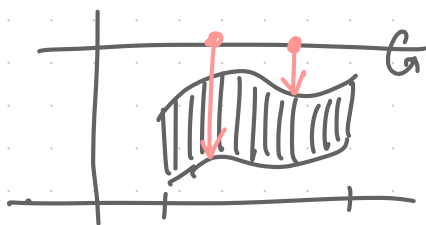


Summary: Method of Disks/Washers

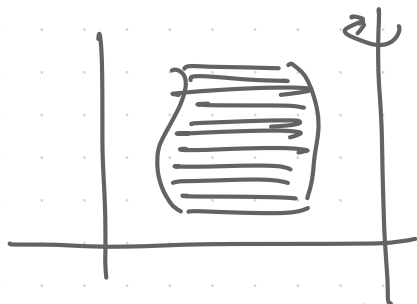
Slice the solid $\perp$ to the axis of revolution

Cross sections are disks ($A = \pi r^2$) or washers ($A = \pi(R^2 - r^2)$).

If you're revolving around a horizontal axis of rev. $\Rightarrow$ integrate in x



If revolving around a vertical axis of rev. $\Rightarrow$ integrate in y

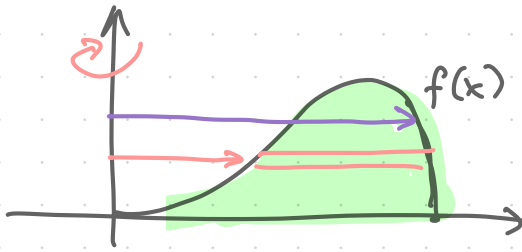


Volume: $V = \int_a^b dV$ where $dV = A(x)dx$
or $dV = A(y)dy$

(Volume is the integral of cross sectional area)

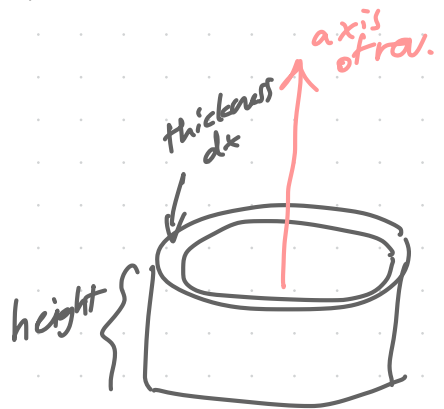
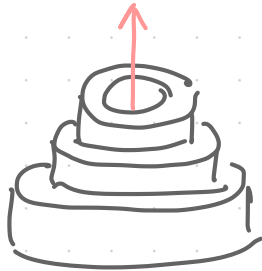
6.3 - Volumes by Cylindrical Shells

Consider the solid formed by revolving this region about the y -axis:

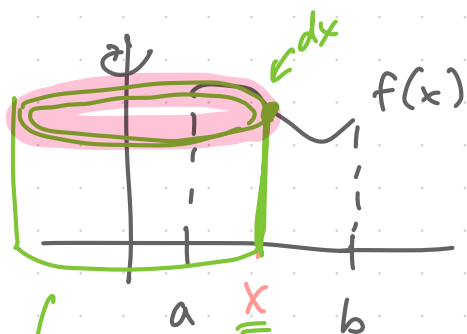


inner and outer radii are given by the same function!

Idea: fill in the solid with hollow cylinders



Consider the region, revolved around the y-axis:



top of shell is a circle



cut and lay out flat

$$f(x) \left\{ \begin{array}{l} dV = 2\pi x f(x) dx \\ \text{width} = \text{circumf. of circle with radius } x = 2\pi x \end{array} \right.$$

width = circumf. of circle with radius $x = 2\pi x$

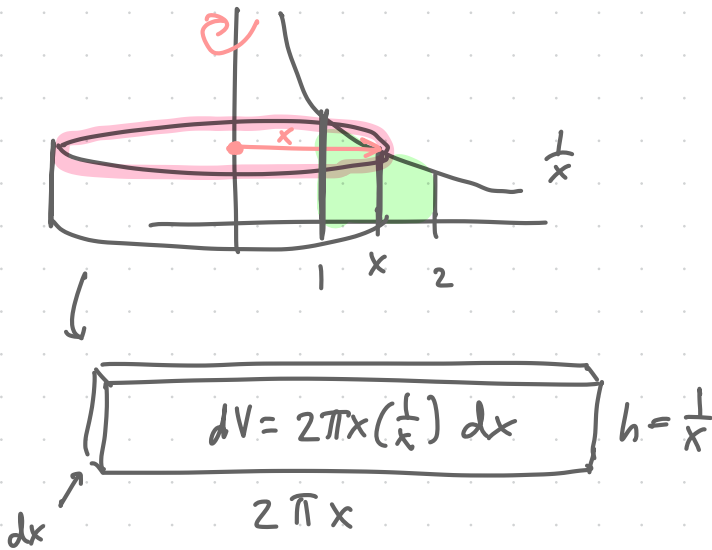
$$V = \int_a^b dV = \int_a^b 2\pi x f(x) dx$$

Generally, $dV = 2\pi$ (radius of a shell) (height of shell) dx/dy

Method of Shells:

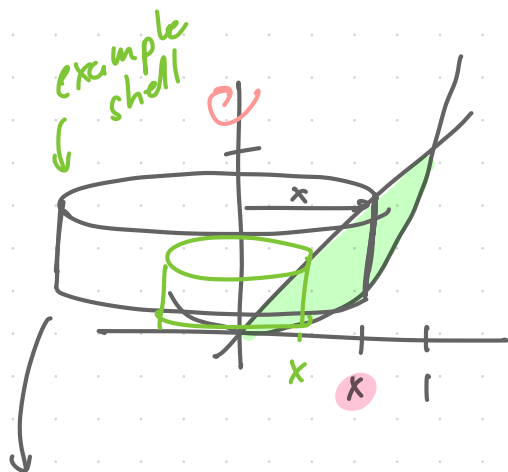
- Slice parallel to the axis of revolution
(vertical axis of rev. $\Rightarrow$ integrate in x)
(horiz. " " " $\Rightarrow$ integrate in y)

example: region bounded by $y = \frac{1}{x}$, $y = 0$,
 $x = 1$ and $x = 2$
rotated about y -axis



$$V = \int_1^2 dV = \int_1^2 2\pi dx = 2\pi x \Big|_1^2 = 2\pi$$

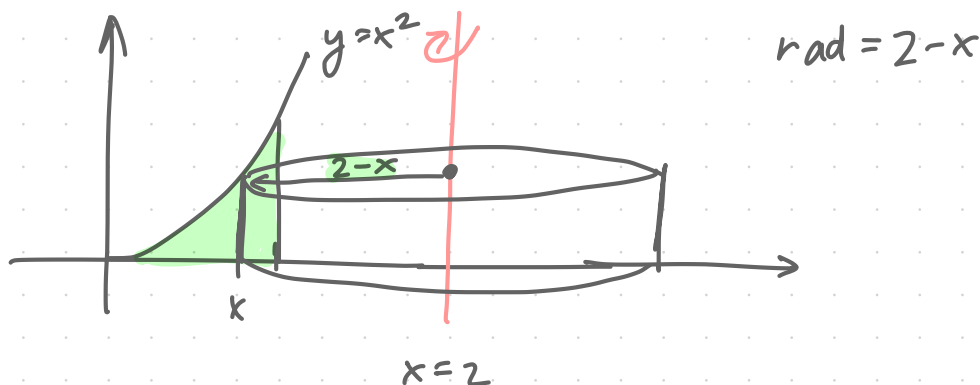
example: region bdd by $y=x$ and $y=x^2$
revolved around y -axis



$$x - x^2 \left\{ \begin{array}{l} dV = 2\pi x(x - x^2) dx \\ 2\pi x \end{array} \right. \uparrow dx$$

$$V = \int_0^1 dV = \int_0^1 2\pi x(x - x^2) dx = \frac{\pi}{6}$$

example: region bdd by $y=x^2$, $y=0$ and $x=1$
revolved around $x=2$

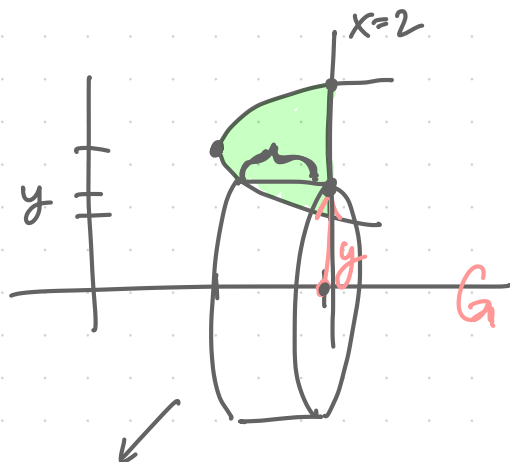


$$x^2 \left\{ \underbrace{dV = 2\pi(2-x)x^2 dx}_{2\pi(2-x)} \right\} \pi dx$$

$$\begin{aligned} V &= \int_0^1 dV = \int_0^1 2\pi(2-x)x^2 dx \\ &= 2\pi \int_0^1 (2x^2 - x^3) dx \\ &= 2\pi \left(2 \cdot \frac{1}{3} x^3 - \frac{1}{4} x^4 \right) \Big|_0^1 \\ &= 2\pi \left(\frac{2}{3} - \frac{1}{4} \right) \end{aligned}$$

example: region bounded by $x = 1 + (y-2)^2$ and $x = 2$
 revolved about x -axis

↑ shift right by 1
 ↑ shift up by 2



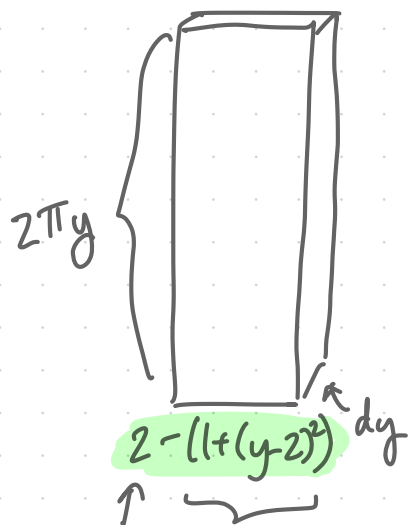
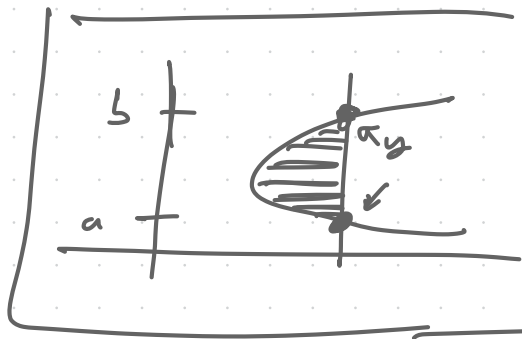
intersection pts:

$$2 = 1 + (y-2)^2$$

$$1 = (y-2)^2$$

$$\pm 1 = y - 2$$

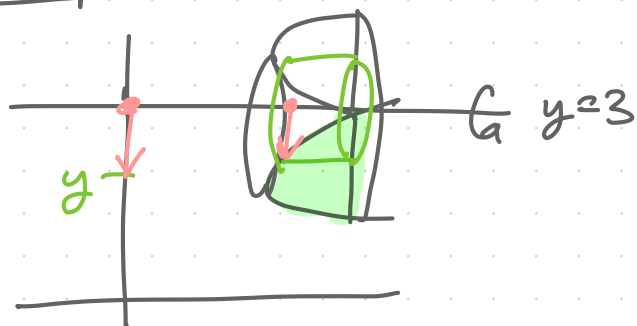
$$y = 2 \pm 1 = 3, 1$$



$$dV = 2\pi y (2 - (1 + (y-2)^2)) dy$$

$$V = \int_1^3 dV = \int_1^3 2\pi y (2 - (y-2)^2) dy$$

example: same region as before, revolved about $y=3$



don't change
bounds, radius, height
will be different

$$\text{height} = 2 - (1 - (y-2)^2)$$

bounds: 1 to 3
for y

$$\text{radius? } 3 - y \geq 0$$

$$dV = 2\pi \cdot \text{rad.} \cdot \text{height}$$

$$= 2\pi (3-y) (2 - (1 - (y-2)^2)) dy$$

$$V = \int_1^3 dV = \int_1^3 2\pi (3-y) (1 - (y-2)^2) dy$$

Radius of a Shell

Only depends on the axis of revolution

y-axis $\Rightarrow$ radius = x



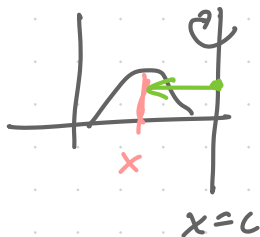
$x = c$ (a vertical line)

$\rightarrow$ axis to the left of region



$$\text{rad} = x - c$$

axis is to the right of your region



$$\text{rad} = c - x$$

x-axis $\Rightarrow$ radius = y

or

$$r = y - c \text{ (below)}$$

or

$$r = c - y \text{ (above region)}$$

