n_y^2 + n_z^2 = \frac{E^{(1)} 8mV^{2/3}}{h^2} = E^{(1)} \quad (2.1)$$

$E^{(1)}$ = energy of ptcl 1

$$n_x, n_y, n_z = 1, 2, 3, \dots$$

2) THE TOTAL energy of N
indisc. ptcls is

$$E = \sum_{i=1}^N E^{(i)}$$

OR

$$\boxed{E^* = \sum_{i=1}^N E_i^* = \sum_{i=1}^{3N} n_i^2} \quad (2.2)$$

3) ^{start} GEOMETRIC APPROACH.

- ⊕ For 1 pt moving in 2-D $n_x^2 + n_y^2 = R^2$
- ⊕ " 1 pt " " 3-D $n_x^2 + n_y^2 + n_z^2 = R^3$
- ⊕ ∴ For N pts moving in 3-D

$$\boxed{\sum_{i=1}^{3N} n_i^2 = R^2} \quad (3.1)$$

⊕ Since $\sum_{i=1}^{3N} n_i^2 = E^*$

then

$$R^2 = E^* \quad (3.2)$$

or $\boxed{R = E^{*1/2}} \quad 3.3$

4) NEXT, AND AGAIN PROCEEDING BY INNOVATION (AS IN STEPS LEADING TO 3.1), WE FIND BY COUNTING discrete state points (n_x, n_y) states $\in R$ in the positive quadrants of 2-D and 3-D spaces (n_x, n_y) and (n_x, n_y, n_z) that roughly, the number

of states bounded by
 a circle⁽²⁻⁰⁾ or sphere⁽³⁻⁰⁾ of
 radius $R, \mathbb{E}$, goes as:

$$\sum (E^{(1)}, V = \frac{\pi R^2}{4}, N=1) \approx \left(\frac{\pi R^2}{4} \right) \frac{1}{4} (2-0)$$

(Vol. 2N sphere, N=1)
 $\downarrow$

$$\sum (E^{(1)}, V = \frac{4}{3} \pi R^3, N=1) \approx \left(\frac{4}{3} \pi R^3 \right) \frac{1}{8} (3-0)$$

(Vol. 3N sphere, N=1)
 $\downarrow$

⊕ FROM THIS PATTERN, we
postulate: ~~the number of states~~ (factor from integral Vol. in pos. quadrant)

$$\sum (E, V_{3N}, N) \approx \text{VOLUME OF } 3N\text{-Dimensional hyper sphere}$$

(positive "quadrant" only)

(4.1)

5) once (4.1) is postulated,
 all that's reqd. is use
 of the machinery in
appendix C (in order to
 calculate the needed
 volume)

a) we infer that

$$U_{3N} \propto R^{3N}$$

(from what we know in
 2- and 3-D)

or
$$U_{3N} = C_{3N} R^{3N} \quad (5.1)$$

(b) ~~that we infer/postulate~~

b) (5.1) $\Rightarrow$
$$dU_{3N} = 3N C_{3N} R^{3N-1} \quad (5.2)$$

~~that~~

c) To find C_{3N} , find a function, ~~which~~
 (or say x), ~~for~~ $f(x)$ whose integral
 over $0 \leq x \leq \infty$ is bounded, ii)

whose integral is non-zero, iii)
 whose product with itself can be related
~~which allows~~
 in straight forward fashion

$$+0 \quad R = (x_1^2 + x_2^2 + \dots + x_{3N}^2)^{1/2}$$

Barris uses $f(x) = e^{-x^2}$

III) | ASIDE 1: ALTERNATE/SIMPLE DERIVATION OF
NOTE 1: since each of

the n_j in $\sum_{j=1}^{3N} n_j^2$ can assume
 values between 1 and n_{max} ,

where $\boxed{n_{max} = E^*}$ (and where

n_{max} occurs when all n 's
 except one are 0), and

since for $n_j \gtrsim$ say 10^3 or
 so, then ~~the~~ n_j values are

closely spaced along the (positive
 n_j axis. ~~thus~~

1.4.17

Note 2: In calculating $\Sigma(E^*, N)$, we're counting

the discrete ~~numbers~~ "tic marks",

$n_1^{(1)} = n_1 = 1$, $n_2^{(1)} = n_4 = 2$, $n_3^{(1)} = n_7 = 3$ etc.
situated
along $3N$ axes. The number

of tics ~~always~~ lying
within the positive "quadrant"
of a $3N$ -dim hypersphere
of Radius $R = \sqrt{E^*}$ is equal
to

$$n_{\max}^{(j)} - n_i^{(j)} = \sqrt{E^*} - 1 \quad (7.1)$$

$[j = 1, 2, \dots, 3N]$

Given the close spacing of
 n 's along any given axis,
and in light of (7.1), we
can express (7.1) as

$$\left. \begin{aligned} n_{\max}^{(j)} &= \sqrt{E^*} = R \\ \int_{n_i^{(j)}}^{n_{\max}^{(j)}} dn^{(j)} &= n_{\max}^{(j)} - 1 \\ &= R - 1 \cong R \end{aligned} \right\} \quad (7.2)$$

[Lec 11.8]

[NOTE 3]:

(7.2) SUGGESTS THAT WE APPROXIMATELY CAN CALCULATE THE VOLUME, V_{3N}

At Q5 (8.0) $V_{3N} \approx \int_{n_1^{(1)}=1}^{n_{\max}^{(1)}=R=\sqrt{E^*}} dn_1^{(1)} dn_2^{(1)} dn_3^{(1)} \dots dn_1^{(N)} dn_2^{(N)} dn_3^{(N)}$

$$\approx (\sqrt{E^*} - 1)^{3N}$$

$$\approx (E^*)^{3N/2}$$

$$\approx \left[\frac{E^* 8m V^{2/3}}{h^2} \right]^{3N/2}$$

$$V_{3N} = \left(\frac{V}{h^3} \right)^N [8mE]^{3N/2} \quad (8.1)$$

⊕ (8.1) differs ^{essentially} from (1.4.17)

BY A FACTOR OF $1/N!$.

THIS MUST BE TIED TO

THE INDISTINGUISHABILITY

OF OUR N IDEAL GAS

PARTICLES. SPECIFICALLY,

LEC. 11.9

IN DOING THE INTEGRAL IN (8.0),
~~EXPLICITLY~~
WE ASSUME THAT ~~THE~~ PARTICLE

j , ~~WHICH~~ AS REPRESENTED

BY THE QUANTUM NUMBERS,

$n_x^{(j)}, n_y^{(j)}, n_z^{(j)}$, IS DISTINGUISHABLE.

IT ISN'T -- THUS, WE CAN

MAKE A STRONG ARGUMENT FOR

INCLUDING A FACTOR

OF $\frac{1}{N!}$, SO, AN IMPROVED

VERSION OF (8.1) IS:

$$(9.1) \quad \sum(N, E, V) \approx \frac{V^3}{h^3} \frac{[8mE]^{3N/2}}{N!}$$

END ASIDE 1

20c. 11.10

IV) AS1082: SIMPLE, ALTERNATIVE
TOTAL
DERIVATION OF "PHASE SPACE
VOLUME FOR AN N-PARTICLE
IDEAL GAS

MUCH SIMPLER
1) THIS IS AN ALTERNATIVE TO
PATHRIA'S CALCULATION
OF W , ~~BY~~ AS GIVEN
BY 2.4.2.

NOTE: PATHRIA'S W (= phase
space vol.) applies to
a $3N$ -dimensional "spherical"
hypervolume OF
surface area

$$S = C_{3N} (3N) R^{3N-1} dR$$

(See eqn. C.3 PATHRIA)

Lec 11.11

where, by App C.6

$$C_{3N} = \pi^{3N/2} / \left(\frac{3N}{2} \right)!$$

(which is shown roughly /
w/out much detail in
last part of these

notes). TWO THICKNESS
OF TWO hypersphere is
 ΔR , where ~~R~~ since

$$R = \sqrt{2ME} = \sqrt{Ex}$$

(see accompanying scratch
notes)

$$dR = \frac{\sqrt{2M}}{\sqrt{E}} dE = \frac{2M}{\sqrt{E}} \Delta$$

NOTE 2 THE PHASE SPACE VOL.
CALCULATED HERE IS A
3N-DIM'L "CUBE" AND ~~AND~~
INCLUDES ALL ~~OTHER~~ PARTICLES

(LOC, 11, 12)

~~max~~ momenta ~~the~~ ~~are~~ lying between

$$\left| -p_{\max}^{(j)} \leq p^{(j)} \leq p_{\max}^{(j)} \right| \quad (12.1)$$

where from the energy relation for an N -ptcl Ideal gas:

$$\left| \frac{1}{2m} \sum_{j=1}^{3N} p_j^2 = E \right| \quad (12.2)$$

$$\left| p_{\max}^{(j)} = \sqrt{2mE} \right| \quad (12.3)$$

i.e. when all momenta except one, say ~~or~~ p_k equal zero then from (12.2)

$$\left| p_k = p_{\max}^{(k)} = \pm \sqrt{2mE} \right|$$

$$\left| k=1, 2, \dots, 3N \right|$$

Lec (11.13)

NOTE: we have to include both roots of (12.2) since particles can (clearly) have negative momenta (in any given direction).

2) In order to calculate the phase space vol. $\equiv \Lambda$, for N ptcls (classical), lying w/in a box of vol. $V = L^3$, and having momenta in any directions satisfying (12.1), we thus set up the following integral:

$$(B.1) \quad \Lambda \equiv \int_0^L dq_1 \int_0^L dq_2 \int_0^L dq_3 \dots \int_0^L dq_{3N-2} \int_0^L dq_{3N-1} \int_{-P_{MAX}}^L dq_{3N} \int_{-P_{MAX}}^L dP_1 \int_{-P_{MAX}}^{P_{MAX}} dP_2 \dots \int_{-P_{MAX}}^{P_{MAX}} dP_{3N}$$

THUS,

$$\Omega = (L^3)^N \cdot (2p_{\max})^{3N}$$

$$= V^N \cdot (2\sqrt{2mE})^{3N}$$

$$\boxed{\Omega = V^N \cdot (8mE)^{3N/2}} \quad (14.1)$$

3) NOW, IN ORDER TO

OBTAIN THE DESIRED

RESULT, $\boxed{\omega_0 = h^{3N}}$

I ~~AM~~ FOLLOW PATTERIA'S

APPROACH AND TAKE

THE RATIO OF ~~AM~~

~~(THE APPROXIMATELY GIVEN)~~

THE TOTAL PHASE SPACE

VOLUME, Ω , AS GIVEN BY

(14.1), TO THE TOTAL NO.

OF QUANTUM STATES, ω IN THIS

$$\Sigma = \Sigma(E, V, N)$$

(Lec. 11.15)

"cubical" (rather than
"spherical") hypervolume,
As given by (9.1). However,
For this to work, I must
either introduce a
factor of $1/N!$ into (14.1)
or assume distinguishable
ptcls and remove the
 $1/N!$ in (9.1). The better
approach, it seems, is
to make an argument
for introducing $1/N!$ into
(14.1) (since the particles
are indistinguishable).

⊕ Possible argument:

a) view the $3N$ momenta, p_i , as
discrete variables

b) now, the energy relation

$$\frac{1}{2m} \sum_{i=1}^{3N} p_i^2 = E \quad \text{or} \quad \boxed{\sum_{i=1}^{3N} p_i^2 = 2mE = E^*} \quad (15.1)$$

[Lec - 11.16]

is the classical analog of the quantum energy reln.

$$\sum_{i=1}^{3N} p_i^2 = E^* = \frac{8mE^{2/3}}{h^2}$$

c) ~~By~~ IF we AGAIN (AS IN QUANTUM CASE) COUNT UP THE NUMBER OF CLASSICAL ENERGY STATES SATISFYING (15.1) we can express

this counting procedure in terms of integrals, i.e.:

$$\left(\begin{array}{l} \text{Vol. of} \\ \text{momentum} \\ \text{part} \\ \text{of} \\ \text{phase} \\ \text{space} \end{array} \right) = \left(\begin{array}{l} \text{no. of} \\ \text{tics} \\ \text{along} \\ p_1 \\ \text{axis} \end{array} \right) \cdot \left(\begin{array}{l} \text{''} \\ p_2 \\ \text{''} \end{array} \right) \cdot \left(\begin{array}{l} \text{''} \\ p_3 \\ \text{''} \end{array} \right) \cdots \left(\begin{array}{l} \text{''} \\ p_{3N} \\ \text{axis} \end{array} \right) \approx \\ (2P_{\max}) \cdot (2P_{\max}) \cdot (2P_{\max}) \cdots (2P_{\max})$$

$\Rightarrow$

$$\begin{aligned} \text{Vol of} \\ \text{momentum} &\approx (2P_{\max})^{3N} \\ \text{part of} \\ \text{phase} &= (2\sqrt{2mE})^{3N} = (8mE)^{3N/2} \\ \text{space} \end{aligned}$$

$$d) \therefore \Omega = \frac{\text{total phase space vol}}{h^{3N}} = (8mE)^{3N/2} \cdot V^N$$

e) so, BASED ON STOPS a) - d)
 BUT we can now already
 see that due to indistinguishability
 we must introduce a
 factor of $1/N!$ into (14.1).

FINAL VERSION OF Ω IS

$$(17.1) \quad \Omega(N, V, E) \cong \frac{V^N (8mE)^{3N/2}}{N!}$$

FINALLY, FORMING $\underline{\Omega(N, V, E)}$
 (eq. 9.1) $\rightarrow \Sigma(N, V, E)$

we get an alternate derivation
 of ω_0 :

$$\omega_0 = \frac{\Omega}{\Sigma} = h^{3N} \quad \left. \begin{matrix} !!! \\ \dots \end{matrix} \right\}$$

Rough / SCRATCH

Replication
of

Pathria's

derivation

of

$$W_0 = h^{3N}$$

Follows

~~A~~

J) BRIDGE YOURS APPROACHES

Quantum

1) Count microstates to get

$$E_1(E_{x1}, n_{x1}, n_{y1}) = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2)$$

CH. used N quantum indices
precis. w/ known energy states

$$S = k \ln \Omega \quad (\text{microcan. ens.})$$

$$\Omega = dE(N, V, \sigma)$$

~~W. S. K. T~~
~~W. S. K. T~~

2) Determine Partition Fun Q
for system

$$A = -kT \ln Q \quad (\text{macrocan. ensemble})$$

~~W. S. K. T~~

$$Q = \sum g_i e^{-\beta \epsilon_i}$$

$$= \int g(\epsilon) e^{-\beta \epsilon} d\epsilon$$

$$1) \quad n_x^2 + n_y^2 + n_z^2 = \frac{E}{\epsilon_1 \frac{8m^2 \hbar^2}{h^2}}$$

$$n_x, n_y, n_z \geq 0$$

$$2) \quad \text{Npols index} \Rightarrow E = \sum_{i=1}^N \epsilon_i$$

$$3) \quad \sum_{i=1}^{3N} n_i^2 = \frac{E \epsilon_1 m \hbar^2}{h^2} = E^* = R^2$$

$\Rightarrow$ No. of states $\sum n^3$ satisfying 3)

4) in $3N$ dim space

$$\equiv \sum (N, \nu, E)$$

a) $R \equiv E^*$ from (3)

b) $V_{3N}(R^{3N})$ (geom)

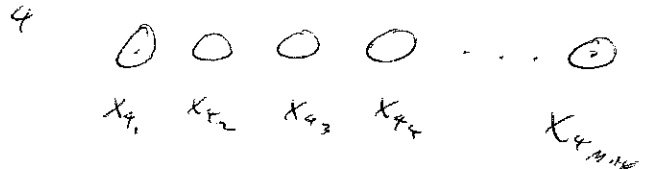
$$\begin{aligned} 1 & \quad \nu = R \\ 2 & \quad \nu = \pi R^2 \\ 3 & \quad \nu = \frac{4}{3} \pi R^3 \end{aligned}$$

$$\begin{aligned} \text{if } E^* &= 25 \\ R &= 5 \\ \Sigma &\approx 25 \end{aligned}$$

or

$$d\nu = 5 dR$$

$$= C_{3N} 3N R^{3N-1} dR$$



$$\text{or c) } V_{3N} \propto E^* \propto R^{3N} \propto \sum (E, \nu, N)$$

$$d) \quad \boxed{\Sigma(E, \nu, N) = V_{3N}}$$

$$\begin{aligned} R^2 &= \sum n_i^2 = E^* \\ R &= (\sum n_i^2)^{1/2} \\ R^3 &= (\sum n_i^2)^{3/2} \end{aligned}$$

$$e) \quad V_N = \int_0^{R_{\max}} S(R) dR = C_1 R^4$$

$$\begin{aligned} 2a) \quad V_{10+1} &= \frac{1}{8} \frac{4}{3} \pi R^3 E^{*3/2} \\ &= \frac{\pi}{6} R^3 \\ &= \frac{\pi}{6} E^{*3/2} \end{aligned}$$

$$2b) \quad \Sigma(E, \nu, 1)$$

2c)



$$\begin{aligned} \Rightarrow R &= 3 \\ E^* &= 9 = R^2 \\ \Sigma &\approx 27 \end{aligned}$$

(A2)

$$1) \boxed{dV_n = C_n R^{n-1} dR}$$

$$2) V_n = \int dV_n = C_n \int R^{n-1} dR$$

$$3) \text{ ~~Ans~~ } \boxed{dV_n = \prod_{i=1}^n dx_i}$$

$$4) \boxed{\pi^{n/2} = \int_{-\infty}^{\infty} e^{-x^2} dx} = 2 \int_0^{\infty} e^{-x^2} dx$$

$$\begin{aligned} \pi &= \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r dr d\theta \Rightarrow \pi = 4 \int_0^{\pi/2} \int_0^{\infty} r e^{-r^2} dr d\theta \\ &= 2\pi \left(\frac{1}{2}\right) e^{-r} \Big|_0^{\infty} = \frac{2\pi}{2} e^{-r^2} \Big|_0^{\infty} \\ &= \frac{\pi}{2} \end{aligned}$$

$$\oint \int_{-\infty}^{\infty} e^{-x^2} dx = C_0 = 2 \int_0^{\infty} e^{-x^2} dx$$

$$\begin{aligned} \therefore C_0^2 &= \int_0^{\infty} \int_0^{2\pi} e^{-r^2} r dr d\theta \\ &= (2\pi) \frac{1}{2} \end{aligned}$$

$$= \pi$$

$$\therefore \boxed{C_0 = \sqrt{\pi}} \checkmark$$

$$\S \text{ ~~Ans~~ } 1) \S 3) \Rightarrow$$

$$\begin{aligned} \pi^{n/2} &= \int_{-\infty}^{\infty} e^{-\underbrace{(x_1^2 + x_2^2 + \dots + x_n^2)}_{R^2}} \underbrace{dx_1 dx_2 \dots dx_n}_{dV_n} \\ &= \int_{-\infty}^{\infty} e^{-R^2} n C_n R^{n-1} dR \end{aligned}$$

(193)

$$\Rightarrow \pi^{1/2} = \int_0^\infty$$

$$\pi = 4 \int_0^\infty \int_0^{\pi/2} d\theta e^{-r^2} r dr$$

$$= 2 \frac{\pi}{2}$$

$$\pi^{1/2} = 2 \int_0^\infty e^{-x^2} dx$$

$$\pi = 4 \int_0^\infty e^{-r^2} r dr \left(\frac{\pi}{2} \right)$$

$$= \frac{2\pi}{2}$$

$$\Rightarrow \pi = \int_0^\infty e^{-r_1^2} r_1 dr_1 \cdot \int_0^\infty e^{-r_2^2} r_2 dr_2$$

$$\dots \int_0^\infty e^{-r_n^2} r_n dr_n$$

$$1 = \int_0^\infty e^{-(R^2 + R^2)} dR$$

$$1 =$$

$$\pi^{n/2} = C_n \int_0^\infty e^{-R^2} R^{n-1} dR$$

$$C_n = \pi^{n/2} / (n/2)!$$

$$\Psi^{3N} = \frac{1}{(2\pi)^{3N}} C_{3N} R^{3N} \approx \Sigma(E, V, N)$$

$$= \frac{1}{(2\pi)^{3N}} \frac{\pi^{3N/2}}{(3N/2)!} \left(\frac{8mV^{2/3}E}{h^2} \right)^{3N/2} R = E \times K$$

$$= \left(\frac{8mV^{2/3}E}{h^2} \right)^{1/2}$$

$$\Sigma(E, V, N) \approx$$

(14)

$$\Sigma(\epsilon, V, N) \approx \left(\frac{V}{h^3} \right)^N \frac{(2\pi m \epsilon)^{3N/2}}{(3N/2)!} \quad (1.4.17)$$

$$V_+ = \frac{\pi R^2}{2^n} = \frac{V_0}{2^n}$$

3-D case:

$$V_+ = \frac{V_0}{2^n}$$

$\Rightarrow$ ~~1) $V \approx \Sigma(N, V, \epsilon)$~~

$$1) V \approx \Sigma(N, V, \epsilon)$$

$$2) dV \approx d\Sigma = \left. \frac{\partial \Sigma}{\partial \epsilon} \right|_{N, V} d\epsilon \quad (N, V \text{ fixed})$$

$$3) \Gamma \approx d\Sigma = \left. \frac{\partial \Sigma}{\partial \epsilon} \right|_{N, V} \Delta$$

$$\Gamma \approx \frac{\partial \Sigma}{\partial \epsilon} = \left(\frac{V}{h^3} \right)^N \frac{(2\pi m)^{3N/2}}{(3N/2)!} \left(\frac{3N}{2} \right) \epsilon^{(3N/2)-1} \left(\frac{\Delta}{\epsilon} \right)$$

$$\Gamma \approx \Sigma(\epsilon, V, N) \left(\frac{3N}{2} \right) \left(\frac{\Delta}{\epsilon} \right)$$

$$4) S = k \ln \Gamma$$

(15)

Sec. 2.4

$$1) \Omega = \int dq \int dp$$

$$2) \int dq = \int_0^L dx_1 \int_0^L dx_2 \int_0^L dx_3 \dots \int_0^L dx_N$$

$$L^3 \cdot L^3 \cdot \dots \cdot L^3$$

$$= V^N$$

$$3) \int dp \equiv p_i =$$

$$4) E = \frac{1}{2m} \sum_{i=1}^{3N} p_i^2$$

$$\therefore \sum_{i=1}^{3N} p_i^2 = E^* = 2mE$$

$$5) \therefore R = \left[\frac{1}{2} \sum_{i=1}^{3N} p_i^2 \right]^{1/2} = E^{1/2} \Rightarrow \sqrt{2mE}$$

$$6) \therefore \frac{dV_E}{dE} = C_0 R^{3N} = C_0 E^{3N/2} \quad \left| \frac{dR}{dE} = \frac{\sqrt{2m}}{2} \frac{1}{E^{1/2}} \right|$$

$$7) dV_E = C_0 \underbrace{3NR^{3N-1}}_{S(R)} dR$$

$$8) \pi^{1/2} = \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$9) \therefore \int dp = \int_{-\infty}^{\infty} dp_1 dp_2 dp_3 \dots dp_{3N} = \Delta E$$

$$\Delta V_E \approx \int_{-\infty}^{\infty} C_0 3N R^{3N-1} dR \xrightarrow{\frac{\sqrt{2m}}{E^{1/2}} \left(\frac{L}{2}\right) \Delta}$$

$$\approx C_0 3N R^{3N-1} \left(\frac{\sqrt{2m}}{E^{1/2}}\right) \left(\frac{L}{2}\right) \Delta$$

A6

$$\Delta \epsilon = \frac{C_0 3N (2m\epsilon)^{3N/2}}{(E^{1/2}) \sqrt{2m} (E^{1/2}) \left(\frac{1}{2}\right) \Delta}$$

$$\Delta \epsilon \approx \int \psi(R) dR$$

$$= \frac{2\pi^{n/2}}{\Gamma(n/2)} R^{n-1} dR$$

$$dR \quad R = E^{1/2} = \sqrt{2m\epsilon}$$

$$dR = \frac{1}{2\sqrt{2m\epsilon}} d\epsilon = \frac{\sqrt{2m}}{2\epsilon^{1/2}} d\epsilon$$

$$\frac{1}{2} \frac{(2m)^{1/2}}{(2m\epsilon)^{1/2}} d\epsilon$$

$$\Delta \epsilon \approx \frac{2\pi^{3N/2}}{\Gamma(3N/2)} \left(\frac{R^{3N}}{R}\right) dR$$

$$\approx \frac{2\pi^{3N/2}}{\Gamma(3N/2)} \frac{(2m\epsilon)^{3N/2}}{(2m\epsilon)^{1/2}} \left(\frac{1}{2}\right) \frac{(2m)^{1/2}}{(2m\epsilon)^{1/2}} \Delta$$

$$n + \frac{1}{2} = \frac{3N}{2}$$

$$n = \frac{3N}{2} - \frac{1}{2}$$

$$\Gamma(n) = \left(\frac{3N}{2} - 1\right)!$$

$$\Delta \epsilon \approx \Delta \left(\frac{m}{2\epsilon}\right)^{1/2} \left[\frac{2\pi^{3N/2} (2m\epsilon)^{\frac{3N-1}{2}}}{\left(\frac{3N}{2} - 1\right)!} \right]$$

$$\omega \approx \sqrt{N} \Delta \epsilon \approx 2.4 \cdot 2$$

$$1) p_x^2 + p_y^2 + p_z^2 = \frac{8mL^2}{h^2} E$$

$$L^3 = V \quad V = L^3 \quad (A7)$$

$$2) E_1 + E_2 \dots E_N = \frac{h^2}{8mV^{2/3}} [p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + \dots + p_{Nx}^2 + \dots]$$

$$3) \Rightarrow \boxed{\frac{E \cdot 8mV^{2/3}}{h^2} = \sum_{i=1}^{3N} p_i^2 = E^* = R^2} \quad p_i = 1, 2, \dots \Rightarrow \boxed{R = E^* k_2}$$

$$4) \sum_{i=1}^{3N} p_i^2 = R^2 = E^* \quad \text{geometric approach}$$

$$5) \Sigma(N, V, E) \approx \frac{4}{3} \pi R^3 \left(\frac{1}{8} \right) = Vol \\ \approx \frac{1}{6} \pi R^3 = \frac{1}{6} \pi E^{3/2}$$

$$6) \Sigma(N, V, E); \quad \text{Area } C$$

$$a) \quad V_n = C_n R^n$$

$$b) \quad dV_n = C_n n R^{n-1} dR$$

$$c) \quad \cancel{Vol} \quad \pi^{1/2} = \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$d) \therefore \pi^{n/2} = \int_{-\infty}^{\infty} e^{-(x_1^2 + x_2^2 + \dots)} \underbrace{dx_1 dx_2 \dots}_{dV_n} \\ = \int_0^{\infty} e^{-R^2} C_n n R^{n-1} dR$$

$$e) \quad \boxed{C_n = \frac{\pi^{n/2}}{(n/2)!}}$$

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$$f) \quad \therefore g) \Rightarrow \quad \mathcal{V}_{3N} \stackrel{\text{or}}{=} \frac{\pi^{3N/2}}{\left(\frac{3N}{2}\right)!} R^{3N} \approx \Sigma(E, \mathcal{V}, N)$$

$$= \frac{\pi^{3N/2}}{\left(\frac{3N}{2}\right)!} [E^*]^{3N/2}$$

$$\therefore \left[\Sigma(E, \mathcal{V}, N) \approx \frac{1}{\left(2^{3N} \left(\frac{3N}{2}\right)!\right)^{1/2}} \frac{\pi^{3N/2}}{h^2} \left[\frac{8m \mathcal{V}^{2/3} E}{h^2} \right]^{3N/2} \right] \checkmark$$

π — to take into account positive and negative

g) Since $\Sigma(E, \mathcal{V}, N) \approx \text{Vol of } 3N\text{-D hypervolume}$

$$= \mathcal{V}_{3N}$$

$$d\Sigma = \frac{\partial \Sigma}{\partial E} \bigg|_{\mathcal{V}} dE$$

h)

$$\boxed{\frac{\partial \Sigma}{\partial E} = \Gamma}$$

$$\left(\begin{array}{l} \text{2-D: } \mathcal{V} = \pi R^2 \\ d\mathcal{V} = 2\pi R dR \\ = C dR \\ \text{3-D: } \mathcal{V} = \frac{4}{3}\pi R^3 \\ \Rightarrow d\mathcal{V} = 4\pi R^2 dR \end{array} \right)$$