Lines: The vector equation of the line through P_0 in direction $\mathbf{v}$ is

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$$

where $\mathbf{r}_0 = \overrightarrow{OP_0}$, the position vector of P_0.

Example: *Find parametric equations for the line through* $P_0 = (1, 2, 3)$ *in direction* $\mathbf{v} = \langle 4, 5, 6 \rangle$.

Plug $\mathbf{r}_0 = \langle 1, 2, 3 \rangle$ and $\mathbf{v} = \langle 4, 5, 6 \rangle$ into the equation above.

$$\mathbf{r}(t) = \langle 1, 2, 3 \rangle + t\langle 4, 5, 6 \rangle = \langle 1 + 4t, 2 + 5t, 3 + 6t \rangle$$

By letting x equal the x-component of this vector, y equal the y-component, etc., we get the parametric equations for the line:

$$\begin{cases} x = 1 + 4t \\ y = 2 + 5t \\ z = 3 + 6t \end{cases} \quad \text{where } t \in \mathbb{R}$$

Planes: The vector equation of a plane through $P_0 = (x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0.$$

Here $\mathbf{r} = \langle x, y, z \rangle$ and $\mathbf{r}_0 = \overrightarrow{OP_0} = \langle x_0, y_0, z_0 \rangle$, the position vector of the point. By expanding out the vector equation, we get the scalar equation of a plane:

$$\underbrace{\langle a, b, c \rangle}_{\mathbf{n}} \cdot \underbrace{\langle x - x_0, y - y_0, z - z_0 \rangle}_{\mathbf{r} - \mathbf{r}_0} = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Example: *Find the scalar equation of the plane through* $P_0 = (1, 2, 3)$ *with normal vector* $\mathbf{v} = \langle 4, 5, 6 \rangle$.

Using the equation above, $4(x - 1) + 5(y - 2) + 6(z - 3) = 0$.

Example: *Find the equation of the plane through* $P = (1, 3, 2)$, $Q = (3, -1, 6)$, $R = (5, 2, 0)$.

Use the points to make vectors that lie in the plane: $\overrightarrow{PQ} = \langle 2, -4, 4 \rangle$ and $\overrightarrow{PR} = \langle 4, -1, -2 \rangle$.

Then $\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 12, 20, 14 \rangle$ is orthogonal to the plane. Using $P = (1, 3, 2)$ as the point, the equation of the plane is

$$12(x - 1) + 20(y - 3) + 14(z - 2) = 0.$$