Robust Algorithm Using Delay for Multi-Agent Systems[1]

Igor B. Furtat*,, Artem N. Nekhoroshikh**, Mikhail S. Tarasov****

**Institute for Problems of Mechanical Engineering Russian Academy of Sciences, 61 Bolshoy ave V.O., St.-Petersburg, 199178, Russia (Tel: +7-812-321-47-66; e-mail: cainenash@mail.ru).*
***ITMO University, 49 Kronverkskiy ave, Saint Petersburg, 197101, Russia.*

Abstract: The paper describes the robust control algorithm for linear multi-agent systems under parametric and structural uncertainties and external unmeasured disturbances. The proposed algorithm is based on left hand side differences for estimation of the derivatives. The resulting algorithm ensures required accuracy of difference between the plant output and the reference signal. The modeling results illustrate the effectiveness of the algorithm.

Keywords: Robust control, time delay, LMI, Lyapunov-Krasovskii functional, descriptor method.

1. INTRODUCTION

Design of simple control systems under parametric uncertainties and external disturbances when only plant output is available for measurement is important problem of control theory and practice. To construct such control schemes many solutions have been proposed in this regard. If the plant relative degree exceeds one then implementation of adaptive and robust control systems requires the estimation of the derivatives of the plant input and output.

The Luenberger observer is widely used to estimate the plant model state vector with known parameters (Luinberger, 1966). The Kalman filter allows to estimates the state vector of a dynamical system under disturbances and noises (Kalman, 1960). Under parametric uncertainty and external disturbances in the paper (Esfandiary and Khalil, 1992) so-called high-gain observer is proposed. Other form of an observer with a high gain is considered in (Slotine et al., 1987). In (Utkin, 1992, Han, 1995) the robust sliding-mode observer is proposed. In (Wang and Gao, 2003) the authors design a nonlinear extended state observer based on a generalization of a high-gain observer and a sliding-mode observer. In (Veluvolu et al., 2011) the observers (Luinberger, 1966, Esfandiary and Khalil, 1992, Utkin, 1992, Han, 1995, Wang and Gao, 2003) are investigated for second order dynamical systems. In paper (Veluvolu et al., 2011) a comparative analysis for each observer under different kinds of parametric uncertainties, external disturbances, and noises are given. Robust observers have found many applications in the synthesis of control systems under uncertainties. In (Atassi and Khalil, 1999) the stabilization of nonlinear dynamical plants is considered by using the control law that depends on estimates of plant output derivatives obtained with a high-gain dynamical observer. The papers (Bazylev, Pyrkin, 2013, Bazylev, Zimenko, Margun et al., 2014, Bazylev, Margun, Zimenko, 2014, Pyrkin, Bobtsov, Kolyubin et al., 2013, Furtat, 2013, Furtat and Putov, 2013, Furtat, 2014,) are used the adaptive control with high gain observers.

Analysis of the above results shows that the designed algorithms use integrators for estimation of the derivatives. Unlike these results we design the observer using left-hand side differences. This observer allows designing the robust control system without using integrators but using delays. For algorithm synthesis the results of (Furtat, 2015) will be used.

The paper is organized as follows. The problem statement is presented in Section 2. In Section 3 we design the control algorithm for multi-agent systems with known relative degree of each agent. In Section 3 we propose the control algorithm for multi-agent systems with unknown relative degree of each agent. In Section 5 we consider simulation results and discuss an efficiency of the proposed scheme. Concluding remarks are given in Section 6. Appendix A gives the proof of the control system.

2. PROBLEM STATEMENT

Let each agent S_i of the multi-agent system S be described by the following equation

$$Q_i(p)y_i(t) = k_i R_i(p)u_i(t) + \sum_{\substack{j=1,\\ j\neq i}}^{N} Y_{ij}(p)y_j(t) + f_i(t), \qquad (1)$$
$$p^{k-1}y(0) = y_{0k}, \quad k = 1,\dots,n, \quad i = 1,\dots,N,$$

[1] The proof of control algorithms was proposed in Appendix A is supported solely by the grant from the Russian Science Foundation (project No. 14-29-00142) in IPME RAS. The algorithm design in Section 3 and simulation results in Section 5 were supported solely by the Russian Federation President Grant (No. 14.W01.16.6325-MD (MD-6325.2016.8)). The other researches were partially supported by grants of RFBR (16-08-00282, 16-08-00686, 14-08-01015), Ministry of Education and Science of Russian Federation (Project 14.Z50.31.0031) and Government of Russian Federation, Grant 074-U01.

where $y_i(t) \in R$ are local outputs, $u_i(t) \in R$ are local inputs, $f_i(t) \in R$ are local uncontrollable bounded disturbances, $Q_i(p)$, $R_i(p)$ and $Y_{ij}(p)$ are linear differential operators with unknown coefficients, $\deg Q_i(p) = n_i$, $\deg R_i(p) = m_i$, $\deg Y_{ij}(p) = n_{ij}$, $n_i > n_{ij}$, $k_i > 0$, $i, j = 1, ..., N$, y_{0k}, $k = 1, ..., n$ are unknown initial conditions, $p = d / dt$.

Assume that the coefficients of $Q_i(p)$, $R_i(p)$, $Y_{ij}(p)$, k_i belong to a known compact set Ξ and the polynomials $R_i(\lambda)$ are Hurwitz, where $i, j = 1 ,..., N$, λ is a complex variable.

The problem is to design the control system such that the following condition holds

$$\left|y_i(t) - y_{mi}(t)\right| < \delta \quad \text{for } t > T, \tag{2}$$

where $i = 1 ,..., N$, $\delta > 0$ is a an accuracy, $T > 0$ is a transient time.

3. ALGORITHM FOR STRUCTURALLY CERTAINTY AGENTS

First, consider the case where orders of the operators $Q_i(p)$, $R_i(p)$ and $Y_{ij}(p)$, $i, j = 1 ,..., N$ are known. Taking into account equations (1) and (2) rewrite the tracking error $e_i(t) = y_i(t) - y_{mi}(t)$ in the form

$$\begin{aligned} Q_i(p)e_i(t) &= k_i R_i(p)u_i(t) + \sum_{\substack{j=1,\\ j\neq i}}^{N} Y_{ij}(p)y_j(t) \\ &+ f_i(t) - Q_i(p)y_{mi}(t), \quad i = 1, \ldots, N. \end{aligned} \tag{3}$$

Since the derivatives of $y_i(t)$ and $u_i(t)$ are not available, then we introduce the control law in the form

$$u_i(t) = -\alpha_i \sum_{\nu=0}^{\gamma_i} d_{\nu i}\bar{e}_i^{(\nu)}(t), \quad i = 1, \ldots, N, \tag{4}$$

where $\alpha_i > 0$ are design parameters, the coefficients $d_{0i}, d_{1i}, ..., d_{\gamma_i, i}$ are chosen such that the polynomial $D_i(\lambda) = d_{\gamma_i, i}\lambda^{\gamma_i} + d_{\gamma_i - 1, i}\lambda^{\gamma_i - 1} + \ldots + d_{1i}\lambda + d_{0i}$ will be Hurwitz, $\gamma_i = n_i - m_i \geq 1$, γ_i is a relative degree of ith agent, $\bar{e}_i^{(\nu)}(t)$ is an estimate of the νth derivative of the signal $e_i(t)$.

Substituting (4) into (3), rewrite (3) as follows

$$F_i(p)e_i(t) = \psi_i(t) + \sum_{\substack{j=1,\\ j\neq i}}^{N} Y_{ij}(p)y_j(t), \quad i = 1, \ldots, N, \tag{5}$$

where $F_i(p) = Q_i(p) + \alpha_i k_i R_i(p)D_i(p)$,

$$\begin{aligned} \psi_i(t) &= f_i(t) - Q_i(p)y_{mi}(t) \\ &+ \alpha_i k_i R_i(p)\left(D_i(p)e_i(t) - \sum_{\nu=0}^{\gamma_i} d_{\nu i}\bar{e}_i^{(\nu)}(t)\right). \end{aligned}$$

Since we know the compact set Ξ, then there exist coefficients α_i and polynomials $D_i(\lambda)$ such that the polynomials $F_i(\lambda)$ are Hurwitz, $i = 1, ..., N$.

For implementation of control law (4) we use the following algorithm

$$\begin{aligned} &\bar{e}_i(t) = e_i(t), \bar{e}_i^{(1)}(t) = \frac{\bar{e}_i(t) - \bar{e}_i(t-h)}{h}, \\ &\bar{e}_i^{(2)}(t) = \frac{\bar{e}_i^{(1)}(t) - \bar{e}_i^{(1)}(t-h)}{h}, \\ &\vdots \\ &\bar{e}_i^{(\gamma_i)}(t) = \frac{\bar{e}_i^{(\gamma_i - 1)}(t) - \bar{e}_i^{(\gamma_i - 1)}(t-h)}{h}, \quad i = 1, \ldots, N. \end{aligned} \tag{6}$$

Substituting (6) into (4), rewrite control law (4) in the form

$$u_i(t) = -\alpha_i \sum_{\nu=0}^{\gamma_i}\left[\frac{d_{\nu i}}{h^\nu}\sum_{j=0}^{\nu}(-1)^j C_\nu^j e(t - jh)\right], \quad i = 1, \ldots, N, \tag{7}$$

where $C_\nu^j = \dfrac{\nu!}{j!\,(\nu - j)!}$.

Rewrite the function $\psi_i(t)$ as the sum $\psi_i(t) = \alpha_i k_i R_i(p)g_i(t) + \varphi_i(t)$, where

$$\varphi_i(t) = f_i(t) - Q_i(p)y_{mi}(t) + \sum_{\substack{j=1,\\ j\neq i}}^{N} Y_{ij}(p)y_{mj}(t),$$

$$g_i(t) = D_i(p)e_i(t) - \sum_{\nu=0}^{\gamma_i} d_{\nu i}\bar{e}_i^{(\nu)}(t).$$

Then equation (5) can be written as

$$\begin{aligned} \dot{\varepsilon}_i(t) &= A_i\varepsilon_i(t) + \alpha_i k_i B_{1i}g_i(t) + B_{2i}\varphi_i(t) \\ &+ \sum_{\substack{j=1,\\ j\neq i}}^{N} B_{3ij}\varepsilon_j(t), \quad e_i(t) = L_i\varepsilon_i(t), \quad i = 1, \ldots, N, \end{aligned} \tag{8}$$

where A_i, B_{1i}, B_{2i}, B_{3ij} and $L_i = [1\ 0\ ...\ 0]$ are matrices obtained at the transition from (5) to (8).

Taking into account control law (7) and the structure of the function $g_i(t)$, rewrite the first equation of system (8) in the form

$$\begin{aligned} \dot{\varepsilon}_i(t) &= A_i\varepsilon_i(t) + \alpha_i k_i B_{1i}\rho_i^T\varepsilon_i(t) - \alpha_i k_i B_{1i} \\ &\times\left[\sum_{\nu=0}^{\gamma_i}\frac{d_{\nu i}}{h^\nu}L_i\varepsilon_i(t) + \sum_{\nu=1}^{\gamma_i}\sum_{j=1}^{\nu}\frac{d_{\nu i}}{h^\nu}(-1)^j C_\nu^j L_i\varepsilon_i(t - jh)\right] \\ &+ \sum_{\substack{j=1,\\ j\neq i}}^{N} B_{3ij}\varepsilon_j(t) + B_{2i}\varphi_i(t), \quad i = 1, \ldots, N, \end{aligned} \tag{9}$$

where the vectors ρ_i are composed of coefficients of the operator $D_i(p)$.

Let us denote

$$\tilde{A}_i = A_i + \alpha_i k_i B_{1i}\left(\rho_i^T - \sum_{v=0}^{\gamma_i}\frac{d_{vi}}{h^v}L_i\right),$$

$$A = \mathrm{diag}\left\{\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_N\right\},$$

$$\tilde{F}_{vj}^{\,i} = \begin{cases} -\alpha_i k_i B_{1i}\dfrac{d_{vi}}{h^v}(-1)^j C_v^j L_i, \text{ if } v \le \gamma_i, \\ \text{O, else,} \end{cases}$$

$$F_{vj} = \mathrm{diag}\left\{\tilde{F}_{vj}^1, \tilde{F}_{vj}^2, ..., \tilde{F}_{vj}^N\right\},\ i, j = 1, ..., N,$$

$$Y = \begin{bmatrix} \mathrm{O} & B_{312} & \dots & B_{31N} \\ B_{321} & \mathrm{O} & \dots & B_{32N} \\ \vdots & \vdots & \ddots & \vdots \\ B_{3N1} & B_{3N2} & \dots & \mathrm{O} \end{bmatrix},$$

$$B = \mathrm{diag}\left\{B_{21}, B_{22}, ..., B_{2N}\right\},$$

$$\varepsilon(t) = \left[\varepsilon_1^T(t), \varepsilon_2^T(t), ..., \varepsilon_N^T(t)\right]^T,$$

$$\varphi(t) = \left[\varphi_1(t), \varphi_2(t), ..., \varphi_N(t)\right]^T,$$

where O is a zero matrix.

Taking into account notations, rewrite equation (9) as follows

$$\dot{\varepsilon}(t) = (A+Y)\varepsilon(t) + \sum_{v=1}^{\gamma}\sum_{j=1}^{v} F_{vj}\varepsilon(t - jh) + B\varphi(t). \qquad (10)$$

Before formulating the main result, let us introduce the following notations:

$$\Psi_{11}^{11} = P_2^T A_s + A_s^T P_2 + \sum_{v=1}^{\gamma}\sum_{j=1}^{v} S_{vj},\ \Psi_{11}^{12} = P - P_2^T + A_s^T P_3,$$

$$\Psi_{11}^{22} = -P_3 - P_3^T + \sum_{v=1}^{\gamma}\sum_{j=1}^{v} jhR_{vj},$$

$$\Psi_{11} = \begin{bmatrix} \Psi_{11}^{11} & \Psi_{11}^{12} \\ * & \Psi_{11}^{22} \end{bmatrix},$$

$$\Psi_{12} = -h\begin{bmatrix} P_2^T\sum_{v=1}^{\gamma}F_{v1} & 2P_2^T\sum_{v=1}^{\gamma}F_{v2} & \dots & \gamma P_2^T\sum_{v=1}^{\gamma}F_{v\gamma} \\ P_3^T\sum_{v=1}^{\gamma}F_{v1} & 2P_3^T\sum_{v=1}^{\gamma}F_{v2} & \dots & \gamma P_3^T\sum_{v=1}^{\gamma}F_{v\gamma} \end{bmatrix},$$

$$\Psi_{22} = -he^{-2\chi h}diag\left\{\sum_{v=1}^{\gamma}R_{v1}, 2\sum_{v=1}^{\gamma}R_{v2}, ..., \gamma\sum_{v=1}^{\gamma}R_{v\gamma}\right\},$$

$$\Psi_{33} = -\beta I,$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & P_2^T B \\ * & \Psi_{22} & \mathrm{O} \\ * & * & -\beta I \end{bmatrix},$$

where $\gamma = \max\limits_{i=1,...,N}\{\gamma_i\}$, $A_s = A + Y + \sum\limits_{v=1}^{\gamma}\sum\limits_{j=1}^{v} F_{vj}$, $P > 0$, $P_2 > 0$, $P_3 > 0$, $S_{vj} > 0$, $R_{vj} > 0$, $v, j = 1, 2, \dots, \gamma$ are matrices of corresponding dimension, I is an identity matrix.

Theorem 1. Given a scalar $\chi > 0$. If there exist constant $\beta > 0$ and matrices $P > 0$, $P_2 > 0$, $P_3 > 0$, $S_{vj} > 0$, $R_{vj} > 0$, $v, j = 1, 2, \dots, \gamma$ such that $\Psi < 0$, then control system (7) ensures goal (2) with an accuracy

$$\delta = \sigma\sqrt{0.5\chi^{-1}\lambda_{\min}^{-1}(P)\beta}, \qquad (11)$$

where $\sigma^2 = \sup\limits_t\left\{\varphi^T(t)\varphi(t)\right\}$.

Proof of Theorem 1 in Appendix A.

4. ALGORITHM FOR STRUCTURALLY UNCERTAINTY AGENTS

Now we consider the case when orders of the operators $Q_i(p)$, $R_i(p)$ and $Y_{ij}(p)$, $i, j = 1, ..., N$ are unknown. However, we assume that we known the upper bounds $\bar{\gamma}_i$ of the relative degrees γ_i, i.e. $\bar{\gamma}_i \ge \gamma_i$, $i = 1, ..., N$.

The goal is to ensure condition (2).

Taking into account equations (1) and (2), rewrite the equation of the tracking error $e_i(t) = y_i(t) - y_{mi}(t)$ in the form (3).

Introduce the control law as follows

$$u_i(t) = -\alpha_i\sum_{v=0}^{\bar{\gamma}_i} d_{vi}\bar{e}_i^{(v)}(t),\quad i = 1, ..., N, \qquad (12)$$

where the coefficients $d_{0i}, d_{1i}, \dots, d_{\bar{\gamma}i}$ are chosen such that the polynomials $D_i(\lambda) = d_{\bar{\gamma}_i,i}\lambda^{\bar{\gamma}_i} + d_{\bar{\gamma}_i-1,i}\lambda^{\bar{\gamma}_i-1} + ... + d_{1,i}\lambda + d_{0,i}$ will be Hurwitz. Substituting (9) into (3), we obtain the equation kind of (5). Obviously, there exist α_i and $D_i(\lambda)$ such that $F_i(\lambda)$ are Hurwitz, $i = 1, ..., N$.

For estimation the derivatives of the signal $e_i(t)$ we use the following algorithm

$$\begin{aligned} \bar{e}_i(t) &= e_i(t), \\ \bar{e}_i^{(1)}(t) &= \frac{\bar{e}_i(t) - \bar{e}_i(t - k_1 h)}{k_1 h}, \\ \bar{e}_i^{(2)}(t) &= \frac{\bar{e}_i^{(1)}(t) - \bar{e}_i^{(1)}(t - k_2 h)}{k_2 h}, \\ &\vdots \\ \bar{e}_i^{(\bar{\gamma}_i)}(t) &= \frac{\bar{e}_i^{(\bar{\gamma}_i-1)}(t) - \bar{e}_i^{(\bar{\gamma}_i-1)}(t - k_{\bar{\gamma}_i} h)}{k_{\bar{\gamma}_i} h},\quad i = 1, ..., N. \end{aligned} \qquad (13)$$

Here $\bar{e}_i^{(v)}(t)$, $v = 0, 1, ..., \bar{\gamma}_i$ are estimates of the vth derivative of the signals $e_i(t)$, $k_1 > k_2 > ... > k_{\bar{\gamma}_i} > 0$. The delay in (13) is reduced in each subsequent equation because it is necessary to increase the accuracy of the estimate of each subsequent derivative.

Consider the set $\Theta_i = \{1, ..., \bar{\gamma}_i\}$, i=1, ..., N. Substituting (13) into (12), we obtain

$$u_i(t) = -\alpha_i \Bigg[d_{0i} e_i(t) + d_{1i} \frac{e_i(t) - e_i(t - k_1 h)}{k_1 h} + \ldots + \frac{d_{\bar{\gamma}_i}}{h^{\bar{\gamma}_i} \prod_{i_1=1}^{\bar{\gamma}_i} k_i} \Bigg(e_i(t) - \sum_{i_1 \in \Theta} e_i(t - k_{i_1} h) + \frac{1}{2!} \sum_{\substack{i_1, i_2 \in \Theta, \\ i_1 \neq i_2}} e_i\left(t - h\left(k_{i_1} + k_{i_2}\right)\right) + \ldots + \frac{(-1)^{\bar{\gamma}_i}}{\bar{\gamma}_i!} \sum_{\substack{i_1, i_2, \ldots, i_{\bar{\gamma}_i} \in \Theta_i, \\ i_1 \neq i_2 \neq \ldots \neq i_{\bar{\gamma}_i}}} e_i\left(t - h\left(k_{i_1} + k_{i_2} + \ldots + k_{i_{\bar{\gamma}_i}}\right)\right) \Bigg) \Bigg], \quad (14)$$

$$i = 1, \ldots, N.$$

As the result, the equation of the closed-loop system is represented by expression (8) where matrices A_i, B_{1i}, B_{2i}, B_{3ij} and L_i are obtained by the transition from (5) to (8) and taking into account, that $D_i(\lambda) = d_{\bar{\gamma}_i} \lambda^{\bar{\gamma}_i} + d_{\bar{\gamma}_i - 1} \lambda^{\bar{\gamma}_i - 1} + \ldots + d_{1i} \lambda + d_{0i}$.

Theorem 2. There exist coefficients $\alpha_i > 0$, $k_1 > k_2 > \ldots > k_{\bar{\gamma}_i} > 0$, i = 1, ..., N and $h > 0$ such that control law (14) ensures goal (2) and boundedness of all signals in the closed-loop system.

The proof of Theorem 2 is similar to the proof of Theorem 1, therefore, the proof of Theorem 2 is not in the paper.

5. EXAMPLES

1) Consider a plant model in the form

$$\begin{aligned} \left(p^3 + q_{11} p^2 + q_{12} p + q_{13}\right) y_1(t) &= k_1 \left(p + r_{11}\right) u_1(t) + n_{12} y_2(t) + f_1(t), \\ \left(q_{20} p^2 + q_{21} p + q_{22}\right) y_1(t) &= k_2 u_2(t) + n_{21} y_2(t) + f_2(t). \end{aligned} \quad (15)$$

The set Ξ is determined by the following inequalities:

$$q_{20} = 1, \ -3 \le q_{1i} \le 3, \ -3 \le q_{2j} \le 3, \ i = 1, 2, 3, \ j = 1, 2,$$

$$1 \le k_1 \le 3, \ 1 \le r_{11} \le 5, \ 1 \le k_2 \le 5, \ 1 \le n_{12} \le 12, \ 1 \le n_{21} \le 12.$$

We assume that $|f_1(t)| \le 10$, $|f_2(t)| \le 10$ and we know equation orders in (15).

Define $d_0 = 1$, $d_1 = 2$, $d_2 = 1$. Then, control law (7) is rewritten as follows

$$\begin{aligned} u_i(t) &= -\alpha_i \Bigg(e_i(t) + 2 \frac{e_i(t) - e_i(t-h)}{h} + 0.05 \frac{e_i(t) - 2e_i(t-h) + e_i(t-2h)}{h^2} \Bigg) \\ &= -\alpha_i h^2 \Big([h^2 + 2h + 0.05] e_i(t) - [2h + 0.1] e_i(t-h) + 0.05 e_i(t-2h) \Big), \quad i = 1, 2. \end{aligned}$$

Let parameters in (15) be presented as follows

$q_{11} = 3$, $q_{12} = 3$, $q_{13} = 3$, $k_1 = 3$, $r_{11} = 3$, $n_{12} = 3$, $f_1(t) = 1 + \sin t$, $y_1(0) = 1$, $\dot{y}_1(0) = 1$, $\ddot{y}_1(0) = 1$;

$q_{21} = 3$, $q_{22} = 3$, $k_2 = 3$, $n_{21} = 3$, $f_1(t) = 2 + 3\sin 3t$, $y_2(0) = 1$, $\dot{y}_2(0) = 1$.

Let $y_{m1}(t) = y_{m2}(t) = 1 + \sin 0.7t$. Fig. 1 shows simulation results on errors $e_i(t) = y_i(t) - y_{mi}(t)$ for $\alpha_i = 0.003$ and $h = 0.01$ (s), $i = 1, 2$. Fig. 2 shows simulation results of errors $e_i(t) = y_i(t) - y_{mi}(t)$ for $\alpha_i = 0.03$ and $h = 0.01$ (s), $i = 1, 2$.

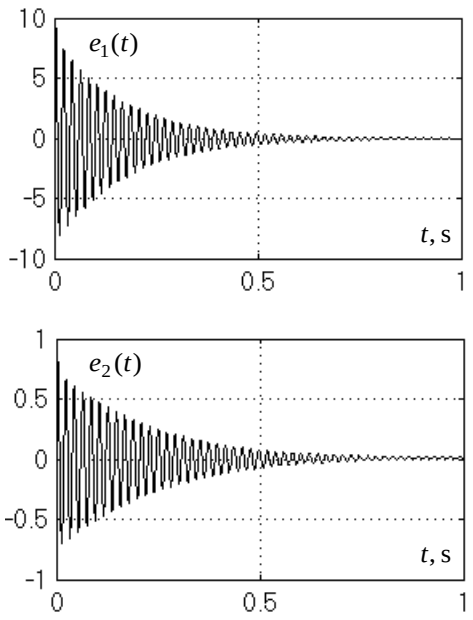


Fig. 1. The transients on tracking errors $e_i(t) = y_i(t) - y_{mi}(t)$ for $\alpha_i = 0.003$ and $h = 0.01$ (s), $i = 1, 2$.

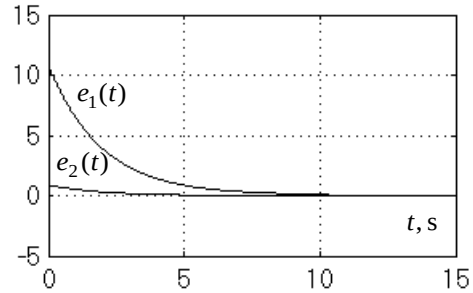


Fig. 2. The transients on tracking errors $e_i(t) = y_i(t) - y_{mi}(t)$ for $\alpha_i = 0.03$ and $h = 0.01$ (s), $i = 1, 2$.

2) Consider a plant model (15), where $q_{20} = 0$ and $q_{21} = 1$. Assume that we unknown equation orders in (15).

Let $\bar{\gamma}_1 = \bar{\gamma}_2 = 2$. Define $h = 0.01$, $k_1 = 1$, $k_2 = 0.1$, $d_0 = 1$, $d_1 = 2$, $d_2 = 1$. Then, control law (7) is defined as follows

$$u_i(t) = -\alpha_i \Big(e_i(t) + 2h^{-1}\big[e_i(t) - e_i(t-h)\big] + 10h^{-2}\big[e_i(t) - e_i(t-h) - e_i(t-0.1h) + e_i(t-1.1h)\big]\Big)$$
$$= -10\alpha_i h^{-2}\big[(0.1h^2 + 0.2h + 1)e_i(t) - (0.2h+1)e_i(t-h) - e_i(t-0.1h) + e_i(t-1.1h)\big), \quad i = 1, 2.$$

Let parameters in (15) be presented as in previous example, only where $q_{20} = 0$, $q_{21} = 1$ and $y_2(0) = 1$.

Fig. 3 shows simulation results of errors $e_i(t) = y_i(t) - y_{mi}(t)$ when $\alpha_i = 0.003$ and $h = 0.01$ (s), $i = 1, 2$.

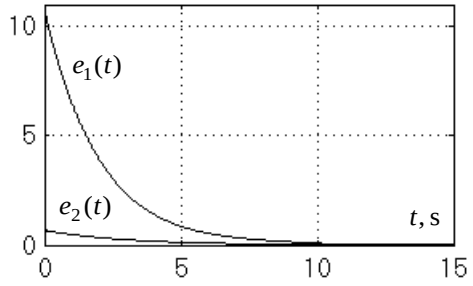


Fig. 3. The transients on tracking errors $e_i(t) = y_i(t) - y_{mi}(t)$ for $\alpha_i = 0.003$ and $h = 0.01$ (s), $i = 1, 2$.

It follows from Fig. 1, 2 that parametric uncertainties and external disturbances are compensated by control system with the required accuracy $\delta = 0.01$ achieved after 11 s. It follows from Fig. 3 the required accuracy $\delta = 0.04$ is achieved after 11 s. Simulation results show that the value δ can be reduced by decrease of the value h.

6. CONCLUSIONS

The robust algorithm for multi-agent dynamical systems under parametric uncertainties and external unmeasured bounded disturbances is proposed. In this paper we use the observer based on left hand side differences for estimation of the derivatives. The algorithm compensates parametric uncertainties and disturbances with a given accuracy. Since the proposed control algorithm uses time delay, then it practical implementation can be based on digital systems. For example, the algorithm can be implemented by using MatLab.

REFERENCES

Atassi A.N. and H.K. Khalil (1999). A separation principle for the stabilization of class of nonlinear systems, *IEEE Trans. Automat. Control*, 44(9), 1672-1687.

Bazylev D, Margun A., Zimenko K., Kremlev A. (2014). UAV equipped with a robotic manipulator, *Proc. 22nd Mediterranean Conf. on Control and Automation, Palermo, Italy*, 1177-1182.

Bazylev D., Pyrkin A. (2013). Stabilization of biped robot standing on nonstationary plane, *Proc. 18th Int. Conf. on Methods and Models in Automation and Robotics, Miedzyzdroje, Poland*, 459-463.

Bazylev D., Zimenko K., Margun A. et al. (2014). Adaptive control system for quadrotor equiped with robotic arm, *Proc. 19th Int. Conf. on Methods and Models in Automation and Robotics, MMAR 2014; Miedzyzdroje; Poland*, 705-710.

Esfandiary F. and H.K. Khalil (1992). Output feedback stabilization of fully linearizable systems, *Int. J. Control*, 56(5), 1007-1037.

Fridman E. (2014). Introduction to Time-Delay Systems. Analysis and Control, *Birkhauser*.

Fridman E. (2014). Tutorial on Lyapunov-based methods for time-delay systems, *European Journal of Control*, 20, 271–283.

Furtat I.B. (2013). Robust synchronization of the structural uncertainty nonlinear network with delay and disturbances, *Proc. 11th IFAC International Workshop on Adaptation and Learning in Control and Signal Processing, ALCOSP 2013*, Caen, France, Vol. 11, Issue PART, 227-232.

Furtat I.B. (2014). Adaptive predictor-free control of a plant with delayed input signal, *Automation and remote control*, 75(1), 139-151.

Furtat I.B. (2015). Robust static control algorithm for linear objects, *Automation and Remote Control*, 76(3), 446-457.

Furtat I.B., V.V Putov (2013). Suboptimal control of aircraft lateral motion, *Proc. 2nd IFAC Workshop on Research, Education and Development of Unmanned Aerial Systems, RED-UAS 2013*, Compiegne, France, Vol. 2, Issue PART 1, 276-282.

Han J. (1995). A class of extended state observers for uncertain system, *Control Decision*, 10(1), 85-88.

Kalman, R.E. (1960). A new approach to linear filtering and prediction problems, *Trans. ASME – J. Basic Engineer*, D(82), 35-45.

Luinberger, D. (1966). Observers for multivariable systems, *IEEE Trans. Automat. Control*, AC-11(2), 190-197.

Slotine J.J.E., J.K. Hedrick, and E.A. Misawa (1987). On sliding observers for nonlinear systems, *J. Dynam. Syst., Measurement, Control*, 109, 245-252.

Utkin V.I. (1992). Sliding-modes in control optimization. Berlin: Springer-Verlag.

Veluvolu K.C., M.Y. Kim, D. Lee (2011). Nonlinear sliding mode high-gain observers for fault estimation, *Int. J. Syst. Sci*., 42(7), 1065-1074.

Wang W. and Z. Gao (2003). A comparison study of advanced state observer design techniques, *Proc. Amer. Control Conf*., 4754-4759.

APPENDIX A.

Proof of Theorem. Consider the following Lemma.

Lemma (Furtat, 2015). Let the system be described by the differential equation

$$\dot{x}(t) = Ax(t) + f(t) + g\big(x(t), x(t-\tau_1), \ldots, x(t-\tau_n), \mu\big), \quad (16)$$

where $x(t)\in R^n$, $A\in R^{n\times n}$ is Hurwitz matrix, $f(t)\in R^n$ is bounded function, $\tau_i > 0$ is a time delay, $i = 1, \ldots, n$, $g(x(t), x(t-\tau_1), \ldots, x(t-\tau_n), \mu)\in R^n$ is a continuous function for all arguments except maybe the case when $\mu = 0$, moreover, $g(x(t), x(t-\tau_1), \ldots, x(t-\tau_n), \mu) \to 0$ when $\mu \to 0$ and $g(0, \ldots, 0, \mu) = 0$. Additionally, the function $g(\cdot)$ satisfies the Lipschitz condition

$$\begin{aligned}&\left|g\left(x_1(t), x_1(t-\tau_1), \ldots, x_1(t-\tau_n), \mu\right)\right.\\&\left.- g\left(x_2(t), x_2(t-\tau_1), \ldots, x_2(t-\tau_n), \mu\right)\right|\\&\le L_0(\mu)\left|x_1(t)-x_2(t)\right| + L_1(\mu)\left|x_1(t-\tau_1)-x_2(t-\tau_1)\right|\\&+\ldots+ L_n(\mu)\left|x_1(t-\tau_n)-x_2(t-\tau_n)\right|,\end{aligned}$$

where $L_0(\mu) > 0$, $L_1(\mu) > 0$, …, $L_n(\mu) > 0$ are Lipschitz constants, moreover, $\lim\limits_{\mu\to 0} L_i(\mu) = 0$, $i = 0, 1, \ldots, n$.

Let $\bar{x}(t)$ be a solution of the equation

$$\dot{\bar{x}}(t) = A\bar{x}(t) + f(t)$$

where $\left|\bar{x}(t)\right| < b < \infty$. Then there exists $\mu_0 > 0$ such that for

$\mu \le \mu_0$ the solution $x(t, \mu)$ of the original system (16) satisfies $|x(t, \mu)| < b$. Moreover, the following condition holds

$$\lim_{\mu\to 0} x(t, \mu) = \bar{x}(t).$$

Let us use Lemma for analysis of equation (8). Since $f_i(t)$ are bounded functions and $y_{mi}(t)$ are smooth bounded functions together with its derivatives, therefore, the functions $\varphi_i(t)$ are bounded, $i = 1, 2, ..., N$. Since $e_i(t)$ is a continuous function, then according to (6)

$$\begin{aligned}&\bar{e}_i^{(1)}(t) \to pe_i(t),\ \ \bar{e}_i^{(2)}(t) \to p^2 e_i(t), \ldots,\\&\bar{e}_i^{(\gamma_i)}(t) \to p^{\gamma_i} e_i(t)\ \text{ when } h\to 0,\ i = 1, 2, ..., N.\end{aligned}$$

Hence, $g_i(t)\to 0$ when $h\to 0$. Since the matrix A_i is Hurwitz, then Lemma conditions will be hold for equation (8). Therefore, all the variables in the system (8) will be bounded.

Now let us show that there exists $h > 0$ such that algorithm (7) ensures condition (2).

Let us use the descriptor method (Fridman, 2014). Using the relation

$$\varepsilon(t-h) = \varepsilon(t) - \int_{t-h}^{t} \dot{\varepsilon}(s)ds,$$

rewrite (10) as follows

$$\dot{\varepsilon}(t) = A_s\varepsilon(t) - \sum_{v=1}^{\gamma}\sum_{j=1}^{v} F_{vj} \int_{t-jh}^{t} \dot{\varepsilon}(s)ds + B\varphi(t). \qquad (17)$$

Consider a Lyapunov-Krasovskii functional in the form

$$\begin{aligned}V = {}&\varepsilon^{\mathrm{T}}(t)P\varepsilon(t) + \sum_{v=1}^{\gamma}\sum_{j=1}^{v}\int_{t-jh}^{t} e^{2\chi(s-t)}\varepsilon^{\mathrm{T}}(s)S_{vj}\varepsilon(s)ds\\&+ \sum_{v=1}^{\gamma}\sum_{j=1}^{v}\int_{-jh}^{0}\int_{t+\theta}^{t} e^{2\chi(s-t)}\dot{\varepsilon}^{T}(s)R_{vj}\dot{\varepsilon}(s)dsd\theta.\end{aligned} \qquad (18)$$

Differentiating (18) along the trajectories of (17), we have

$$\begin{aligned}\dot{V} = {}&2\varepsilon^T(t)P\dot{\varepsilon}(t) + 2\left[\varepsilon^T(t)P_2^T + \dot{\varepsilon}^T(t)P_3^T\right]\\&\times\left[-\dot{\varepsilon}(t) + A_s\varepsilon(t) - \sum_{v=1}^{\gamma}\sum_{j=1}^{v} F_{vj}\int_{t-jh}^{t}\dot{\varepsilon}(s)ds + B\varphi(t)\right]\\&- 2\chi\sum_{v=1}^{\gamma}\sum_{j=1}^{v}\int_{t-jh}^{t} e^{2\chi(s-t)}\varepsilon^T(s)S_{vj}\varepsilon(s)ds\\&+ \varepsilon^T(t)\sum_{v=1}^{\gamma}\sum_{j=1}^{v} S_{vj}\varepsilon(t)\\&- \sum_{v=1}^{\gamma}\sum_{j=1}^{v} e^{-2jh}\varepsilon^T(t-jh)S_i\varepsilon(t-jh)\\&- 2\chi\sum_{v=1}^{\gamma}\sum_{j=1}^{v}\int_{-jh}^{0}\int_{t+\theta}^{t} e^{2\chi(s-t)}\dot{\varepsilon}^T(s)R_{vj}\dot{\varepsilon}(s)dsd\theta\\&+ \dot{\varepsilon}^T(t)\sum_{v=1}^{\gamma}\sum_{j=1}^{v} jhR_{vj}\dot{\varepsilon}(t)\\&- \sum_{v=1}^{\gamma}\sum_{j=1}^{v} jh\int_{t-jh}^{t} e^{2\chi(s-t)}\dot{\varepsilon}^T(s)R_{vj}\dot{\varepsilon}(s)ds.\end{aligned} \qquad (19)$$

Consider the following notations:

$$\omega_1(t) = col\left\{\varepsilon(t), \dot{\varepsilon}(t)\right\},$$

$$\omega_2(t) = \frac{1}{h}col\left\{\int_{t-h}^{t}\dot{\varepsilon}(s)ds, \frac{1}{2}\int_{t-2h}^{t}\dot{\varepsilon}(s)ds, \ldots, \frac{1}{\gamma}\int_{t-\gamma h}^{t}\dot{\varepsilon}(s)ds\right\},$$

$$\omega(t) = col\left\{\omega_1(t), \omega_2(t), \varphi(t)\right\}.$$

Differentiating (18), we find

$$W = \dot{V} + 2\chi V - \beta|\varphi|^2 \le \omega^T\Psi\omega.$$

Given $\chi > 0$. There exist constant $\beta > 0$ and matrices $P > 0$, $P_2 > 0$, $P_3 > 0$, $S_{vj} > 0$, $R_{vj} > 0$, $v, j = 1, 2, \ldots, \gamma$ such that $\Psi < 0$, then $W < 0$. Therefore, given $\sigma > 0$, where $\sigma^2 = \sup\limits_t\left\{\varphi^T(t)\varphi(t)\right\}$, the ellipsoid

$$K_\infty = \left\{\varepsilon\in R^n : \varepsilon^T P\varepsilon \le 0.5\chi^{-1}\sigma^2\beta\right\} \qquad (20)$$

is an exponentially attractive.

From (20) find the upper bound for ε as follows

$$|\varepsilon| \le \sigma\sqrt{0.5\chi^{-1}\lambda_{\min}^{-1}(P)\beta}. \qquad (21)$$

Thereby, the value δ in (2) can be chosen by variation of parameters of (12) and (15). For example, the value δ in (2) can be reduced by decreasing the value h. Theorem is proved.