In order to investigate the effect of elementary row operations on determinants, we will use *elementary matrices*. An elementary matrix is a matrix obtained by performing a single elementary row operation on the identity matrix I_n.

1. Compute EA for each of the elementary matrices below, then describe the row operation that multiplication by E performed on A where

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$$

(a) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

2. For each of the following row operations, write down an elementary matrix that performs that operation.

   (a) Scale row 3 by $\frac{1}{2}$

   (b) Swap row 1 and row 3

   (c) Replace row 2 by row 2 minus 2 times row 1

3. Suppose that we start with the 3×3 matrix A and perform the following sequence of row operations:

   (1) Multiply row 1 by -2 and add to row 2.

   (2) Multiply row 1 by 4 and add to row 3.

   (3) Swap rows 2 and 3.

   (4) Scale row 2 by $1/2$.

   We arrive at the upper triangular matrix

$$U = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{bmatrix},$$

   so that $E_4E_3E_2E_1A = U$.

   (a) Write the matrices E_1, E_2, E_3, and E_4 that perform the four row operations.

   (b) Use $\det(E_1)$, $\det(E_2)$, $\det(E_3)$, $\det(E_4)$, and $\det(U)$ to find $\det(A)$ without finding A.

   (c) Find the matrix A. Check your answer from part (b) by computing $\det(A)$ directly.