

I) DENSITY OF STATES, $g(E)$

A) For Quantum Problems where states are somewhat separated / discrete, we have the partition fn:

here g_i is referred to as the "degeneracy" of state E_i

$$Q(V, N, T) = \sum_i g_i e^{-\beta E_i}$$

B) For Quantum Problems where states are very closely separated, e.g. problems involving

$$V \geq 10^2 \text{ nm}, \text{ and/or } N \gg 1,$$

$$\text{and/or } \lambda_{\text{thermal}} \sim \frac{h}{\sqrt{2mKT}} \quad (z \ll 1);$$

(where latter follows from:

$$i) \quad p^2 \sim 2mE \sim \hbar^2 k^2$$

characteristic
K. E.

quantum
momentum z

i) Recognize that $e^{-\beta E}$ is still proportional to the probability of observing our system in (quantum) state E_j .

ii) Define $g(E)dE$ as the number of quantum states in the interval $(E, E+dE)$; so that

iii) $\boxed{P(E)dE \propto g(E)e^{-\beta E}dE} \quad (2.1)$

i.e., the probability of observing our system in the interval

$$(E, E+dE) \text{ is } \propto \left(\begin{array}{c} \text{No. of} \\ \text{Quantum} \\ \text{States} \\ \text{in} \\ (E, E+dE) \end{array} \right) * \left(\begin{array}{c} \text{probability} \\ \text{density} \\ @ \\ E \end{array} \right)$$

iv) INTRODUCE PROPORTIONALITY CONSTANT INTO (2.1) AND USE $\int_0^\infty P(E)dE = 1$

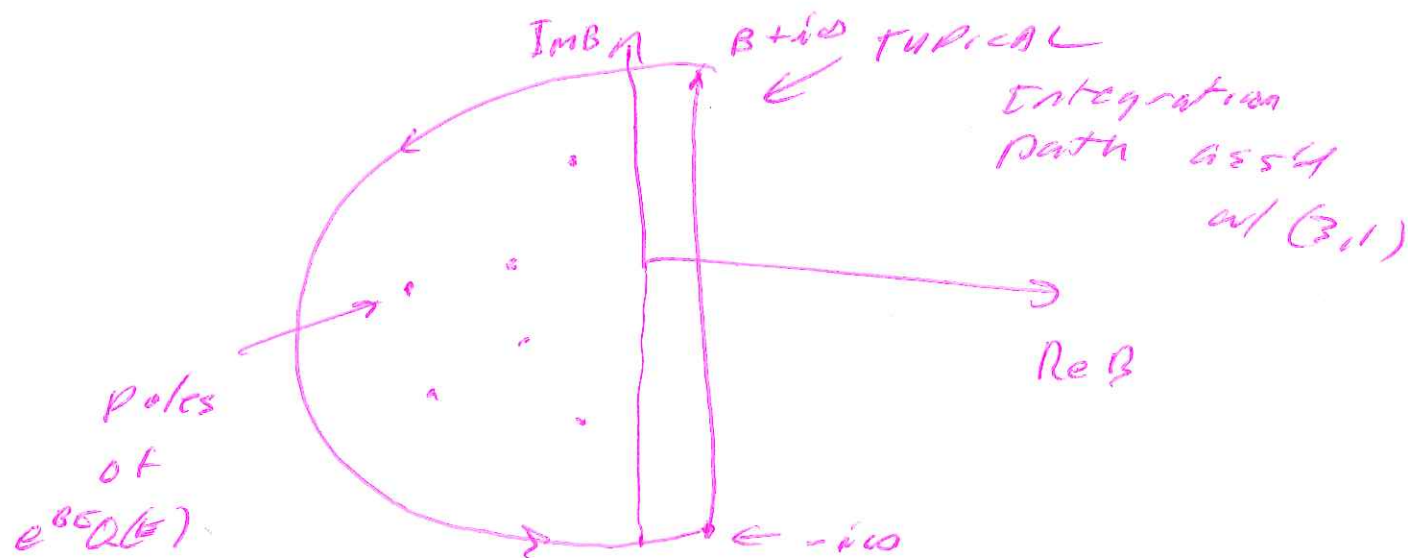
TO GET

$$\boxed{\begin{aligned} P(E)dE &= \frac{g(E)e^{-\beta E}dE}{Q} \\ Q &= \int_0^\infty g(E)e^{-\beta E}dE \end{aligned}} \quad (2.2)$$

V) From (2.2), Q can be
incorporated and calculated as
 the Laplace transform of
 the density of states,

$g(E) \left(= \begin{array}{l} \text{no. of} \\ \text{quantum} \\ \text{states} \\ \text{in vicinity} \\ \text{of } E \end{array} \right)$; thus

$$g(E) = \frac{1}{2\pi i} \int_{B-i\infty}^{B+i\infty} e^{BE} Q(B) dB \quad (3.1)$$



⊕ Use the Euler Formulae
 (more soon) to evaluate
 (3.1).

⊕ $Q(B)$ must be known

c) RELATIONSHIP BETWEEN

$g(E)$ and $\Omega = \Omega(E, V, N) =$ NO OF QUANTUM STATES IN A SYSTEM HAVING TOTAL ENERGY E , N PARTICLES, VOL V

i) FROM CH. 2:

$$\Omega(N, V, E, \Delta) \cong \frac{\partial \Sigma}{\partial E} \Delta \quad (1.4.17a)$$

$\Sigma =$ total NUMBER OF QUANTUM STATES w/ energy $\leq E$

$\Delta =$ typically uncertainty in measuring E ; $\left[\frac{\Delta}{E} \ll 1 \right]$

$\Omega =$ total number of QUANTUM STATES (FOR OUR N -PARTICLE SYSTEM) w/in $(E, E+\Delta)$

(AS WE DID IN LECTURE 18 (2015))

ii) ARGUE THAT FOR SMALL Δ , i.e., $\Delta/E \ll 1$, that

$\left[\Omega \right]$, which is the NO. of Ω states w/in a HYPERSPHERE SHELL OF RADIUS

LEC 20 SP4 15 (5)

$R = \frac{\sqrt{8m\hbar^{2/3}E}}{h}$ for N non-interacting ptcls in V , w/ $E = \text{TOTAL energy}$,

& NO THICKNESS $\Delta R = \frac{\partial R}{\partial E} \Delta =$

$$\frac{\sqrt{8m\hbar^{2/3}}}{h} \Delta$$

(5.11) $\boxed{\Gamma \approx \Omega}$

* $\left[\begin{array}{l} \text{NOTE, AS } \partial R / \partial E \propto 1/E \\ \rightarrow 0, \Gamma \rightarrow \Omega \end{array} \right]$
 i.e., for all practical purposes, for $E/E \ll 1$
 (5.11a) $\boxed{\Gamma = \Omega}$

(2c) ARGUE THAT FOR A THIN HYPERBOLIC SHELL, the NO. OF QUANTUM STATES w/in this shell (E) IS APPROXIMATELY = to the NO. OF QUANTUM STATES ON THE SURFACE OF THIS SPHERE $\approx \Omega$.

(*) FOR STRONGER STATEMENT

iii) FINALLY, since

$$\Omega \approx \Gamma \approx \frac{\partial \Sigma}{\partial E} \Delta =$$

NO OF QUANTUM STATES w/in $(E, E+\Delta)$

AND since

$$g(E)dE = \text{NO. OF QUANTUM STATES w/in } (E, E+dE)$$

lec 20
spg 15

And since geometrically:

⑥
hypersphere shell

$\Gamma =$ no. of Q states within

$\mathcal{R} =$ no. of Q states on hypersphere
surface (of radius $R = \frac{\sqrt{8m^3 E}}{h}$
for quantum ideal gas)

$$\frac{\Delta R}{R} = \frac{\Delta}{E} = \frac{dE}{E} \rightarrow 0 \quad \text{AS } \frac{dE}{E} \text{ becomes small}$$

then

w/

$$\boxed{g(E) = \Gamma \cong \mathcal{R}} \quad \times$$

$$\boxed{g(E) = \Gamma = \mathcal{R}} \quad \times$$

$$\text{AS } \frac{dE}{E} \rightarrow 0 \quad \times$$

(6.1)

D) A NEW, ALTERNATIVE

WAY TO CALCULATE S :

i) since $S = k \ln R$

and $R = g$ then we
also have:

$$\boxed{S = k \ln g} \quad (2.1)$$