

80. Use the midpoint of each of four 2-day intervals. Let $t = 0$ correspond to July 18 and note that the inflow rate, $r(t)$, is in ft^3/s .

$$\text{Amount of water} = \int_0^8 r(t) dt \approx [r(1) + r(3) + r(5) + r(7)] \frac{8-0}{4} \approx [6401 + 4249 + 3821 + 2628](2) = 34,198.$$

Now multiply by the number of seconds in a day, $24 \cdot 60^2$, to get 2,954,707,200 ft^3 .

81. Let $P(t)$ denote the bacteria population at time t (in hours). By the Net Change Theorem,

$$P(1) - P(0) = \int_0^1 P'(t) dt = \int_0^1 (1000 \cdot 2^t) dt = \left[1000 \frac{2^t}{\ln 2} \right]_0^1 = \frac{1000}{\ln 2} (2^1 - 2^0) = \frac{1000}{\ln 2} \approx 1443.$$

Thus, the population after one hour is $4000 + 1443 = 5443$.

82. Let $M(t)$ denote the number of megabits transmitted at time t (in hours) [note that $D(t)$ is measured in megabits/second]. By the Net Change Theorem and the Midpoint Rule,

$$\begin{aligned} M(8) - M(0) &= \int_0^8 3600D(t) dt \approx 3600 \cdot \frac{8-0}{4} [D(1) + D(3) + D(5) + D(7)] \\ &\approx 7200(0.32 + 0.50 + 0.56 + 0.83) = 7200(2.21) = 15,912 \text{ megabits} \end{aligned}$$

83. Power is the rate of change of energy with respect to time; that is, $P(t) = E'(t)$. By the Net Change Theorem and the Midpoint Rule, the electric energy used on that day is

$$\begin{aligned} E(24) - E(0) &= \int_0^{24} P(t) dt \approx \frac{24-0}{12} [P(1) + P(3) + P(5) + \cdots + P(21) + P(23)] \\ &\approx 2(11 + 10.5 + 11 + 14 + 15.1 + 15.5 + 15.1 + 15 + 15 + 16.1 + 15 + 13) \\ &= 2(166.3) = 332.6 \text{ gigawatt-hours} \end{aligned}$$

84. (a) From Exercise 4.1.78(a), $v(t) = 0.00146t^3 - 0.11553t^2 + 24.98169t - 21.26872$.

$$(b) h(125) - h(0) = \int_0^{125} v(t) dt = [0.000365t^4 - 0.03851t^3 + 12.490845t^2 - 21.26872t]_0^{125} \approx 206,407 \text{ ft}$$

5.5 The Substitution Rule

- Let $u = 2x$. Then $du = 2 dx$ and $dx = \frac{1}{2} du$, so $\int \cos 2x dx = \int \cos u \left(\frac{1}{2} du\right) = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2x + C$.
- Let $u = -x^2$. Then $du = -2x dx$ and $x dx = -\frac{1}{2} du$, so $\int x e^{-x^2} dx = \int e^u \left(-\frac{1}{2} du\right) = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$.
- Let $u = x^3 + 1$. Then $du = 3x^2 dx$ and $x^2 dx = \frac{1}{3} du$, so

$$\int x^2 \sqrt{x^3 + 1} dx = \int \sqrt{u} \left(\frac{1}{3} du\right) = \frac{1}{3} \frac{u^{3/2}}{3/2} + C = \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (x^3 + 1)^{3/2} + C.$$
- Let $u = \sin \theta$. Then $du = \cos \theta d\theta$, so $\int \sin^2 \theta \cos \theta d\theta = \int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} \sin^3 \theta + C$.
- Let $u = x^4 - 5$. Then $du = 4x^3 dx$ and $x^3 dx = \frac{1}{4} du$, so

$$\int \frac{x^3}{x^4 - 5} dx = \int \frac{1}{u} \left(\frac{1}{4} du\right) = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |x^4 - 5| + C.$$

6. Let $u = 1 + \frac{1}{x}$. Then $du = -\frac{1}{x^2} dx$ and $\frac{1}{x^2} dx = -du$, so

$$\int \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx = \int \sqrt{u} (-du) = -\frac{2}{3} u^{3/2} + C = -\frac{2}{3} \left(1 + \frac{1}{x}\right)^{3/2} + C.$$

7. Let $u = \sqrt{t}$. Then $du = \frac{1}{2\sqrt{t}} dt$ and $\frac{1}{\sqrt{t}} dt = 2 du$, so $\int \frac{\cos \sqrt{t}}{\sqrt{t}} dt = \int \cos u (2 du) = 2 \sin u + C = 2 \sin \sqrt{t} + C$.

8. Let $u = z - 1$. Then $z = u + 1$ and $du = dz$, so

$$\begin{aligned} \int z \sqrt{z-1} dz &= \int (u+1) \sqrt{u} du = \int (u^{3/2} + u^{1/2}) du = \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{5} (z-1)^{5/2} + \frac{2}{3} (z-1)^{3/2} + C \end{aligned}$$

9. Let $u = 1 - x^2$. Then $du = -2x dx$ and $x dx = -\frac{1}{2} du$, so

$$\int x \sqrt{1-x^2} dx = \int \sqrt{u} \left(-\frac{1}{2} du\right) = -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1-x^2)^{3/2} + C.$$

10. Let $u = 5 - 3x$. Then $du = -3 dx$ and $dx = -\frac{1}{3} du$, so

$$\int (5-3x)^{10} dx = \int u^{10} \left(-\frac{1}{3} du\right) = -\frac{1}{3} \cdot \frac{1}{11} u^{11} + C = -\frac{1}{33} (5-3x)^{11} + C.$$

11. Let $u = -t^4$. Then $du = -4t^3 dt$ and $t^3 dt = -\frac{1}{4} du$, so $\int t^3 e^{-t^4} dt = \int e^u \left(-\frac{1}{4} du\right) = -\frac{1}{4} e^u + C = -\frac{1}{4} e^{-t^4} + C$.

12. Let $u = 1 + \cos t$. Then $du = -\sin t dt$ and $\sin t dt = -du$, so

$$\int \sin t \sqrt{1 + \cos t} dt = \int \sqrt{u} (-du) = -\frac{2}{3} u^{3/2} + C = -\frac{2}{3} (1 + \cos t)^{3/2} + C.$$

13. Let $u = \frac{\pi}{3} t$. Then $du = \frac{\pi}{3} dt$ and $dt = \frac{3}{\pi} du$, so

$$\int \sin\left(\frac{\pi t}{3}\right) dt = \int \sin u \left(\frac{3}{\pi} du\right) = \frac{3}{\pi} \cdot (-\cos u) + C = -\frac{3}{\pi} \cos\left(\frac{\pi}{3} t\right) + C.$$

14. Let $u = 2\theta$. Then $du = 2 d\theta$ and $d\theta = \frac{1}{2} du$, so $\int \sec^2 2\theta d\theta = \int \sec^2 u \left(\frac{1}{2} du\right) = \frac{1}{2} \tan u + C = \frac{1}{2} \tan 2\theta + C$.

15. Let $u = 4x + 7$. Then $du = 4 dx$ and $dx = \frac{1}{4} du$, so $\int \frac{dx}{4x+7} = \int \frac{1}{u} \left(\frac{1}{4} du\right) = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |4x+7| + C$.

16. Let $u = 4 - y^3$. Then $du = -3y^2 dy$ and $y^2 dy = -\frac{1}{3} du$, so

$$\int y^2 (4 - y^3)^{2/3} dy = \int u^{2/3} \left(-\frac{1}{3} du\right) = -\frac{1}{3} \cdot \frac{3}{5} u^{5/3} + C = -\frac{1}{5} (4 - y^3)^{5/3} + C.$$

17. Let $u = 1 + \sin \theta$. Then $du = \cos \theta d\theta$, so

$$\int \frac{\cos \theta}{1 + \sin \theta} d\theta = \int \frac{1}{u} du = \ln |u| + C = \ln |1 + \sin \theta| + C = \ln(1 + \sin \theta) + C \quad [\text{since } 1 + \sin \theta \text{ is nonnegative}].$$

18. Let $u = z^3 + 1$. Then $du = 3z^2 dz$ and $\frac{1}{3} du = z^2 dz$, so

$$\int \frac{z^2}{z^3+1} dz = \int \frac{1}{u} \left(\frac{1}{3} du\right) = \frac{1}{3} \ln |u| + C = \frac{1}{3} \ln |z^3+1| + C.$$

19. Let $u = \cos \theta$. Then $du = -\sin \theta d\theta$ and $\sin \theta d\theta = -du$, so

$$\int \cos^3 \theta \sin \theta d\theta = \int u^3 (-du) = -\frac{1}{4}u^4 + C = -\frac{1}{4}\cos^4 \theta + C.$$

20. Let $u = -5r$. Then $du = -5 dr$ and $dr = -\frac{1}{5} du$, so $\int e^{-5r} dr = \int e^u (-\frac{1}{5} du) = -\frac{1}{5}e^u + C = -\frac{1}{5}e^{-5r} + C$.

21. Let $x = 1 - e^u$. Then $dx = -e^u du$ and $e^u du = -dx$, so

$$\int \frac{e^u}{(1 - e^u)^2} du = \int \frac{1}{x^2} (-dx) = -\int x^{-2} dx = -(-x^{-1}) + C = \frac{1}{x} + C = \frac{1}{1 - e^u} + C.$$

22. Let $u = 1/x$. Then $du = -\frac{1}{x^2} dx$ and $\frac{1}{x^2} dx = -du$, so $\int \frac{\sin(1/x)}{x^2} dx = \int \sin u (-du) = \cos u + C = \cos(1/x) + C$.

23. Let $u = 3ax + bx^3$. Then $du = (3a + 3bx^2) dx = 3(a + bx^2) dx$, so

$$\int \frac{a + bx^2}{\sqrt{3ax + bx^3}} dx = \int \frac{\frac{1}{3} du}{u^{1/2}} = \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} \cdot 2u^{1/2} + C = \frac{2}{3} \sqrt{3ax + bx^3} + C.$$

24. Let $u = 3t^2 + 6t - 5$. Then $du = (6t + 6) dt = 6(t + 1) dt$ and $(t + 1) dt = \frac{1}{6} du$, so

$$\int \frac{t + 1}{3t^2 + 6t - 5} dt = \int \frac{1}{u} \left(\frac{1}{6} du \right) = \frac{1}{6} \ln |u| + C = \frac{1}{6} \ln |3t^2 + 6t - 5| + C.$$

25. Let $u = \ln x$. Then $du = \frac{dx}{x}$, so $\int \frac{(\ln x)^2}{x} dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$.

26. Let $u = \cos x$. Then $du = -\sin x dx$ and $-du = \sin x dx$, so

$$\int \sin x \sin(\cos x) dx = \int \sin u (-du) = (-\cos u)(-1) + C = \cos(\cos x) + C.$$

27. Let $u = \tan \theta$. Then $du = \sec^2 \theta d\theta$, so $\int \sec^2 \theta \tan^3 \theta d\theta = \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}\tan^4 \theta + C$.

28. Let $u = x + 2$. Then $du = dx$ and $x = u - 2$, so

$$\int x\sqrt{x+2} dx = \int (u-2)\sqrt{u} du = \int (u^{3/2} - 2u^{1/2}) du = \frac{2}{5}u^{5/2} - 2 \cdot \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C.$$

29. Let $u = x^2 + \frac{2}{x}$. Then $du = \left(2x - \frac{2}{x^2}\right) dx = 2\left(x - \frac{1}{x^2}\right) dx$ and $\left(x - \frac{1}{x^2}\right) dx = \frac{1}{2} du$, so

$$\int \left(x - \frac{1}{x^2}\right) \left(x^2 + \frac{2}{x}\right)^5 dx = \int u^5 \left(\frac{1}{2} du\right) = \frac{1}{2} \cdot \frac{1}{6}u^6 + C = \frac{1}{12}\left(x^2 + \frac{2}{x}\right)^6 + C.$$

30. Let $u = ax + b$. Then $du = a dx$ and $dx = (1/a) du$, so

$$\int \frac{dx}{ax + b} = \int \frac{(1/a) du}{u} = \frac{1}{a} \int \frac{1}{u} du = \frac{1}{a} \ln |u| + C = \frac{1}{a} \ln |ax + b| + C.$$

31. Let $u = 2 + 3e^r$. Then $du = 3e^r dr$ and $e^r dr = \frac{1}{3} du$, so

$$\int e^r (2 + 3e^r)^{3/2} dr = \int u^{3/2} \left(\frac{1}{3} du\right) = \frac{1}{3} \cdot \frac{2}{5}u^{5/2} + C = \frac{2}{15}(2 + 3e^r)^{5/2} + C.$$

32. Let $u = \arcsin x$. Then $du = \frac{1}{\sqrt{1-x^2}} dx$, so $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx = \int e^u du = e^u + C = e^{\arcsin x} + C$.

33. Let $u = \tan \theta$. Then $du = \sec^2 \theta d\theta$, so $\int \frac{\sec^2 \theta}{\tan \theta} d\theta = \int \frac{1}{u} du = \ln |u| + C = \ln |\tan \theta| + C$.

34. Let $u = \tan x$. Then $du = \sec^2 x dx$, so

$$\int \frac{\sec^2 x}{\tan^2 x} dx = \int \frac{1}{u^2} du = \int u^{-2} du = -1u^{-1} + C = -\frac{1}{\tan x} + C = -\cot x + C.$$

$$\text{Or: } \int \frac{\sec^2 x}{\tan^2 x} dx = \int \left(\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} \right) dx = \int \csc^2 x dx = -\cot x + C$$

35. Let $u = \arctan x$. Then $du = \frac{1}{x^2 + 1} dx$, so $\int \frac{(\arctan x)^2}{x^2 + 1} dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\arctan x)^3 + C$.

36. Let $u = \arctan x$. Then $du = \frac{1}{1 + x^2} dx = \frac{1}{x^2 + 1} dx$, so

$$\int \frac{1}{(x^2 + 1) \arctan x} dx = \int \frac{1}{u} du = \ln |u| + C = \ln |\arctan x| + C.$$

37. Let $u = 5^t$. Then $du = 5^t \ln 5 dt$ and $5^t dt = \frac{1}{\ln 5} du$, so

$$\int 5^t \sin(5^t) dt = \int \sin u \left(\frac{1}{\ln 5} du \right) = -\frac{1}{\ln 5} \cos u + C = -\frac{1}{\ln 5} \cos(5^t) + C.$$

38. Let $u = 1 + \sin^2 \theta$. Then $du = 2 \sin \theta \cos \theta d\theta$ and $\sin \theta \cos \theta d\theta = \frac{1}{2} du$, so

$$\begin{aligned} \int \frac{\sin \theta \cos \theta}{1 + \sin^2 \theta} d\theta &= \int \frac{1}{u} \left(\frac{1}{2} du \right) = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |1 + \sin^2 \theta| + C \\ &= \frac{1}{2} \ln(1 + \sin^2 \theta) + C \quad [\text{since } 1 + \sin^2 \theta \text{ is positive}] \end{aligned}$$

39. Let $u = 1 + 5t$. Then $du = 5 dt$ and $dt = \frac{1}{5} du$, so

$$\int \cos(1 + 5t) dt = \int \cos u \left(\frac{1}{5} du \right) = \frac{1}{5} \sin u + C = \frac{1}{5} \sin(1 + 5t) + C.$$

40. Let $u = \frac{\pi}{x}$. Then $du = -\frac{\pi}{x^2} dx$ and $\frac{1}{x^2} dx = -\frac{1}{\pi} du$, so

$$\int \frac{\cos(\pi/x)}{x^2} dx = \int \cos u \left(-\frac{1}{\pi} du \right) = -\frac{1}{\pi} \sin u + C = -\frac{1}{\pi} \sin \frac{\pi}{x} + C$$

41. Let $u = \cot x$. Then $du = -\csc^2 x dx$ and $\csc^2 x dx = -du$, so

$$\int \sqrt{\cot x} \csc^2 x dx = \int \sqrt{u} (-du) = -\frac{u^{3/2}}{3/2} + C = -\frac{2}{3} (\cot x)^{3/2} + C.$$

42. Let $u = 2^t + 3$. Then $du = 2^t \ln 2 dt$ and $2^t dt = \frac{1}{\ln 2} du$, so

$$\int \frac{2^t}{2^t + 3} dt = \int \frac{1}{u} \left(\frac{1}{\ln 2} du \right) = \frac{1}{\ln 2} \ln |u| + C = \frac{1}{\ln 2} \ln(2^t + 3) + C.$$

43. Let $u = \sinh x$. Then $du = \cosh x dx$, so $\int \sinh^2 x \cosh x dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3} \sinh^3 x + C$.

44. Let $u = 1 + \tan t$. Then $du = \sec^2 t \, dt$, so

$$\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}} = \int \frac{\sec^2 t \, dt}{\sqrt{1 + \tan t}} = \int \frac{du}{\sqrt{u}} = \int u^{-1/2} du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{1 + \tan t} + C.$$

45. $\int \frac{\sin 2x}{1 + \cos^2 x} dx = 2 \int \frac{\sin x \cos x}{1 + \cos^2 x} dx = 2I$. Let $u = \cos x$. Then $du = -\sin x \, dx$, so

$$2I = -2 \int \frac{u \, du}{1 + u^2} = -2 \cdot \frac{1}{2} \ln(1 + u^2) + C = -\ln(1 + u^2) + C = -\ln(1 + \cos^2 x) + C.$$

Or: Let $u = 1 + \cos^2 x$.

46. Let $u = \cos x$. Then $du = -\sin x \, dx$ and $\sin x \, dx = -du$, so

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{-du}{1 + u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C.$$

47. $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$. Let $u = \sin x$. Then $du = \cos x \, dx$, so $\int \cot x \, dx = \int \frac{1}{u} du = \ln |u| + C = \ln |\sin x| + C$.

48. Let $u = \ln t$. Then $du = \frac{1}{t} dt$, so $\int \frac{\cos(\ln t)}{t} dt = \int \cos u \, du = \sin u + C = \sin(\ln t) + C$.

49. Let $u = \sin^{-1} x$. Then $du = \frac{1}{\sqrt{1-x^2}} dx$, so $\int \frac{dx}{\sqrt{1-x^2} \sin^{-1} x} = \int \frac{1}{u} du = \ln |u| + C = \ln |\sin^{-1} x| + C$.

50. Let $u = x^2$. Then $du = 2x \, dx$, so $\int \frac{x}{1+x^4} dx = \int \frac{\frac{1}{2} du}{1+u^2} = \frac{1}{2} \tan^{-1} u + C = \frac{1}{2} \tan^{-1}(x^2) + C$.

51. Let $u = 1 + x^2$. Then $du = 2x \, dx$, so

$$\begin{aligned} \int \frac{1+x}{1+x^2} dx &= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx = \tan^{-1} x + \int \frac{\frac{1}{2} du}{u} = \tan^{-1} x + \frac{1}{2} \ln |u| + C \\ &= \tan^{-1} x + \frac{1}{2} \ln |1+x^2| + C = \tan^{-1} x + \frac{1}{2} \ln(1+x^2) + C \quad [\text{since } 1+x^2 > 0]. \end{aligned}$$

52. Let $u = 2 + x$. Then $du = dx$, $x = u - 2$, and $x^2 = (u - 2)^2$, so

$$\begin{aligned} \int x^2 \sqrt{2+x} \, dx &= \int (u-2)^2 \sqrt{u} \, du = \int (u^2 - 4u + 4) u^{1/2} \, du = \int (u^{5/2} - 4u^{3/2} + 4u^{1/2}) \, du \\ &= \frac{2}{7} u^{7/2} - \frac{8}{5} u^{5/2} + \frac{8}{3} u^{3/2} + C = \frac{2}{7} (2+x)^{7/2} - \frac{8}{5} (2+x)^{5/2} + \frac{8}{3} (2+x)^{3/2} + C \end{aligned}$$

53. Let $u = 2x + 5$. Then $du = 2 \, dx$ and $x = \frac{1}{2}(u - 5)$, so

$$\begin{aligned} \int x(2x+5)^8 \, dx &= \int \frac{1}{2}(u-5)u^8 \left(\frac{1}{2} du\right) = \frac{1}{4} \int (u^9 - 5u^8) \, du \\ &= \frac{1}{4} \left(\frac{1}{10} u^{10} - \frac{5}{9} u^9\right) + C = \frac{1}{40} (2x+5)^{10} - \frac{5}{36} (2x+5)^9 + C \end{aligned}$$

54. Let $u = x^2 + 1$ [so $x^2 = u - 1$]. Then $du = 2x \, dx$ and $x \, dx = \frac{1}{2} du$, so

$$\begin{aligned} \int x^3 \sqrt{x^2+1} \, dx &= \int x^2 \sqrt{x^2+1} x \, dx = \int (u-1) \sqrt{u} \left(\frac{1}{2} du\right) = \frac{1}{2} \int (u^{3/2} - u^{1/2}) \, du \\ &= \frac{1}{2} \left(\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2}\right) + C = \frac{1}{5} (x^2+1)^{5/2} - \frac{1}{3} (x^2+1)^{3/2} + C \end{aligned}$$

[continued]

Or: Let $u = \sqrt{x^2 + 1}$. Then $u^2 = x^2 + 1 \Rightarrow 2u du = 2x dx \Rightarrow u du = x dx$, so

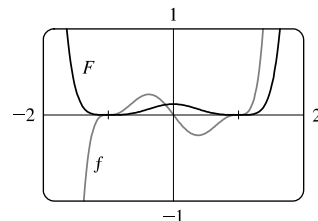
$$\begin{aligned}\int x^3 \sqrt{x^2 + 1} dx &= \int x^2 \sqrt{x^2 + 1} x dx = \int (u^2 - 1) u \cdot u du = \int (u^4 - u^2) du \\ &= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} (x^2 + 1)^{5/2} - \frac{1}{3} (x^2 + 1)^{3/2} + C\end{aligned}$$

Note: This answer can be written as $\frac{1}{15} \sqrt{x^2 + 1} (3x^4 + x^2 - 2) + C$.

55. $f(x) = x(x^2 - 1)^3$. $u = x^2 - 1 \Rightarrow du = 2x dx$, so

$$\int x(x^2 - 1)^3 dx = \int u^3 \left(\frac{1}{2} du\right) = \frac{1}{8} u^4 + C = \frac{1}{8} (x^2 - 1)^4 + C$$

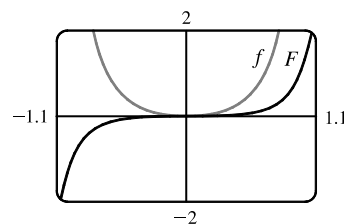
Where f is positive (negative), F is increasing (decreasing). Where f changes from negative to positive (positive to negative), F has a local minimum (maximum).



56. $f(\theta) = \tan^2 \theta \sec^2 \theta$. $u = \tan \theta \Rightarrow du = \sec^2 \theta d\theta$, so

$$\int \tan^2 \theta \sec^2 \theta d\theta = \int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} \tan^3 \theta + C$$

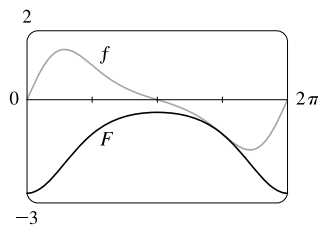
Note that f is positive and F is increasing. At $x = 0$, $f = 0$ and F has a horizontal tangent.



57. $f(x) = e^{\cos x} \sin x$. $u = \cos x \Rightarrow du = -\sin x dx$, so

$$\int e^{\cos x} \sin x dx = \int e^u (-du) = -e^u + C = -e^{\cos x} + C$$

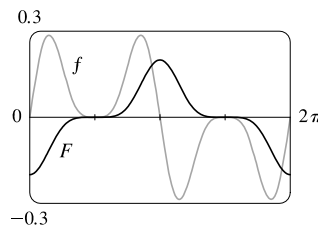
Note that at $x = \pi$, f changes from positive to negative and F has a local maximum. Also, both f and F are periodic with period 2π , so at $x = 0$ and at $x = 2\pi$, f changes from negative to positive and F has a local minimum.



58. $f(x) = \sin x \cos^4 x$. $u = \cos x \Rightarrow du = -\sin x dx$, so

$$\int \sin x \cos^4 x dx = \int u^4 (-du) = -\frac{1}{5} u^5 + C = -\frac{1}{5} \cos^5 x + C$$

Note that at $x = \pi$, f changes from positive to negative and F has a local maximum. Also, both f and F are periodic with period 2π , so at $x = 0$ and at $x = 2\pi$, f changes from negative to positive and F has a local minimum.



59. Let $u = \frac{\pi}{2}t$, so $du = \frac{\pi}{2} dt$. When $t = 0$, $u = 0$; when $t = 1$, $u = \frac{\pi}{2}$. Thus,

$$\int_0^1 \cos(\pi t/2) dt = \int_0^{\pi/2} \cos u \left(\frac{2}{\pi} du\right) = \frac{2}{\pi} [\sin u]_0^{\pi/2} = \frac{2}{\pi} (\sin \frac{\pi}{2} - \sin 0) = \frac{2}{\pi} (1 - 0) = \frac{2}{\pi}$$

60. Let $u = 3t - 1$, so $du = 3 dt$. When $t = 0$, $u = -1$; when $t = 1$, $u = 2$. Thus,

$$\int_0^1 (3t - 1)^{50} dt = \int_{-1}^2 u^{50} \left(\frac{1}{3} du\right) = \frac{1}{3} \left[\frac{1}{51} u^{51}\right]_{-1}^2 = \frac{1}{153} [2^{51} - (-1)^{51}] = \frac{1}{153} (2^{51} + 1)$$

61. Let $u = 1 + 7x$, so $du = 7 dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = 8$. Thus,

$$\int_0^1 \sqrt[3]{1+7x} dx = \int_1^8 u^{1/3} \left(\frac{1}{7} du\right) = \frac{1}{7} \left[\frac{3}{4} u^{4/3} \right]_1^8 = \frac{3}{28} (8^{4/3} - 1^{4/3}) = \frac{3}{28} (16 - 1) = \frac{45}{28}$$

62. Let $u = \frac{1}{2}t$, so $du = \frac{1}{2} dt$. When $t = \frac{\pi}{3}$, $u = \frac{\pi}{6}$; when $t = \frac{2\pi}{3}$, $u = \frac{\pi}{3}$. Thus,

$$\begin{aligned} \int_{\pi/3}^{2\pi/3} \csc^2\left(\frac{1}{2}t\right) dt &= \int_{\pi/6}^{\pi/3} \csc^2 u (2 du) = 2 \left[-\cot u \right]_{\pi/6}^{\pi/3} = -2 \left(\cot \frac{\pi}{3} - \cot \frac{\pi}{6} \right) \\ &= -2 \left(\frac{1}{\sqrt{3}} - \sqrt{3} \right) = -2 \left(\frac{1}{3}\sqrt{3} - \sqrt{3} \right) = \frac{4}{3}\sqrt{3} \end{aligned}$$

63. Let $u = \cos t$, so $du = -\sin t dt$. When $t = 0$, $u = 1$; when $t = \frac{\pi}{6}$, $u = \sqrt{3}/2$. Thus,

$$\int_0^{\pi/6} \frac{\sin t}{\cos^2 t} dt = \int_1^{\sqrt{3}/2} \frac{1}{u^2} (-du) = \left[\frac{1}{u} \right]_1^{\sqrt{3}/2} = \frac{2}{\sqrt{3}} - 1.$$

64. Let $u = 2 + \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}} dx$ and $\frac{1}{\sqrt{x}} dx = 2 du$. When $x = 1$, $u = 3$; when $x = 4$, $u = 4$. Thus,

$$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}} dx = \int_3^4 \sqrt{u} (2 du) = \left[\frac{4}{3} u^{3/2} \right]_3^4 = \frac{4}{3} (4^{3/2} - 3^{3/2}) = \frac{4}{3} (8 - 3\sqrt{3}) \text{ or } \frac{32}{3} - 4\sqrt{3}.$$

65. Let $u = 1/x$, so $du = -1/x^2 dx$. When $x = 1$, $u = 1$; when $x = 2$, $u = \frac{1}{2}$. Thus,

$$\int_1^2 \frac{e^{1/x}}{x^2} dx = \int_1^{1/2} e^u (-du) = -[e^u]_1^{1/2} = -(e^{1/2} - e) = e - \sqrt{e}.$$

66. Let $u = e^x$. Then $du = e^x dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = e$. Thus,

$$\int_0^1 \frac{e^x}{1+e^{2x}} dx = \int_1^e \frac{1}{1+u^2} du = \left[\arctan u \right]_1^e = \arctan e - \arctan 1 = \arctan e - \frac{\pi}{4}.$$

67. $\int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x) dx = 0$ by Theorem 7(b), since $f(x) = x^3 + x^4 \tan x$ is an odd function.

68. Let $u = \sin x$, so $du = \cos x dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,

$$\int_0^{\pi/2} \cos x \sin(\sin x) dx = \int_0^1 \sin u du = [-\cos u]_0^1 = -(\cos 1 - 1) = 1 - \cos 1.$$

69. Let $u = 1 + 2x$, so $du = 2 dx$. When $x = 0$, $u = 1$; when $x = 13$, $u = 27$. Thus,

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} u^{-2/3} \left(\frac{1}{2} du\right) = \left[\frac{1}{2} \cdot 3u^{1/3} \right]_1^{27} = \frac{3}{2} (3 - 1) = 3.$$

70. Assume $a > 0$. Let $u = a^2 - x^2$, so $du = -2x dx$. When $x = 0$, $u = a^2$; when $x = a$, $u = 0$. Thus,

$$\int_0^a x \sqrt{a^2 - x^2} dx = \int_{a^2}^0 u^{1/2} \left(-\frac{1}{2} du\right) = \frac{1}{2} \int_0^{a^2} u^{1/2} du = \frac{1}{2} \cdot \left[\frac{2}{3} u^{3/2} \right]_0^{a^2} = \frac{1}{3} a^3.$$

71. Let $u = x^2 + a^2$, so $du = 2x dx$ and $x dx = \frac{1}{2} du$. When $x = 0$, $u = a^2$; when $x = a$, $u = 2a^2$. Thus,

$$\int_0^a x \sqrt{x^2 + a^2} dx = \int_{a^2}^{2a^2} u^{1/2} \left(\frac{1}{2} du\right) = \frac{1}{2} \left[\frac{2}{3} u^{3/2} \right]_{a^2}^{2a^2} = \left[\frac{1}{3} u^{3/2} \right]_{a^2}^{2a^2} = \frac{1}{3} \left[(2a^2)^{3/2} - (a^2)^{3/2} \right] = \frac{1}{3} (2\sqrt{2} - 1) a^3$$

72. $\int_{-\pi/3}^{\pi/3} x^4 \sin x \, dx = 0$ by Theorem 7(b), since $f(x) = x^4 \sin x$ is an odd function.

73. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$$\int_1^2 x \sqrt{x-1} \, dx = \int_0^1 (u+1) \sqrt{u} \, du = \int_0^1 (u^{3/2} + u^{1/2}) \, du = \left[\frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right]_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{16}{15}.$$

74. Let $u = 1 + 2x$, so $x = \frac{1}{2}(u - 1)$ and $du = 2 \, dx$. When $x = 0$, $u = 1$; when $x = 4$, $u = 9$. Thus,

$$\begin{aligned} \int_0^4 \frac{x \, dx}{\sqrt{1+2x}} &= \int_1^9 \frac{\frac{1}{2}(u-1)}{\sqrt{u}} \frac{du}{2} = \frac{1}{4} \int_1^9 (u^{1/2} - u^{-1/2}) \, du = \frac{1}{4} \left[\frac{2}{3} u^{3/2} - 2u^{1/2} \right]_1^9 \\ &= \frac{1}{6} [(27 - 9) - (1 - 3)] = \frac{20}{6} = \frac{10}{3} \end{aligned}$$

75. Let $u = \ln x$, so $du = \frac{dx}{x}$. When $x = e$, $u = 1$; when $x = e^4$, $u = 4$. Thus,

$$\int_e^{e^4} \frac{dx}{x \sqrt{\ln x}} = \int_1^4 u^{-1/2} \, du = 2 \left[u^{1/2} \right]_1^4 = 2(2 - 1) = 2.$$

76. Let $u = (x - 1)^2$, so $du = 2(x - 1) \, dx$. When $x = 0$, $u = 1$; when $x = 2$, $u = 1$. Thus,

$$\int_0^2 (x - 1) e^{(x-1)^2} \, dx = \int_1^1 e^u \left(\frac{1}{2} du \right) = 0 \text{ since the limits are equal.}$$

77. Let $u = e^z + z$, so $du = (e^z + 1) \, dz$. When $z = 0$, $u = 1$; when $z = 1$, $u = e + 1$. Thus,

$$\int_0^1 \frac{e^z + 1}{e^z + z} \, dz = \int_1^{e+1} \frac{1}{u} \, du = \left[\ln |u| \right]_1^{e+1} = \ln |e + 1| - \ln |1| = \ln(e + 1).$$

78. Let $u = \sqrt{x}$. Then $u^2 = x$, $2u \, du = dx$, $du = \frac{1}{2\sqrt{x}} \, dx$, and $\frac{1}{\sqrt{x}} \, dx = 2 \, du$. When $x = 1$, $u = 1$; when $x = 4$, $u = 2$.

Thus,

$$\begin{aligned} \int_1^4 \frac{1}{(x+1)\sqrt{x}} \, dx &= \int_1^2 \frac{1}{u^2+1} (2 \, du) = 2 \left[\arctan u \right]_1^2 = 2(\arctan 2 - \arctan 1) \\ &= 2 \left(\arctan 2 - \frac{\pi}{4} \right) = 2 \arctan 2 - \frac{\pi}{2} \end{aligned}$$

79. Let $u = 1 + \sqrt{x}$, so $du = \frac{1}{2\sqrt{x}} \, dx \Rightarrow 2\sqrt{x} \, du = dx \Rightarrow 2(u - 1) \, du = dx$. When $x = 0$, $u = 1$; when $x = 1$,

$u = 2$. Thus,

$$\begin{aligned} \int_0^1 \frac{dx}{(1+\sqrt{x})^4} &= \int_1^2 \frac{1}{u^4} \cdot [2(u-1) \, du] = 2 \int_1^2 \left(\frac{1}{u^3} - \frac{1}{u^4} \right) \, du = 2 \left[-\frac{1}{2u^2} + \frac{1}{3u^3} \right]_1^2 \\ &= 2 \left[\left(-\frac{1}{8} + \frac{1}{24} \right) - \left(-\frac{1}{2} + \frac{1}{3} \right) \right] = 2 \left(\frac{1}{12} \right) = \frac{1}{6} \end{aligned}$$

80. Let $u = 1 + x^{3/4}$. Then $x^{3/4} = u - 1$, $du = \frac{3}{4} x^{-1/4} \, dx$, and $x^{-1/4} \, dx = \frac{4}{3} \, du$. When $x = 1$, $u = 2$; when $x = 16$, $u = 9$.

Thus,

$$\begin{aligned} \int_1^{16} \frac{x^{1/2}}{1+x^{3/4}} \, dx &= \int_2^9 \frac{x^{3/4} \cdot x^{-1/4}}{1+x^{3/4}} \, dx = \int_2^9 \frac{u-1}{u} \left(\frac{4}{3} \, du \right) = \frac{4}{3} \int_2^9 \left(1 - \frac{1}{u} \right) \, du = \frac{4}{3} \left[u - \ln |u| \right]_2^9 \\ &= \frac{4}{3} [(9 - \ln 9) - (2 - \ln 2)] = \frac{4}{3} (7 - \ln 9 + \ln 2) = \frac{4}{3} (7 + \ln \frac{2}{9}) \end{aligned}$$