

I) DERIVATION OF $S = K \ln \Omega$

A) i) $E^0 = E_1 + E_2$

ii) $N = N_1 + N_2$

1	N_1, V_1, E_1
2	N_2, V_2, E_2

$\{N_1, N_2, V_1, V_2 \text{ fixed}\}$

iii) $\Omega_1 = \Omega_1(E_1, N_1, V_1) = \Omega(E_1)$ for N_1, V_1 fixed

iv) $\Omega_2 = \Omega_2(E_2) = \Omega_2(E^0 - E_1) = \Omega_2(E_1)$

since E^0, N_2, V_2 fixed

v) $\therefore \Omega^0 = \Omega_1(E_1) \cdot \Omega_2(E^0 - E_1) = \Omega^0(E_1)$

vi) POST 2: @ equil., Ω^0 is MAXIMIZED as SUBJECT to given V, N 's

vii) $\therefore \frac{d\Omega^0}{dE_1} = \Omega_2 \frac{\partial \Omega_1}{\partial E_1} \bigg|_{E_1} + \Omega_1 \frac{\partial \Omega_2}{\partial E_2} \bigg|_{E_2} (-1) \stackrel{\left(\frac{dE_2}{dE_1} = -1\right)}{=} 0$

viii) $\Rightarrow$

$$\left[\frac{\partial \ln \Omega_1}{\partial E_1} \bigg|_{E_1} = \frac{\partial \ln \Omega_2}{\partial E_2} \bigg|_{E_2} \right] \quad (1.0)$$

or $\left[\beta_1 = \beta_2 \right] \quad (1.0a)$
@ equil.

B) NOW TRY TO CONVERT $\Omega = \Omega(E, V, N)$ TO A MACROSCOPIC THERMO VARIABLE

i) FOCUS ON S . SINCE

S IS ALSO A FN. OF N, V, E .

PROOF: a) write the 1st Tds

reln:

$$\left[Tds = dE + PdV - \mu dN \right] \quad (1.1)$$

b) (1.1) shows that $S = S(E, V, N)$

ii) Thus, $ds = S_{1E/4N} dE + S_{1N/4E} dN + S_{1V/4N} dV$ in general

$$= S_{1E/4N} dE \quad \text{For } N, V \text{ fixed}$$

iii) (2.1) $\boxed{ds = \frac{1}{T} dE}$ from Maxwell's eqns

c) Express Ω as;

$\Omega = \Omega(E, V, N)$

Thus,

(2.2) $\boxed{d \ln \Omega = \frac{\partial \ln \Omega}{\partial E} dE} \quad (\text{For } V, N \text{ fixed})$

d) $\frac{(2.1)}{(2.2)} \Rightarrow \boxed{\frac{ds}{d \ln \Omega} = \frac{(V_T)}{\beta}}$

e) KEY STOP / Assumption:
Assume / guess THAT

(2.3) $\boxed{ds \propto d \ln \Omega}$

$\Rightarrow \boxed{ds = K d \ln \Omega} \quad (2.4)$

$\boxed{K = \text{Boltzmann const}}$

F)

INTEGRATE (2.4) FROM

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SYSTEM QUANTUM GROUND

STATE (AND FOR SIMPLICITY,

ASSUME $\Omega = 1$ WHEN SYS

IS IN GROUND STATE, I.E.,

THERE IS ONLY ONE QUANTUM

STATE @ $T = 0K$. ALSO,

DEFINING STATE THAT THE

ENTROPY, S , @ $T = 0K$

IS 0. THUS,

$$S(N, V, E) = k \ln \Omega(N, V, E)$$

II) QUICK CONSISTENCY CHECK

A) FROM (1.00), @ EQUILIBRIUM.

$$B_1 = B_2 \Leftrightarrow \left(\frac{\partial \ln \Omega_1}{\partial E_1} = \frac{\partial \ln \Omega_2}{\partial E_2} \right)$$

B) ^{BUT,} FROM $S = k \ln \Omega$

$$\begin{aligned} \left. \frac{\partial \ln \Omega_1}{\partial E_1} \right|_{N,V} &= \frac{1}{k} \left. \frac{\partial S_1}{\partial E_1} \right|_{N,V} \xrightarrow{\text{MAXWELL RELN}} \frac{1}{kT_1} \\ \left. \frac{\partial \ln \Omega_2}{\partial E_2} \right|_{N,V} &= \frac{1}{k} \left. \frac{\partial S_2}{\partial E_2} \right|_{N,V} = \frac{1}{kT_2} \end{aligned}$$

C) $\therefore$ @ EQUILIBRIUM

$$\frac{1}{T_1} = \frac{1}{T_2} \quad \text{ie} \quad \boxed{T_1 = T_2}$$

$\therefore$ EVERYTHING SEEMS ^{AT LEAST} CONSISTENT

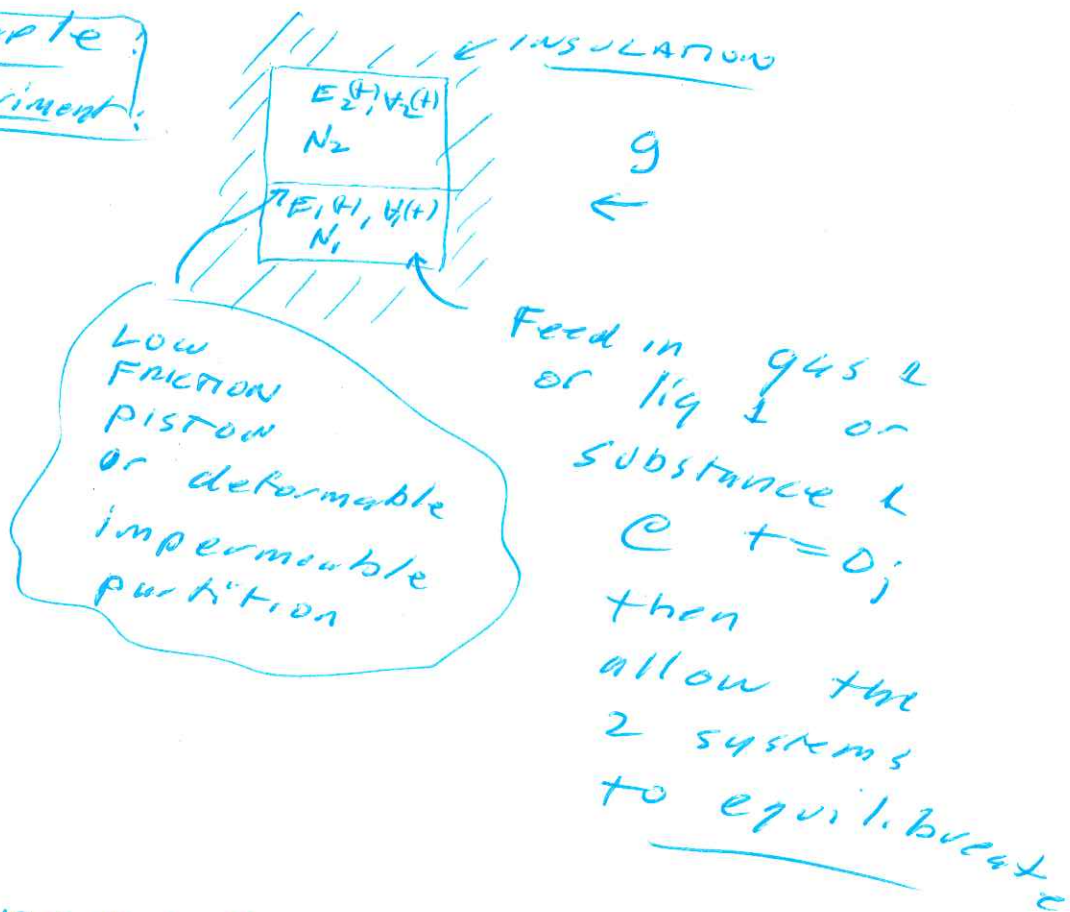
III) DERIVATION OF FURTHER CONNECTIONS BETWEEN $\Omega = \Omega(E, V, N)$ AND MACROSCOPIC THERMODYNAMICS SYSTEM

A) NEXT THOUGHT EXPERIMENT,

CONSIDER THE CASE WHERE
THE 2 SYSTEMS CAN

BOTH exchange energy, AND
EXPONENTIAL VOLUME changes.

Example:
Experiment:



Here, $N^0 = N_1 + N_2$; N_1, N_2 fixed
 $E^0 = E_1 + E_2$; E^0 fixed
 $V^0 = V_1 + V_2$; V^0 fixed

i) $\Omega_1 = \Omega_1(E_1, V_1)$

$\Omega_2 = \Omega_2(E_2, V_2) = \Omega_2(E^0 - E_1, V^0 - V_1) = \Omega_2(E_1, V_1)$

ii) $\therefore \Omega^0 = \Omega_1(E_1, V_1) \Omega_2(E_2, V_2) = \Omega^0(E_1, V_1)$

iii) ~~at~~ when the two systems equilibrate, Ω^0 is MAXIMIZED

SUBJECT TO GIVEN $\lambda^0, \epsilon^0, \& N^0$, or

$$\frac{\partial \Omega^0}{\partial \epsilon_1} = 0 \quad (6.14)$$

$$\text{AND} \quad \frac{\partial \Omega^0}{\partial \epsilon_2} = 0 \quad (6.15)$$

$$\text{OR } (6.14) \Rightarrow \left[\frac{\partial \Omega_1}{\partial \epsilon_1} \Big|_{N_1, \epsilon_1} \Omega_2 + \Omega_1 \frac{\partial \Omega_2}{\partial \epsilon_1} \Big|_{N_2, \epsilon_2} \frac{d\epsilon_2}{d\epsilon_1} = 0 \right] \quad (6.23)$$

$$(6.15) \Rightarrow \left[\frac{\partial \Omega_1}{\partial \epsilon_2} \Big|_{N_1, \epsilon_1} \Omega_2 + \Omega_1 \frac{\partial \Omega_2}{\partial \epsilon_2} \Big|_{N_2, \epsilon_2} \frac{d\epsilon_2}{d\epsilon_1} = 0 \right] \quad (6.24)$$

$$\therefore (6.24) \Rightarrow \left[\frac{1}{\Omega_1} \frac{\partial \Omega_1}{\partial \epsilon_1} \Big|_{N_1, \epsilon_1} = \frac{1}{\Omega_2} \frac{\partial \Omega_2}{\partial \epsilon_2} \Big|_{N_2, \epsilon_2} \right] \quad (\text{@ equiv.})$$

$$\text{OR} \quad (6.3) \quad \left[\frac{\partial \ln \Omega_1}{\partial \epsilon_1} \Big|_{N_1, \epsilon_1} = \frac{\partial \ln \Omega_2}{\partial \epsilon_2} \Big|_{N_2, \epsilon_2} \right] \quad (")$$

$$\text{OR} \quad \left[B_1 = B_2 \right] \quad (")$$

LIKELIHOOD,

$$\text{OR } (6.4) \quad \left[\frac{\partial \ln \Omega_1}{\partial \epsilon_1} \Big|_{N_1, \epsilon_1} = \frac{\partial \ln \Omega_2}{\partial \epsilon_2} \Big|_{N_2, \epsilon_2} \right] \quad (\text{@ equiv.})$$

$$\left[M_1 = M_2 \right] \quad (")$$

III) CONT'D

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(A7)

B) AGAIN, SINCE $[N, V, \text{ and } E]$

ARE THE NATURAL ^{MACROSCOPIC} INDEPENDENT
VARIABLES FOR Ω (THIS
IS DETERMINED BY EXPERIENCE)

AND SINCE THESE ARE ALSO
THE NATURAL INDEP. VARIABLES

FOR $[S]$ LET'S TRY THE
SAME STEPS ^{ARGUMENT} AS BEFORE. (w/
THE GOAL OF MAKING CONNECTIONS
BETWEEN THE MICROSCOPIC PARAMETERS
 $\beta \equiv \frac{\partial \ln \Omega}{\partial E} \Big|_{N, V}$ AND $\eta \equiv \frac{\partial \ln \Omega}{\partial V} \Big|_{N, E}$
AND MACROSCOPIC THERMO
VARIABLES).

i) $S = S(E, V, N)$

(AGAIN, THIS IS IMPLIED
BY THE 1ST TDS

(FUNDAMENTAL THERMO)

$$\text{rel'n: } TdS = dE + PdV - \mu dN$$

ii) $\therefore ds = \frac{\partial S}{\partial E} \Big|_{N, V} dE + \frac{\partial S}{\partial V} \Big|_{E, N} dV + \frac{\partial S}{\partial N} \Big|_{E, V} dN$ (Maxwell rel. $1/T$ P/T (Maxwell rel.) (genl))

iii) or here, w/ N fixed for both systems:

$$ds = \frac{1}{T} dE + \frac{P}{T} dV \quad (8.1)$$

iv) Now, from $[S = K \ln \Omega]$, we have:

a) $\frac{\partial S}{\partial E} \Big|_{N, V} = K \frac{\partial \ln \Omega}{\partial E} \Big|_{N, V}$

or

$$\beta = \frac{1}{KT}$$

(8.2) $\Leftarrow$ (objective achieved)

b) $\frac{\partial S}{\partial V} \Big|_{E, N} = K \frac{\partial \ln \Omega}{\partial V} \Big|_{E, N}$

or

$$\eta = \frac{P}{KT}$$

(8.3) $\Leftarrow$ (objective achieved)

v) CONSISTENCY CHECK

ii) we showed above that when the two systems reach equilibrium;

$$(9.14) \quad \boxed{E_1 = E_2} \Leftrightarrow \left. \frac{\partial \ln \Omega_1}{\partial E_1} \right|_{N_1, V_1} = \left. \frac{\partial \ln \Omega_2}{\partial E_2} \right|_{N_2, V_2}$$

and

$$(9.15) \quad \boxed{M_1 = M_2} \Leftrightarrow \left. \frac{\partial \ln \Omega_1}{\partial M_1} \right|_{N_1, E_1} = \left. \frac{\partial \ln \Omega_2}{\partial M_2} \right|_{N_2, E_2}$$

iii) using (8.2) & (8.3) in (9.14) & (9.15):

$$\Rightarrow \frac{1}{T_1} = \frac{1}{T_2} \Rightarrow$$

$$\boxed{T_1 = T_2 \text{ @ EQUILIBRIUM} \Rightarrow \underline{\underline{\text{CONSISTENT}}}}$$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow$$

$$\boxed{P_1 = P_2 \text{ @ EQUIL.} \Rightarrow \underline{\underline{\text{CONSISTENT}}}}$$