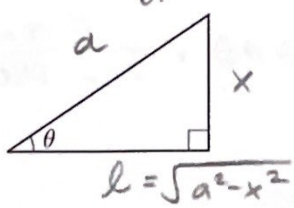
1. (a) Suppose $\sin\theta = \frac{x}{a}$. Label the triangle below with x and a. Find the length of the remaining side.

← opposite

← hypotenuse



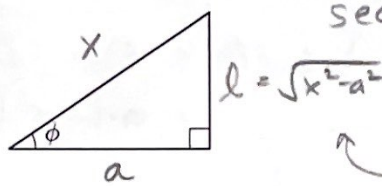
$$x^2 + l^2 = a^2$$
$$l^2 = a^2 - x^2$$
$$l = \pm\sqrt{a^2-x^2}$$

(b) Evaluate $\cos\theta$, $\tan\theta$ and $\sec\theta$ using the triangle from part (a).

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{a^2-x^2}}{a} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{a}{\sqrt{a^2-x^2}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{x}{\sqrt{a^2-x^2}}$$

2. (a) Suppose $\sec\phi = \frac{x}{a}$. Label the triangle below with x and a. Find the length of the remaining side.



$$\sec\varphi = \frac{x}{a} = \frac{\text{hyp}}{\text{adj}}$$

$$a^2 + l^2 = x^2$$
$$l^2 = x^2 - a^2$$
$$l = \pm\sqrt{x^2-a^2}$$

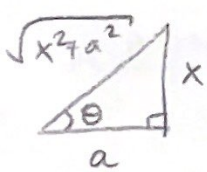
(b) Evaluate $\sin\phi$, $\cos\phi$ and $\tan\phi$ using the triangle from part (a).

$$\sin\varphi = \frac{\sqrt{x^2-a^2}}{x}, \quad \cos\varphi = \frac{a}{x}, \quad \tan\varphi = \frac{\sqrt{x^2-a^2}}{a}$$

3. Evaluate $\sin(\underbrace{\arctan(\tfrac{x}{a})}_{\theta})$.

arctan $(\frac{x}{a})$ is the angle θ such that $\tan\theta = \frac{x}{a}$

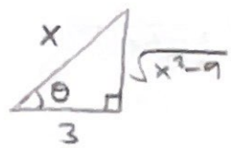
Using $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$, we get this triangle.



So $\sin\left(\arctan\left(\frac{x}{a}\right)\right) = \sin\theta = \frac{x}{\sqrt{x^2+a^2}}$.

4. Determine the appropriate trig substitution for computing the following integrals and draw the associated triangles. You do not need to evaluate the integrals.

(a) $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx = \int \frac{3\tan\theta}{(3\sec\theta)^3}\cdot 3\sec\theta\tan\theta\, d\theta = \frac{1}{3}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta$



$$\boxed{\begin{array}{l} x = 3\sec\theta \\ dx = 3\sec\theta\tan\theta\,d\theta \end{array}}$$

$$\frac{\sqrt{x^2-9}}{3} = \tan\theta$$

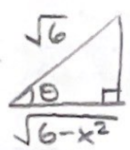
(b) $\displaystyle\int \frac{x}{\sqrt{x^2+4}}\,dx$

u-sub is easier! $\boxed{\begin{array}{l} u = x^2+4 \\ du = 2x\,dx \end{array}}$ $\displaystyle\int \frac{1}{\sqrt{u}}\cdot\frac{1}{2}\,du = \sqrt{u} + C$

$= \sqrt{x^2+4} + C$

(c) $\displaystyle\int \frac{1}{\sqrt{6-x^2}}\,dx = \int \frac{\sqrt{6}\cos\theta}{\sqrt{6}\cos\theta}\,d\theta = \int 1\,d\theta = \theta + C$

$= \arcsin\left(\frac{x}{\sqrt{6}}\right) + C$

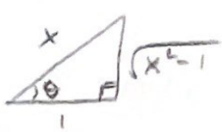


$$\boxed{\begin{array}{l} x = \sqrt{6}\sin\theta \\ dx = \sqrt{6}\cos\theta\,d\theta \end{array}}$$

And $\sqrt{6-x^2}/\sqrt{6} = \cos\theta \Rightarrow \sqrt{6-x^2} = \sqrt{6}\cos\theta$

(d) $\displaystyle\int \frac{1}{(x^2-1)^{3/2}}\,dx = \int \frac{1}{(\sqrt{x^2-1})^3}\,dx = \int \frac{1}{\tan^3\theta}\cdot\sec\theta\tan\theta\,d\theta$

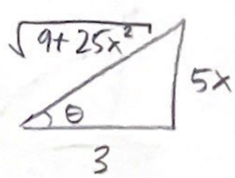
$= \displaystyle\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$



$$\boxed{\begin{array}{l} x = \sec\theta \\ dx = \sec\theta\tan\theta\,d\theta \end{array}}$$

$$\tan\theta = \sqrt{x^2-1}$$

(e) $\displaystyle\int \frac{x^2}{\sqrt{9+25x^2}}\,dx = \int \frac{x^2}{\sqrt{9+(5x)^2}}\,dx = \int \frac{\left(\frac{3}{5}\tan\theta\right)^2\cdot\frac{3}{5}\sec^2\theta}{3\sec\theta}\,d\theta$



$\tan\theta = \dfrac{5x}{3} \Rightarrow \boxed{\begin{array}{l} x = \frac{3}{5}\tan\theta \\ dx = \frac{3}{5}\sec^2\theta\,d\theta \end{array}}$

And $\sec\theta = \dfrac{\sqrt{9+25x^2}}{3}$