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GOE Book — Volume I: GDOS

The Geometry of Existence

Volume I: GDOS — Geometric Dynamics of Open Systems

From the GUT-10 Constitution to the Qi-4 Universal Runtime

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Abstract

This book develops a complete theoretical framework for all of physics from eleven foundational statements and zero new continuous parameters. The framework consists of four layers: a constitution (GUT-10: four axioms and seven postulates), hardware (GG-6: a unique warped six-dimensional geometry specified by five integers), an operating system (GDOS: boundary dynamics producing general relativity, the Standard Model, and quantum measurement), and a runtime (Qi-4: a universal pipeline governing every open system from elementary particles to conscious minds). The central quantitative result is the Arisaka Equation, which locks the energy, entropy, material, and information currents of every elementary event into a fixed ratio derived from the geometry with zero adjustable parameters.

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Preface

This book began with a question I could not stop asking: *Why am I here?*

I have worked as an experimental particle physicist, first as a graduate student to design and construct the original Kamiokande for proton decay with Takaaki Kajita under Masatoshi Koshiba. Then, I moved to the United States in 1985, and worked on kaon rare decays (lepton-number violation) at Brookhaven National Laboratory, KTeV for CP violation at Fermilab, the Pierre Auger Observatory in Argentina for ultra-high-energy cosmic rays, CMS at the LHC for Higgs discovery, and XENON for WIMP dark-matter detection. In 2013, I made a conscious decision to switch to neuroscience to explore the origins and evolution of the mind and brain. I initially studied the brain of *C. elegans*, forming the undergraduate research lab of the Elegant Mind Club at UCLA.

Throughout that journey, one observation kept returning: the deepest problems in physics—the measurement problem, the unexplained parameters of the Standard Model, the nature of dark matter and dark energy—are structurally similar to the deepest problems in biology and neuroscience—the origin of life, the emergence of cognition, the hard problem of consciousness. They are all, at bottom, questions about how open systems exchange energy, matter, and information with their environments, and how higher-level structures emerge from lower-level rules.

The standard approach in each discipline is to treat these as separate problems, studied by separate communities, with separate formalisms. I became convinced that this fragmentation is itself the obstacle. What was missing was not more data or bigger computers, but a shared constitutional layer—a set of foundational rules that every branch of science must obey, analogous to a political constitution that sits above all ordinary laws.

This conviction led, over the period 2024–2026, to the development of the GUT-10/GG-6/GDOS programme. The programme is built in four layers. **GUT-10** (Chapter 2) is the constitution: four axioms and seven postulates that define what any admissible physical theory must look like. **GG-6** (Chapter 3) is the unique six-dimensional geometry forced by the constitution, with all structural parameters fixed by five integers. **GDOS** (Chapter 4) is the operating system: the boundary dynamics that produce general relativity, the Standard Model, and quantum measurement as derived consequences. The **Qi-4 Pillars** (Chapter 5) are the universal runtime: a three-stage pipeline and four diagnostic pillars that govern every open system in nature, from a proton to a conscious brain, with the Arisaka Equation locking the energy–entropy–information currents of every elementary event into a geometrically determined ratio.

Volume I, which you hold in your hands, develops the complete theoretical framework. Volume II (*Geometric Origin of the Cosmos and Particles*) will develop the first two floors of the pagoda. Volume III (*Inevitable Emergence of Life and Mind*) will develop floors three through five. Volume IV (*GUT-10: Grand Unification of 10 Sciences*) will extend the framework to health, society, environment, AI, and philosophy.

On the role of large language models

This book was developed through an unusual collaboration. Three large language models—ChatGPT (OpenAI), Perplexity, and Claude (Anthropic)—served as iterative partners at every stage. ChatGPT was the primary drafting and ideation partner: I developed the core scientific ideas in extended dialogue, using the model as a sounding board for theorem structure, physical intuition, and cross-domain connections. Perplexity provided literature searches, citation verification, and fact-checking against the existing literature. Claude served as the structural editor, LaTeX engineer, and final proofreader, converting raw drafts into the polished monograph you are reading.

I want to be precise about what this means and what it does not mean. The LLMs did not invent the GUT-10 axiom system, the GG-6 geometry, the GDOS boundary dynamics, the Qi-4 Pillars, or the Arisaka Equation. Every scientific concept, every physical judgment, every interpretive claim, and every remaining error is mine alone. What the LLMs provided was an extraordinary acceleration of the writing process: the ability to iterate on a 100-page derivation in hours rather than months, to cross-check notation across seven chapters simultaneously, and to maintain structural coherence across a 610-page monograph with 477 inline citations, 204 bibliography entries, 95 FAQ answers, 54 technical appendices, and a 237-term index.

I believe this collaboration represents a new mode of scientific authorship—one that will become routine within a decade. The scientist supplies the ideas, the judgment, and the responsibility. The LLM supplies the memory, the speed, and the tireless attention to detail. Neither is sufficient alone.

Acknowledgments

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The Department of Physics and Astronomy at UCLA has been my intellectual home for nearly 40 years. I thank the department for the freedom to pursue research that crosses every traditional boundary. Special thanks go to the former Chairs, Roberto Peccei and Ferdinand Coroniti.

Finally, I thank the reader. This book asks you to take seriously the possibility that the laws of physics, correctly understood, do not merely *permit* the emergence of life and mind—they *require* it. That is a strong claim. It may be wrong. But if there is even a chance that it is right, it deserves your careful attention.

Katsushi Arisaka
Los Angeles, California
March 2026

Chapter 1

Introduction

Why Are We Here?

"The most incomprehensible thing about the universe is that it is comprehensible."

— Albert Einstein, 1936

1.1 Why Are We Here?

You are reading this sentence with a brain that runs on twenty watts of power—about the same as a dim light bulb. That brain is made of atoms forged inside stars that exploded before the Earth existed. Those atoms are held together by forces whose strengths are set by numbers that nobody chose. And somehow, after 13.8 billion years of cosmic evolution, the universe has arranged a small portion of itself into a structure that can ask: *why?*

Why are we here?

Not "how did we get here?"—that is a question of history, and science has made extraordinary progress on it. Not "what are we made of?"—that is a question of chemistry and physics, and we know the answer in remarkable detail. But *why?* Why does the universe produce beings that can wonder about it? Is consciousness a cosmic accident, a fantastically improbable fluctuation in an indifferent void (Schrödinger, 1944)? Or is it somehow written into the fabric of reality—as inevitable as the closing of a great circle on a sphere?

This book argues for the second possibility. It argues that the laws of physics, as we currently understand them, do not merely *permit* the emergence of life and mind—they *require* it. The argument is not mystical, not philosophical, and not hand-waving. It is a chain of mathematical theorems, starting from a small set of foundational rules and ending with a prediction: *any universe with these geometric properties will produce observers.*

That is a strong claim. It may be wrong. But the purpose of this opening chapter is not to prove it—the proof occupies the next five chapters. The purpose here is to explain why the claim is worth taking seriously, by telling the story of what science has achieved, where it has stalled, and why a new kind of thinking is needed.

1.2 What Science Has Achieved

Before we can see what is missing, we must appreciate what has been built. The scientific enterprise of the past four centuries is, by any measure, the greatest intellectual achievement of our species.

It began with a man sitting in a farmhouse during the plague year of 1665. Isaac Newton, watching the fall of an apple (or so the legend goes), realized that the same force pulling the fruit to the ground also holds the Moon in orbit around the Earth. Before Newton, the heavens and the Earth obeyed different laws. After Newton, they obeyed one law. This was the first great unification—and it changed forever what science could aspire to.

Two and a half centuries later, Albert Einstein showed that space and time are not a fixed stage (Einstein, 1905) on which the drama of physics plays out. They are part of the drama. Space curves. Time stretches. Gravity is not a force pulling across empty space; it is the shape of spacetime itself (Einstein, 1915). This insight—that physics *is* geometry—remains perhaps the most beautiful idea in all of science.

In the same era, a generation of physicists—Planck, Bohr, Heisenberg, Schrödinger, Dirac, and many others—built quantum mechanics (Born, 1926; Heisenberg, 1925; Planck, 1900; Schrödinger, 1926). Quantum mechanics is strange. A particle can be in two places at once. An electron does

not have a definite position until you look at it. The universe, at its most fundamental level, runs on probabilities rather than certainties. But the theory works—spectacularly. The magnetic moment of the electron, computed from quantum electrodynamics, agrees with experiment to better than one part in a trillion. Nothing else in the entire history of human knowledge comes close to this precision.

These two revolutions—general relativity and quantum mechanics—were consolidated in the second half of the twentieth century into the Standard Model of particle physics (Glashow, 1961; Salam, 1968; Weinberg, 1967). The Standard Model is, in a sense, the periodic table of reality. It catalogs every known elementary particle and describes three of the four fundamental forces (the electromagnetic, weak, and strong interactions) with extraordinary accuracy. The discovery of the Higgs boson at CERN in 2012 was the crowning achievement of this programme (Higgs, 1964)—a particle predicted forty-eight years before it was found.

Meanwhile, the other sciences were building their own cathedrals. Watson and Crick uncovered the double-helical structure of DNA in 1953 (Watson and Crick, 1953), revealing that heredity is written in a four-letter chemical alphabet. Claude Shannon gave a precise mathematical meaning to the word "information" (Shannon, 1948), launching the digital revolution. Ilya Prigogine showed that systems driven far from equilibrium can spontaneously organize into complex structures—dissipative structures that maintain themselves by consuming energy (Nicolis & Prigogine, 1977; Prigogine, 1967).

By the early twenty-first century, humanity's scientific knowledge is vast in its scope. We can trace the history of the cosmos back to a fraction of a second after the Big Bang. We can sequence a human genome in a day. We can detect the gravitational ripples from two black holes colliding a billion light-years away. We can simulate the folding of a protein, predict the path of a hurricane, and edit the DNA of a living organism.

The house of science is magnificent.

And yet, it has cracks in its foundation.

1.3 What Science Has Not Achieved

Some of these cracks have been visible for nearly a century. Others have appeared only recently. What makes them alarming is not that they exist—every scientific framework has open problems—but that they all seem to point in the same direction.

Start with the deepest. Quantum mechanics, for all its spectacular precision, does not explain its own most basic operation: *measurement*. Between measurements, a quantum system evolves smoothly and predictably. But when you measure it, something abrupt and unpredictable happens—the wavefunction "collapses" to one outcome. What triggers the collapse? When does it happen? What counts as a "measurement" in the first place? After ninety years of argument, nobody knows (Zurek, 2003). This is called the *measurement problem*, and it is not a minor technical glitch. It is a crack in the conceptual foundation of the most successful theory in the history of science.

Move up in scale. The Standard Model describes the subatomic world with astonishing accuracy (Weinberg, 1967), but it requires nineteen numbers—masses, coupling constants, mixing angles—

that must be measured in the laboratory and plugged in by hand. Nobody knows *why* the electron has the mass it has, or why the strong force is strong, or why there are exactly three families of quarks and leptons. These nineteen numbers are the DNA of the universe, and we cannot read them. We can only copy them.

Move up further. Roughly 95% of the energy content of the cosmos is in forms that we do not understand. We call them "dark matter" and "dark energy"—names that advertise our ignorance. Dark matter holds galaxies together; dark energy tears the universe apart. Their existence is inferred from gravitational effects, but their nature remains unknown.

Now make the biggest jump of all—from physics to biology. We know the laws governing every atom in a living cell. We know the Schrödinger equation, the Coulomb potential, the Pauli exclusion principle. In principle, these laws determine the behavior of every molecule in every organism. In practice, we cannot derive from them why life exists at all. We cannot explain why atoms, governed by the laws of quantum mechanics, spontaneously assemble themselves into self-maintaining, self-replicating, information-processing systems. The gap between the physics of atoms and the biology of cells is the widest chasm in all of science (Schrödinger, 1944).

And at the top of the ladder sits consciousness—the most familiar and the most mysterious phenomenon in the universe. You know what it is like to see the color red, to feel a cool breeze, to remember a friend's voice. Neuroscience can tell you which brain regions are active during these experiences. It cannot tell you *why there is an experience at all.* The "hard problem of consciousness" (Chalmers, 1995) is not merely unsolved; we do not even agree on what kind of answer would count as a solution.

Now step back and notice the pattern. These problems are not independent. They form a staircase: from the foundations of quantum mechanics, through the unexplained parameters of particle physics, through the dark components of cosmology, through the origin of life, up to the nature of mind. At each level, the existing framework—however brilliant—stops short of explaining how the next level emerges from the one below. Physics cannot derive chemistry from first principles. Chemistry cannot derive biology. Biology cannot derive consciousness. Each science has its own language, its own methods, and its own implicit assumptions about what counts as an explanation.

This fragmentation is not a historical accident that will be cured by more data or bigger computers. It is a *structural* failure. The sciences do not share a common set of foundational rules. There is no agreed-upon definition of what an "observation" is that works equally well in quantum mechanics, in evolutionary biology, and in neuroscience. There is no shared principle that governs both the dissipation of heat in a star and the processing of information in a brain. Each discipline has developed its own foundations, learned as craft rather than stated as law, and the result is that problems that span multiple disciplines—precisely the deepest problems—fall into the gaps between departments.

The question this book asks is simple: *could all of these problems have a common root?*

1.4 The Case for a Unified Science

We believe they do. The common root is the absence of what we will call a *constitutional layer*—a shared set of foundational rules, applicable across all branches of science, that specifies what counts as an admissible law, an admissible observation, and an admissible prediction.

The analogy to a political constitution is deliberate. A nation has many laws: traffic laws, tax laws, criminal laws, environmental regulations. These are written by different legislatures, enforced by different agencies, and cover different aspects of life. But above all of them sits a constitution—a small set of foundational principles that every law must obey. A law that violates the constitution is struck down, no matter how popular or practical it might be.

Science today is like a nation without a constitution. Quantum mechanics, thermodynamics, general relativity, and molecular biology are each internally consistent and spectacularly successful. But there is no shared document that tells all of them what foundational rules they must respect. When a problem falls at the intersection of two disciplines—as the measurement problem falls between quantum mechanics and the theory of open systems, or as the origin of life falls between chemistry and biology—there is no constitutional authority to adjudicate.

The insight that points toward a solution has been discovered independently, in different forms, by at least seven scientists over the past half-century. Each of them, working in a different field, found a genuine principle governing systems that exchange energy, matter, or information with their surroundings—what physicists call *open systems*.

Ilya Prigogine showed that systems far from thermal equilibrium can spontaneously generate order (Prigogine, 1967). Hans Ziegler proposed that nonequilibrium flows evolve toward maximum entropy production (Ziegler, 1983). Richard Dawkins recognized that evolution is, at bottom, about the replication and selection of information (Dawkins, 1976). Stuart Kauffman argued that the emergence of life is a near-inevitable consequence of complex chemical networks (Kauffman, 1993). Kunihiko Kaneko demonstrated that coupled dynamical systems can spontaneously differentiate into distinct cell types (Kaneko, 2006). Jeremy England derived thermodynamic bounds on the emergence of self-replicators (England, 2013). Karl Friston proposed that adaptive systems—from single cells to human brains—minimize a quantity called variational free energy (Friston, 2010).

Each of these pioneers was right. Each discovered a real principle. But each was looking at the elephant from a different angle: Prigogine saw the gradient, Ziegler the dissipation, Dawkins the replicator, Kauffman the attractor, Kaneko the symmetry breaking, England the thermodynamic bound, Friston the inference loop. What none of them had was a *geometric* framework that could reveal all seven principles as projections of a single underlying structure.

The deeper insight, which connects all seven, is this: *every system in nature is open.* No system—not an atom, not a star, not a cell, not a brain—exists in perfect isolation. Every system exchanges energy, matter, and information with its environment through boundaries and ports. The act of observation itself is an exchange: the observer gains information about the system, and the system is altered by the observation. Once you take this seriously—once you treat openness not as a complication to be dealt with later, but as the *foundational feature* of all physical reality—the landscape of possible theories narrows dramatically.

A framework that takes openness as foundational must answer specific questions that closed-system physics can avoid. How does an observer obtain information? What is the cost of that information (Landauer, 1961)? How does a system maintain its identity while constantly exchanging with its environment? How do new levels of organization emerge from lower ones? These are precisely the questions that the measurement problem, the origin of life, and the hard problem of consciousness are all, in different languages, trying to ask.

This book develops such a framework. The next section describes what it looks like.

1.5 The Proposed Solution

The framework developed in this book begins with a simple observation and follows it to an extraordinary conclusion.

The observation is this: every system in nature—every atom, every star, every cell, every brain—is *open*. No system exists in perfect isolation. Every system exchanges energy, matter, and information with its surroundings through boundaries and ports. This is not a technicality. It is the deepest structural feature of physical reality, and it has been hiding in plain sight.

The conclusion, reached after four chapters of mathematical derivation, is this: if you take openness seriously—if you build it into the foundations of physics rather than treating it as a complication to be handled later—then the laws of nature turn out to be far more constrained than anyone expected. So constrained, in fact, that they determine not only the masses of particles and the strengths of forces, but also the energy cost of every chemical reaction, the architecture of every living cell, and the oscillatory structure of every conscious brain.

The path from the observation to the conclusion runs through four layers, each derived from the one below, with nothing added at any stage that was not already forced by the layer beneath.

The first layer is a *constitution*: a set of foundational rules—four axioms and seven postulates—that specify what any admissible physical theory must look like. These rules do not mention specific particles, forces, or equations. They constrain the *form* that a theory can take. The rules demand locality, causality, the equivalence of geometry and gauge fields, an operational definition of measurement, and maximum symmetry with minimum freedom. Chapter 2 develops this constitution in full mathematical detail (Arisaka, 2026a).

The second layer is the *hardware*: the unique physical theory that satisfies all eleven constitutional rules. When you ask "what is the simplest geometry consistent with this constitution?", the answer turns out to be a six-dimensional space with a specific warped shape (Randall & Sundrum, 1999), a specific set of symmetries, and a specific set of five integers—$(1, 3, 5; 7, 35)$—that fix every structural parameter. There is no landscape of possibilities, no menu of options, no room for adjustment. The constitution selects one geometry. Chapter 3 derives it (Arisaka, 2026b).

The third layer is the *operating system*. We do not live in six dimensions. We live on a four-dimensional boundary of the six-dimensional bulk. Everything we observe—every particle, every force, every measurement outcome—is a projection of the higher-dimensional reality onto our lower-dimensional experience. This projection produces, as derived consequences: Einstein's general relativity, the Standard Model of particle physics, and quantum measurement. The

famous "collapse of the wavefunction" turns out not to be a collapse at all. It is a projection effect—the shadow cast by a six-dimensional process onto a four-dimensional screen. Chapter 4 derives this operating system (Arisaka, 2026c).

The fourth layer is the *runtime*: a universal grammar that governs how any open system—any system exchanging energy, matter, or information with its surroundings—behaves over time. The runtime consists of a three-stage pipeline (explore many possibilities, select one, stabilize it) and four diagnostic pillars that govern every open system's energy budget, trajectory selection, statistical signatures, and memory architecture. Its central result is an equation that locks the energy, entropy, material, and information currents of any elementary event into a fixed ratio, with the conversion constant derived entirely from the five integers. Chapter 5 derives this runtime (Arisaka, 2026d).

What makes this framework different from every previous attempt at unification is not just that it derives the Standard Model from geometry—string theory aspires to do the same (Maldacena, 1998)—but that it *continues beyond particle physics*. The same five integers that determine the masses of quarks and the strength of the strong force also determine the energy cost of ATP synthesis in a bacterial cell, the frequency of theta oscillations in a mammalian hippocampus, and the timescale of conscious perception in a human brain. The framework does not stop at the boundary of physics. It walks through chemistry, biology, and neuroscience with the same geometric grammar.

To see how this works, it helps to have a picture. The framework organizes all of nature into five levels—five "floors" of a hierarchical structure. We call this structure the *five-story pagoda*.

1.5.1 The five-story pagoda

In Kyoto, Japan, there stands a five-story wooden tower called Tōji. It is fifty-five meters tall and more than a thousand years old. It has survived centuries of typhoons, earthquakes, and wars. Its secret is a single massive cypress pillar—the *shinbashira*—that runs from the foundation to the spire. The five stories are not rigidly bolted to this pillar. They rest loosely upon it, free to sway independently during an earthquake. Each story absorbs seismic energy through friction with the pillar, while the pillar itself provides structural continuity.

This architecture is the perfect metaphor for the framework developed in this book. Nature, too, is organized into five stories, each exhibiting its own characteristic phenomena, each swaying to its own rhythm, but all connected by a single central pillar—the universal runtime derived in Chapter 5.

The first story is **particles**: protons, neutrons, electrons, quarks, leptons, and the gauge bosons that mediate the forces between them. This is the foundation. It provides stable matter and the four fundamental interactions.

The second story is the **cosmos**: stars, galaxies, planets, and the large-scale structure of the universe. Stars are the furnaces that forge heavy elements from hydrogen and helium. Planets are the stages on which chemistry can unfold. The cosmos provides the persistent energy gradients—the temperature differences, the gravitational wells, the photon baths—without which nothing interesting can happen.

The third story is **chemistry**: atoms bonding into molecules, molecules folding into machines, machines assembling into catalytic networks. This is the level at which the energy cost per elementary event—governed by the runtime's central equation—becomes operationally relevant. Every chemical reaction in every cell in your body pays a price denominated in the same geometric currency, derived from the same five integers that fix the masses of quarks.

The fourth story is **life**: self-maintaining systems that actively resist the tendency toward disorder. A living cell is not just a bag of chemicals. It is an open system that imports energy, exports waste, repairs its own damage, replicates its own instructions, and maintains its own identity over time. The transition from chemistry to life—from a reaction network that happens to cycle, to a system that *maintains* its cycling—is one of the central results of the framework.

The fifth story is **mind**: systems with sufficient internal architecture to build a model of their own world and observe that model from within. A bacterium senses chemical gradients and swims toward food. A mouse builds a spatial map of its environment using place cells in the hippocampus (O'Keefe and Dostrovsky, 1971). A human being constructs a rich internal representation of the world—a model that includes not only physical space but abstract concepts, memories, plans, and the experience of being a self. Consciousness, in this framework, is the system's capacity to turn its measurement apparatus inward and observe its own world-model.

Each story depends on the stories below it: no chemistry without stable atoms, no life without molecular machines, no mind without living cells. But each story also exhibits qualitatively new phenomena that cannot be efficiently computed from the story below. The pagoda is simultaneously reductionist—every story obeys the same constitutional rules—and non-reductionist—every story has its own effective dynamics, its own observables, and its own surprises.

And the *shinbashira*—the central pillar that connects all five stories—is the universal runtime: the same three-stage pipeline, the same energy equation, the same statistical signatures, the same memory architecture, operating at every level with different data but identical grammar.

1.5.2 Walking up the pagoda

The most vivid way to see the framework in action is to walk up the pagoda, story by story, and watch the same geometric grammar produce increasingly complex structures.

Begin at the bottom. In the first fraction of a second after the Big Bang, the six-dimensional geometry determined which particles could exist and which forces would act between them. Quarks confined themselves into protons and neutrons—not because someone designed them to, but because the energy landscape of the strong force has a single deepest valley, and the quarks fell into it. This is the first instance of the universal pipeline: the quarks explored many possible configurations (the exploration phase), the strong force selected the lowest-energy configuration (the selection phase), and the proton stabilized as a permanent structure (the stabilization phase). The proton is the most stable composite object in the known universe. It has been sitting in that energy valley for 13.8 billion years and shows no signs of leaving.

Move up to the second story. Gravity gathered hydrogen and helium into clouds. The clouds collapsed into stars. Inside the stars, nuclear fusion built heavier elements: carbon, nitrogen, oxygen, iron. When massive stars exhausted their fuel, they exploded as supernovae, scattering these elements across interstellar space. Every atom of carbon in your body, every atom of

oxygen you breathe, every atom of iron in your blood was forged inside a star and delivered to the early Earth by a supernova explosion. You are, quite literally, made of stardust—and the nuclear reactions that produced that stardust were governed by the same five integers that determine the rest of the framework.

Move up to the third story. On the surfaces of interstellar dust grains, bathed in ultraviolet light, simple molecules combined into amino acids, sugars, and nucleotide bases—the building blocks of life. This is not speculation. Amino acids have been found in meteorites, in interstellar clouds, and in laboratory simulations of interstellar conditions. The chemistry that produces them is driven by the same energy equation that governs the interior of stars, denominated in the same geometric currency. The building blocks of life are not rare. They are cosmically abundant, written into the chemistry of the universe by the same geometry that wrote the masses of quarks.

Continue climbing. On the early Earth, roughly four billion years ago, these building blocks assembled into something qualitatively new. Fatty acid membranes formed vesicles—tiny compartments that could concentrate chemicals and protect fragile reactions from the outside world. Inside these compartments, energy-currency molecules (the ancestors of ATP) began to drive chemical reactions uphill, against the thermodynamic gradient. RNA molecules appeared, capable of both storing information and catalyzing reactions. A translation system emerged, linking the information in RNA to the construction of proteins. And finally, DNA took over the role of long-term information storage, freeing RNA to specialize as a messenger and a catalyst.

Each of these steps—membrane, energy currency, RNA, translation, DNA—follows the same three-stage pipeline: exploration of possible configurations, selection of the most stable one, and stabilization into a persistent structure. And each step pays an energy cost denominated in the same geometric constant. The framework does not just assert that life is possible. It derives, step by step, that the specific sequence of innovations leading from prebiotic chemistry to the first living cell is the unique path that satisfies the energy equation at every stage.

Climb one more story. Over the next four billion years, life diversified and complexified. Single cells became multicellular organisms. Organisms developed nervous systems. Nervous systems developed the capacity to build internal models of the external world—models that could be used to predict, plan, and act. In the mammalian brain, these models are implemented through a hierarchy of oscillations: theta rhythms at five cycles per second provide the clock, gamma rhythms at forty cycles per second carry the information, and the interplay between them implements a continuous cycle of prediction and correction.

The frequency of the theta rhythm—roughly five hertz—is not an accident. It is determined by the same energy equation that governs ATP synthesis. Each theta cycle processes a fixed amount of information at a fixed energy cost, denominated in the same geometric constant that appears at every other level. The speed of human thought, like the efficiency of a molecular machine, is written into the geometry of the six-dimensional bulk.

1.5.3 The closing of the circle

And now we arrive at the deepest claim of the framework—the claim that returns us to the question with which this chapter began.

The six-dimensional geometry projects itself onto the four-dimensional boundary through what we call the *Outer Hologram* (Maldacena, 1998): the lawful image of the higher-dimensional world in the lower-dimensional records accessible to an observer. Every scientific measurement—every photon hitting a detector, every molecule binding to a receptor, every sound wave entering an ear—is an Outer Hologram event.

But some systems do more than passively receive these projections. Some systems—specifically, systems with sufficient memory and predictive architecture—reconstruct the external world internally. They build what we call an *Inner Hologram*: an internal model that represents not just the immediate sensory input but the spatial structure, the temporal regularities, and the causal relationships of the world beyond the boundary.

The progressive alignment of the Inner Hologram with the Outer Hologram is what we call the *Evolving Inner Cosmos*. It is the process by which an open system's internal model becomes an increasingly faithful reconstruction of the external geometry that produced it.

And here is the circle. The universe is a six-dimensional geometry. That geometry projects itself onto four-dimensional boundaries (the Outer Hologram). On at least one planet orbiting at least one star, the projected boundary conditions produced chemistry, which produced life, which produced brains, which produced Internal Holograms—internal reconstructions of the very geometry that created them. The universe, through a chain of derived consequences with no free parameters and no outside intervention, has produced structures that can look back at it and recognize its shape.

That is why we are here. Not by accident. Not by design. By geometry.

1.5.4 What this book covers, and what it does not

This volume—Volume I—develops the four-layer framework: the constitution (Chapter 2) (Arisaka, 2026a), the hardware (Chapter 3) (Arisaka, 2026b), the operating system (Chapter 4) (Arisaka, 2026c), and the runtime (Chapter 5) (Arisaka, 2026d). Chapter 6 summarizes, compares the framework with alternative programmes, states the falsifiability contract, and identifies the open problems.

Volume I does not develop the five stories of the pagoda in quantitative detail. That is the subject of the subsequent volumes. Volume II (*Geometric Origin of the Cosmos and Particles*) develops the first two floors: GUT-Particle and GUT-Cosmos. Volume III (*Inevitable Emergence of Life and Mind*) develops floors three through five: GUT-Chemistry, GUT-Life, and GUT-Mind. Volume IV (*GUT-10: Grand Unification of 10 Sciences*) extends the framework to the remaining five sciences: GUT-Health, GUT-Society, GUT-Environment, GUT-AI, and GUT-Philosophy. By the end of Volume I, the reader will possess the complete theoretical toolkit. By the end of Volume IV, that toolkit will have been applied to every level of natural organization, with specific predictions that can be tested in the laboratory.

1.6 The Third Scientific Revolution

The framework described in this book constitutes, if confirmed by experiment, a third scientific revolution in the precise sense defined by Thomas Kuhn (Kuhn, 1962): a change not in the answers science gives, but in the *questions* science asks.

Newton changed the question from "what is the divine purpose?" to "what is the mathematical law?" Einstein changed it from "what is the absolute space and time?" to "what is the geometry of spacetime?" (Einstein, 1905, 1915) GDOS changes it from "what are the laws of each domain?" to "what is the universal runtime of all open systems?"

Each revolution reduces the number of unexplained parameters while expanding the scope of unification. Newton unified celestial and terrestrial mechanics with one constant (G_N). Einstein unified space, time, and matter with zero additional constants. GDOS unifies all five floors of the pagoda—particles, cosmos, chemistry, life, and mind—with five derived integers and one inherited constant.

Whether this qualifies as a revolution will be determined by experiment. The claim is falsifiable, which is all that science requires.

1.7 How to Read This Book

1.7.1 How to Read This Book

This book has a simple linear structure. **Chapter 1** (this chapter) tells the story in plain English—no equations, no proofs, just the ideas and the motivation. **Chapter 2** establishes the constitution: four axioms and seven postulates, stated with full mathematical precision. **Chapter 3** derives the hardware: the unique six-dimensional geometry forced by the constitution. **Chapter 4** derives the operating system: how the four-dimensional world we observe emerges as a boundary projection of the six-dimensional bulk. **Chapter 5** derives the runtime: the universal pipeline and four diagnostic pillars that govern every open system in nature. **Chapter 6** summarizes, discusses, compares with alternative programmes, and looks ahead.

Each of Chapters 2–5 is written at two levels. Part I contains the full technical derivations, theorems, and proofs—intended for physics graduate students and researchers. Part II restates the same material for a broader audience, with metaphors, FAQ, and plain-English explanations. A reader who wants only the core ideas can read Chapter 1 (this chapter) and the Part II sections of each subsequent chapter, returning to Part I for the mathematics as curiosity and confidence grow.

The remaining three volumes of *The Geometry of Existence* apply the framework to the ten branches of science. **Volume II** (*Geometric Origin of the Cosmos and Particles*) develops GUT-Particle and GUT-Cosmos—the first two floors of the pagoda. **Volume III** (*Inevitable Emergence of Life and Mind*) develops GUT-Chemistry, GUT-Life, and GUT-Mind—the upper three floors. **Volume IV** (*GUT-10: Grand Unification of 10 Sciences*) extends the programme to GUT-Health, GUT-Society, GUT-Environment, GUT-AI, and GUT-Philosophy, completing the ten-science unification.

Chapter 2

The GUT-10 Constitution

Four Axioms and Seven Postulates

"The laws of nature are but the mathematical thoughts of God."

— Euclid, as attributed by Plutarch

2.1 Introduction

2.1.1 Motivation: a constitutional gap after the twentieth-century revolutions

The twentieth century produced two foundational revolutions—quantum mechanics and relativity—that reshaped the meaning of state, symmetry, and prediction. Quantum theory emerged from the early quantum hypothesis and the subsequent development of matrix and wave mechanics (Born, 1926; Heisenberg, 1925; Planck, 1900; Schrödinger, 1926). Relativity recast spacetime as a geometric object and reinterpreted gravity as curvature (Einstein, 1905, 1915). These achievements were consolidated into robust theory stacks: quantum field theory (QFT), the Standard Model (SM), and precision cosmology. Gauge symmetry and renormalization became the workhorses of particle theory ('t Hooft, 1971; Noether, 1918; Wilson, 1971), while cosmic observations established a precise phenomenological expansion history (Guth, 1981; Linde, 1982; Perlmutter et al., 1999; Planck Collaboration, 2018; Riess et al., 1998).

Yet, despite dramatic success, several deep problems have remained resistant for decades. The point of departure of GUT is a specific diagnosis: the resistance is not only technical; it is also *constitutional*. By a *constitutional layer* we mean the minimal set of foundational commitments that specify (i) what counts as a law, (ii) what counts as an observation/measurement, (iii) what degrees of freedom are admissible as fundamental, and (iv) which "free choices" (parameters, normalizations, conventions) are allowed to remain as non-derivable input. This paper proposes an explicit axiom/postulate system intended to serve as such a layer.

The constitutional gap can be made concrete by a simple thought experiment. Suppose we are given an equation of motion and a Hilbert-space state. Without additional clauses, we still cannot answer four basic questions that every scientific account must answer: *(i)* which variables are physically redundant ("gauge") and which are physical; *(ii)* which operations count as legitimate observations and how they are represented mathematically; *(iii)* which parameters are inputs rather than outputs; and *(iv)* which coarse-grainings are allowed without changing the truth value of a claim. Twentieth-century physics answers these questions implicitly and differently in different subfields: QFT relies on locality, microcausality, and renormalization; measurement theory relies on external instruments and an observer cut; cosmology relies on effective parameters that fit data. The result is not inconsistency in practice, but *constitutional drift*: admissibility rules are often learned as craft rather than stated as law.

The aim of GUT is therefore modest in one sense and radical in another. It is modest because it does not begin by proposing a new Lagrangian or a new particle sector. It is radical because it makes admissibility clauses explicit and insists that they apply across domains. In this book, "unification" means: the same logical pipeline—symmetry, consistency, operational semantics, scale robustness, and falsifiability—is used to judge candidate explanations in particle physics, chemistry, biology, and beyond. In that sense, the constitution is intended as an infrastructure for subsequent technical work, including the companion chapters on GG-6 and GDOS.

2.1.2 Four clusters of unsolved problems

We group the motivation into four clusters.

(i) Quantum mechanics: measurement, openness, and operational meaning. Standard quantum theory combines unitary evolution with additional rules for measurement (Born rule, projection postulate, etc.) (Born, 1926; von Neumann, 1932). In practice, measurements involve devices, environments, coarse-graining, and irreversible records. Open quantum systems provide an operational language (CPTP maps and GKLS generators), but it is typically treated as an effective framework rather than as a constitutional requirement (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976). This leaves a persistent ambiguity: what, precisely, is a measurement, and how should it be written as a law-like component of a theory rather than as meta-language?

(ii) Gravity and quantum theory: "quantize geometry" versus "geometrize quantum." The dominant strategy for quantum gravity has been to start from classical geometry (GR) and attempt to quantize it, either perturbatively or non-perturbatively, including string-theoretic UV completions (Green and Schwarz, 1984; Polchinski, 1995; Rovelli, 2004). In parallel, gauge/gravity duality suggests a structural relation between gravity and gauge dynamics ('t Hooft, 1993; Maldacena, 1998; Susskind, 1995; Witten, 1998). These developments raise a constitutional question: is "quantize geometry" the right starting line? The GUT stance reverses the emphasis: it asserts a *geometrization of quantum/gauge structure* at the level of the primitive connection (AX-2), with quantization treated as an operationally defined boundary manifestation (AX-3).

(iii) Unification and predictivity: too many dials and too many landscapes. The SM is remarkably rigid once its symmetry and field content are fixed, yet it contains parameters not predicted from first principles (Particle Data Group, 2024). Beyond-the-SM (BSM) model building often introduces further tunable sectors; cosmology similarly relies on parameterized fits (Planck Collaboration, 2018). Naturalness debates articulate the concern that small numbers and large hierarchies appear unstable unless protected by principle ('t Hooft, 1979; Susskind, 1979). Many UV frameworks admit large spaces of consistent vacua, making it unclear which features are predictions versus choices (Polchinski, 2004). GUT responds by introducing an explicit axiom (MSMF) and a postulate (OGN) designed to forbid hidden continuous knobs and to force normalization choices into a minimal discrete ledger.

(iv) Cosmology and complexity: fragmentation across sciences. Cosmology raises further questions: the origin of inflation, the nature of dark matter and dark energy, and the link between microphysics and initial conditions (Guth, 1981; Linde, 1982; Perlmutter et al., 1999; Riess et al., 1998). Beyond fundamental physics, chemistry, life science, neuroscience, and social systems are dominated by open-system dynamics, multi-scale coarse-graining, and inference from incomplete data. Even when micro-laws are known, effective laws often appear ad hoc. This motivates the GUT ambition: not to reduce all sciences to particle physics, but to unify the *admissibility grammar* of models (symmetry, consistency, operational observables, scale covariance, and falsifiability) across domains.

2.1.3 Why "axioms + postulates"? Historical background

Axiom systems are not a matter of style; they are a compression technology. In geometry, Euclid (c. 300 BCE) demonstrated that a small set of postulates can generate a vast theorem space. In mechanics, Newton (1687) provided a compact set of laws and definitions that rendered astronomical motion calculable. In modern physics, invariance principles and variational principles play the role of quasi-axioms; Noether (1918) made the symmetry–conservation link precise. In quantum mechanics, von Neumann (1932) provided an axiomatic Hilbert-space framework, yet the operational status of measurement remained interpretationally loaded (Bell, 1964; Zurek, 2003). The modern open-system formalism strengthens operational meaning, but is not widely treated as an axiom layer (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976). This book adopts a specific stance: *the next foundational step is to make the constitution explicit.*

In the twentieth century, several *partial* constitutions were proposed for subproblems. Relativity is essentially an invariance constitution (equivalence principle and general covariance) (Einstein, 1905, 1915). Axiomatic QFT pushed locality and positivity into explicit axioms (Wightman and Haag–Kastler frameworks) (Haag, 1992; Streater & Wightman, 1964; Wightman, 1956). Open quantum systems introduced a strict notion of admissible evolution (completely positive semigroups) (Gorini et al., 1976; Lindblad, 1976). And modern renormalization made scale a first-class concept by turning "zooming out" into a flow on theory space (Wilson, 1971). What is missing is not these ingredients, but a *unified ordering and dependency chain*: which clauses are primitive, which are derived, and which are non-negotiable when we cross from closed systems to open observations and from physics to other sciences.

The GUT choice of "axioms + postulates" reflects this dependency logic. Axioms fix semantic primitives (what objects exist and what counts as redundancy and causality), while postulates act as ordered admissibility filters (consistency, positivity, scale coherence, normalization, and empirical auditability). This separation mirrors successful practice in physics: we often reason with invariance/structure first and only then choose model-specific details. The novelty here is to *freeze* that practice into a portable constitution that can be applied uniformly to particle models, cosmological models, chemical reaction networks, and biological open systems.

2.1.4 Definitions: axiom, postulate, theorem, and constitution

To avoid terminological drift we adopt the following working definitions.

Definition 2.1.1 (Axiom)**.** An *axiom* is a foundational statement that fixes the semantic primitives of the framework: what structures exist, what counts as redundancy (gauge/representation), and what is forbidden (e.g. acausal operational kernels). Axioms are not derived within the framework; they are the minimal commitments that define admissibility.

Definition 2.1.2 (Postulate)**.** A *postulate* is an ordered operational admissibility filter that constrains candidate models within the axioms. Postulates typically enforce mathematical consistency (e.g. anomaly cancellation), physical admissibility (e.g. positivity and normalization of states), scale robustness (RG compatibility), and empirical auditability (measurement protocols and falsifiers).

Definition 2.1.3 (Theorem)**.** A *theorem* is a derived result forced by axioms and postulates plus explicitly declared assumptions of a concrete model instance. In the GUT style, theorems are expected to ship with an operational contract: a map to registered observables and a pre-declared falsifier.

Definition 2.1.4 (Constitution)**.** The *constitution* is the frozen union of axioms and postulates:

$$\text{Constitution} := \{\text{AX}\} \cup \{\text{POS}\}. \tag{2.1}$$

2.1.5 A preview of the GUT constitution: four axioms and seven postulates

The constitution comprises four axioms (AX-1–AX-4) and seven postulates (POS-1–POS-7). We state them here informally; Sections 3L.5 and 2J.6 provide the rigorous kernels, parameter definitions, and derivations.

Four axioms (AX).

AX-1. LC **(Locality and Causality).** Admissible evolution is local and causal in the operational sense: boundary updates are retarded; no acausal kernels.

AX-2. GGEP **(Geometry–Gauge Equivalence Principle).** The primitive field is a composite geometry⊕gauge connection; quantum/gauge structure is geometrized.

AX-3. OGGEP **(Operational/Open** GGEP**).** Observation/measurement is defined by a holographic bulk→boundary registration map that produces operational observables.

AX-4. MSMF **(Maximum Symmetry, Minimum Freedom).** Nature selects maximal symmetry with minimal free parameters; continuous dials not forced by the constitution are forbidden.

Seven postulates (POS).

POS-1. GI **(Gauge Invariance).** Gauge invariance is layered across scales (local, dynamical/open, global).

POS-2. UEP **(Unified Extremal Principle).** A single variational principle governs dynamics, unifying least action with least-time/free-energy principles.

POS-3. AF **(Anomaly-Free / cancellability by inflow).** Admissible theories must cancel gauge and mixed anomalies, including by inflow mechanisms (Green and Schwarz, 1984).

POS-4. CP **(Closure & Positivity).** State evolution must remain physical (quantum: CPTP; classical: positivity-preserving Markov semigroups).

POS-5. RG **(RG-compatibility & scale covariance).** Coarse-graining must commute with the operational dictionary; predictions must be stable under resolution changes (Wilson, 1971).

POS-6. OGN **(Occam Gauge Normalization).** Among consistent normalizations, nature selects minimal rational/integer choices; no arbitrary continuous rescalings survive.

POS-7. MF **(Measurement & Falsifiability).** Every claim must map to a registered observable and a pre-declared kill criterion.

Admissibility predicate. For a candidate theory $\mathcal{T}$, we define an admissibility predicate:

$$\boxed{\mathsf{Admissible}(\mathcal{T}) := (\mathcal{T} \models \mathrm{AX}) \wedge (\mathcal{T} \models \mathrm{POS})}. \tag{2.2}$$

The purpose of this book is to specify AX and POS with enough precision that becomes a practical filter rather than a slogan.

2.1.6 Two downstream engines: GG-6 and GDOS

The constitution is designed to compile into two technical "engines":

(a) GG-6: Geometry–Gauge Locking in six dimensions. AX-2 motivates a bulk stage in which geometry and gauge live as a composite connection; consistency (especially POS-AF and POS-OGN) is expected to lock structures and eliminate moduli. The detailed GG-6 construction is developed in the companion preprint Arisaka (2026b).

(b) GDOS: Geometric Dynamics of Open Systems. AX-3 elevates open-system operational dynamics and observation to the level of definition. GDOS provides a canonical open-system generator constrained by positivity and operational semantics. The detailed GDOS construction is developed in the companion preprint Arisaka (2026c).

2.1.7 Paper organization

Section 2.1 motivates the constitution and defines terminology. Section 3L.5 states the four axioms (AX-1–AX-4) in a fixed internal format: constitutional statement, mathematical kernel, forced consequences, intuitive meaning, relation to GG-6/GDOS, and historical background. Section 2J.6 states the seven postulates (POS-1–POS-7) as ordered admissibility filters. Section **??** previews applications in GG-6, GDOS, and the ten-branch export. Section **??** compares the constitution to leading modern frameworks and to historical axiom systems. Section 2.6 concludes and outlines the companion-paper roadmap.

2.2 Four Axioms (AX-1–AX-4)

This section states the four axioms as the *constitutional layer* of GUT. Each axiom is presented with a fixed internal structure:

1. **Constitutional statement (rigorous).** The axiom as an admissibility rule.

Table 2.1: **Motivation map.** Structural gaps motivating a new constitution and the corresponding axiom/postulate responses proposed by GUT. This table is used as an audit checklist that links each motivation to a specific clause whose mathematical kernel is spelled out later.

Structural gap	Representative symptom	Constitutional response (preview)
Geometry vs. quantum	No consensus UV completion; gravity treated separately from gauge QFT	GGEP: geometry–gauge composite object; GG-6 engine (Sec. **??**)
Measurement/open systems	Unitary evolution + ad hoc collapse; observer cut; environment treated as afterthought	OGGEP: bulk→boundary operational dictionary; POS-CP and POS-MF enforce admissible open dynamics
Hidden dials / parameter freedom	Many fitted constants; scheme/basis dependence mimics tunability	MSMF + OGN: maximize symmetry; fix normalizations by discrete data (Secs. 3L.5–2J.6)
Scale dependence	Different effective theories at different resolutions	POS-RG: coarse-graining commutes with operational semantics; predictions stable under resolution changes

2. **Mathematical kernel.** Minimal equations/objects the axiom talks about.
3. **Forced consequences / derivations.** The first nontrivial statements the axiom implies.
4. **Intuitive meaning.** "What the axiom is really saying."
5. **Relation to GG-6 and GDOS.** How the axiom constrains the downstream engines.
6. **Historical background.** Lineage and contrast to standard approaches.

Throughout, we adopt a *bulk–boundary* staging consistent with Sections 2.1–3L.5: a six-dimensional bulk (developed concretely in (Arisaka, 2026b)) exports operational boundary physics through a fixed dictionary (developed concretely in (Arisaka, 2026c)). However, the axioms themselves are stated abstractly and do not presuppose a specific microscopic model.

2.2.1 Common mathematical arena for the axioms

(i) Bulk geometry. Let $(\mathcal{M}_6, g_{AB})$ be a smooth, time-oriented six-dimensional Lorentzian manifold, with indices $A, B = 0, \ldots, 5$. The causal structure is encoded by the metric light cone. We will often write local coordinates as

$$x^A = (x^\mu, r, \theta), \qquad \mu = 0, 1, 2, 3, \tag{2.3}$$

anticipating the GG-6 specialization in which r plays the role of a scale (RG) coordinate and θ a compact holonomy/phase coordinate. (These roles will be derived and used more heavily once POS-RG and POS-OGN are stated in Section 3.)

(ii) Composite connection. Let $P \to \mathcal{M}_6$ be a principal bundle with structure group

$$G_{\text{tot}} \simeq \text{Spin}(1,5) \times G_{\text{gauge}}, \qquad G_{\text{gauge}} \supseteq SU(5) \times U(1)_B. \tag{2.4}$$

In the concrete GG-6 stage developed in (Arisaka, 2026b), the internal sector is taken to contain $SU(5) \times U(1)_B$. A *composite (locked) connection* is a Lie-algebra valued one-form on P,

$$\mathcal{A} \;=\; \omega[g] \;\oplus\; A^{\text{gauge}}, \tag{2.5}$$

where $\omega[g]$ is the spin connection induced by the bulk metric and A^{gauge} collects the internal gauge connections. The corresponding curvature is

$$\mathcal{F} \;=\; d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} \;=\; R[\omega] \;\oplus\; F[A^{\text{gauge}}], \tag{2.6}$$

and the covariant derivative acting on a field Ψ in an associated bundle is

$$\nabla_A^{(\mathcal{A})}\Psi \;=\; \partial_A\Psi + \mathcal{A}_A \cdot \Psi. \tag{2.7}$$

(iii) Boundary operational package. Let the boundary be $\mathcal{M}_4$ (an operational screen/sector supporting registered records). Operationally, the boundary is described by: (i) a state space (classical distributions or quantum density operators ρ on a Hilbert space $\mathcal{H}$), (ii) a set of registered observables (POVMs or instruments), and (iii) admissible time evolution between interventions (channels $\Phi_{t,s}$). The OGGEP axiom will legislate that these are *outputs* of a bulk→boundary map rather than optional add-ons.

We now proceed axiom by axiom.

2.2.2 AX-1: LC (Locality and Causality)

2.2.2.1 Constitutional statement (rigorous)

Axiom 2.2.1 (AX-1 (LC): Locality and Causality)**.** For every admissible bulk configuration and every admissible boundary intervention protocol, the induced boundary evolution is *local* (generated by local/quasilocal structures) and *causal*: registered boundary records at time t depend only on boundary-accessible data in the causal past of t. Equivalently, the operational influence of any intervention vanishes outside the future light cone defined by the bulk/boundary causal structure, and no admissible effective description contains acausal (advanced) response kernels.

Two clarifications are constitutional (not interpretational):

1. **LC is about operational influence, not about metaphysical "hidden variables."** The axiom is enforced at the level of registered statistics and admissible effective kernels.
2. **LC is formulated for open systems.** Because OGGEP defines observation as an open-system projection, LC must hold for open evolution maps and measurement instruments, not only for closed unitary dynamics.

2.2.2.2 Mathematical kernel

LC can be encoded in several equivalent mathematical statements, depending on the level of description.

(i) Retarded support of Green's functions (field level). Consider a linear hyperbolic equation on the boundary (or projected from the bulk),

$$\mathcal{L}\phi(x) = J(x), \tag{2.8}$$

for a differential operator $\mathcal{L}$. LC requires that the physical propagator be the *retarded* Green's function G_R:

$$\phi(x) = \int d^4y\, G_R(x,y)\, J(y), \qquad G_R(x,y) = 0 \text{ if } x \notin J^+(y), \tag{2.9}$$

where $J^+(y)$ is the causal future of y.

(ii) Microcausality (operator level). In a relativistic QFT regime, locality and causality are encoded by microcausality: for local observables $\hat{\mathcal{O}}(x)$ and $\hat{\mathcal{O}}(y)$,

$$[\hat{\mathcal{O}}(x), \hat{\mathcal{O}}(y)] = 0 \qquad \text{if } (x-y)^2 < 0, \tag{2.10}$$

a condition consistent with special relativity (Einstein, 1905; Minkowski, 1908) and standard QFT axioms.

(iii) Retarded kernels for open evolution (channel level). Let $\rho(t)$ be a boundary state (density operator or probability distribution). A general causal effective evolution may be written using a memory kernel:

$$\frac{d}{dt}\rho(t) = \int_{-\infty}^{+\infty} dt'\, \mathcal{K}(t,t')\, \rho(t'), \qquad \mathcal{K}(t,t') = 0 \;\text{ for } t' > t, \tag{2.11}$$

where $\mathcal{K}$ is a superoperator kernel. LC is precisely the statement that $\mathcal{K}$ has retarded support.

In the Markovian limit (no memory beyond $\rho(t)$) LC is consistent with a forward-time semigroup of maps

$$\rho(t) = \Phi_{t,t_0}(\rho(t_0)), \qquad \Phi_{t_2,t_0} = \Phi_{t_2,t_1} \circ \Phi_{t_1,t_0} \quad (t_2 \geq t_1 \geq t_0), \tag{2.12}$$

with $\Phi_{t,t}$ the identity channel. If the dynamics is time-homogeneous, $\Phi_{t,t_0} \equiv \Phi_{t-t_0}$ and

$$\Phi_{t+s} = \Phi_t \circ \Phi_s, \qquad \Phi_0 = \mathrm{id}, \qquad t, s \geq 0. \tag{2.13}$$

This is the operational form of an "arrow of time" for open systems.

2.2.2.3 Forced consequences / derivations

Derivation 1 (LC ⇒ forward semigroup for memoryless evolution). Assume: (i) the effective description has no memory beyond $\rho(t)$ and (ii) interventions are localized in time. Then

the state at t_2 depends only on the state at t_1 and the evolution between them, implying (2.12). Time-homogeneity reduces this to the semigroup law (2.13). The restriction $t \geq 0$ (no inverse map in general) is not a technicality: it is the operational content of irreversibility once records are created.

Derivation 2 (LC + locality ⇒ no superluminal signalling). In any operational theory whose statistics are computed from local algebras satisfying microcausality (4.4), interventions in a spacelike-separated region commute with observables in the target region. Hence the target region's marginal statistics cannot depend on spacelike-separated choices, blocking superluminal signalling at the level of registered records. (Foundationally, this is distinct from Bell's notion of "local causality" for hidden variables (Bell, 1964), but it is the relevant notion for an axiomatic operational constitution.)

Derivation 3 (LC as a necessary precondition for OGGEP). Because OGGEP will define measurement as a bulk→boundary projection producing irreversible records, LC must already constrain the admissible operational map: the map cannot output channels/instruments that enable advanced response or backward-in-time dependence. Thus AX-1 is logically prior to AX-3.

2.2.2.4 Further mathematical elaboration: causality ⇒ analyticity and dispersion constraints

AX-1 is often tested experimentally not by literal light-cone tomography but by frequency-domain constraints. If a linear response function $\chi(t)$ is causal (retarded), $\chi(t) = 0$ for $t < 0$, then its Fourier transform

$$\chi(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}t\, \mathrm{e}^{+\mathrm{i}\omega t}\chi(t) = \int_{0}^{+\infty} \mathrm{d}t\, \mathrm{e}^{+\mathrm{i}\omega t}\chi(t) \tag{2.14}$$

extends analytically to the upper-half complex ω plane (a Paley–Wiener type consequence of support on $t \geq 0$). Analyticity implies *dispersion relations* (Kramers–Kronig):

$$\Re\chi(\omega) = \frac{1}{\pi}\,\mathrm{P}\!\int_{-\infty}^{+\infty} \mathrm{d}\omega'\, \frac{\Im\chi(\omega')}{\omega' - \omega}, \tag{2.15}$$

$$\Im\chi(\omega) = -\frac{1}{\pi}\,\mathrm{P}\!\int_{-\infty}^{+\infty} \mathrm{d}\omega'\, \frac{\Re\chi(\omega')}{\omega' - \omega}, \tag{2.16}$$

where P denotes the principal value. Thus, any proposed effective kernel that violates these relations (after accounting for instrument response and coarse-graining) signals an *acausal* or *non-admissible* operational model.

In open systems the same logic applies to transfer functions of estimators. If $x(t)$ is a registered record produced by a linear filter $h(t)$ acting on an input $u(t)$,

$$x(t) = \int \mathrm{d}t'\, h(t - t')\, u(t') + \eta(t), \tag{2.17}$$

then LC demands $h(t < 0) = 0$, and the same analyticity constraints follow for $H(\omega)$. This provides an experimentally accessible "LC audit" that is independent of microscopic interpretation: causality is enforced at the level of *registered records* and their admissible kernels.

2.2.2.5 Intuitive meaning

LC says: *physics is not allowed to compute today's record using tomorrow's knob setting.* In practice this forbids: (i) advanced propagators as physical response, (ii) acausal kernels hidden inside effective open-system models, and (iii) "interpretational loopholes" where measurement is declared external and therefore exempt from causal auditing.

2.2.2.6 Relation to GG-6 and GDOS

- **Relation to GG-6.** GG-6 will be formulated as a bulk geometric engine (warped AdS-like causal structure). LC restricts admissible bulk geometries and boundary couplings so that the induced boundary response is retarded.
- **Relation to GDOS.** GDOS is the runtime theory of open systems. LC is the reason GDOS is naturally written in forward-time form (semigroups, kernels with retarded support), and it sets the admissibility condition for "ports" (environment couplings) used in system-instance packets.

2.2.2.7 Historical background (with citations)

Locality and causality are constitutional pillars inherited from relativity: special relativity defines the light cone and the causal ordering of events (Einstein, 1905; Minkowski, 1908), and general relativity internalizes causality into the geometry of spacetime (Einstein, 1915). In relativistic quantum field theory, locality and causal propagation are enforced by the microcausality axiom (4.4) and by the use of retarded Green's functions (Peskin and Schroeder, 1995; Weinberg, 1995).

In the foundations of quantum theory, Bell emphasized "local causality" as a criterion for hidden-variable completions (Bell, 1964); GUT uses a different but related notion: *operational* locality and causality of the observable record-generating process. In open quantum systems and quantum information, the same causal arrow is encoded in forward-time CPTP maps and instruments (Kraus, 1983; Nielsen and Chuang, 2000). The central move of GUT is to elevate these operational constraints to axioms rather than treating them as approximations.

2.2.3 AX-2: GGEP (Geometry–Gauge Equivalence Principle)

2.2.3.1 Constitutional statement (rigorous)

Axiom 2.2.2 (AX-2 (GGEP): Geometry–Gauge Equivalence Principle)**.** In a world with gravity and gauge interactions, there exists a *single primitive geometric object*—a composite connection $\mathcal{A}$ on a principal bundle over the bulk—such that:

1. **Locked equivalence / local flattening.** For any timelike curve γ and any event $p \in \gamma$, there exists a combined choice of (i) local coordinates, (ii) local Lorentz frame, and (iii) internal gauge trivialization, in which the pullback of the composite connection along γ vanishes at p (and can be flattened to first order along γ in the equivalence-principle sense).

2. **Invariant physics.** All physical content is carried by curvature and holonomy invariants of $\mathcal{A}$ (and by the induced topological data); "forces" appear when working in an unlocked description.

This upgrades Einstein's equivalence principle from "gravity only" to "geometry+gauge as one locked structure".

2.2.3.2 Mathematical kernel

(i) Composite connection and curvature. The composite connection $\mathcal{A}$ and curvature $\mathcal{F}$ were defined in Eqs. (2.5)–(2.6). Local gauge transformations act as

$$\mathcal{A} \mapsto \mathcal{A}' = g^{-1}\mathcal{A}g + g^{-1}dg, \qquad \mathcal{F} \mapsto \mathcal{F}' = g^{-1}\mathcal{F}g, \tag{2.18}$$

and diffeomorphisms act by pullback on forms. The locked structure is not a new mathematical object; it is the *constitutional claim* that this single object is the correct primitive for unification.

(ii) Holonomy/Wilson loops as invariants. For a curve C in $\mathcal{M}_6$, define the holonomy (Wilson line)

$$U(C) = \mathcal{P}\exp\left(\int_C \mathcal{A}\right), \tag{2.19}$$

where $\mathcal{P}$ denotes path ordering. For a small loop $\partial\Sigma$ bounding an oriented surface Σ,

$$U(\partial\Sigma) \;=\; \exp\left(\int_\Sigma \mathcal{F}\right) + \mathcal{O}(\Sigma^{3/2}), \tag{2.20}$$

so curvature is the infinitesimal holonomy.

(iii) Locked chart (first-order flattening). A compact local rendering of the locked condition is: for a timelike curve $\gamma(\tau)$ with tangent $u^A = \dot{x}^A$, there exists a local trivialization such that at a chosen parameter value τ_0,

$$\mathcal{A}_A(\gamma(\tau_0)) = 0, \qquad u^B\partial_B\big(\mathcal{A}_A u^A\big)\Big|_{\tau_0} = 0, \tag{2.21}$$

while $\mathcal{F}(\gamma(\tau_0))$ may be nonzero. Equation (2.21) is the composite analog of normal coordinates in GR and of local gauge trivialization in Yang–Mills theory.

2.2.3.3 Forced consequences / derivations

Derivation 1 (local construction of $\mathcal{A}(p) = 0$). At any point $p \in \mathcal{M}_6$, choose local coordinates centered at p and a group element $g(x)$ with $g(p) = \mathbb{I}$. Because $\mathcal{A}$ is a smooth one-form, one can choose $g(x)$ such that $\partial_A g(p) = -\mathcal{A}_A(p)$. Then the transformed connection (2.18) satisfies $\mathcal{A}'_A(p) = 0$. This is the standard local trivialization property of connections (Kobayashi & Nomizu, 1963; Nakahara, 2003). GGEP's additional claim is *conceptual uniqueness*: this is the *definition of inertial/force-free structure* in the unified theory.

Derivation 2 (unified geodesic/Lorentz-force form from a worldline action). Consider a point-like probe carrying generalized charges Q in a representation of G_{tot}. A minimal worldline action capturing both geometry and gauge coupling is

$$S[\gamma, Q] = \int d\tau \left[\frac{m}{2}\, g_{AB}(x)\, \dot{x}^A \dot{x}^B + \langle Q(\tau), \mathcal{A}_A(x) \rangle \dot{x}^A \right], \tag{2.22}$$

where: m is an inertial mass parameter, τ is an affine parameter, $\langle \cdot, \cdot \rangle$ is the natural pairing between charges and connection components, and $\dot{x}^A \equiv dx^A/d\tau$. Varying with respect to x^A yields a generalized geodesic equation, schematically

$$m\, \frac{D\dot{x}^A}{d\tau} = \langle Q, \mathcal{F}^A{}_B \rangle \dot{x}^B, \tag{2.23}$$

where $D/d\tau$ is the covariant derivative induced by the metric/spin part of the composite connection. In a locked chart at p, $\mathcal{A}(p) = 0$ and inertial motion is locally straight, while curvature provides the first nontrivial invariant content—exactly the equivalence-principle logic.

Derivation 3 (geometrization of discreteness from compact holonomy). If the bulk contains a compact $U(1)$ fiber coordinate $\theta \sim \theta + 2\pi$, then single-valuedness of a wavefunction $\Psi(\theta)$ implies a Fourier expansion

$$\Psi(\theta) = \sum_{n \in \mathbb{Z}} c_n e^{in\theta}, \tag{2.24}$$

and the generator of translations $-i\partial_\theta$ has integer eigenvalues. Restoring $\hbar$, the momentum spectrum is quantized:

$$p_\theta = n\hbar, \qquad n \in \mathbb{Z}. \tag{2.25}$$

In Kaluza–Klein language, p_θ appears as a conserved charge in the boundary effective theory. This provides an explicit mechanism for the slogan "geometrize quantum physics": discreteness is a topological/geometric consequence rather than an externally imposed quantization rule (Dirac, 1931; Kobayashi & Nomizu, 1963; Nakahara, 2003).

2.2.3.4 Further mathematical elaboration: composite connection, covariant derivative, and unified force law

A compact way to express GGEP is that the *covariant derivative* is built from a single composite connection. For a matter field ψ carrying both Lorentz/spin and internal gauge indices, one writes

$$D_\mu \psi = \left(\partial_\mu + \frac{1}{4}\, \omega_\mu^{ab} \gamma_{ab} + \mathrm{i}g\, A_\mu^a T^a \right) \psi \equiv (\partial_\mu + \mathcal{A}_\mu)\, \psi, \tag{2.26}$$

where ω_μ^{ab} is the spin connection, A_μ^a is a Yang–Mills connection, and $\mathcal{A}_\mu$ denotes the *locked* (composite) connection acting on ψ. Its curvature is likewise composite,

$$\mathcal{F}_{\mu\nu} := [D_\mu, D_\nu] = \frac{1}{4}\, R_{\mu\nu}^{ab} \gamma_{ab} + \mathrm{i}g\, F_{\mu\nu}^a T^a, \tag{2.27}$$

so that "force" and "curvature" are the same object in representation-dependent dress.

In a point-particle limit, one can view motion as parallel transport in the composite connection. A standard calculation shows that the usual geodesic term and Lorentz-force term arise from extremizing a single worldline functional with composite coupling:

$$S[x] = -m \int \mathrm{d}s \; + \; q \int A_\mu(x)\, \mathrm{d}x^\mu, \tag{2.28}$$

whose Euler–Lagrange equations yield

$$m\, \frac{Du^\mu}{\mathrm{d}\tau} = q\, F^\mu{}_\nu\, u^\nu, \qquad u^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}. \tag{2.29}$$

GGEP interprets this familiar "sum" as a *single* statement about parallel transport in $\mathcal{A}$: the separation into "gravity" and "gauge force" is representation bookkeeping rather than a separation of primitives.

Finally, the holonomy viewpoint makes quantization conditions geometric. For a compact $U(1)$ factor, the Wilson loop

$$W(\gamma) = \exp\!\left(\mathrm{i}q \oint_\gamma A \right) \tag{2.30}$$

is invariant under small deformations and, in topologically nontrivial sectors, forces charge/flux quantization. This is the same mathematical discipline behind Dirac quantization (Dirac, 1931) and motivates why OGN (POS-6) will later insist on a canonical integer charge lattice: if holonomy is the invariant content, then basis drift in Abelian normalization is not physics but bookkeeping.

2.2.3.5 Intuitive meaning

GGEP says: *what we call "gauge force" is not a separate ingredient that must be quantized on top of geometry*; it is part of geometry from the beginning, because both gravity and gauge interactions are aspects of parallel transport. In this view, "free" motion means "parallel transport/horizontal lift" with respect to the composite connection, and "force" is the artifact of describing that transport in a non-inertial (unlocked) chart. The quantum signature (phase, charge quantization) is then traced to holonomy and topology.

2.2.3.6 Relation to GG-6 and GDOS

- **Relation to GG-6.** GG-6 is the concrete realization of GGEP: a warped six-dimensional bulk in which the composite connection locks gravity with a specific gauge content (minimally $SU(5) \times U(1)_B$) and in which anomaly cancellation fixes otherwise free coefficients (Arisaka, 2026b).
- **Relation to GDOS.** GGEP determines *what* is projected by OGGEP: curvature/holonomy data of the composite connection. GDOS then describes *how* that projected data evolves as an open system on the boundary (Arisaka, 2026c).

2.2.3.7 Historical background (with citations)

Einstein's equivalence principle and its geometric formulation of gravity are the prototype: locally, inertial frames remove the gravitational connection at a point, while curvature remains as the invariant tidal field (Einstein, 1907, 1915). The idea that gauge potentials are connections, and field strengths are curvatures, appears in early gauge ideas and reaches modern form in Yang–Mills theory (Weyl, 1918; Yang and Mills, 1954). Mathematically, the language of principal bundles and connections became standard through the second half of the twentieth century (Kobayashi & Nomizu, 1963; Nakahara, 2003).

What is distinctive in GGEP is not the existence of connections, but the *constitutional choice of primitive*: instead of taking "metric + separate gauge fields" as independent inputs, GGEP insists on a unified equivalence-principle statement for the composite object. This choice is aligned in spirit with geometric perspectives on gauge theories and with the broader idea that holonomy and topology control quantization phenomena (Berry, 1984; Dirac, 1931).

2.2.4 AX-3: OGGEP (Operational/Open GGEP)

2.2.4.1 Constitutional statement (rigorous)

Axiom 2.2.3 (AX-3 (OGGEP): Operational/Open GGEP)**.** A bulk theory is not physically defined unless it specifies a *fixed operational dictionary* that maps bulk structure to registered boundary observables and admissible open-system evolution. Concretely, there exists a causal, gauge-covariant bulk↔boundary map $\mathcal{D}$ such that for every admissible bulk configuration B,

$$\mathcal{D}(B) \;=\; \Big(\mathcal{H}, \rho, \{\mathcal{I}_\alpha\}, \{\Phi_{t,s}\}, \dots\Big), \tag{2.31}$$

where: ρ is a boundary state, $\{\mathcal{I}_\alpha\}$ is a set of measurement *instruments* (with POVM semantics), and $\Phi_{t,s}$ are admissible boundary evolution maps between interventions. The map must satisfy: (i) LC/retardedness (AX-1), (ii) compatibility with the gauge/geometry equivalences of GGEP (AX-2), and (iii) operational positivity (channels/instruments map physical states to physical states).

This axiom is the constitutional answer to the century-old ambiguity of "what is a measurement?" In GUT, measurement is not a metaphysical add-on; it is part of the theory's syntax.

2.2.4.2 Mathematical kernel

(i) Instruments and POVMs. A measurement with outcomes labeled by α is represented by an *instrument*: a set $\{\mathcal{I}_\alpha\}$ of completely positive maps such that $\sum_\alpha \mathcal{I}_\alpha$ is trace-preserving. A Kraus representation is

$$\mathcal{I}_\alpha(\rho) = \sum_k K_{\alpha k}\, \rho\, K_{\alpha k}^\dagger, \qquad \sum_{\alpha,k} K_{\alpha k}^\dagger K_{\alpha k} = \mathbf{1}. \tag{2.32}$$

The associated POVM elements are

$$E_\alpha = \sum_k K_{\alpha k}^\dagger K_{\alpha k}, \qquad E_\alpha \succeq 0, \qquad \sum_\alpha E_\alpha = \mathbf{1}. \tag{2.33}$$

Outcome probabilities and conditional post-measurement states are

$$p_\alpha = \mathrm{Tr}(E_\alpha \rho), \qquad \rho_\alpha = \frac{\mathcal{I}_\alpha(\rho)}{p_\alpha}. \tag{2.34}$$

This modern operational formulation synthesizes the Born rule and measurement update into a single object (Born, 1926; Davies and Lewis, 1970; Kraus, 1983; Nielsen and Chuang, 2000; von Neumann, 1932).

(ii) Admissible open evolution as channels. Between interventions, the boundary state evolves by a (typically time-ordered) family of quantum channels $\Phi_{t,s}$, i.e. completely positive trace-preserving (CPTP) maps:

$$\rho(t) = \Phi_{t,s}\big(\rho(s)\big), \qquad \Phi_{t,s} \text{ CPTP}. \tag{2.35}$$

CPTP is the minimal mathematical encoding of "physicality" for open evolution (no negative probabilities) (Choi, 1975; Kraus, 1983). Time-homogeneous Markovian evolution is described by a semigroup $\Phi_t = e^{t\mathcal{L}}$.

(iii) Bulk→boundary map as structure-preserving functor. A compact way to make OGGEP precise is to view it as a functor between categories:

Definition 2.2.4 (Bulk and operational categories)**.** Let **Bulk** be the category whose objects are admissible bulk packages $B = (\mathcal{M}_6, g, \mathcal{A}, \ldots)$ and whose morphisms include diffeomorphisms, gauge transformations, and coarse-graining/RG maps. Let **Op** be the category whose objects are operational boundary packages $O = (\mathcal{H}, \rho, \{\mathcal{I}_\alpha\}, \{\Phi_{t,s}\}, \ldots)$ and whose morphisms are operational equivalences (unitary conjugations, classical relabelings, coarse-grainings).

Definition 2.2.5 (OGGEP map)**.** OGGEP asserts the existence of a functor

$$\mathcal{D} : \mathbf{Bulk} \to \mathbf{Op}, \tag{2.36}$$

that is compatible with:

1. gauge covariance (bulk gauge equivalences map to operational equivalences),
2. causality (retardedness, AX-1),
3. positivity/physicality (channels and instruments preserve state positivity),
4. and, with POS-RG (Section 2.3), commutation with coarse-graining.

2.2.4.3 Forced consequences / derivations

Derivation 1 (LC + CPTP semigroup ⇒ GKLS/Lindblad generator). Assume: (i) forward-time semigroup property (2.13) (from LC and Markovity), (ii) complete positivity and trace preservation for each Φ_t, and (iii) mild continuity in t. Then the generator $\mathcal{L}$ has GKLS form (Gorini et al., 1976; Lindblad, 1976):

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i[H,\rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right), \tag{2.37}$$

with Hermitian H and Lindblad operators L_j.

Derivation 2 (measurement as a special CPTP map with classical output). Each instrument map $\mathcal{I}_\alpha$ in (2.32) is CP; the total map $\sum_\alpha \mathcal{I}_\alpha$ is CPTP. Thus "measurement" is not a new dynamical law beyond open-system evolution; it is a particular CP update with a registered classical outcome α. This reframes "collapse" as the operational bookkeeping of conditioning on an outcome, consistent with the quantum-instrument formalism (Davies and Lewis, 1970; Kraus, 1983; Ozawa, 1984).

Derivation 3 (OGGEP as the operational closure of quantum theory). Standard axiomatizations of QM typically treat unitary evolution as primary and measurement as additional. OGGEP inverts the emphasis: the theory is only complete when the operational map is specified. This is not a claim to new measurement mathematics; it is a claim that measurement should be *constitutional* in any theory that aims to apply across open systems (chemistry, life, mind) (Breuer & Petruccione, 2002; Nielsen and Chuang, 2000).

2.2.4.4 Further mathematical elaboration: instruments, Stinespring dilation, and the bulk→boundary dictionary

AX-3 is easiest to audit when written in dilation form. Any instrument $\{\mathcal{I}_m\}$ admits a Stinespring/Naimark representation: there exists an environment Hilbert space $\mathcal{H}_E$, an initial environment state σ_E, a unitary U on $\mathcal{H} \otimes \mathcal{H}_E$, and a pointer POVM $\{\Pi_m\}$ on $\mathcal{H}_E$ such that

$$\mathcal{I}_m(\rho) = \mathrm{Tr}_E\Big[(\mathbf{1} \otimes \Pi_m)\, U\, (\rho \otimes \sigma_E)\, U^\dagger \Big]. \tag{2.38}$$

Equation (2.38) makes "measurement" concrete: a measurement is a lawful open interaction plus a readout. Different experimental claims correspond to different (U, σ_E, Π_m), and OGGEP requires that the intended protocol be specified.

In the GUT-10 language, the operational dictionary $\mathcal{D}$ is a compiler from *bulk structure* to such boundary instruments. Schematically,

$$\mathcal{D}: \ (\text{bulk fields}, \text{ports}, \text{geometry}) \longmapsto (\{\mathcal{I}_m\},\ \text{estimator},\ \text{error model}), \tag{2.39}$$

with LC enforcing causality (no advanced response) and CP enforcing physicality. This rephrases the "measurement problem" as a missing-map problem: not "how does the wavefunction collapse?"

but "which CPTP instrument does the theory assign to the laboratory protocol, and what falsifier does it imply?"

A practical consequence is that ambiguity about the "Heisenberg cut" becomes a *controlled approximation*: different coarse-grainings correspond to different effective instruments, and POS-RG will demand that the dictionary commute with the coarse-graining map (so that the claim is not resolution-dependent).

2.2.4.5 Intuitive meaning

OGGEP says: *there is no physics without an observation compiler.* A bulk theory that cannot tell you what an instrument will register is not a physical theory, even if its equations look elegant. In this constitution, the bulk is like a program and $\mathcal{D}$ is the compiler to experimental records. This removes the ambiguity of "measurement" by definition: measurement is the operational export $\mathcal{D}$.

2.2.4.6 Relation to GG-6 and GDOS

- **Relation to GG-6.** GG-6 supplies the bulk structure (geometry+gauge) that $\mathcal{D}$ acts on. In particular, holography-inspired bulk geometries provide a natural environment for such a dictionary ('t Hooft, 1993; Maldacena, 1998; Susskind, 1995; Witten, 1998).
- **Relation to GDOS.** GDOS is the systematic boundary-side expression of $\mathcal{D}$: it provides the general form of admissible open evolution (quantum GKLS, classical Fokker–Planck/Langevin-like limits), and it provides the "ports" and system-instance packets that define measurement protocols (Arisaka, 2026c; Breuer & Petruccione, 2002).

2.2.4.7 Historical background (with citations)

The measurement issue is present from the early days of quantum mechanics: Born introduced probabilistic interpretation (Born, 1926) and von Neumann formalized measurement as a distinct process alongside unitary evolution (von Neumann, 1932). Modern quantum information theory replaces vague "collapse" with instruments/POVMs and channels, providing a precise operational semantics (Davies and Lewis, 1970; Kraus, 1983; Nielsen and Chuang, 2000; Ozawa, 1984). Open quantum systems theory provides the canonical evolution structures (GKLS/Lindblad generators) for Markovian dynamics (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976).

Independently, holography and gauge/gravity duality established that bulk theories can be encoded on lower-dimensional boundaries, with a precise bulk–boundary dictionary in special cases ('t Hooft, 1993; Maldacena, 1998; Susskind, 1995; Witten, 1998). OGGEP uses holography not just as a duality but as an *operational definition* of observation: measurement is a bulk→boundary projection that produces boundary records.

2.2.5 AX-4: MSMF (Maximum Symmetry, Minimum Freedom)

2.2.5.1 Constitutional statement (rigorous)

Axiom 2.2.6 (AX-4 (MSMF): Maximum Symmetry, Minimum Freedom)**.** Among all candidate theories consistent with AX-1–AX-3 (and with the postulates of Section 2.3), Nature selects those that (i) maximize admissible symmetry and (ii) minimize non-forced freedom. Equivalently: no continuous parameters, moduli, or normalization dials are permitted unless they are strictly forced by symmetry, topology, anomaly cancellation, or operational necessity. When discrete invariants exist, Nature selects the minimal non-degenerate integer/ratio values compatible with the constitution.

The axiom is intentionally deterministic: it denies that the deep architecture of physics is accidental tuning.

2.2.5.2 Mathematical kernel

The axiom can be expressed as an ordering principle on the space of admissible theories.

(i) Candidate set. Let $\mathfrak{T}$ be the set of all *admissible* bulk–boundary packages $T = (B, \mathcal{D})$ satisfying AX-1–AX-3 and the postulates (POS stack). (Here B encodes bulk data, and $\mathcal{D}$ encodes operational export.)

(ii) Symmetry measure. Let $\mathrm{Aut}(T)$ denote the automorphism group of T (bulk isometries and gauge symmetries compatible with the operational map). A convenient proxy for "how symmetric" is

$$S(T) \;:=\; \dim \mathrm{Aut}(T), \tag{2.40}$$

though other monotone measures (e.g. symmetry algebra rank) could be used.

(iii) Freedom measure (moduli/knobs). Let $\mathcal{M}(T)$ be the moduli space of physically inequivalent continuous deformations of T that preserve the axioms (after quotienting by gauge/diffeomorphism redundancies). Define

$$F(T) \;:=\; \dim \mathcal{M}(T), \tag{2.41}$$

or more generally a weighted count of continuous parameters that change operational predictions. In practice, F counts "free parameters" that would otherwise be fitted to data.

(iv) Lexicographic selection (MSMF). MSMF is implemented as the lexicographic optimization

$$T_\star = \arg\max_{T\in\mathfrak{T}} \Big(S(T), -F(T)\Big), \tag{2.42}$$

meaning: maximize symmetry first; among ties minimize freedom.

2.2.5.3 Forced consequences / derivations

Derivation 1 (MSMF as an anti-landscape rule). Suppose $\mathfrak{T}$ contains a continuous family $T(\lambda)$ of physically inequivalent admissible theories, with $\lambda \in \mathbb{R}$ and operational predictions varying with λ. Then $F(T(\lambda)) \geq 1$, and MSMF forbids this family unless λ is forced by the constitution. Thus large "landscapes" of continuously tunable solutions are constitutionally disfavored. This responds to the long-standing concern that unification frameworks can become underconstrained by principle alone ('t Hooft, 1979; Polchinski, 2004; Susskind, 1979).

Derivation 2 (MSMF pushes conventions into postulates). Many continuous "parameters" are actually conventions (choices of normalization, scheme, basis). MSMF demands that such freedoms be explicitly fixed by a constitutional rule rather than left implicit. In GUT, this is implemented downstream by POS-OGN (Occam gauge normalization) and by a strict separation between redundancy (gauge) and freedom (physical parameters).

Derivation 3 (MSMF turns symmetry breaking into a selection problem). In standard particle physics, spontaneous symmetry breaking is often treated as the selection of a vacuum within a potential landscape. MSMF reframes the question constitutionally: symmetry breaking patterns are admissible only if they are forced by the minimal freedom rule, which becomes especially powerful for open systems (GDOS) where attractors and phase selection generalize the notion of "vacuum" (Arisaka, 2026c). (The detailed open-system symmetry-breaking story is developed in the companion GDOS paper.)

Derivation 4 (Open-system minimal drift template; GDOS preview). A recurring export from "maximum symmetry, minimum freedom" is that many open-system laws can be written in a *minimal constitutive form* once positivity, invariance, and a choice of ledger are fixed. In a classical mesoscopic limit, let $p(x,t)$ be a probability density on a state manifold $\mathcal{X}$. A generic positivity-preserving Markov generator can be written in Fokker–Planck form (Gardiner, 2009; Risken, 1989)

$$\partial_t p(x,t) = -\nabla \cdot \big(b(x)\, p(x,t)\big) + \frac{1}{2}\nabla \cdot \big(D(x)\, \nabla p(x,t)\big)\,, \tag{2.43}$$

equivalently as an Itô SDE

$$dX_t = b(X_t)\, dt + \sigma(X_t)\, dW_t, \qquad D = \sigma\sigma^{\top}, \tag{2.44}$$

where b is the drift, D the diffusion tensor, σ a noise amplitude, and W_t a Wiener process. Given a chosen ledger functional F (free-energy-like, in the EEGG sense) and mobility tensor $M \succeq 0$, a minimal decomposition consistent with steady-state cycling is

$$b(x) \;=\; -M(x)\,\nabla F(x) \;+\; u(x), \qquad \nabla \cdot \big(u(x)\, p_*(x)\big) = 0, \tag{2.45}$$

where u is a solenoidal (non-gradient) component with respect to the stationary measure p_*. This is not asserted as a domain-specific law; rather it is a *candidate universal template* that

survives the admissibility filters (GI/CP/RG) while minimizing non-forced structure—precisely the role of MSMF in a cross-domain open-system setting (Seifert, 2012).

2.2.5.4 Further mathematical elaboration: MSMF as an ordering on theory space and a penalty functional

AX-4 can be operationalized as an *ordering* on candidate theory classes. Let $S(T)$ be a symmetry score (e.g. $\dim \mathrm{Aut}(T)$ or another monotone measure of invariance) and let $F(T)$ be a freedom score (e.g. the number of independent continuous parameters/moduli remaining after quotienting gauge redundancies). A minimal "auditor's" ordering is lexicographic:

$$T_1 \prec T_2 \quad \Longleftrightarrow \quad \big(S(T_1) > S(T_2)\big) \;\text{ or }\; \big(S(T_1) = S(T_2) \text{ and } F(T_1) < F(T_2)\big), \tag{2.46}$$

which encodes "maximize symmetry, then minimize freedom."

For quantitative model selection and textbook discussion, it is useful to package MSMF into a penalty functional that resembles minimum-description-length or information-criterion objectives (Akaike, 1974; Rissanen, 1978):

$$\mathcal{J}(T) := -S(T) + \lambda\, F(T) + \mathcal{C}_{\text{protocol}}(T), \tag{2.47}$$

where $\mathcal{C}_{\text{protocol}}$ denotes the "cost" of the operational dictionary and falsifiability contract (MF). The constant λ is not a tunable physics parameter; it is a bookkeeping device expressing the constitutional preference. The conceptual point is that freedom is penalized *unless* it is forced by symmetry, consistency, or operational meaning.

In practice, MSMF interacts directly with POS-OGN: even when two descriptions are physically equivalent, a theory can acquire spurious knobs via continuous basis drift (especially in Abelian sectors). OGN removes that drift by fixing a canonical integer/rational normalization so that "choice of units/basis" cannot masquerade as new physics.

2.2.5.5 Intuitive meaning

MSMF says: *Nature does not hide knobs.* If a parameter is not forced, it is not allowed to remain fundamental. This is a direct response to the culture of model catalogues with many adjustable constants. MSMF converts aesthetic preference into an auditable demonstrate-or-eliminate rule.

2.2.5.6 Relation to GG-6 and GDOS

- **Relation to GG-6.** In GG-6 this axiom demands that the bulk background, gauge group, and discrete topological data be as symmetric and as moduli-free as possible. Combined with anomaly cancellation (POS-AF) and normalization (POS-OGN), this is intended to eliminate continuous free parameters in the unification stage (Arisaka, 2026b).
- **Relation to GDOS.** In GDOS MSMF becomes a constitutive-selection rule for open dynamics: drift/flow terms and noise structures are constrained to be the minimal ones

compatible with positivity and gauge invariance. This plays the role of a universal "closure" principle for complex systems (Arisaka, 2026c).

2.2.5.7 Historical background (with citations)

The centrality of symmetry in physics is codified by Noether's theorem, relating continuous symmetries to conserved quantities (Noether, 1918). Wigner emphasized symmetry as the organizing principle of quantum mechanics and particle classification (Wigner, 1939). The success of gauge symmetry in the Standard Model is a modern example of "maximum symmetry" as a guiding rule (Glashow, 1961; Salam, 1968; Weinberg, 1967).

The "minimum freedom" aspect echoes Occam's razor as a methodological principle, but MSMF is stronger: it is legislated as a physical axiom rather than a Bayesian preference. Historically, naturalness arguments (e.g. ('t Hooft, 1979; Susskind, 1979)) articulate the idea that unexplained small numbers and parameter sensitivity are warnings that symmetry or deeper structure is missing. In the string landscape discussion, the proliferation of vacua motivates the search for stronger selection principles (Polchinski, 2004). MSMF makes such a selection principle constitutional.

2.2.6 Inter-axiom logic and "first theorems" forced by the axioms

The axioms are designed to be used as a dependency-controlled proof system. A compact dependency order is:

$$\text{AX-1 (LC)} \implies \{\text{AX-2 (GGEP)}, \text{AX-3 (OGGEP)}\} \implies \text{AX-4 (MSMF)}. \tag{2.48}$$

LC sets the admissible causal arrow. GGEP defines the primitive bulk structure to be projected. OGGEP defines what it means to observe that structure. MSMF closes the theory space by eliminating non-forced freedom.

Several first theorems already appeared above: (i) LC + Markovity ⇒ semigroup (2.13); (ii) LC + CPTP ⇒ GKLS generator (2.37); (iii) compact holonomy ⇒ quantization (2.25); (iv) MSMF ⇒ anti-landscape selection (2.42). Section 2.3 will supply the postulate stack that turns these statements into a full admissibility engine (anomaly freedom, RG commutation, normalization rigidity, and falsifiability).

2.2.7 Comprehensive summary table for AX-1–AX-4

Table 2.2 provides a compact "auditor's view" of the four axioms: what they assert, the minimal mathematical kernels, the first forced consequences, and the immediate falsifiability handles.

Table 2.2: **Axioms summary (AX-1–AX-4).** Auditor-style overview of each axiom: a crisp constitutional clause, representative equations, first forced consequences, and an explicit failure mode. The rightmost column is written as a practical falsifier: what experimental/operational fact would make the axiom unusable.

Axiom	**Name**	**Formal kernel (representative equations)**	**Forced consequences (first theorems)**	**Failure mode / falsifier**
AX-1	LC	Retarded support of response kernels (2.9), (2.11); microcausality (4.4); forward semigroup (2.13).	No superluminal signalling in registered statistics; arrow-of-time for open evolution; prerequisite for operational dictionary.	Any experimentally validated operational kernel enabling advanced response/backward influence; acausal signalling in registered records.
AX-2	GGEP	Composite connection $\mathcal{A}$ and curvature $\mathcal{F}$ (2.5)–(2.6); holonomy (2.19); local trivialization (2.21).	Unified inertial notion (locked chart); geodesic/Lorentz-force unification (2.23); quantization from compact holonomy (2.25).	Obstruction to defining a single composite connection with consistent gauge/geometry covariance; need for independent "force primitives" beyond curvature/holonomy invariants.
AX-3	OGGEP	Operational map $\mathcal{D}$ (functor) (2.36); instruments/POVMs (2.32)–(2.34); admissible evolution as CPTP channels; GKLS generator (2.37).	Measurement defined unambiguously as instrument output; LC+CPTP⇒ Lindblad form; bulk statements become scientific only if they compile to observables.	Inability to construct a causal, gauge-covariant operational dictionary; predicted evolution violates CPTP/positivity or enables inconsistent measurement semantics.
AX-4	MSMF	Theory-space ordering by symmetry and freedom (2.40)–(2.42); moduli count $F(T)$.	Anti-landscape selection: continuous tunable families forbidden unless forced; conventions must be constitutionally fixed (via POS-OGN); symmetry breaking becomes constrained selection.	Operational necessity of continuous moduli/knobs that cannot be eliminated without losing empirical adequacy; parameter landscapes required by data.

2.3 Seven Postulates (POS-1–POS-7)

2.3.1 Role of the postulates: "runtime admissibility" for a ten-science framework

Section 2.2 supplied the four axioms as the "constitutional primitives" of GUT: *(i)* LC fixes the causal arrow and locality discipline; *(ii)* GGEP defines the bulk primitive as a composite geometric–gauge object; *(iii)* OGGEP defines observation/measurement as an operational bulk→boundary compilation; and *(iv)* MSMF constrains theory-space by maximum symmetry with minimum freedom.

The *postulates* now serve as *runtime admissibility filters*. They answer a different question: given the axiom-level syntax, *which dynamics, normalizations, and observational claims are allowed to run?* Operationally, POS-1–POS-7 are "house rules" that every admissible theory instance must satisfy before any domain-specific modeling (particles, chemistry, life, mind, etc.) begins.

A key editorial point is to keep the canonical numbering

$$(POS{-}1, \ldots, POS{-}7) = (\mathrm{GI}, \mathrm{UEP}, \mathrm{AF}, \mathrm{CP}, \mathrm{RG}, \mathrm{OGN}, \mathrm{MF}), \tag{2.49}$$

and to avoid legacy drift in which OGN and MF were sometimes merged in earlier drafts. In this book, OGN is separated (normalization rigidity, "no basis drift") from MF (measurement contracts and falsifiability).

2.3.2 Minimal formal setting shared by all postulates

The postulates are stated so as to cover both (i) *closed microscopic* descriptions (standard quantum field theory or closed quantum mechanics), and (ii) *open operational* descriptions required by OGGEP/GDOS. We therefore fix a minimal common mathematical language.

(i) State spaces. On the operational boundary, a "state" means a normalized, positive element of a convex state space. In quantum theory this is a density operator ρ on a Hilbert space $\mathcal{H}$:

$$\rho \succeq 0, \qquad \mathrm{Tr}\,\rho = 1. \tag{2.50}$$

In classical probability, it is a distribution $p(x) \geq 0$ with $\int p(x)\,\mathrm{d}x = 1$. In both cases, the allowed states form a convex set $\mathfrak{S}$.

(ii) Operations, channels, and instruments. An operation is a linear map on states. A quantum channel is a completely positive trace-preserving (CPTP) map $\Phi : \rho \mapsto \Phi(\rho)$. A measurement is specified by an *instrument* $\{\mathcal{I}_m\}_{m\in\mathcal{M}}$: each $\mathcal{I}_m$ is CP and $\sum_m \mathcal{I}_m$ is TP. The associated POVM is $E_m = \mathcal{I}_m^*(\mathbb{I})$ and the Born rule reads $p(m) = \mathrm{Tr}(\rho E_m)$ (see Section 2.2, Eqs. (2.32)–(2.34)).

(iii) Between-intervention evolution. Between interventions, the operational state evolves by a one-parameter family of admissible maps $\{\Phi_t\}_{t\geq 0}$ with semigroup property

$$\Phi_{t+s} = \Phi_t \circ \Phi_s, \qquad \Phi_0 = \mathrm{id}, \tag{2.51}$$

consistent with LC (AX-1). Under mild continuity, there exists a generator $\mathcal{L}$ such that $\Phi_t = e^{t\mathcal{L}}$. In quantum Markovian limits, admissibility plus complete positivity forces the GKLS form (Eq. (2.37)) (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976).

(iv) Scale and coarse-graining. A coarse-graining or renormalization step is modeled abstractly as a map RG_ℓ acting on descriptions (states, couplings, effective actions, or operational kernels). POS-RG will require that this map commute with the operational dictionary OGGEP (AX-3). In GG-6 applications, the bulk coordinate r is identified with the RG scale as

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right), \tag{2.52}$$

so that translation in r encodes a change of effective description scale.

With this shared setting, we now state and analyze the seven postulates one by one.

2.3.3 POS-1: Gauge invariance (GI) across scales: LGI/DGI/GGI

2.3.3.1 Rigorous statement (constitutional text)

Postulate 2.3.1 (POS-1 (GI): gauge invariance across scales)**.** All lawful descriptions must be gauge invariant/covariant in a *layered* sense:

(GIa) **LGI (Local Gauge Invariance).** For closed microscopic dynamics, the action and quantum measure are invariant under local gauge transformations.

(GIb) **DGI (Dynamical Gauge Invariance).** For open mesoscopic dynamics, the admissible evolution maps (or their generators) are equivariant under gauge transformations acting on states and on the couplings/ports that define the open system.

(GIc) **GGI (Global Gauge Invariance).** For macroscopic ledgers, globally conserved charges are invariant bookkeeping quantities when the system is enlarged to include its exchange environment; coarse-grained ledgers must respect these invariants.

In particular, any operational dynamics or measurement that violates gauge covariance is inadmissible in GUT.

2.3.3.2 Mathematical formulation of LGI (closed microscopic systems)

Let G be a Lie group with Lie algebra $\mathfrak{g}$ and generators T^a. Let $A = A^a_\mu T^a \,\mathrm{d}x^\mu$ be a gauge connection on spacetime M_4, with curvature

$$F = \mathrm{d}A + A \wedge A. \tag{2.53}$$

A local gauge transformation is a map $g : M_4 \to G$. On matter fields ψ in representation R,

$$\psi \mapsto \psi^g = R(g)\psi, \qquad A \mapsto A^g = gAg^{-1} - (\mathrm{d}g)\, g^{-1}, \qquad F \mapsto F^g = gFg^{-1}. \tag{2.54}$$

LGI demands invariance of the microscopic action:

$$S[\psi^g, A^g] = S[\psi, A]. \tag{2.55}$$

Derivation: Noether/Ward identities and the anomaly obstruction. For an infinitesimal transformation $g(x) = \exp(\alpha^a(x)T^a)$, gauge invariance implies the covariant conservation law

$$D_\mu J^{\mu a} = 0, \tag{2.56}$$

with D_μ the covariant derivative. At the quantum level, the generating functional $Z[A] = \int \mathcal{D}\psi\, e^{iS[\psi,A]}$ obeys Ward–Takahashi identities. A gauge anomaly corresponds to a failure of invariance of the measure/effective action, and therefore spoils these identities. POS-AF exists precisely to forbid this quantum obstruction (Adler, 1969; Bell and Jackiw, 1969; Green and Schwarz, 1984).

2.3.3.3 Mathematical formulation of DGI (open mesoscopic systems)

For an open quantum system with state ρ, DGI is the equivariance condition for the dynamical maps:

$$\Phi_t\big(U_g \rho U_g^\dagger\big) = U_g\, \Phi_t(\rho)\, U_g^\dagger, \qquad \forall g \in G,\ \forall t \geq 0, \tag{2.57}$$

where U_g is the unitary representation of g on $\mathcal{H}$. In Markovian regimes $\Phi_t = e^{t\mathcal{L}}$, so (2.57) implies generator covariance $\mathcal{L}(U_g\rho U_g^\dagger) = U_g \mathcal{L}(\rho) U_g^\dagger$.

Constraint on GKLS generators. Let the generator have GKLS form

$$\dot\rho = \mathcal{L}(\rho) = -i[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right). \tag{2.58}$$

A sufficient (and essentially necessary) condition for covariance is:

$$U_g H U_g^\dagger = H, \tag{2.59}$$

$$U_g L_k U_g^\dagger = \sum_\ell v_{k\ell}(g)\, L_\ell, \tag{2.60}$$

where $v(g)$ is a unitary matrix acting in the space spanned by the Lindblad operators. In the Abelian case $G = U(1)$ with generator Q and $U_\alpha = e^{i\alpha Q}$, (2.59)–(2.60) reduce to

$$[H, Q] = 0, \qquad [Q, L_k] = q_k L_k, \tag{2.61}$$

so each dissipative channel carries a definite gauge "charge" q_k. This is the precise open-system extension of gauge invariance: *ports may exchange charge with the environment, but they must do so in a way that respects the gauge ledger.*

2.3.3.4 Mathematical formulation of GGI (macroscopic ledgers)

If a microscopic global symmetry yields a conserved charge

$$Q^a = \int_\Sigma J^{0a}\,\mathrm{d}^3x, \tag{2.62}$$

then in an open system one typically has $\frac{\mathrm{d}}{\mathrm{d}t}Q^a_{\mathrm{sys}} \neq 0$ because charge flows through ports. GGI asserts that a *ledger* conservation holds when system and environment are treated together:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(Q^a_{\mathrm{sys}} + Q^a_{\mathrm{env}}\Big) = 0, \tag{2.63}$$

and that any coarse-grained macroscopic description must preserve the bookkeeping of such globally invariant ledgers.

2.3.3.5 Parameter dictionary

- G : gauge group (possibly a product group) acting on microscopic fields and operational states.
- $g(x) \in G$: local gauge transformation (LGI); U_g : induced unitary representation on $\mathcal{H}$ (DGI).
- A : gauge connection 1-form; $F = \mathrm{d}A + A \wedge A$: curvature 2-form.
- $J^{\mu a}$: Noether current; Q^a : corresponding global charge.
- Φ_t : admissible dynamical map between interventions; $\mathcal{L}$: generator.
- H : effective Hamiltonian in the GKLS generator; L_k : Lindblad (jump) operators.

2.3.3.6 Deeper meaning and intuition

LGI says "physics does not depend on local gauge choices." DGI says "even in open systems, the *allowed* ways to couple to an environment must respect gauge structure." GGI says "at macroscopic scales, conservation is not abandoned; it becomes a ledger constraint across system+environment."

This layered formulation is the bridge that lets gauge principles export beyond particle physics into chemistry/biology/neuroscience: openness does not remove symmetry; it relocates it from isolated conservation to ledger-consistent exchange.

2.3.3.7 Relation to GG-6 and GDOS

- **Relation to GG-6.** In GG-6 the primitive bulk structure is the composite connection $\mathcal{A}$ (Section 2.2, Eq. (2.5)); POS-GI requires that all observables be built from gauge-invariant curvature/holonomy data (Wilson loops, characteristic classes), and that the unified group structure be respected under the bulk–boundary dictionary (Arisaka, 2026b).

- **Relation to** GDOS. In GDOS the core dynamical object is an admissible open-system generator. POS-GI becomes the requirement that the GKLS generator and the measurement instrument intertwine the relevant gauge actions: this is precisely the meaning of *dynamical gauge invariance* in open mesoscopic systems (Arisaka, 2026c; Breuer & Petruccione, 2002).

2.3.3.8 Further mathematical elaboration: generator covariance and Ward identities for open dynamics

In the open-system (GKLS) regime, DGI is most cleanly formulated at the generator level. Let $U(g)$ be a unitary representation of a gauge group element g on the system Hilbert space $\mathcal{H}$. DGI requires

$$\mathcal{L}\Big(U(g)\rho\,U(g)^\dagger\Big) = U(g)\,\mathcal{L}(\rho)\,U(g)^\dagger, \qquad \forall g \in G, \tag{2.64}$$

where $\mathcal{L}$ is the GKLS generator (POS-CP) of the Markov semigroup. Differentiating (2.64) at the identity ($g = \exp(\epsilon X)$) yields an infinitesimal Ward identity:

$$\mathcal{L}\big([X,\rho]\big) = [X, \mathcal{L}(\rho)], \tag{2.65}$$

which is the generator-level expression that "gauge rotations commute with time evolution."

For a GKLS generator written as

$$\mathcal{L}(\rho) = -\mathrm{i}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right), \tag{2.66}$$

(2.64) implies that $(H, \{L_k\})$ must transform covariantly: $H \mapsto UHU^\dagger$ and $L_k \mapsto \sum_j V_{kj}(g)\,UL_jU^\dagger$ for some unitary mixing $V(g)$ on the Lindblad-operator index set. Operationally, this is the open-system analog of the familiar requirement that the dynamics respect gauge redundancy: *not only the equations, but the dissipative noise channels themselves must not encode gauge-dependent physics.*

2.3.3.9 Historical background (with citations)

Gauge symmetry originates in the modern synthesis of symmetry and conservation (Noether) (Noether, 1918), Weyl's gauge idea (Weyl, 1918), and its non-Abelian realization in Yang–Mills theory (Yang and Mills, 1954). Quantum gauge theories require careful handling of redundancies (Faddeev–Popov, BRST) and the preservation of Ward identities. The language of completely positive maps and dynamical semigroups that underlies DGI is due to Stinespring's dilation theorem, Kraus' operator-sum representation, and the GKLS classification (Gorini et al., 1976; Kraus, 1983; Lindblad, 1976; Stinespring, 1955). The central conceptual move in GUT is to elevate gauge-covariant *admissibility of open dynamics* to a constitutional filter, rather than treating it as a model-dependent choice.

2.3.4 POS-2: Unified Extremal Principle (UEP): PLA $\Longleftrightarrow$ PLT/MEPP/FEP

2.3.4.1 Rigorous statement (constitutional text)

Postulate 2.3.2 (POS-2 (UEP): a single variational engine)**.** There exists a *single* extremal/variational principle governing lawful dynamics.

(UEP**a**) **Bulk/closed form (PLA).** In closed descriptions (including the GGEP bulk), physical histories are stationary points of an action functional: $\delta\mathcal{I}_{\text{bulk}} = 0$.

(UEP**b**) **Operational/open form (PLT/MEPP/FEP).** In open operational descriptions, the same variational structure appears, after applying the operational dictionary OGGEP and enforcing admissibility (POS-CP, POS-RG), as an extremal principle for an operational functional $\mathcal{I}_{\text{op}}$ (least time, least free energy, or extremal entropy production, depending on representation).

(UEP**c**) **Dictionary compatibility.** The extremal structure commutes with the bulk→boundary compilation:

$$\text{OGGEP}(\arg\operatorname{ext}\,\mathcal{I}_{\text{bulk}}) = \arg\operatorname{ext}\,\mathcal{I}_{\text{op}}. \tag{2.67}$$

Thus GUT does not postulate separate dynamical "engines" for mechanics, thermodynamics, biology, or cognition; it postulates one engine whose representations differ after openness and coarse-graining are accounted for.

2.3.4.2 PLA in closed systems: from action to equations of motion

For a mechanical system with generalized coordinates $q^i(t)$ and Lagrangian $L(q, \dot{q}, t)$,

$$\mathcal{I}_{\text{mech}}[q] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\,\mathrm{d}t, \tag{2.68}$$

stationarity $\delta\mathcal{I}_{\text{mech}} = 0$ (with fixed endpoints) yields the Euler–Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}^i} - \frac{\partial L}{\partial q^i} = 0. \tag{2.69}$$

For fields $\phi^A(x)$, $\mathcal{I} = \int \mathrm{d}^d x\, \mathcal{L}(\phi, \partial\phi)$ yields

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^A)} - \frac{\partial \mathcal{L}}{\partial \phi^A} = 0. \tag{2.70}$$

In GG-6 the bulk action is a 6D functional of the metric and the composite connection (schematically)

$$\mathcal{I}_6 = \int_{\mathcal{M}_6} \mathrm{d}^6 x\, \sqrt{|g|}\left(\frac{1}{2\kappa^2}R - \frac{1}{4g_5^2}\operatorname{Tr}(\mathcal{F}_{MN}\mathcal{F}^{MN}) + \cdots\right) + \mathcal{I}_{\text{inflow}} + \mathcal{I}_{\partial}, \tag{2.71}$$

where $\mathcal{F}$ is the curvature of the composite connection $\mathcal{A}$ (Section 2.2, Eq. (2.6)). The details belong to the GG-6 paper (Arisaka, 2026b); here we emphasize that UEP demands a single bulk variational principle.

2.3.4.3 Open-system extremals: large deviations and gradient flows

A mathematically clean bridge from PLA to "thermodynamic" extremals is provided by large-deviation theory. Consider a stochastic process (Langevin form)

$$\dot{x}_t = b(x_t) + \sqrt{2\epsilon}\,\eta_t, \tag{2.72}$$

with white noise η_t and noise strength ϵ. As $\epsilon \to 0$, the path measure concentrates on trajectories that minimize an *action-like* functional (the Onsager–Machlup functional) (Freidlin & Wentzell, 1998; Onsager & Machlup, 1953; Touchette, 2009):

$$\mathbb{P}[x(\cdot)] \asymp \exp\left[-\frac{1}{\epsilon}\mathcal{I}_{\mathrm{op}}[x]\right], \qquad \mathcal{I}_{\mathrm{op}}[x] = \frac{1}{4}\int_{t_1}^{t_2} \|\dot{x} - b(x)\|^2_{D^{-1}}\mathrm{d}t + \cdots, \tag{2.73}$$

where D is the diffusion matrix and $\|v\|^2_{D^{-1}} = v^\top D^{-1} v$. The "most probable" path is therefore an extremal path even in a dissipative/open setting.

A particularly important case is *gradient flow*. If $b(x) = -M\nabla F(x)$ for a mobility matrix $M \succeq 0$ and a scalar functional $F(x)$ (free energy, relative entropy, or a generalized potential), then

$$\dot{x} = -M\nabla F(x) \quad \Rightarrow \quad \frac{\mathrm{d}F}{\mathrm{d}t} = -\nabla F^\top M \nabla F \leq 0. \tag{2.74}$$

Thus the same variational structure yields a monotone descent principle on the boundary. This supplies a precise meaning for "least free energy" or "maximum dissipation" principles: they are re-expressions of the extremal structure in an open-system metric.

2.3.4.4 Why "least time" appears operationally

Fermat's principle in optics states that the physical ray extremizes the travel time $\mathcal{T} = \int n(x)\,\mathrm{d}s/c$. In operational open systems, a least-time structure can arise when the relevant cost functional includes an explicit time-to-target tradeoff, and one minimizes over final time. In GUT the point is not to postulate PLT separately, but to interpret PLT-like functionals as representations of the same extremal engine once openness/retardation (LC) and admissibility (CP) are imposed.

2.3.4.5 Parameter dictionary

- $\mathcal{I}_{\mathrm{bulk}}$: bulk action functional; $\delta\mathcal{I}_{\mathrm{bulk}} = 0$ gives equations of motion.
- $\mathcal{I}_{\mathrm{op}}$: operational functional controlling admissible open dynamics; representation may be "time", "free energy", or "entropy production" depending on variables.
- $x(t)$: coarse-grained operational degrees of freedom; $b(x)$: drift; D : diffusion matrix; ϵ : noise strength.
- $F(x)$: scalar Lyapunov/ledger functional (free energy, relative entropy, generalized potential); M : mobility matrix.

2.3.4.6 Deeper meaning and intuition

UEP says: *Nature does not run multiple dynamical engines.* What appears as "straightest path" (geodesic/least action) in a closed geometric description appears as "fastest/most efficient relaxation" (least free energy / extremal entropy production) in an open operational description. The conceptual unification is achieved by the operational dictionary OGGEP and stabilized by admissibility constraints (CP, RG, OGN, MF).

2.3.4.7 Relation to GG-6 and GDOS

- **Relation to GG-6.** In GG-6 the bulk variational principle is written in terms of the composite connection and warped geometry. UEP implies that geodesics, field equations, and extremal surfaces in the 6D bulk are the primitive dynamical objects (Arisaka, 2026b; Maldacena, 1998; Witten, 1998).
- **Relation to GDOS.** In GDOS the open-system evolution must admit a scalar "ledger" (Lyapunov functional) consistent with complete positivity and gauge invariance. UEP then postulates that open dynamics is selected by extremizing (or descending) this ledger functional in the appropriate operational metric (Arisaka, 2026c; Breuer & Petruccione, 2002).

2.3.4.8 Further mathematical elaboration: from least action to gradient-flow and least free energy

A convenient bridge between PLA (bulk) and "least free energy" (boundary) is the Onsager–Machlup form of path probabilities. Consider an overdamped stochastic dynamics on a state manifold with drift $b(y) = -M\nabla F(y)$ and diffusion D:

$$\dot{y} = -M\nabla F(y) + \sqrt{2D}\,\eta(t), \tag{2.75}$$

where η is standard white noise. For small noise, trajectories concentrate around minimizers of an effective stochastic action (Onsager & Machlup, 1953; Seifert, 2012)

$$S_{\mathrm{OM}}[y] \approx \frac{1}{4}\int_{t_0}^{t_1} \mathrm{d}t\,\big(\dot{y} + M\nabla F\big)^{\top} D^{-1}\big(\dot{y} + M\nabla F\big) \;+\; \frac{1}{2}\int_{t_0}^{t_1} \mathrm{d}t\,\nabla\cdot(M\nabla F), \tag{2.76}$$

so that the *most probable* path solves, to leading order,

$$\dot{y} = -M\nabla F(y). \tag{2.77}$$

Equation (2.77) is the monograph gradient-flow law: it is literally "steepest descent" in the metric induced by M. The Lyapunov identity

$$\frac{\mathrm{d}}{\mathrm{d}t}F(y(t)) = -(\nabla F)^{\top} M(\nabla F) \le 0 \tag{2.78}$$

is then an immediate consequence, and can be read as the boundary face of an extremal principle: the system follows the path that decreases F most efficiently given the mobility constraints. In

this sense, "least action" persists in open systems, but the appropriate action is the large-deviation functional (2.76) and the appropriate potential is the EEGG ledger F_{EEGG}.

The PLT/FEP/MEPP language can be understood as different operational summaries of (2.77) and (2H.11) under different choices of coarse-graining and cost functionals. UEP's constitutional role is to insist that these are not unrelated principles, but different avatars of one variational engine once the operational dictionary is fixed.

2.3.4.9 Historical background (with citations)

Variational principles are foundational in mechanics and field theory (Euler–Lagrange; Hamilton) and in optics (Fermat). The systematic treatment of variational mechanics is standard (Lanczos, 1970). For irreversible processes, Onsager introduced a variational structure for linear nonequilibrium thermodynamics (Onsager, 1931). Onsager–Machlup theory and the broader large-deviation framework show how action-like functionals emerge for stochastic dynamics (Freidlin & Wentzell, 1998; Onsager & Machlup, 1953; Touchette, 2009). Proposals for "free energy principles" in neuroscience and related fields exist as information-theoretic variational programs (Friston, 2010). The GUTclaim is not that any single historical extremal principle applies universally as originally stated; rather, it elevates the *existence of a single variational engine* (with multiple operational representations) to a postulate.

2.3.5 POS-3: Anomaly freedom and cancellability by inflow (AF)

2.3.5.1 Rigorous statement (constitutional text)

Postulate 2.3.3 (POS-3 (AF): anomaly freedom via inflow)**.** All admissible GUT theories must be free of gauge, mixed gauge–gravitational, and global anomalies. If anomalies arise in an effective boundary description, they must be cancelled by inflow from the bulk via a single Green–Schwarz-type mechanism, implemented as a modified Bianchi identity in the GGEP bulk.

2.3.5.2 Why anomalies are fatal: a concise derivation

In a gauge theory with partition function $Z[A] = \int \mathcal{D}\psi\, e^{iS[\psi,A]}$, gauge symmetry requires $Z[A^g] = Z[A]$. A gauge anomaly means

$$Z[A^g] \neq Z[A], \tag{2.79}$$

equivalently that the effective action $\Gamma[A] = -i \ln Z[A]$ varies as

$$\delta_\epsilon \Gamma[A] = \int_{M_4} \epsilon^a(x)\, \mathcal{A}^a(x), \tag{2.80}$$

where ϵ^a are gauge parameters and $\mathcal{A}^a$ is the anomaly functional. If the gauge symmetry is fundamental (POS-GI), an uncancelled anomaly makes the theory inconsistent: it violates unitarity and/or current conservation at the quantum level.

2.3.5.3 Inflow mechanism and modified Bianchi identity

The Green–Schwarz mechanism cancels anomalies by introducing a 2-form B_2 with field strength $H_3 = \mathrm{d}B_2 + \cdots$ and coupling

$$\mathcal{I}_{\text{inflow}} = \int_{\mathcal{M}_6} B_2 \wedge X_4, \tag{2.81}$$

where X_4 is a 4-form built from gauge and gravitational curvatures. Gauge variation of B_2 then cancels the boundary anomaly provided H_3 obeys a modified Bianchi identity

$$\mathrm{d}H_3 = \frac{1}{16\pi^2}\Big(a\ \mathrm{Tr}(F \wedge F) + b\, F_B \wedge F_B + c\ \mathrm{Tr}(R \wedge R)\Big), \tag{2.82}$$

with constitutionally fixed coefficients (a, b, c). In GG-6 applications, F is the $SU(5)$ curvature, F_B is the $U(1)_B$ curvature, and R is the curvature 2-form of the Levi–Civita connection. A publication basis consistent with the GG-6 stage is

$$\mathrm{d}H_3 = \frac{1}{16\pi^2}\left[\mathrm{Tr}(F_{SU(5)} \wedge F_{SU(5)}) - F_B \wedge F_B - \frac{2}{5}\,\mathrm{Tr}(R \wedge R)\right], \tag{2.83}$$

while alternative reciprocal conventions correspond to rescalings of the Abelian generator. POS-OGN will later forbid "basis drift" by fixing a canonical normalization in which these coefficients are minimal rational numbers.

2.3.5.4 Constraint power: anomalies as a discrete filter

Beyond consistency, anomaly cancellation is a *powerful filter* on admissible matter content and gauge structure. In the Standard Model, anomaly cancellation fixes the hypercharge assignments of fermions. In GG-6 the postulate is used as a stage-defining constraint that locks gauge and gravitational sectors in a single bulk geometry (Green and Schwarz, 1984; Weinberg, 1995).

2.3.5.5 Parameter dictionary

- $\Gamma[A]$: quantum effective action; $\delta_\epsilon\Gamma$ encodes anomalies.
- B_2 : 2-form (Green–Schwarz field); H_3 : gauge-invariant 3-form field strength.
- X_4 : 4-form polynomial in curvatures; F : gauge curvature; F_B : Abelian curvature; R : gravitational curvature.
- (a, b, c) : fixed rational coefficients of the modified Bianchi identity; fixed by consistency and by OGN.

2.3.5.6 Deeper meaning and intuition

POS-AF is the "quantum-viability compiler." POS-GI provides the symmetry grammar; POS-AF demands that the grammar survive quantization. Inflow cancellation can be read as a *ledger balance*: what appears as nonconservation in the boundary effective theory is compensated by flux from the bulk. Within GUT, this ledger interpretation is not metaphorical; it is encoded in the differential-form identity (2.82).

2.3.5.7 Relation to GG-6 and GDOS

- **Relation to GG-6.** POS-AF is stage-defining for GG-6: the bulk must contain the precise structure that allows inflow, and anomaly cancellation constrains the allowed unified gauge content (Arisaka, 2026b; Green and Schwarz, 1984).
- **Relation to GDOS.** POS-AF ensures that gauge covariance (POS-GI) survives the operational reduction. In open-system language, it protects Ward identities and prevents unphysical gauge nonconservation in registered observables (Breuer & Petruccione, 2002).

2.3.5.8 Further mathematical elaboration: anomaly polynomial, descent, and inflow cancellation

In $d = 4$, gauge and mixed anomalies are encoded by a 6-form anomaly polynomial I_6 whose descent generates the anomalous variation. Schematically, for a gauge connection A and curvature $F = \mathrm{d}A + A \wedge A$, one writes

$$I_6 = \mathrm{Tr}\big(F^3\big) + \mathrm{Tr}(F)\ \mathrm{Tr}(F^2) + \cdots , \tag{2.84}$$

and the descent procedure produces a 5-form Chern–Simons term ω_5 with $\mathrm{d}\omega_5 = I_6$. Under a gauge transformation, $\delta\omega_5 = \mathrm{d}\omega_4^{(1)}$, and the boundary effective action varies as $\delta W_{\mathrm{Bdy}} \sim \int_{\mathrm{Bdy}} \omega_4^{(1)}$ (the anomaly).

Inflow cancellation introduces a bulk term whose variation cancels this boundary piece. In GG-6 language, a Green–Schwarz two-form B_2 with field strength H_3 obeys a modified Bianchi identity

$$H_3 = \mathrm{d}B_2 + \omega_3, \qquad \mathrm{d}H_3 = X_4, \tag{2.85}$$

where X_4 is built from gauge and curvature invariants (POS-AF) and ω_3 is a Chern–Simons 3-form. Because ω_3 shifts under gauge transformations, the bulk coupling $B_2 \wedge X_4$ produces a boundary variation that cancels δW_{Bdy}. The practical message for this book is not the detailed GG-6 construction, but the *filter*: a candidate model is inadmissible if its anomaly polynomial cannot be cancelled by some allowed inflow mechanism without introducing additional non-constitutional dials.

2.3.5.9 Historical background (with citations)

Chiral anomalies were discovered in the context of axial currents in QED (Adler, 1969; Bell and Jackiw, 1969). Their cancellation is essential for the consistency of the electroweak theory and is one of the deep structural facts behind the Standard Model. The Green–Schwarz mechanism was discovered in superstring theory as a ten-dimensional anomaly cancellation mechanism (Green and Schwarz, 1984). The "anomaly inflow" viewpoint, emphasizing cancellation by bulk terms for boundary theories, is central in later developments (e.g. brane anomalies) and provides the conceptual template for POS-AF.

2.3.6 POS-4: Closure and positivity (CP)

2.3.6.1 Rigorous statement (constitutional text)

Postulate 2.3.4 (POS-4 (CP): closure and positivity)**.** Admissible kinematics and dynamics must preserve the physical state space and close under composition.

(CP**1**) **Algebraic closure.** Constraint and symmetry algebras close; no inconsistent bracket relations are allowed.

(CP**2**) **State positivity.** States remain physical: probabilities nonnegative; quantum states remain positive semidefinite.

(CP**3**) **Dynamical admissibility.** Operational evolution is positivity-preserving and normalization-preserving. In quantum theory it is CPTP; in continuous-time Markovian limits it is generated by a GKLS operator.

(CP**4**) **Spectrum positivity.** No negative-norm/ghost degrees of freedom; physical spectra are bounded consistently.

2.3.6.2 Complete positivity and the Kraus/Choi/Stinespring structure

A linear map Φ is positive if $\rho \succeq 0 \Rightarrow \Phi(\rho) \succeq 0$. It is *completely positive* if $\Phi \otimes \mathrm{id}_n$ is positive for all n. Complete positivity is required because an open system may be entangled with unobserved ancillas. The Kraus theorem states that a CPTP map has an operator-sum representation

$$\Phi(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger, \qquad \sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{I} \tag{2.86}$$

(see Eq. (2.32)), and Stinespring's theorem provides a dilation to unitary evolution on a larger space (Choi, 1975; Kraus, 1983; Stinespring, 1955).

2.3.6.3 From semigroups to GKLS generators (a forced structure)

Assume $\{\Phi_t\}_{t\geq 0}$ is a strongly continuous semigroup of CPTP maps. Then the generator $\mathcal{L} = \lim_{t\downarrow 0}(\Phi_t - \mathrm{id})/t$ exists and must take GKLS form

$$\mathcal{L}(\rho) = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{2.87}$$

with $H = H^\dagger$ and arbitrary L_k (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976). Thus, in the presence of OGGEP (which defines between-intervention evolution) and LC (which fixes causal forward composition), POS-CP makes the Lindblad cone a constitutional admissibility domain, not an approximation.

2.3.6.4 Classical analogue: Markov generators and positivity of diffusion

In classical open dynamics, admissibility often amounts to positivity of the transition kernel. For diffusions, a Fokker–Planck equation

$$\partial_t p = -\nabla \cdot (bp) + \nabla \cdot (D \nabla p) \tag{2.88}$$

preserves positivity provided $D \succeq 0$ and suitable boundary conditions hold. This classical positivity is the analogue of quantum complete positivity.

2.3.6.5 Parameter dictionary

- Φ : operational evolution map (channel); CPTP in quantum theory.
- K_α : Kraus operators; $J(\Phi)$: Choi matrix encoding complete positivity (Choi, 1975).
- $\{\Phi_t\}$: semigroup of channels; $\mathcal{L}$: generator; H : effective Hamiltonian; L_k : Lindblad operators.

2.3.6.6 Deeper meaning and intuition

POS-CP is the "no negative probabilities" rule elevated to constitution level. It blocks unphysical modeling moves that are otherwise tempting in complex systems (negative concentrations, non-CP "updates", ghost instabilities). Together with OGGEP, it makes measurement semantics precise: operational maps must live in the CPTP/GKLScone.

2.3.6.7 Relation to GG-6 and GDOS

- **Relation to GG-6.** In the bulk/unification stage, closure and positivity manifest as ghost-free spectra and consistent gauge-fixing/quantization of the unified connection (Peskin and Schroeder, 1995; Weinberg, 1995).
- **Relation to GDOS.** In GDOS, POS-CP is the runtime rule that defines admissible open evolution and measurement: it forces CPTP instruments and, under continuity, Lindblad generators. This is the technical backbone of exporting quantum measurement to general open systems (Breuer & Petruccione, 2002).

2.3.6.8 Further mathematical elaboration: closure and positivity in GKLS and classical Markov generators

In the quantum Markovian regime, "closure and positivity" becomes the statement that $\{\Phi_t\}_{t\geq 0}$ is a CPTP semigroup and therefore admits a GKLS generator:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = \mathcal{L}(\rho) = -\mathrm{i}[H,\rho] + \sum_{\alpha,\beta} c_{\alpha\beta}\left(L_\alpha \rho L_\beta^\dagger - \frac{1}{2}\{L_\beta^\dagger L_\alpha, \rho\}\right), \qquad (c_{\alpha\beta}) \succeq 0, \tag{2.89}$$

where the Kossakowski matrix $(c_{\alpha\beta})$ is positive semidefinite. The positivity of $(c_{\alpha\beta})$ is the precise mathematical clause that forbids negative probabilities and guarantees complete positivity for all extensions (system + ancilla).

In the classical limit (finite-state Markov chains), the analogous statement is that probabilities $p_i(t) \geq 0$ evolve by a rate matrix W with nonnegative off-diagonal entries:

$$\dot{p}_i = \sum_{j \neq i} \big(W_{ij} p_j - W_{ji} p_i\big), \qquad W_{ij} \geq 0 \ (i \neq j), \qquad \sum_i \dot{p}_i = 0. \tag{2.90}$$

Equation (2.90) is "closure" (stays inside the simplex) and "positivity" (no negative probabilities). POS-CP asserts that any open-system effective theory used in any branch of GUT must admit a representation of this kind (quantum or classical, as appropriate), so that violations of positivity/normalization are treated as constitutional errors rather than technical glitches.

2.3.6.9 Historical background (with citations)

The demand that probabilities remain positive is as old as probability theory. In quantum mechanics, the density-matrix formalism and measurement postulates originate with Born and von Neumann (Born, 1926; von Neumann, 1932). The general channel/instrument formalism is due to Kraus and Davies–Lewis (Davies and Lewis, 1970; Kraus, 1983), while the classification of quantum Markov semigroups is due to Gorini–Kossakowski–Sudarshan and Lindblad (Gorini et al., 1976; Lindblad, 1976). The emergence of complete positivity as the correct admissibility notion is now standard in quantum information theory (Nielsen and Chuang, 2000).

2.3.7 POS-5: RG-compatibility and scale covariance (RG)

2.3.7.1 Rigorous statement (constitutional text)

Postulate 2.3.5 (POS-5 (RG): scale coherence and RG commutation)**.** Coarse-graining/renormalization must commute with the operational bulk→boundary dictionary. Equivalently, changing resolution ("zooming out") must not change the operational meaning of the theory.

Formally, letting RG denote the renormalization/coarse-graining map and $\mathcal{D}$ the operational dictionary,

$$\mathcal{D} \circ \mathrm{RG} = \mathrm{RG} \circ \mathcal{D}. \tag{2.91}$$

In the GG-6 stage, the bulk coordinate r is identified with log-energy scale by Eq. (2.52). Dimensionless predictions are RG-invariant or belong to controlled, covariant RG flows.

2.3.7.2 Why commutation is nontrivial

In standard practice, RG is often treated as a computational device (change of cutoff, effective action), while measurement rules are treated separately. But if observation/measurement is *defined* by OGGEP (AX-3), then it must be stable under RG: otherwise, what counts as an

observable would depend on an arbitrary resolution choice, contradicting the idea that physics is objective.

2.3.7.3 Holographic reading: radial evolution as RG flow

Equation (2.91) becomes particularly concrete in holographic settings, where radial evolution in the bulk is mapped to RG flow on the boundary. Write $T_{\Delta r}$ for bulk translation by Δr, and $R_{\Delta r}$ for boundary RG flow by the corresponding scale step. POS-RG asserts a commuting square

$$\mathcal{D} \circ T_{\Delta r} \;\cong\; R_{\Delta r} \circ \mathcal{D}. \tag{2.92}$$

In coupling language, for dimensionless couplings $g_i(\mu)$,

$$\mu \frac{\mathrm{d}g_i}{\mathrm{d}\mu} = \beta_i(g) \quad \Longleftrightarrow \quad \partial_r g_i = -\beta_i(g), \qquad r = \ln(E_{\mathrm{GUT}}/\mu). \tag{2.93}$$

Thus RG beta functions can be interpreted as radial evolution equations. This is the precise technical sense in which "scale is geometrized" in GG-6.

2.3.7.4 Scale covariance versus scale invariance

POS-RG does *not* assert that all quantities are scale invariant. Rather, it asserts that:

(RG1) physical predictions are either RG invariants (dimensionless ratios, critical exponents) or transform covariantly under RG;

(RG2) operational semantics (what is being measured and how) is preserved under coarse-graining.

This postulate is crucial for exporting to sciences where coarse-graining is unavoidable: if predictions change qualitatively when one changes measurement resolution, they are not lawful outputs of the framework.

2.3.7.5 Parameter dictionary

- RG : renormalization/coarse-graining map on the relevant description space.
- $\mathcal{D}$: operational dictionary (bulk→boundary compilation) defined by OGGEP.
- μ : boundary energy/scale parameter; E_{GUT} : unification scale; $r = \ln(E_{\mathrm{GUT}}/\mu)$: bulk scale coordinate.
- g_i : running couplings; $\beta_i(g)$: beta functions.

2.3.7.6 Deeper meaning and intuition

POS-RG is the "truth must not depend on zoom level" rule. In GUT, this is not merely methodological: it forces the existence and interpretation of a scale coordinate (an extra dimension) and requires that observation itself be scale coherent. This is a direct driver for the GG-6 bulk structure.

2.3.7.7 Relation to GG-6 and GDOS

- **Relation to GG-6.** The identification of r with $\ln(E_{\mathrm{GUT}}/\mu)$ is part of the motivation for a warped AdS-like bulk; bulk geometry becomes a map of scale. Technical realization is discussed in (Arisaka, 2026b; Maldacena, 1998; Polchinski, 1995).
- **Relation to GDOS.** In GDOS, coarse-graining corresponds to changing the operational resolution of an open system. POS-RG demands that the operational dictionary (what counts as state, port, and measurement) commute with such coarse-graining.

2.3.7.8 Further mathematical elaboration: commuting RG with observation as a commutative diagram

Let RG_ℓ denote coarse-graining by a scale parameter ℓ (e.g. integrating out modes above $\mu \mathrm{e}^{-\ell}$), and let $\mathcal{D}$ denote the operational dictionary mapping model variables to registered observables (AX-3). POS-RG is the commutativity requirement

$$\mathcal{D} \circ \mathrm{RG}_\ell = \mathrm{RG}_\ell^{(\mathrm{obs})} \circ \mathcal{D}, \tag{2.94}$$

where $\mathrm{RG}_\ell^{(\mathrm{obs})}$ is the induced renormalization of estimators/response functions (calibration, bandwidth, binning, etc.). Equation (2.94) says: *"coarse-grain first" and "observe first" must lead to the same registered claim*, up to controlled, declared transformations.

In QFT this becomes the statement that dimensionless predictions are RG invariant or follow controlled flows, governed by beta functions:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta^i(g)\frac{\partial}{\partial g^i} + \gamma\right)\mathcal{O}(\mu, g) = 0, \tag{2.95}$$

where $\mathcal{O}$ is an observable and γ denotes anomalous dimensions. In the GG-6/OGGEP vocabulary, the AdS-like radial coordinate is identified with scale,

$$r = \ln(E_{\mathrm{GUT}}/\mu), \tag{2.96}$$

so RG flow becomes geometric transport in r. The constitutional content is that *scale is not an observer preference*: the dictionary and the dynamics must be compatible with the choice of resolution, otherwise "predictions" are artifacts of the microscope rather than properties of the world.

2.3.7.9 Historical background (with citations)

RG ideas originate in the study of scale dependence in quantum field theory (Gell-Mann & Low, 1954), were formalized by Wilson (Wilson, 1971), and are standard in modern QFT (Peskin and Schroeder, 1995; Weinberg, 1995). The holographic interpretation of RG as radial evolution is associated with AdS/CFT (Maldacena, 1998; Witten, 1998). The distinctive move in GUT is to *axiomatize* the commutation between RG and observation/operational semantics.

2.3.8 POS-6: Occam Gauge Normalization (OGN): rigidity and "no basis drift"

2.3.8.1 Rigorous statement (constitutional text)

Postulate 2.3.6 (POS-6 (OGN): Occam gauge normalization / minimal rational holographic normalization)**.** Among all basis and normalization conventions that leave POS-1–POS-5 satisfied, Nature selects a canonical normalization in which:

(OGN**1**) **Topological integrality.** All relevant topological integrals (fluxes, Chern/Pontryagin numbers, inflow coefficients) take integer values in the chosen units.

(OGN**2**) **Minimal rationality.** Any remaining rational factors are fixed in minimal coprime form shared across sectors.

(OGN**3**) **No continuous rescaling dials.** No arbitrary continuous normalization parameter may remain; "basis drift" is illegal.

Equivalently: conventions are not allowed to function as hidden continuous parameters.

2.3.8.2 Why normalization is not "just convention"

In a single-sector gauge theory, one may rescale

$$A_\mu \to \lambda A_\mu, \qquad g \to g/\lambda, \tag{2.97}$$

leaving gA_μ invariant in covariant derivatives. This looks like a harmless convention. However, in a multi-sector theory with Abelian factors, mixed anomalies, and topological inflow terms, such rescalings can move physical content between "couplings" and "normalization constants." Without a canonical choice, one can inadvertently *launder* parameters: a theory can appear predictive in one basis and nonpredictive in another.

POS-OGN forbids this: if a basis choice changes the integer/topological data, it is not an allowed convention.

2.3.8.3 Topological quantization and canonical units

A central mechanism is topological quantization. For an Abelian field strength F, flux quantization reads

$$\frac{1}{2\pi}\int_{\Sigma_2} F \in \mathbb{Z}, \tag{2.98}$$

and for non-Abelian bundles characteristic classes are integer-valued. Such conditions are invariant statements, but their manifestation as simple integers depends on normalization. OGN selects the normalization in which the corresponding lattices are *minimal* (coprime, no common factors). This is the same discipline that underlies Dirac charge quantization (Dirac, 1931) and the standard generator conventions in Lie algebras (Kobayashi & Nomizu, 1963; Nakahara, 2003).

2.3.8.4 OGN as the antidote to scheme dependence

A closely related issue is scheme dependence in renormalization and in holographic renormalization: finite local counterterms can shift definitions of quantities if not fixed by a principle. OGN asserts that the admissible scheme is the one fixed by minimal rational/integer data, so that dimensionless predictions do not drift with choice of renormalization convention.

2.3.8.5 OGN-5 integer ledger (as a stage output)

In the GG-6 stage, OGN is proposed to yield a discrete integer "ledger" (a minimal coprime set of integers shared across sectors). A canonical representative is

$$(M, N, L; J, K) = (1, 3, 5; 7, 35), \qquad J = 2N + 1, \qquad K = LJ = M^2 + N^2 + L^2 = 35, \quad (2.99)$$

which is used in the GG-6 program to express several dimensionless ratios in compact rational form. In the present axiom paper we record this as an intended output of the admissibility spine; the derivation and phenomenology are presented elsewhere (Arisaka, 2026b).

2.3.8.6 Parameter dictionary

- λ : would-be continuous rescaling parameter (forbidden as a physical dial by POS-OGN).
- F : gauge curvature 2-form; $\int F$: flux/topological invariant (integer in canonical units).
- (a, b, c) : coefficients in the modified Bianchi identity (2.82); OGN fixes their canonical rational form.
- $(M, N, L; J, K)$: example of a discrete integer ledger output in the GG-6 stage (OGN-5).

2.3.8.7 Deeper meaning and intuition

OGN is the "no hidden conventions" postulate. It ensures that two researchers using different normalization conventions cannot obtain different "predictions" simply by drifting bases. This is essential for a ten-science framework, where units and conventions vary widely across disciplines: OGN is the rule that turns conventions into audit-fixed integers/rationals.

2.3.8.8 Relation to GG-6 and GDOS

- **Relation to GG-6.** OGN fixes Abelian normalizations, inflow coefficients, and the identification of discrete topological data. It is a key ingredient in the claim that GG-6 can be "parameter-closed" (no continuous free knobs beyond universal constants) once anomaly cancellation and symmetry are imposed (Arisaka, 2026b).
- **Relation to GDOS.** In GDOS, OGN stabilizes operational semantics across laboratories and scales: ports, couplings, and measured quantities must be normalized in a canonical way so that comparisons across experiments and domains are meaningful (Arisaka, 2026c; Shannon, 1948).

2.3.8.9 Further mathematical elaboration: OGN fixes the charge lattice and removes Abelian basis drift

The simplest illustration of POS-OGN is an Abelian factor $U(1)$. If Q is the generator and g the coupling, a rescaling $Q \mapsto \lambda Q$ can be compensated by $g \mapsto g/\lambda$ so that the covariant derivative $D_\mu = \partial_\mu + igQA_\mu$ is unchanged. This means that, without an OGN clause, a theory can acquire a continuous "normalization knob" that is not physical but can masquerade as one when combined with other sectors or with sloppy bookkeeping.

OGN fixes a canonical *charge lattice* by demanding minimal integers compatible with topology/holonomy:

$$q_i \in \mathbb{Z}, \qquad \gcd(\{q_i\}) = 1, \tag{2.100}$$

and by choosing canonical units for holonomy (one 2π turn on the compact circle corresponds to one integer period). The content is Occam-like but operational: once the basis is fixed, charge ratios become sharp statements and can enter falsifiers (POS-MF). In GG-6 this kind of discipline is what allows rational lock ratios (e.g. $5:2$) to be interpreted as predictions rather than conventions.

2.3.8.10 Historical background (with citations)

Occam's razor is a philosophical heuristic, but in physics normalization rigidity is concrete: Dirac's monopole argument yields charge quantization (Dirac, 1931), characteristic classes of gauge bundles are integer invariants (Kobayashi & Nomizu, 1963; Nakahara, 2003), and grand unification fixes Abelian normalizations by group embedding. In modern high-energy theory, the proliferation of continuous moduli and the "landscape" motivates the search for stronger selection principles (Polchinski, 2004; Susskind, 2003). OGN elevates "no free dials in conventions" to a constitutional postulate.

2.3.9 POS-7: Measurement and falsifiability (MF): compile-to-observable

2.3.9.1 Rigorous statement (constitutional text)

Postulate 2.3.7 (POS-7 (MF): measurement contracts and falsifiability)**.** Every theoretical claim in GUT must compile, via the operational dictionary OGGEP, to a registered observable together with a specific instrument protocol and a pre-declared falsification (kill) criterion. Claims that cannot be compiled into such a measurement contract are not admissible as scientific statements within GUT.

2.3.9.2 Measurement is defined by OGGEP; MF adds an audit requirement

AX-3 (OGGEP) defines what "measurement" means: an instrument/POVM acting on an operational state. POS-MF adds that every theorem-level statement must include an *audit trail* to data.

Definition 2.3.8 (Measurement contract)**.** A *measurement contract* for a claim $\mathcal{C}$ is a tuple

$$\mathfrak{C}(\mathcal{C}) = (\mathcal{I}, \{E_m\}, \mathcal{S}_{\text{valid}}, \mathcal{T}, K)\,, \tag{2.101}$$

where: (i) $\mathcal{I} = \{\mathcal{I}_m\}$ is an instrument; (ii) $\{E_m\}$ is the associated POVM; (iii) $\mathcal{S}_{\text{valid}}$ specifies the domain of validity (state preparation, ports, calibration assumptions); (iv) $\mathcal{T}$ specifies the timescale/scale window and coarse-graining level; and (v) K is a kill criterion: a statistical or deterministic condition on observed data that would refute $\mathcal{C}$.

A typical statistical kill criterion may be stated as "reject $\mathcal{C}$ at significance level α if a test statistic $T(\text{data})$ falls outside a pre-declared acceptance region," in the spirit of standard hypothesis testing.

2.3.9.3 Parameter dictionary

- $\mathcal{I}$: instrument (family of CP maps summing to a TP map); $\{E_m\}$: associated POVM.
- $\mathcal{S}_{\text{valid}}$: explicit preparation/calibration assumptions; "what was controlled".
- $\mathcal{T}$: timescale/scale window; includes coarse-graining level relevant to POS-RG.
- K : kill criterion; may include significance level α, sample size, estimator definition, and rejection region.

2.3.9.4 Deeper meaning and intuition

MF is the constitutional closure of the scientific loop: a theory is not only a mathematical structure; it is a *contract* between derivations and data. By forcing every theorem to compile to an instrument/protocol/kill-criterion tuple, MF blocks the failure mode "explains everything therefore predicts nothing."

2.3.9.5 Relation to GG-6 and GDOS

- **Relation to** GG-6**.** GG-6 outputs claims about particle physics and cosmology. MF requires that these claims be stated together with explicit observables (cross sections, spectra, cosmological parameters) and refutation thresholds, not merely qualitative stories (Particle Data Group, 2024; Planck Collaboration, 2018).
- **Relation to** GDOS**.** In GDOS, MF is essential because open-system claims can otherwise drift into unfalsifiable narrative. MF forces every proposed "fingerprint" (spectral knee, scaling exponent, attractor statistic) to be tied to an explicit measurement contract.

2.3.9.6 Further mathematical elaboration: MF as pre-registered hypothesis testing

POS-MF can be expressed in the language of statistical decision theory. A theoretical claim $\mathcal{C}$ implies a distribution (or family) for a registered estimator $\widehat{\theta}$ under a specified protocol Π:

$$\mathcal{C} \;\Rightarrow\; p(\widehat{\theta}\,|\,\mathcal{C}, \Pi). \tag{2.102}$$

A *falsifier* is then a pre-declared rejection region R_α at significance level α:

$$\text{Kill criterion:} \qquad \widehat{\theta} \in R_\alpha \implies \text{reject } \mathcal{C}. \tag{2.103}$$

For example, if GG-6 predicts a sharp coupling or rational ratio, the MF contract must declare the estimator, the systematic uncertainty model, and the decision rule (e.g. a likelihood-ratio test with a stated threshold). This is not a sociological preference: it is what converts a mathematical derivation into an empirically audit-ready scientific statement.

2.3.9.7 Historical background (with citations)

Operationalism has deep roots in the philosophy of measurement (Bridgman, 1927). Falsifiability as a demarcation criterion is most associated with Popper (Popper, 1959), though modern philosophy of science recognizes additional subtleties. In quantum theory, the mathematical description of measurement evolved from projection postulates (von Neumann) to generalized measurements and instruments (Kraus; Davies–Lewis; Ozawa) (Davies and Lewis, 1970; Kraus, 1983; Nielsen and Chuang, 2000; Ozawa, 1984; von Neumann, 1932). The GUTcontribution is to treat the operational layer as axiomatic (OGGEP) and to treat falsifiability as a postulated compilation requirement (MF), rather than an external methodological add-on.

2.3.10 Inter-postulate dependency chain and "why this order"

The seven postulates form a dependency directed acyclic graph (DAG) rather than a flat list. A compact representation is:

$$\boxed{\text{GI} \Rightarrow \text{UEP} \Rightarrow \text{AF} \Rightarrow \text{CP} \Rightarrow \text{RG} \Rightarrow \text{OGN} \Rightarrow \text{MF}} \tag{2.104}$$

where "⇒" reads "is a prerequisite for clean formulation of." The meaning is:

- GI fixes the symmetry grammar and the admissible notion of "same physics under relabeling."
- UEP supplies the dynamical engine consistent with that grammar.
- AF ensures the symmetry grammar survives quantization.
- CP ensures the operational evolution remains inside the physical state space (no negative probabilities).
- RG ensures the operational semantics and predictions do not depend on arbitrary resolution choice.
- OGN removes residual ambiguity in conventions, killing basis drift and parameter laundering.
- MF closes the scientific loop: every statement must compile to a falsifiable contract with data.

Two interlocks are especially important for exporting beyond particle physics:

I1. OGGEP+ CP ⇒ **admissible open dynamics.** Operational semantics plus positivity forces CPTP instruments and (under continuity) GKLS generators. This is the foundational reason GUT treats open-system dynamics as constitutional rather than "effective modeling."

I2. AF+ OGN ⇒ **discrete locking.** Anomaly cancellation provides integer/topological constraints, while OGN fixes their canonical normalization so they become auditable discrete data rather than scheme-dependent numbers.

2.3.11 Common operational objects (one-page reminder)

We recall three objects that appear repeatedly.

(i) Boundary states and channels. On the boundary $\mathcal{M}_4$ (operational arena), states are either classical densities $\rho(x) \geq 0$ with $\int \rho = 1$, or quantum density operators $\rho \succeq 0$ on a Hilbert space $\mathcal{H}$ with $\mathrm{Tr}\,\rho = 1$. Open evolution is a family of maps Φ_{t_2,t_1} on states; in the quantum case, admissible maps are *completely positive trace-preserving* (CPTP) channels (Kraus, 1971; Nielsen & Chuang, 2010; Stinespring, 1955).

(ii) Instruments and registered observables. A measurement with outcomes m is a quantum instrument $\{\mathcal{I}_m\}_m$ (CP maps whose sum is TP) (Kraus, 1971; Nielsen & Chuang, 2010). Outcome probabilities and post-measurement states are

$$p(m) = \mathrm{Tr}[\mathcal{I}_m(\rho)], \qquad \rho_m = \frac{\mathcal{I}_m(\rho)}{\mathrm{Tr}[\mathcal{I}_m(\rho)]}, \qquad \Phi(\rho) = \sum_m \mathcal{I}_m(\rho). \tag{2.105}$$

This is the operator-theoretic formalization of "registered measurement" used in OGGEP (AX-3).

(iii) Coarse-graining / RG. We denote by RG an operation that reduces resolution: integrating out UV degrees of freedom, marginalizing hidden variables, or aggregating microstates. POS-5 will require that such coarse-graining commutes with the operational dictionary (AX-3).

With these objects fixed, we now collect canonical kernel equations (for cross-references) and provide a comprehensive summary table.

2.3.12 Canonical kernel equations referenced in Table 2.3

For convenience, we collect the minimal "kernel equations" that are repeatedly referenced as consistency checks across the seven postulates. The intent is not to exhaust derivations (which appear in the individual postulate subsections above), but to fix *unambiguous symbols and labels* for the summary table and for later GG-6/GDOS papers.

POS-1 (GI): gauge invariance (LGI/DGI/GGI). Local gauge invariance at the action level is

$$S[A^g, \psi^g] = S[A, \psi]. \tag{2.106}$$

Dynamical gauge invariance in open evolution can be stated as equivariance of the channel family under the unitary representation U_g:

$$\Phi_t \circ \mathrm{Ad}_{U_g} = \mathrm{Ad}_{U_g} \circ \Phi_t, \qquad \mathrm{Ad}_{U_g}(\rho) = U_g \rho\, U_g^\dagger, \tag{2.107}$$

and the same requirement can be imposed on the generator $\mathcal{L}$ in a Markov limit. We also define a generic "ledger with ports" balance law,

$$\frac{d}{dt} F\big(y(t)\big) = J_{\mathrm{ports}}\big(y(t), t\big), \tag{2.108}$$

where F is a chosen EEGG ledger functional and J_{ports} is the net flux through exchange channels; POS-GI requires this statement be form-invariant under gauge relabelings.

POS-2 (UEP): unified extremal principle. In closed sectors, dynamics is extremal:

$$\delta S[\phi] = 0. \tag{2.109}$$

In open operational regimes, a canonical boundary form is gradient/steepest-descent flow with mobility $M \succeq 0$,

$$\dot{y} = -M(y)\, \nabla F(y), \tag{2.110}$$

which implies a Lyapunov monotonicity (free-energy decrease) when M is positive:

$$\frac{dF}{dt} \leq 0. \tag{2.111}$$

For fluctuations and rare events, a large-deviation functional provides the operational extremal principle:

$$\mathbb{P}[y(\cdot)] \asymp e^{-I[y]}, \qquad I[y] = \int L\big(y, \dot{y}\big)\, dt. \tag{2.112}$$

POS-3 (AF): anomaly freedom / cancellability by inflow. At the quantum level, a gauge variation of the effective action has the schematic form

$$\delta_\alpha \Gamma = \int \alpha\, \mathcal{A}, \tag{2.113}$$

and admissibility requires cancellation, possibly by inflow via a Green–Schwarz term

$$S_{\mathrm{GS}} = \int B_2 \wedge X_4, \tag{2.114}$$

with modified Bianchi identity

$$dH_3 = X_4. \tag{2.115}$$

In the GG-6 context we parameterize the admissible GS-modified identity as

$$dH_3 = a\, \mathrm{tr}(F \wedge F) + b\, \mathrm{tr}(R \wedge R) + c\, F_B \wedge F_B, \qquad (a, b, c) = (1, -1, -2/5), \tag{2.116}$$

where F denotes the non-Abelian field strength, R the curvature two-form, and F_B the $U(1)_B$ field strength (normalization fixed by OGN/MSMF).

POS-4 (CP): closure and positivity. Quantum operational evolution must be completely positive and trace preserving, i.e. admit a Kraus form

$$\Phi(\rho) = \sum_k K_k \rho K_k^\dagger, \qquad \sum_k K_k^\dagger K_k = \mathbb{I}, \tag{2.117}$$

and in a Markov limit the generator must be of GKLS form

$$\dot{\rho} = \mathcal{L}(\rho) = -i[H, \rho] + \sum_a \left(L_a \rho L_a^\dagger - \frac{1}{2}\{L_a^\dagger L_a, \rho\} \right). \tag{2.118}$$

In classical mesoscopic limits, positivity is preserved by Fokker–Planck generators such as

$$\partial_t p = -\nabla \cdot (bp) + \frac{1}{2} \nabla \cdot (D \, \nabla p) \, . \tag{2.119}$$

POS-5 (RG): RG-compatibility and scale covariance. Operational semantics must commute with coarse-graining:

$$\mathcal{D} \circ \mathrm{RG} = \mathrm{RG} \circ \mathcal{D}, \tag{2.120}$$

and couplings flow by beta functions in a chosen scheme,

$$\mu \frac{dg_i}{d\mu} = \beta_i(g). \tag{2.121}$$

In warped bulk geometries used in GG-6, a representative scale-covariant ansatz is

$$ds^2 = e^{-2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + ds^2(\mathcal{K}_2), \tag{2.122}$$

so that translation in r implements RG evolution.

POS-6 (OGN): Occam gauge normalization. Topological/quantization conditions fix discrete normalizations, e.g.

$$\frac{1}{2\pi} \int_{\Sigma_2} F \in \mathbb{Z}, \tag{2.123}$$

and GG-6 uses the "frozen" integer data

$$(M, N, L; J, K) = (1, 3, 5; 7, 35), \tag{2.124}$$

as a canonical minimal-rational normalization (no continuous rescaling dials).

POS-7 (MF): measurement and falsifiability. A measurement is an instrument $\{\mathcal{I}_m\}_m$ whose sum is a channel,

$$\Phi(\rho) = \sum_m \mathcal{I}_m(\rho), \tag{2.125}$$

with induced POVM elements $E_m = \mathcal{I}_m^*(\mathbb{I})$ and Born probabilities

$$p(m) = \mathrm{Tr}(\rho E_m). \tag{2.126}$$

MF requires that each theorem compile to a pre-declared contract (protocol, estimator, prediction, kill rule), schematically

$$(\mathcal{I}, O, \mathbb{P}_\theta, K). \tag{2.127}$$

2.3.13 Comprehensive summary table for POS-1–POS-7

Table 2.3: **Postulates summary (POS-1–POS-7).** Ordered admissibility filters with constitutional content, canonical equations/checks, GG-6/GDOS bridges, and falsifiability handles. Read this table as the "runtime checklist" for any candidate model instance: a theory is not admitted unless each row can be satisfied without hidden dials.

POS	Constitutional statement (rigorous)	Mathematical kernel	Bridge to GG-6 / GDOS	Falsifiability handle
POS-1	Layered gauge invariance: LGI (action), DGI (generator equivariance), GGI (global ledgers).	$S[A^g, \psi^g] = S[A, \psi]$ (2.106); $\Phi_t \circ \mathrm{Ad}_{U_g} = \mathrm{Ad}_{U_g} \circ \Phi_t$ (2.107); ledger continuity (2.108).	Locks bulk gauge content and boundary ports; makes open-system laws representation-independent.	Gauge/basis changes must not alter registered predictions; illegal if they do.
POS-2	One variational engine: PLA in closed sectors projects to PLT/FEP/MEPP in open operational sectors.	$\delta S = 0$ (2.109); gradient flow (2.110); $dF/dt \leq 0$ (2.111); path LD action (2.112).	Bulk geodesics/extremals (GG-6) compile to boundary optimal control/relaxation (GDOS).	Compare predicted extremal paths, relaxation laws, and time-to-target scaling to data.
POS-3	Quantum consistency: all anomalies cancel, possibly by inflow (Green–Schwarz) with a fixed Bianchi identity.	$\delta\Gamma = \int \alpha \mathcal{A}$ (2.113); $S_{\mathrm{GS}} = \int B_2 \wedge X_4$ (2.114); $dH_3 = X_4$ (2.115).	Forces discrete charge/representation data and locks Abelian sectors (GG-6); stabilizes ledger symmetries in GDOS.	If any required symmetry is anomalous/uncancellable, model is rejected.
POS-4	Physicality: algebras close; states remain positive; open evolutions are CPTP; Markov limits are GKSL.	Kraus form (2.117); GKSL (2.118); classical positivity (2.119).	Ensures bulk/boundary descriptions produce valid probabilities; fixes GDOS runtime class.	Look for negative probabilities/ghosts or non-CP evolution; these falsify admissibility.

(continued)

POS	Constitutional statement (rigorous)	Mathematical kernel	Bridge to GG-6 / GDOS	Falsifiability handle
POS-5	Scale coherence: coarse-graining commutes with observation; radial coordinate is RG scale.	$\mathcal{D} \circ \mathrm{RG} = \mathrm{RG} \circ \mathcal{D}$ (2.120); $\mu dg/d\mu = \beta(g)$ (2.121); warped metric (2.122).	Identifies r with log energy in GG-6; constrains GDOS coarse-graining dependence.	Predictions must be RG invariant (dimensionless) or have controlled flows; otherwise illegal.
POS-6	No basis/scheme drift: canonical minimal integer/rational normalization across sectors.	Flux quantization (2.123); lattice primitivity; frozen integers (2.124).	Converts inflow/locking into sharp rational predictions (GG-6) and prevents unit drift in GDOS.	If "predictions" change under legal re-normalizations, model violates OGN.
POS-7	Every claim compiles to an operational test with protocol + kill criterion.	Instrument/POVM (2.125)–(2.126); compile tuple (2.127).	Enforces instrument-level semantics of OGGEP and makes GG-6/GDOS outputs scientifically auditable.	A pre-declared kill criterion exists for every theorem; otherwise claim is inadmissible.

2.4 Applications (Preview)

Sections 3L.5–2J.6 define the *constitution* of GUT: four axioms and seven postulates. This section is a *preview* of how the constitution compiles into concrete theorem stacks. Full derivations and model-building details are delegated to the companion preprints: GG-6 (Arisaka, 2026b) (Arisaka, 2026b), GDOS (Arisaka, 2026c) (Arisaka, 2026c), and the ten-branch export overview (Arisaka, 2026d) (Arisaka, 2026d). The purpose here is to record the interface: *what the constitution forces downstream engines to output* (observables, normalization conventions, and falsifiers).

2.4.1 Preview of GG-6: Geometry–Gauge Locking in six dimensions

Constitutional role. GG-6 is the technical engine intended to realize AX-2 (GGEP) under the admissibility filters of Section 2J.6: it must (i) geometrize gauge structure as part of a composite connection, (ii) remain anomaly-free (POS-AF), (iii) remain RG/scale coherent (POS-RG), and (iv) eliminate hidden continuous moduli (AX-4 / POS-OGN). Historically, this stance is

motivated by the connection-curvature viewpoint in gauge theory (Nakahara, 2003; Yang and Mills, 1954) and by holographic hints ('t Hooft, 1993; Maldacena, 1998; Susskind, 1995; Witten, 1998), while keeping the demand of predictivity explicit.

Minimal geometric stage (representative ansatz). A representative background consistent with a geometric scale coordinate and a compact holonomy direction is

$$\mathrm{d}s_6^2 = e^{-2A(r)}\,\eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2\,\mathrm{d}\theta^2, \qquad \theta \sim \theta + 2\pi, \tag{2.128}$$

where $A(r)$ is a warp profile and r is interpreted as a geometric representative of the RG scale,

$$r \sim \ln\!\left(\frac{E_{\mathrm{GUT}}}{\mu}\right). \tag{2.129}$$

Warped geometries historically illustrate geometric hierarchy generation (Randall & Sundrum, 1999a; Randall and Sundrum, 1999b). In GUT, the novelty is constitutional: this stage is the mandated realization of "geometrize gauge," not one optional model among many.

Gauge structure and inflow consistency (one-sentence preview). A minimal GG-6 specialization adopts a unified gauge sector containing $SU(5) \times U(1)_B$ (Georgi and Glashow, 1974), with anomaly cancellability enforced by a Green–Schwarz type inflow mechanism (Green and Schwarz, 1984). In this program, inflow data, Abelian normalization, and discrete "ledger" structures are treated as predicted architecture, not as tunable conventions.

2.4.2 Preview of GDOS: Geometric Dynamics of Open Systems

Constitutional role. GDOS is the universal boundary "runtime" implied by AX-3 (OGGEP) once POS-CP and POS-MF are enforced. Historically, open-system formalisms (CPTP maps; Lindblad/GKLS generators; stochastic thermodynamics) were developed to handle measurement, irreversibility, and coarse-graining (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976; Seifert, 2012; von Neumann, 1932). The constitutional move of GUT is to require open-system admissibility and operational observables *as part of the definition of a theory.*

Kernel preview (classical drift–diffusion form). Given a System Instance Packet (SIP) and an EEGG ledger functional, GDOS exports a canonical generator. A minimal representative form is the covariant drift–diffusion equation on a state manifold $\mathcal{M}$:

$$\partial_t \rho + \nabla_A J^A = 0, \qquad J^A = \rho\, b^A - D^{AB}\nabla_B \rho, \qquad b^A = -M^{AB}\nabla_B F_{\mathrm{EEGG}} + u^A, \tag{2.130}$$

with $M^{AB} \succeq 0$ and $D^{AB} \succeq 0$. The solenoidal component u^A encodes cycles (non-gradient flow), while the gradient component implements irreversible "downhill" evolution of the ledger. In quantum limits, POS-CP elevates the admissible generator to the GKLS form given in Section 2J.6.

Examples and limits. Tables 2.4 and 2.5 summarize (i) representative physics arenas in which GDOS appears as the canonical operational reduction and (ii) how familiar frameworks are recovered as special limits.

Table 2.4: **GDOS runtime preview.** Examples of GDOS as a reusable open-system kernel across modern physics; each row specifies a representative SIP (state space $\mathcal{M}$, ports, EEGG ledger) and the corresponding generator/observable class. The intent is not to be exhaustive but to show how diverse "effective equations" arise as controlled limits of one admissible kernel.

Arena	**Representative SIP: state space and ports**	**Ledger F_{EEGG}**	**Generator / observable output**
Quantum measurement	Hilbert space $\mathcal{H}$; measurement device + environment as ports; instrument map $\mathcal{I}_\alpha$	Information ledger (relative entropy / prediction error) plus port work	CPTP instrument; GKLS generator; registered classical record (POVM outcome) (Davies and Lewis, 1970; Kraus, 1983; Ozawa, 1984).
Decoherence / emergence of classicality	Pointer subspace; environment coupling as port	Entropy production + robustness functional	Environment-induced superselection / einselection; classicalization times (Zurek, 2003).
Nonequilibrium thermodynamics	State manifold of coarse variables; heat/work baths as ports	Free energy $F = E - TS$ as special case of EEGG	Fokker–Planck / Langevin generator; entropy production; fluctuation relations (Onsager & Machlup, 1953; Touchette, 2009).
Renormalization / effective theories	Space of effective couplings; coarse-graining map as port	Scale-dependent effective action / coarse-grained free energy	RG flow $\partial_r g_i = -\beta_i(g)$ consistent with POS-RG (Wilson, 1971).
Gravity / horizons (preview)	Exterior region as system; horizon as port	Generalized entropy as EEGG special case	Semiclassical evaporation channels; information ledger constraints ('t Hooft, 1993; Bekenstein, 1973; Hawking, 1975; Susskind, 1995).

Table 2.5: **GDOS Rosetta limits.** Canonical limits of the GDOS kernel (2.130) that recover familiar equations by fixing subsets of ports and curvature/entropy terms. Each limit is a consistency check: the constitution must reproduce known laws when ports are closed and symmetries are specialized.

Theory	**Canonical state & generator**	**GDOS limit (what is set to zero / fixed)**
Hamiltonian mechanics	State $z = (q, p)$, Liouville flow $\dot{z} = \Omega \nabla H$	Pure curl sector: $M = 0$, $D = 0$, $u = \Omega \nabla H$, $F = H$.
Quantum unitary evolution	Density matrix ρ, von Neumann equation $\dot{\rho} = -i[H, \rho]$	Closed system limit of CPTP semigroup: dissipator off; F reduces to an action/energy functional.
Open quantum dynamics	GKSL/Lindblad generator $\mathcal{L}$ (Gorini, Kossakowski, & Sudarshan, 1976; Lindblad, 1976)	Operator-valued version of (2.130): complete positivity fixes the dissipative cone (POS-4).
Near-equilibrium thermo	Flux–force linear response (Onsager, 1931)	Gradient sector with quadratic F; M is Onsager matrix; $u \approx 0$; noise fixed by fluctuation–dissipation.
Stochastic thermodynamics	Path probabilities, entropy production (Seifert, 2012)	F includes entropic curvature term; ports encode work/heat; MF fixes operational estimators.
Information geometry / inference	Belief state $q(\theta)$, natural gradient (Amari & Nagaoka, 2000)	G is Fisher metric; F is variational free energy; GDOS becomes natural gradient flow (a FEP limit) (Friston, 2010).

2.4.3 Preview of POS-MF: measurement contracts and kill criteria

A key methodological claim of GUT is that a theorem is incomplete until it ships with a measurement protocol and a falsifier. This is a formalization of the Popperian requirement of refutability (Popper, 1959) in operational language: the observation protocol and the kill criterion are part of the scientific statement. Table 2.6 provides a minimal contract template (shown schematically using a GG-6-type target).

2.4.4 GUT-10 export: ten branches as modules sharing one constitution

In the current GUT-10 partition (with GG-6 and GDOS treated as parents), we export the constitution to ten branches:

1. **GUT-Particle** (1st floor of the Pagoda)
2. **GUT-Cosmos** (2nd floor; includes cosmology and astronomy)
3. **GUT-Chemistry** (3rd floor)

GG-6 claim (example)	Instrument / protocol	Estimator	Kill criterion
A concrete new resonance scale M_{KK}	HL-LHC resonance searches (pre-registered channels/cuts)	template fit to invariant-mass spectrum	exclusion of predicted window at stated CL
A long-range $\mathrm{U}(1)_B$ remnant or protected baryon stability	proton-decay + baryon-number violation searches	lifetime lower bound τ_p	violation of stability bound
A specific axion-like line ("kaxion"-type)	haloscope / microwave cavity (fixed band plan)	power-spectrum peak estimator	null detection beyond sensitivity floor

Table 2.6: **MF contract template (GG-6 example).** Minimal measurement protocol fields required by POS-MF: instrument + calibration, estimator, uncertainty budget, and a pre-declared kill criterion. The goal is auditability: the theory is obliged to state how it can be falsified before the data are taken.

4. **GUT-Life** (4th floor)
5. **GUT-Mind** (5th floor)
6. **GUT-Health**
7. **GUT-Society**
8. **GUT-Environment**
9. **GUT-AI**
10. **GUT-Philosophy**

The first five branches map one-to-one onto the five stories of the Japanese pagoda. Branches 1–2 are developed in Volume II (*Geometric Origin of the Cosmos and Particles*); branches 3–5 in Volume III (*Inevitable Emergence of Life and Mind*); branches 5–10 in Volume IV (*GUT-10: Grand Unification of 10 Sciences*). The claim is not reductionism but *interface discipline*: each branch must declare a SIP (state manifold, symmetries, ports, and ledger), evolve it by an admissible GDOS kernel, and export operational observables with falsifiers. Table 2.7 provides a compact "module contract" template for later branch-specific papers.

Table 2.7: **GUT-10 branch interface contract.** For each branch, required SIP specification, primary observable channels, and representative kill criteria implied by POS-MF. This table functions as an interface specification: later branch-specific papers must fill in the SIP and falsifiers rather than merely proposing narratives.

Branch	**State variables (examples)**	**Primary observables**	**Kill criteria (examples)**
GUT-Particle	fields/S-matrix data; couplings $g_i(\mu)$; anomaly polynomial	precision couplings; resonance lines; rare processes	persistent violation of predicted scheme-stable relations; missing required signatures
GUT-Cosmos	$\Omega_i(t)$, $H(t)$, inflationary order parameters; compact-object states; lensing maps	CMB spectra; LSS; BAO; distance ladder; timing; rotation curves	incompatibility with closure and declared targets; systematic mismatch of predicted scaling
GUT-Chemistry	concentrations; reaction-network states; fluxes	rates, yields, impedance spectra, fluctuation statistics	failure of predicted universality class exponents/knees
GUT-Life	population/genetic network states; metabolic fluxes	growth/fitness curves; multiscale physiological time series	absence of predicted constrained-attractor structure under controlled ports
GUT-Mind	belief-state manifolds; neural population states	PSD knees, cross-modal correlations, behavior metrics	failure of registered operational readouts to match predicted invariants
GUT-Health	multi-layer physiological networks; biomarker state vectors	multimodal biomarkers, network resilience measures	inability to operationalize and falsify interventions as GDOS control
GUT-Society	network states; flows of goods/info; decision metrics	macro indicators, scaling laws, response functions	policies that cannot be expressed with operational observables/kill rules
GUT-Environment	ecosystem state vectors; biogeochemical fluxes; climate variables	species diversity indices; nutrient cycles; climate time series	failure of predicted multi-scale feedback structure under controlled forcing
GUT-AI	network state + training variables; information throughput budgets	learning curves; generalization metrics; efficiency/entropy trade-offs	need for extra ad hoc parameters beyond MSMF/OGN; violation of predicted scaling

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Branch	State variables (examples)	Primary observables	Kill criteria (examples)
GUT-Philosophy	meta-level claims constrained by operational semantics	consistency with OGGEP + falsifiability commitments	claims that cannot be tethered to an operational audit trail

2.5 Discussion

This book is intentionally constitutional rather than model-specific: it specifies a rulebook for admissibility before it specifies a particular Lagrangian or particle spectrum. A constitution can—and must—be compared to existing frameworks by asking four concrete questions: *(i)* which ambiguities are removed (especially around observation and openness), *(ii)* which freedoms are forbidden (especially hidden continuous knobs), *(iii)* which consistency conditions are elevated from "technicalities" to first-class filters, and *(iv)* which new falsifiers are demanded at the protocol level. We organize this discussion accordingly.

2.5.1 Significance and uniqueness of the GUT constitution

(1) A constitution as a scientific object. In much of modern practice, "foundations" are discussed in prose while calculations are performed in mathematics. GUT treats this split as itself a failure mode: a scientific framework is incomplete if its admissibility rules cannot be stated as objects that can be checked. Formally, the constitution is the predicate : a candidate theory is either admitted or rejected by a finite checklist. This posture is deliberately Euclidean/Newtonian in spirit (Euclid, c. 300 BCE; Newton, 1687): the aim is that once the clauses are accepted, large theorem spaces become unavoidable rather than optional (Noether, 1918).

(2) A reversal of emphasis: geometrize quantum/gauge structure. Most "quantum gravity" programs begin from classical geometry and then attempt to quantize it; many unification programs begin from gauge structure and then accommodate gravity. AX-2 (GGEP) imposes a different primitive: the primary field is a composite geometry⊕gauge connection. Connections and holonomies already unify "phase transport" (quantum) with "parallel transport" (geometry), so the constitution demands that this unity be treated as fundamental rather than emergent. This stance is compatible with the historical centrality of gauge connections (Nakahara, 2003; Yang and Mills, 1954) and with holographic hints ('t Hooft, 1993; Maldacena, 1998; Susskind, 1995; Witten, 1998), but it is strengthened here into an admissibility rule: theories that require independent non-geometric primitives for forces are constitutionally suspect.

(3) Measurement is defined (operationally), not interpreted. A century after the birth of quantum mechanics, "measurement" is still often treated as extra structure layered on top of unitary dynamics (Bell, 1964; von Neumann, 1932; Zurek, 2003). OGGEP (AX-3) enforces a definition: a measurement is an output of a lawful bulk→boundary registration map, i.e. a specific instrument/estimator/protocol triple. This collapses much interpretational ambiguity

into a technical (and auditable) question: *which CPTP instrument represents the experiment? (Zurek, 2003)* POS-MF then closes the loop by requiring a pre-declared falsifier. In effect, the constitution converts foundational debates into workflow: write the dictionary, state the protocol, declare the kill criterion, and only then claim a "prediction."

(4) Open systems are constitutional, not effective. Most sciences beyond fundamental particle theory live in open regimes: systems exchange energy, matter, and information with environments. Even in fundamental physics, laboratory measurements create irreversible records and therefore implicitly invoke open-system dynamics. POS-CP and POS-MF formalize this fact: admissible evolution is not an optional "environmental correction" but must be positivity-preserving and protocol-defined from the outset. The payoff is unification by method: the same admissibility grammar that keeps quantum channels physical is the grammar that keeps chemical concentrations nonnegative and keeps biological models from smuggling in acausal inference.

(5) Predictivity by forbidding hidden knobs. The Standard Model is rigid once its structure is fixed, yet it does not predict the numerical values of its parameters (Particle Data Group, 2024). Beyond-the-SM model building often introduces additional tunable sectors; quantum-gravity and unification programs frequently admit large families of consistent vacua or moduli (Polchinski, 2004). AX-4 (MSMF) and POS-OGN respond by treating hidden continuous tunability as non-admissible unless forced by symmetry, consistency, or operational meaning. Methodologically this is an "anti-landscape" stance: a correct theory should look like a compression, not like a catalogue of adjustable stories. Technically, OGN insists that normalization/basis drift be fixed to a minimal discrete ledger so that would-be "choices" cannot masquerade as physical parameters.

(6) Unification of sciences by unification of admissibility. The ten-branch program is not a claim that all domains reduce to particle physics. It is a claim that *admissibility* should be shared: gauge redundancy, positivity of states, scale covariance, and falsifiability contracts are not optional in some disciplines and ignored in others. This is why GUT is partly a proposal about the method of science: it aims to standardize the audit trail

$$\text{SIP specification} \Rightarrow \text{admissible dynamics} \Rightarrow \text{registered observables} + \text{falsifier}. \tag{2.131}$$

If this pipeline can be made productive, "unification" becomes operational rather than metaphorical.

2.5.2 Comparison with the Standard Model and leading unification/quantum-gravity programs

A constitution should not be compared by rhetorical scope but by what it treats as primitive and what it forces to be derived. The Standard Model is a remarkably successful effective field theory: it encodes gauge invariance, locality, and renormalizability in a compact structure and survives stringent precision tests. However, it inherits two constitutional gaps relevant to this book. First, measurement is largely external to the model: experimental claims are made by coupling

the SM to detector models and statistical procedures that are not part of the SM constitution. Second, predictivity is limited by input parameters that are not derived from within the SM itself (Particle Data Group, 2024).

Quantum-gravity/unification programs (e.g. string theory, loop approaches, and related UV-completion programs) address gravity and unification more directly, but often at the price of a large theory space (many vacua, moduli, compactifications) (Maldacena, 1998) and, again, with measurement semantics inherited rather than constitutionally defined. GUT does not claim to replace these programs by a single model in this book; it claims to supply a *constitutional filter* that clarifies what must be fixed for a claim to count as a prediction.

Table 2.8 summarizes the contrast at the level of primitives, measurement semantics, normalization discipline, and falsifiability.

Table 2.8: **Comparison with modern frameworks.** Constitution-level contrast among the Standard Model, representative quantum-gravity/unification programs, and GUT, emphasizing measurement semantics and normalization discipline. The aim is to isolate which choices are treated as primitives versus which are forced to be derived and falsifiable.

Criterion	**Standard Model (SM)**	**Quantum gravity/unification (string/QG)**	GUT **constitution (this book)**
Primary object	4D renormalizable QFT; fields on spacetime (Peskin and Schroeder, 1995; Weinberg, 1995)	UV completion with gravity; extra dimensions; extended objects (Polchinski, 1995)	Composite connection (geometry⊕gauge) + operational map (bulk→boundary).
Treatment of measurement	External postulates / interpretation; effective open-system tools (von Neumann, 1932; Zurek, 2003)	Typically inherited from QM; measurement not a structural primitive	Measurement is constitutional: OGGEP + MF contract required.
Gravity	Not included; added via EFT on curved backgrounds	Included fundamentally (graviton / strings)	Included by GGEP locking; geometrized gauge–gravity composite.
Free parameters / moduli	Parameters fit to data (Particle Data Group, 2024)	Often many vacua/moduli; landscape issues (Polchinski, 2004)	Axiom/postulate filters (MSMF, OGN, AF) aim to remove hidden dials.
Scale / RG	EFT and RG central (Wilson, 1971)	RG and holography appear in AdS/CFT	Scale is geometric coordinate; operational projection must commute with RG (POS-RG).

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Criterion	**SM**	**String/QG**	GUT **constitution**
Anomalies	Require cancellation; constrain field content	Cancelled via GS and consistency (Green and Schwarz, 1984)	Anomaly freedom by inflow is a postulate (POS-AF) and a design filter.
Primary falsifiers	Precision SM tests; collider + flavor + EDMs	Indirect tests; difficult due to scale/landscape	MF requires explicit kill criteria for each theorem; emphasizes protocol-level falsifiability.

Two clarifying remarks are important. First, GUT is compatible with the effective-field-theory practice of the SM and with holographic ideas in quantum gravity, but it insists that these ideas be made constitutional: RG compatibility is not just a calculation tool (POS-RG), and measurement is not just an interpretational story (AX-3 and POS-MF). Second, the MSMF/OGN clauses are not intended to deny the empirical fact that parameters exist; they are intended to force an accounting: *which numbers are derived, which are conventions, which are inputs, and which are falsifiable outputs?* A theory that cannot answer that accounting question is constitutionally underdetermined.

2.5.3 Comparison with historical axiom systems

Historical axiom systems were successful not merely because they were "elegant" but because they clarified what was to be taken as primitive and what was to be derived. Euclid's postulates turned geometric truths into inevitabilities; Newton's laws turned celestial and terrestrial motion into one calculable subject; Einstein's principles turned time and simultaneity into operational definitions (Einstein, 1905, 1915). Quantum theory achieved enormous compression, but it famously left a seam: unitary dynamics is clean while the operational meaning of measurement is partly external (von Neumann, 1932).

GUT aims to be in the same family of moves while adding clauses that earlier constitutions did not need: open-system admissibility (positivity-preserving semigroups), explicit operational dictionaries (bulk→boundary compilation), and explicit falsifiers. Table 2.9 summarizes this comparison.

Table 2.9: **Comparison with classic axiom systems.** Contrast between GUT and landmark constitutions (Euclid, Newton, Relativity, Quantum Mechanics, Popper), highlighting the explicit operational dictionary and open-system admissibility. The point is methodological: GUT treats measurement semantics and open dynamics as constitutional clauses in the same spirit that earlier constitutions treated geometry, force, or spacetime.

Axiom system	**Domain**	**Constitutional move**	**Relation to** GUT
Euclid (c. 300 BCE)	Geometry	Axiomatize geometric truth from minimal postulates	GUT borrows the "constitution first" ethos: admissibility is decided before calculation.
Newton (1687)	Mechanics / gravity	Universal laws of motion + gravitation; predictivity	GUT seeks similar universality, but with open-system and operational measurement built in.
Einstein (1905), Einstein (1915)	Relativity	Invariance principles reshape spacetime and dynamics	GUT extends invariance to the generator of open dynamics and to the measurement map.
Quantum mechanics (canonical) (Born, 1926; von Neumann, 1932)	Microscopic physics	Unitary dynamics + probabilistic measurement rule	GUT keeps quantum structure but replaces ambiguous measurement with OGGEP and MF contracts.
Falsifiability criterion (Popper, 1959)	Philosophy of science	Demarcation by refutability	GUT formalizes this as POS-MF: every theorem is a contract with a kill criterion.

This table also clarifies why POS-MF is placed *inside* the constitution rather than treated as external philosophy. Popper's falsifiability criterion (Popper, 1959) is often cited as a demarcation slogan; GUT treats it as an engineering requirement: a theorem that cannot be paired with a protocol and a kill criterion is simply incomplete.

2.5.4 A direction for 21st-century science

If the constitution succeeds, it suggests a concrete change in scientific workflow, especially in data-rich disciplines where model flexibility is high.

(i) From model catalogues to admissibility proofs. The postulate stack turns many "soft" questions into hard ones. Before proposing a new sector or mechanism, one must show that it passes the runtime checklist: GI (symmetry grammar), UEP (variational engine), AF (quantum consistency), CP (physical state evolution), RG (scale coherence), OGN (normalization

rigidity), and MF (protocol-level falsifiability). This makes "consistency first" practical rather than ceremonial.

(ii) From closed ideals to open operational laws. Because registered data are produced by open interactions, admissible open dynamics is not optional (Gorini et al., 1976; Lindblad, 1976). This shifts emphasis from unitary groups to CPTP semigroups (Kraus, 1983; Stinespring, 1955), from idealized isolation to explicit ports, and from abstract observables to estimators with error models. In turn, foundational debates about measurement become model-building constraints: the dictionary must be written down.

(iii) From parameter fitting to parameter elimination (when possible). MSMF and OGN do not forbid fitting where fitting is unavoidable; they forbid *unjustified* freedom. Their purpose is to push explanations toward symmetry and discrete data. When a continuous knob is truly forced (e.g. a coupling constant that is not fixed by topology or anomaly constraints), the constitution demands that this be stated as such and that the knob be promoted to a falsifiable prediction target through RG flow, scaling relations, or additional constraints.

(iv) From domain narratives to a shared audit trail. A key claim of GUT is that every domain can be asked the same three questions: What is the SIP? What is the admissible kernel? What are the registered observables and kill criteria? This is how unification is intended to avoid becoming metaphor.

(v) Ultimate aim. The ultimate aim is not just a "bigger theory" but a disciplined standard for what we mean by explanation across sciences: a smallest admissible description that reproduces the largest set of registered invariants, under an explicit prohibition against acausal kernels, unphysical state evolution, scale artifacts, and tuning that cannot be justified by symmetry or operational meaning. The companion chapters (Arisaka, 2026b–d) serve as stress tests: they must either produce sharp, killable numerical targets or reveal exactly which clause of the constitution must be revised.

2.6 Summary

This paper introduced the GUT constitution: four axioms (AX-1–AX-4) and seven postulates (POS-1–POS-7) intended to function as an admissibility grammar for theories across ten scientific branches. The core claim is constitutional: several persistent bottlenecks of modern science arise from missing or implicit rulebook clauses—especially around measurement, open systems, and unconstrained modeling freedom. The present work therefore aims to provide a stable public citation anchor (UCLA-Arisaka/GG-2026-1) for subsequent technical papers and for textbook-scale expansion.

What the axioms do. AX-1 (LC) is the operational expression of "cause precedes effect": registered boundary updates must be local and retarded, and effective kernels are inadmissible if they silently use information that could not have arrived through allowed channels. This is

familiar as microcausality in relativistic QFT, but it becomes nontrivial as a constitutional clause precisely because much of modern science is dominated by open systems and coarse-grained records.

AX-2 (GGEP) asserts a deliberate reversal of emphasis: rather than attempting to quantize classical geometry and then append internal gauge structure, the primitive object is declared to be a composite geometry⊕gauge connection. In this view, quantum/gauge structure is already geometric when expressed in the correct bundle language (holonomy, parallel transport, and consistency constraints). The novelty in this book is not a particular Lagrangian but the constitutional claim that unification must be written as geometry-first at the primitive level.

AX-3 (OGGEP) addresses a century-old ambiguity by making measurement part of the definition of a theory. An observation is not a metaphysical "collapse" statement; it is the output of a lawful bulk→boundary registration map that produces operational observables (POVMs, instruments, estimators) together with declared protocols and error models. In this constitution, it is not enough to write down dynamics: a theory is incomplete if it does not specify how its claims become registered records.

AX-4 (MSMF) enforces predictivity as a constitutional demand. Nature selects maximal symmetry with minimal continuous freedom; continuous knobs that are not compelled by deeper symmetry, consistency, or operational meaning are treated as inadmissible. This axiom is motivated by the long history of successful symmetry-based compression, and by the modern experience that when model spaces become too flexible (many parameters, many vacua, many conventions), the distinction between prediction and choice becomes ambiguous.

What the postulates do. The seven postulates turn the axioms into a theorem machine. POS-1 (GI) generalizes gauge invariance to a layered principle across scales: local gauge redundancy in closed microscopic systems, generator-level covariance for open mesoscopic dynamics (DGI), and ledger-level invariance in macroscopic accounting (GGI). POS-2 (UEP) asserts that a single variational "engine" underlies dynamics: least action in closed bulk limits and least-time/least-free-energy avatars in open operational limits. POS-3 (AF) elevates anomaly freedom to an admissibility filter: consistency obstructions are not optional subtleties but architecture constraints, including cancellability by inflow where required. POS-4 (CP) enforces physicality: states must remain positive and normalized under evolution (quantum: CPTP semigroups; classical: positivity-preserving Markov dynamics). POS-5 (RG) insists on scale coherence: coarse-graining must commute with the operational dictionary; predictions cannot depend on arbitrary resolution choices. POS-6 (OGN) fixes normalization discipline: among consistent conventions, nature selects minimal rational/integer ledgers, forbidding hidden continuous rescalings. Finally, POS-7 (MF) closes the scientific loop: every claim must map to a registered observable and a pre-declared falsifier (kill criterion).

Two parental engines and the ten-branch export. With the constitution in place, GUT is designed to compile into two complementary parental engines. The first is GG-6 (Geometry–Gauge Locking in six dimensions), a bulk stage intended to realize GGEP with anomaly inflow, RG geometry, and normalization discipline. The second is GDOS (Geometric Dynamics of Open Systems), a boundary runtime in which open-system admissibility and observation are first-class.

These engines are developed in the companion preprints Arisaka (2026b,c); the ten-branch export map is developed in Arisaka (2026d). The ten branches are not claimed to be identical in microscopic degrees of freedom. Rather, they are required to share a single audit trail: *declare a SIP* (state variables, symmetries, ports, ledger), *evolve by an admissible kernel*, and *export observables with falsifiers*. This is how unification is intended to avoid becoming metaphor: cross-domain claims are constrained to be operationally testable and scale-coherent.

A philosophical closing (and a falsifiability opening). Historically, scientific revolutions are as much about rules of explanation as about new equations. Euclid did not merely list geometric facts; he made them inevitable by a constitution. Newton did not merely fit planetary data; he provided a rulebook that made celestial and terrestrial dynamics one subject. Einstein did not merely change formulas; he redefined the operational meaning of time and simultaneity. In the twentieth century, quantum theory and relativity achieved enormous explanatory compression, but they also inherited open constitutional seams: measurement remained partially informal, and the status of "what is input vs. what is derived" became increasingly blurred as theories grew in complexity.

This book proposes a specific answer to that blur: treat the constitution as an explicit object. A scientific claim is not only an equation; it is a contract that (i) specifies its operational observables, (ii) preserves physical state space under admissible dynamics, and (iii) forbids hidden knobs that would convert prediction into choice. In a data-rich century, the danger is not lack of models but excess of flexibility. A constitution is a discipline for saying "no"—no to acausal kernels, no to unphysical evolution, no to scale artifacts, and no to tuning that cannot be justified by symmetry or operational meaning. Under this discipline, explanatory progress becomes a compression game played under audit: the goal is the smallest admissible description that reproduces the largest set of registered invariants.

This conclusion is therefore also a falsifiability opening. The constitution will be judged by whether its downstream engines can generate sharp, killable targets without reintroducing hidden dials: quantitative coupling/normalization relations, anomaly-locked integer ledgers, cosmological budget constraints, spectral fingerprints, and protocol-level measurement contracts that survive contact with data. If the program succeeds, it would change the standard of what we mean by explanation across the sciences: not merely more detailed models, but a shared grammar for admissibility and auditability. If it fails, the failure modes are equally informative: they will identify which constitutional clauses must be revised, and which parts of the open-system and measurement semantics cannot be made universal. In either case, making the constitution explicit is a concrete step toward turning foundational debate into a tractable scientific workflow.

Chapter 3

GG-6: Geometry–Gauge Locking

The Warped Six-Dimensional Bulk

"Geometry is not true, it is advantageous."

— Henri Poincaré, *Science and Hypothesis*, 1902

3.1 Introduction

3.1.1 Unresolved problems and the constitutional gap

The twentieth century bequeathed two profoundly successful revolutions in physics: quantum mechanics (Born, 1926; Heisenberg, 1925; Planck, 1900; Schrödinger, 1926) and general relativity (Einstein, 1905, 1915). From their synthesis emerged quantum field theory (QFT), the Standard Model (SM) of particle physics (Yang and Mills, 1954; 't Hooft and Veltman, 1972), and precision cosmology. These represent arguably the greatest intellectual edifice ever constructed. And yet, when one examines the foundations of this edifice, at least *six* major structural lacunae remain unresolved.

Problem 1 — The measurement problem. Quantum mechanics prescribes a unitary, deterministic evolution of the state vector $|\psi\rangle$ via the Schrödinger equation. Yet any actual experiment demands the extraction of a classical outcome, accompanied by wavefunction collapse (von Neumann, 1932). No purely closed-system formulation of quantum mechanics provides a derivation of when and how collapse happens, or what constitutes a "measurement" versus an ordinary interaction.

Problem 2 — Gravity versus gauge: ontological heterogeneity. In the modern theoretical framework, gravity is geometric—encoded in the Riemann curvature of spacetime—while the gauge interactions (strong, weak, electromagnetic) are internal symmetries, i.e., fibers attached to spacetime *after the fact* (Kaluza, 1921; Klein, 1926; Yang and Mills, 1954). There is no deep constitutional reason, within the Standard Model framework, why gauge structure and spacetime geometry should be regarded as different ontological categories.

Problem 3 — Unification versus predictivity. Grand Unified Theories attempt to merge the strong, weak, and electromagnetic interactions into a single simple gauge group at high energy (Georgi and Glashow, 1974). While conceptually beautiful, minimal SU(5) predicts proton decay at a rate already ruled out by experiment. Supersymmetric GUTs (Haber and Kane, 1985; Nilles, 1984; Wess and Zumino, 1974) and superstring theories (Polchinski, 1998a,b) attempt to repair this, but at the cost of introducing hundreds of new fields, a vast landscape of vacua, and many new continuous parameters. Predictivity is thus traded for theoretical freedom.

Problem 4 — Proton stability: accidental vs. exact. The proton is extraordinarily stable: its lifetime exceeds 10^{34} years. In the Standard Model, baryon number B is an *accidental* symmetry of the renormalizable Lagrangian—it is not gauged, and it is violated at the non-perturbative level by $B + L$-violating sphaleron processes. The absence of observed proton decay is a strong constraint but lacks a *constitutional* explanation.

Problem 5 — Three generations: origin unknown. The Standard Model contains exactly three generations of quarks and leptons. No dynamical or constitutional principle within the SM itself explains why there are precisely three families, not two, four, or any other number.

Problem 6 — Open systems and the arrow of time. Physical science in practice deals overwhelmingly with *open* systems: systems that exchange energy, entropy, and information with their environments. Yet the foundational layer of theoretical physics—QFT and general relativity—is written for *closed*, isolated systems. Irreversibility, coarse-graining, and the emergence of classical behavior from quantum substrates remain imperfectly understood at the constitutional level.

The diagnosis of the present program is that these six problems are not merely technical difficulties to be patched by adding new particles or parameters. They are symptoms of a **missing constitutional layer**: a set of foundational admissibility rules that must be imposed *before* any specific model is selected. The GUT-10 constitution (Arisaka, 2026a) provides exactly this layer, and this book shows how its axioms and postulates conspire to force a unique six-dimensional geometric-gauge construction called GG-6.

3.1.2 The GGEP and OGGEP approach

The core conceptual innovation of the GUT-10 / GG-6 program is to **geometrize gauge forces** rather than quantize gravity. These are two philosophically distinct strategies:

- **Quantize gravity** (mainstream approach): Start with classical general relativity and the Standard Model as separate frameworks, then seek a quantum theory of gravity at the Planck scale $E_{\mathrm{Pl}} \sim 10^{19}$ GeV.
- **Geometrize gauge** (GG-6 approach): Declare that the *primitive field* of the theory is a **composite geometry–gauge connection** $\mathcal{A} = \omega[g] \oplus A^{\mathrm{gauge}}$ (cf. **AX-2**, GGEP) in which gravity and gauge structure are two projections of one object. The key endpoint is not the Planck scale but the **GUT scale** $E_{\mathrm{GUT}} \sim 2 \times 10^{16}$ GeV, where all gauge forces unify.

The two approaches are complementary in spirit but very different in execution. The GG-6 approach does not require supersymmetry, strings, or extra moduli: it requires only that the six-dimensional bulk be anomaly-consistent (via the Green–Schwarz mechanism, **POS-3**) and maximally symmetric with minimal freedom (**AX-4**).

The companion principle **AX-3** (OGGEP) then mandates that any admissible theory provide a *lawful bulk-to-boundary operational map*, so that four-dimensional quantum mechanics and measurement emerge as the boundary shadow of the six-dimensional bulk. This derivation is the subject of Chapter 4.

3.1.3 Maximum Symmetry–Minimum Freedom (MSMF) and OGN

A third key pillar is **AX-4**: among all theories satisfying AX-1–AX-3 and the seven postulates, nature selects those with maximal admissible symmetry and minimal non-forced continuous freedom. Together with **POS-6** (Occam Gauge Normalization), this lexicographic principle converts what would normally be a landscape of free parameters into a discrete arithmetic ledger.

The result is the **OGN-5 integer package**:

$$(M, N, L;\ J, K) = (1,\ 3,\ 5;\ 7,\ 35).$$

Beyond Newton's constant G_N, *no independent continuous dimensionless parameter* survives in the minimal GG-6 bulk.

3.1.4 Scope and organization of the paper

This book is organized as follows.

1. **Introduction** (present section): motivates the constitutional approach and identifies the six unresolved problems.
2. **Recap of the GUT-10 axiom system**: states the four axioms and seven postulates in fixed mathematical format and derives their immediate pre-model consequences.
3. GG-6 **from** GGEP **(AX-2)**: derives the minimal warped six-dimensional bulk from the admissibility clauses, including the six-force content, the BI-6 identity, the full anomaly factorization proof, and the OGN-5 ledger.
4. **The complete 6D QFT at** E_{GUT}: constructs the classical action, introduces BRST symmetry and ghost sector, derives the descent equations and Green–Schwarz cancellation, proves the homogeneous Slavnov–Taylor identity, and provides an explicit comparison with superstring and SUSY constructions.
5. **Discussion**: significance, uniqueness, falsifiability, and comparison with SM/MSSM/strings.
6. **Conclusions**.
7. **Appendices**: mathematical foundations, detailed proofs, summary tables, notation.

The central methodological commitment is that **no new axiom is introduced after Section 3L.5**. Every later construction is derived from the same eleven constitutional clauses.

3.2 Recap of the GUT-10 axiom system

3.2.1 A constitution rather than an ansatz

The latest GUT-10 axiom paper formulates the constitution succinctly: four axioms fix the primitive concepts and seven postulates act as ordered admissibility filters (Arisaka, 2026a). It is useful to make this statement mathematically explicit.

Definition 3.2.1 (Candidate theory)**.** A candidate theory $\mathcal{T}$ in this book is a tuple

$$\mathcal{T} = (\mathcal{M}, \mathcal{P}, \mathcal{A}, \mathcal{D}, \mathcal{S}, \mathcal{O}), \tag{3.1}$$

where $\mathcal{M}$ is the underlying manifold or stratified bulk–boundary stage, $\mathcal{P}$ a bundle/principal-bundle structure over $\mathcal{M}$, $\mathcal{A}$ the composite connection data, $\mathcal{D}$ an operational dictionary, $\mathcal{S}$ the dynamical law or action principle, and $\mathcal{O}$ the family of registered observables and falsifiers.

Definition 3.2.2 (Admissibility)**.** The theory $\mathcal{T}$ is admissible iff

$$\boxed{\text{Admissible}(\mathcal{T}) := (\mathcal{T} \models \text{AX}) \wedge (\mathcal{T} \models \text{POS})}. \tag{3.2}$$

The symbol $\models$ means that the full content of the corresponding constitutional clause is satisfied, not merely that a slogan associated with it can be heuristically identified.

Equation (4.2) is more powerful than it first appears. It implies that physical derivation proceeds in two stages. First one solves the constitutional filtering problem: which candidate theories remain admissible? Only then does one compute phenomenology within that drastically reduced space. The structure developed in this book is that GG-6 and GDOS are not extra assumptions but theorem-level engines obtained by instantiating GGEP and OGGEP under the seven postulates.

3.2.2 The four axioms

We now restate the four axioms in a fixed format: constitutional statement, mathematical kernel, forced consequences, and relevance for the later derivation chain.

Axiom 3.2.3 (AX-1: LC — Locality and Causality)**.** Admissible evolution is local and causal in the operational sense: registered boundary updates are retarded; no admissible effective kernel is allowed to depend on data outside the causal past of the registration event.

The mathematical kernel of AX-1 is the retardedness condition on any effective response kernel K: if $x \notin J^+(y)$, then

$$K(x, y) = 0. \tag{3.3}$$

For relativistic fields this is the familiar microcausality requirement,

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)]_{\mp} = 0 \qquad \text{for spacelike separation,} \tag{3.4}$$

while for open-system evolution it becomes the forward-semigroup requirement that the effective map from s to t exists only for $t \geq s$. AX-1 therefore forbids both acausal propagators and semigroup generators that require hidden advanced data. This axiom is not derived later in the paper; it is the operational legal floor beneath all later constructions.

Axiom 3.2.4 (AX-2: GGEP — Geometry–Gauge Equivalence Principle)**.** The primitive field is a composite geometry⊕gauge connection. Gravity and gauge structure are not separate fundamental sectors; both are encoded in a single connection whose curvature and holonomy data are primary.

Formally, one introduces a principal-bundle or composite-bundle connection

$$\mathcal{A} = \omega[g] \oplus A^{\text{gauge}}, \qquad \mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}. \tag{3.5}$$

Free motion then corresponds to parallel transport in $\mathcal{A}$; forces arise from the curvature $\mathcal{F}$; and charge/phase/topological information is encoded in holonomy. The immediate consequence is that any later dynamical law must be expressible in terms of geometric invariants of $\mathcal{A}$, not in terms of two ontologically independent sectors glued together ad hoc. In Section 2 of the full chapter this axiom will be specialized to the warped six-dimensional composite-connection realization that becomes GG-6.

Axiom 3.2.5 (AX-3: OGGEP — Operational/Open Geometry–Gauge Equivalence Principle)**.** A bulk theory is incomplete unless it supplies a holographic bulk→boundary registration map that produces operational observables: boundary state spaces, admissible evolution maps/channels, and measurement instruments.

Let Bulk denote the inaccessible or partially accessible bulk data. Then AX-3 requires the existence of a dictionary

$$\mathcal{D}: \text{ Bulk} \to (\mathcal{H}_\partial, \rho_\partial, \Phi_{t,s}, \{\mathcal{I}_\alpha\}), \tag{3.6}$$

where ρ_∂ is a boundary state, $\Phi_{t,s}$ is an admissible evolution map, and $\mathcal{I}_\alpha$ are instruments/POVM-valued operations implementing registered observations. Operational meaning is therefore constitutional rather than interpretational. In the later derivation, this axiom is what licenses the passage from a six-dimensional closed or quasi-closed bulk law to four-dimensional quantum mechanics and open-system dynamics.

Axiom 3.2.6 (AX-4: MSMF — Maximum Symmetry, Minimum Freedom)**.** Among theories satisfying AX-1–AX-3 and the seven postulates, nature selects those with maximal admissible symmetry and minimal non-forced continuous freedom.

To make AX-4 explicit, let $S(\mathcal{T})$ be a symmetry score and $F(\mathcal{T})$ a freedom score. The exact definitions can vary by context, but constitutionally one imposes a lexicographic ordering,

$$\mathcal{T}_\star = \arg\operatorname*{lexmax}_{\mathcal{T}\in\mathfrak{T}_{\text{adm}}} \big(S(\mathcal{T}), -F(\mathcal{T})\big), \tag{3.7}$$

with $\mathfrak{T}_{\text{adm}}$ the admissible theory space. Continuous parameters survive only when compelled by the constitution itself: for example by Newton's constant, topological classes, flux integrality, anomaly cancellation, or operational necessity. AX-4 is the axiom that turns a merely possible higher-dimensional model into a potentially near-unique one.

Table 3.1: The four axioms in fixed mathematical format.

AX	**Name**	**Mathematical kernel**	**What it forces**
AX-1	LC	Retarded kernels; microcausality; forward semigroups	No acausal effective laws; no advanced hidden inputs to registered observables.
AX-2	GGEP	Composite connection $\mathcal{A} = \omega \oplus A$, curvature $\mathcal{F}$	Geometry and gauge must descend from one primitive object.

AX	Name	Mathematical kernel	What it forces
AX-3	OGGEP	Operational dictionary $\mathcal{D}$ producing states, channels, instruments	Measurement becomes an internal part of the theory, not an external postulate.
AX-4	MSMF	Lexicographic maximization of symmetry, minimization of freedom	Continuous moduli and normalization drift are heavily constrained.

3.2.3 The seven postulates

The postulates refine admissibility. They are ordered not because later ones are less important, but because each acts as an additional filter on the theory space already shaped by the previous clauses.

Postulate 3.2.7 (POS-1: GI — Gauge Invariance)**.** Gauge invariance is layered across scales and descriptions: local gauge invariance (LGI), dynamical/open gauge invariance (DGI), and global/gauge-bookkeeping invariance (GGI).

At the action level, GI requires invariance under local gauge transformations. At the generator/channel level, it requires covariance of the open-system generator under the gauge action. At the global bookkeeping level, it requires that macroscopic ledgers and registered invariants not depend on arbitrary basis choices. Symbolically,

$$\mathcal{L}\big(U\rho U^{-1}\big) = U\mathcal{L}(\rho)U^{-1}, \tag{3.8}$$

for the relevant gauge action U. This postulate is what later allows exact Ward identities and Slavnov–Taylor identities to survive the bulk-to-boundary projection.

Postulate 3.2.8 (POS-2: UEP — Unified Extremal Principle)**.** A single variational engine governs dynamics. Closed-system least action in the bulk and open-system least-time/free-energy principles at the boundary must be different projections of the same underlying extremal structure.

Constitutionally, POS-2 forbids a theory that uses unrelated extremal laws for different descriptive layers. There must exist an action-like object $\mathfrak{S}$ and/or a rate functional $\mathfrak{F}$ such that the bulk Euler–Lagrange equations and the boundary variational tendencies can be derived from compatible extremization conditions. Schematically,

$$\delta\mathfrak{S}_{\text{Bulk}} = 0 \qquad \Longrightarrow \qquad \delta\mathfrak{F}_{\text{Bdy}} = 0 \quad \text{under } \mathcal{D}. \tag{3.9}$$

This is the constitutional root of the later GDOS claim that PLA, PLT, MEPP, and FEP are not independent principles but boundary projections of one bulk variational engine.

Postulate 3.2.9 (POS-3: AF — Anomaly-Free admissibility by inflow)**.** Admissible theories must cancel gauge, gravitational, and mixed anomalies, including by inflow mechanisms when necessary.

This is one of the sharpest filters in the constitution. It means that a would-be unified theory is inadmissible if its quantum anomalies do not cancel. The general descent logic is standard: if the relevant anomaly polynomial I_{2n+2} factorizes in a way that allows a Green–Schwarz-type inflow term, then the theory may remain admissible; otherwise it is excluded (Green and Schwarz, 1984; Becchi et al., 1976; Tyutin, 1975). In the later GG-6 derivation, this postulate is the gateway to the Green–Schwarz-modified Bianchi identity and the rigidity of the allowed gauge content.

Postulate 3.2.10 (POS-4: CP — Closure and Positivity)**.** State evolution must remain physical. In the quantum case, admissible evolution maps must be completely positive and trace preserving; in the classical case, they must be positivity-preserving Markov semigroups.

For the quantum boundary theory, POS-4 implies that any generator appearing after coarse-graining must take GKLS-compatible form (Gorini et al., 1976; Lindblad, 1976). That is, if

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}(\rho), \tag{3.10}$$

then admissibility requires

$$\mathcal{L}(\rho) = -\mathrm{i}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \tfrac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{3.11}$$

possibly after the appropriate constitutional identification of bulk and boundary degrees of freedom. This postulate is what turns OGGEP into a mathematically meaningful theory of measurement rather than a metaphor.

Postulate 3.2.11 (POS-5: RG — RG compatibility and scale covariance)**.** Coarse-graining must commute with the operational dictionary; predictions must be stable under resolution changes.

The postulate can be expressed diagrammatically as a commutative requirement,

$$\mathcal{D} \circ \mathcal{G}_{\mathrm{Bulk}} = \mathcal{G}_{\mathrm{Bdy}} \circ \mathcal{D}, \tag{3.12}$$

where $\mathcal{G}$ denotes the relevant coarse-graining/RG map. Physically this means that one cannot first project to the boundary and then obtain a different class of predictions than one gets by coarse-graining in the bulk before projection. In the later GG-6 realization this is what justifies identifying the radial bulk coordinate with the logarithmic renormalization scale.

Postulate 3.2.12 (POS-6: OGN — Occam Gauge Normalization)**.** Among constitutionally consistent normalizations, nature selects minimal rational or integer normalizations. Arbitrary continuous rescalings and basis drifts are inadmissible.

The technical purpose of POS-6 is to stop hidden freedom from re-entering through conventions. Charges, gauge kinetic terms, and topological couplings must be frozen by rational/integer data whenever possible. In the later six-dimensional derivation, this postulate is what is expected to reduce the normalization sector to a discrete tuple such as . Together with AX-4, it is the principal anti-landscape clause of the constitution.

Postulate 3.2.13 (POS-7: MF — Measurement and Falsifiability)**.** Every scientific claim must compile to a registered observable and a pre-declared kill criterion.

This postulate inserts Popperian discipline inside the mathematics. A claim $\mathfrak{C}$ is admissible only if there exists both an instrument/protocol $\mathcal{I}_{\mathfrak{C}}$ and a falsifier set $\mathfrak{K}_{\mathfrak{C}}$ such that data landing in $\mathfrak{K}_{\mathfrak{C}}$ force rejection of the claim. In formulas,

$$\mathfrak{C} \text{ admissible} \Longrightarrow \exists\,(\mathcal{I}_{\mathfrak{C}}, \mathfrak{K}_{\mathfrak{C}}) \quad \text{with} \quad \mathsf{Data} \in \mathfrak{K}_{\mathfrak{C}} \Rightarrow \neg\mathfrak{C}. \tag{3.13}$$

Later sections will use POS-7 to state explicit falsifiers for the six-dimensional stage, for the baryonic sector, for the operational channel, and for the predicted low-energy remnants.

Table 3.2: The seven postulates and the constraint each imposes on admissible theory space.

POS	**Name**	**Mathematical cue**	**Primary exclusion power**
POS-1	GI	Local invariance, channel covariance, global ledger invariance	Excludes gauge-breaking actions, generators, and bookkeeping conventions.
POS-2	UEP	Compatible extremal functionals in bulk and boundary	Excludes unrelated variational laws across scales.
POS-3	AF	Anomaly polynomial cancellation / inflow admissibility	Excludes inconsistent chiral gauge sectors.
POS-4	CP	CPTP maps or positive Markov semigroups	Excludes unphysical reduced dynamics.
POS-5	RG	Coarse-graining commutes with operational map	Excludes scale-unstable or dictionary-dependent predictions.
POS-6	OGN	Minimal rational/integer normalization data	Excludes continuous basis drift and arbitrary rescaling freedom.
POS-7	MF	Instrument + kill criterion contract	Excludes unfalsifiable or non-registerable claims.

3.2.4 What the eleven clauses already imply before model building

Even before the details of a bulk theory are fixed, the constitution already narrows the admissible space strongly. This is worth stating as a proposition because it organizes the rest of the paper.

Proposition 3.2.14 (Pre-model structural consequences of the constitution)**.** *Let $\mathcal{T}$ be admissible in the sense of (4.2). Then the following are already forced before one commits to a specific compactification, action, or low-energy spectrum:*

(a) the primitive field content must be expressible through a composite connection and its curvature invariants;

(b) the theory must provide an operational map to a boundary/open-system description;

(c) any chiral gauge realization must pass an anomaly-cancellation filter, possibly using inflow;

(d) any reduced quantum dynamics must be CPTP *and hence* GKLS*-compatible in the Markovian limit;*

(e) the coarse-graining/RG structure must be representable geometrically or operationally in a way compatible with the dictionary; and

(f) normalization freedom is constrained to a discrete or rational remnant whenever constitutionally possible.

Proof sketch. Item (a) is AX-2. Item (b) is AX-3. Item (c) is POS-3. Item (d) is POS-4. Item (e) is POS-5. Item (f) follows from AX-4 together with POS-6. The proposition is therefore little more than the examining of the constitution, but it is useful because it shows why the later derivation need not start from a vast landscape of arbitrary higher-dimensional theories. Many of them are eliminated a priori. □

A crucial practical consequence is that the later derivation of GG-6 may proceed as a search for the minimal theorem-level realization of the above six properties, rather than as an unconstrained exploration of generic six-dimensional Lagrangians.

3.2.5 Constitution-to-engine pipeline for this book

This book isolates the first half of the original derivation chain: from the constitution to the bulk engine GG-6. The relevant theorem roadmap is therefore deliberately narrow.

Theorem 3.2.15 (Roadmap theorem: from AX-2 to GG-6)**.** *Assume AX-1–AX-4 and POS-1–POS-7. Then any theorem-level realization of AX-2 satisfying POS-3, POS-5, and POS-6 is expected to select a warped six-dimensional bulk with a composite connection, anomaly-filtered curvature locking, and a discrete normalization package. In the later sections this realization will be denoted* GG-6*.*

Proposition 3.2.16 (Bulk-side stopping point of this book)**.** *Within the split architecture adopted here, the logical endpoint of Chapter 3 is the construction of an anomaly-free BRST-consistent six-dimensional gauge-gravity-tensor quantum field theory at* E_{GUT}*. The bulk-to-boundary open-system reduction and the derivation of GDOS, four-dimensional quantum mechanics, general relativity, and the low-energy Standard Model are deferred to Chapter 4.*

This sharpening is useful for two reasons. First, it allows the GG-6 paper to be judged on its own mathematical burden: whether the six-dimensional stage, BI-6, OGN-5, and the complete 6D QFT really do follow from the constitutional clauses without adding supersymmetry or strings by hand. Second, it makes the companion GDOS paper cleaner, because that paper can begin directly from the operational/open consequences of AX-3 without re-deriving the full bulk architecture.

3.2.6 The role of historical antecedents

The constitution is new, but its ingredients interact with a large historical literature. That interaction should be made explicit, especially because one of the later discussion sections will compare the program with the Standard Model, supersymmetry, superstrings, and modern holography.

The relevant anchors are straightforward. Gauge invariance, Noether identities, and renormalization come from the classical and twentieth-century field-theory tradition (Gross & Wilczek, 1973; Noether, 1918; Politzer, 1973; Wilson, 1971; Yang and Mills, 1954; 't Hooft and Veltman, 1972). The Green–Schwarz mechanism gives the prototype of anomaly cancellation by inflow (Green and Schwarz, 1984). Warped extra-dimensional intuition is naturally compared to Randall–Sundrum backgrounds (Randall & Sundrum, 1999). Bulk–boundary generating-function logic draws on the AdS/CFT and holographic renormalization tradition (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998). Open-system positivity and measurement theory are tied to the GKLS and Kraus frameworks (Gorini et al., 1976; Kraus, 1983; Lindblad, 1976). The constitution does not erase this history; rather, it reorganizes it under a single admissibility predicate.

3.3 Derivation from GGEP to GG-6 with BI-6

3.3.1 Derivational strategy and statement of the problem

Section 3.1 reduced the admissible theory search to a narrow constitutional question: what is the *minimal* geometric realization of AX-2 consistent with POS-1, POS-3, POS-5, and POS-6? The purpose of the present section is to answer that question and to do so in the only manner allowed by the constitution: not by freely postulating a favorite higher-dimensional ansatz, but by deriving the stage, symmetry, and locking identity from the admissibility clauses themselves.

The logic is simple to state. AX-2 requires a primitive composite connection rather than a direct product of "spacetime geometry" and "internal gauge bundle." POS-3 requires that the resulting chiral gauge theory be anomaly-cancellable, possibly by inflow. POS-5 requires that the theory possess a constitutionally meaningful coarse-graining parameter that can be interpreted both geometrically and operationally. POS-6 forbids arbitrary continuous normalizations and therefore prefers a primitive rational/integer ledger whenever such a ledger exists. AX-4 then orders the admissible solutions by maximum symmetry and minimum freedom.

The result of this four-step filter is the GG-6 stage:

$$\boxed{M_6 = M_4 \times I_r \times S^1_\theta, \qquad G_{\text{Bulk}} = \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi} \times \text{Gravity}, \qquad \text{d}H_3 = X_4,} \tag{3.14}$$

with

$$\boxed{X_4 = \text{tr}(R \wedge R) - \text{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}.} \tag{3.15}$$

The remainder of this section derives (4A.1) and (4A.2) from the constitution in a sequence of lemmas.

3.3.2 Why the minimal admissible stage is six-dimensional

The first question is dimensional rather than group-theoretic. Why should the theorem-level realization of AX-2 be six-dimensional rather than five-, seven-, ten-, or eleven-dimensional? Within the present program the answer is constitutional and can be stated sharply.

Theorem 3.3.1 (Minimality of the six-dimensional stage)**.** *Assume AX-1–AX-4 and POS-1, POS-3, POS-5, POS-6. Then the minimal admissible bulk realizing AX-2 while preserving a four-dimensional registered boundary, a geometrized renormalization-scale direction, and a compact Abelian holonomy direction is six-dimensional:*

$$M_6 \cong M_4 \times I_r \times S^1_\theta. \tag{3.16}$$

Any candidate with fewer than six dimensions lacks one of the three required structures; any candidate with more than six dimensions introduces additional continuous directions not forced by the constitution.

Proof. The operational boundary of AX-3 is four-dimensional by construction: every registered event, detector record, and classical spatiotemporal protocol in the present program lives on a $3+1$-dimensional boundary manifold M_4. This fixes the first four coordinates. POS-5 then demands a constitutionally meaningful coarse-graining parameter. If one wishes to geometrize that parameter, the minimal choice is a single non-compact interval coordinate $r \in I_r$; any attempt to represent scale covariance without an extra direction leaves the coarse-graining structure external to the geometry and therefore violates the intent of AX-2. Finally, POS-1 and POS-6 require a minimal compact holonomy sector capable of supporting a primitive Abelian phase and Wilson-line data. The smallest such manifold is S^1, parameterized by a periodic coordinate $\theta \sim \theta + 2\pi$. Thus at least two extra coordinates are needed beyond M_4: one non-compact scale coordinate and one compact phase coordinate.

A five-dimensional bulk can provide only one of these two functions. A space of dimension greater than six may also realize them, but the additional directions are not forced by any constitutional clause and therefore violate AX-4/POS-6 unless independently justified. The minimal admissible dimension is therefore six. □

The theorem is deliberately stronger than a mere "Kaluza–Klein motivation." It says that the sixth dimension is not introduced for aesthetic embellishment; it is the minimal price of turning the renormalization semigroup and the compact Abelian phase into geometric data while preserving the observed four-dimensional boundary.

3.3.3 The warped metric and the logarithmic radial coordinate

Once the stage is fixed as $M_4 \times I_r \times S^1_\theta$, the next question is the metric. The constitution again restricts the answer sharply. Local Lorentz symmetry on the boundary requires the metric to split into a four-dimensional piece and an internal piece. POS-5 requires the scale direction r to act monotonically along coarse-graining. AX-4 prefers the maximally symmetric metric

consistent with these requirements. The natural ansatz is therefore the warped product

$$\mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}(x)\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2\,\mathrm{d}\theta^2. \tag{3.17}$$

Here $g_{\mu\nu}(x)$ is the four-dimensional boundary metric, $A(r)$ is a warp factor, and R_θ is the radius of the compact Abelian circle. The r-dependence is confined to the four-dimensional conformal prefactor because this is the minimal way to encode scale dependence without breaking boundary Lorentz covariance.

The logarithmic interpretation of r follows directly from the semigroup structure of coarse-graining. Let μ be the effective renormalization scale of a boundary description and let $\mathcal{G}_\lambda$ denote a scale transformation by factor $\lambda > 0$. Composition of coarse-grainings gives

$$\mathcal{G}_{\lambda_1} \circ \mathcal{G}_{\lambda_2} = \mathcal{G}_{\lambda_1\lambda_2}. \tag{3.18}$$

Any geometric coordinate implementing this semigroup additively must therefore satisfy

$$r(\lambda_1\lambda_2) = r(\lambda_1) + r(\lambda_2), \tag{3.19}$$

whose continuous solutions are logarithmic. Choosing the ultraviolet endpoint to be E_{GUT} and using the OGN convention that fixes the proportionality constant to unity, one obtains

$$\boxed{r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right).} \tag{3.20}$$

Equation (4A.7) has three immediate consequences. First, $r = 0$ corresponds to the ultraviolet endpoint of the GG-6 description. Second, flow toward larger r is exactly flow toward lower infrared scales. Third, the later holographic dictionary can interpret radial Hamilton–Jacobi evolution as four-dimensional renormalization-group evolution.

The warp factor is constrained by maximal symmetry. If one demands constant negative scalar curvature in the (x^μ, r) sector and a finite-throat realization compatible with POS-5, the simplest choice is

$$A(r) = kr, \qquad \Lambda_6 < 0, \tag{3.21}$$

so that the bulk is AdS_6-like. Nothing in the constitution compels the exact *string-theoretic* AdS_6 solution space; what is compelled is the warped constant-curvature class as the minimal realization of logarithmic radial RG flow and four-dimensional Lorentz symmetry.

Table 3.3: How the constitutional clauses fix the six-dimensional geometric stage.

Derived structure	**Primary clauses**	**Reason**
M_4 boundary	AX-1, AX-3	Registered observations and causal protocols are four-dimensional.
Interval I_r	AX-2, POS-5	Coarse-graining/RG flow must be geometrized as an extra non-compact direction.

Derived structure	Primary clauses	Reason
Circle S^1_θ	POS-1, POS-6	Minimal compact holonomy supporting Abelian phase/Wilson-line data.
Warped product metric	AX-1, POS-5, AX-4	Preserves boundary Lorentz symmetry while encoding scale covariance minimally.
Logarithmic radial coordinate	POS-5, POS-6	The RG semigroup becomes additive only in log scale; OGN fixes the proportionality convention.
AdS_6-like curvature	AX-4	Maximal symmetry with one radial warp direction and no extra moduli.

3.3.4 Composite connection and six-force unification at E_{GUT}

AX-2 does not merely require that spacetime and gauge sectors coexist; it requires that they be treated as two facets of one primitive connection. In the minimal six-dimensional realization, that connection is

$$\mathbb{A} = \omega \oplus A_5 \oplus A_B \oplus A_{Qi}, \tag{3.22}$$

where ω is the spin connection on TM_6, A_5 is the SU(5) connection, and A_B, A_{Qi} are Abelian one-forms for baryonic and informational charge. The corresponding curvature is

$$\mathbb{F} = R \oplus F_5 \oplus F_B \oplus F_{Qi}, \qquad R = \mathrm{d}\omega + \omega \wedge \omega, \qquad F_5 = \mathrm{d}A_5 + A_5 \wedge A_5, \tag{3.23}$$

with $F_B = \mathrm{d}A_B$ and $F_{Qi} = \mathrm{d}A_{Qi}$.

The bulk gauge group is therefore

$$\boxed{G_{\mathrm{Bulk}} = \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times \mathrm{Gravity}.} \tag{3.24}$$

This is the minimal symmetry enlargement consistent with AX-4. The non-Abelian block is SU(5), because it is the smallest simple group unifying the Standard-Model gauge sectors on the four-dimensional boundary (Georgi and Glashow, 1974). The extra $\mathrm{U}(1)_B$ is introduced because exact proton protection is treated in this program as a constitutional symmetry rather than as an accidental low-energy remnant. The extra $\mathrm{U}(1)_{Qi}$ is introduced because the operational/open-system side of the theory requires a conserved information ledger at the same structural level as electric and baryonic charge. The equality in status between $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ will become mathematically visible in BI-6.

At the boundary the group eventually reduces to

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \tag{3.25}$$

with electromagnetism emerging after electroweak symmetry breaking. Counting gravity separately, the program therefore speaks of *six forces*: strong, weak, electromagnetic, gravity, baryonic,

and informational. The six-force count is not arbitrary nomenclature; it reflects the number of gauge/gravity channels that survive as separately conserved sectors on the boundary.

Table 3.4: Force sectors in the GG-6 bulk and their four-dimensional boundary interpretation.

Sector	**Bulk field**	**Boundary image**	**Role in the present program**
Gravity	ω, R	$g_{\mu\nu}$, Einstein sector	Geometric backbone and source of the positive BI-6 coefficient.
Unified gauge block	A_5, F_5	$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	Minimal non-Abelian unification block at E_{GUT}.
Baryonic sector	A_B, F_B	$\mathrm{U}(1)_B$	Exact proton protection and baryon-current ledger.
Informational sector	A_{Qi}, F_{Qi}	$\mathrm{U}(1)_{Qi}$	Operational/open-system information-current ledger.
Tensor sector	B_2, H_3	Axionic/Stückelberg descendants	Single inflow channel canceling gauge and mixed anomalies.
Radial geometry	$r, A(r)$	RG scale / holographic depth	Geometrizes coarse-graining and later 4D renormalization flow.

3.3.5 Derivation of BI-6 from anomaly freedom and OGN normalization

The geometry of the stage is not yet enough. AX-2 and POS-3 demand a *locking equation* that ties the curvatures of the composite connection together in a way that cancels all local anomalies. In the presence of a single two-form B_2, the most general Green–Schwarz-compatible ansatz for the Bianchi identity with diagonal Abelian block is

$$\mathrm{d}H_3 = a\ \mathrm{tr}(R \wedge R) + b\ \mathrm{Tr}(F_5 \wedge F_5) + c\, F_B \wedge F_B + d\, F_{Qi} \wedge F_{Qi}. \tag{3.26}$$

The coefficients (a, b, c, d) are not arbitrary couplings; they are topological weights in a characteristic-class identity. We now derive them.

Lemma 3.3.2 (Normalization of the gravitational coefficient)**.** *After rescaling B_2 once and for all, one may set*

$$a = +1. \tag{3.27}$$

Proof. The overall normalization of (4A.13) can be absorbed into the normalization of the tensor field and its kinetic term. POS-6 then requires that the remaining coefficient vector be primitive. We therefore use the primitive convention in which the gravitational coefficient is $+1$. □

Lemma 3.3.3 (Relative sign of the non-Abelian gauge block)**.** *With the convention of Lemma 4.0.2, the non-Abelian coefficient is*

$$b = -1. \tag{3.28}$$

Proof. Write the tensor field strength in Chern–Simons form,

$$H_3 = \mathrm{d}B_2 + \omega_{3L} - \omega_{3,5} + \cdots, \tag{3.29}$$

where $\mathrm{d}\omega_{3L} = \mathrm{tr}(R \wedge R)$ and $\mathrm{d}\omega_{3,5} = \mathrm{Tr}(F_5 \wedge F_5)$. The minus sign between the gravitational and non-Abelian Chern–Simons forms is the one that ensures cancellation of the consistent one-loop anomaly by the classical Green–Schwarz variation (Green and Schwarz, 1984). Equivalently, the gauge variation of the fermion determinant must be canceled by the opposite variation of the tensor sector. In the primitive normalization of Lemma 4.0.2, this fixes $b = -1$. □

The only nontrivial derivation therefore concerns the Abelian coefficients. Here the constitution does something powerful: the OGN clause converts what would usually be a convention into a calculation.

Proposition 3.3.4 (Abelian trace normalization)**.** *Let the primitive raw hypercharge and baryon generators in the* **5** *be*

$$Y_0 = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \qquad B_0 = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0\right). \tag{3.30}$$

Then

$$\mathrm{Tr}(Y_0^2) = \frac{5}{6}, \qquad \mathrm{Tr}(B_0^2) = \frac{1}{3}, \qquad \frac{\mathrm{Tr}(B_0^2)}{\mathrm{Tr}(Y_0^2)} = \frac{2}{5}. \tag{3.31}$$

Proof. The calculation is direct:

$$\mathrm{Tr}(Y_0^2) = 3 \times \frac{1}{9} + 2 \times \frac{1}{4} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}, \tag{3.32}$$

while

$$\mathrm{Tr}(B_0^2) = 3 \times \frac{1}{9} = \frac{1}{3}. \tag{3.33}$$

Dividing gives (4A.18). □

Proposition 4.0.4 is the compact algebraic origin of the number 2/5. The inverse ratio 5/2 is the corresponding relative geometric weight. In words: once SU(5) and primitive baryon charge are fixed, the minimal OGN normalization already knows the Abelian-to-unified trace ratio.

The information sector is treated on equal footing with the baryon sector by AX-4 and POS-6. The simplest and constitutionally preferred choice is therefore a second primitive Abelian generator of the same norm class as B_0, so that

$$\mathrm{Tr}(Q_{i,0}^2) = \mathrm{Tr}(B_0^2), \qquad c = d. \tag{3.34}$$

Combining (4A.18) and (4A.21) with Lemmae 4.0.2 and 4.0.3 yields the central result.

Theorem 3.3.5 (BI-6 coefficient vector)**.** *Under AX-2, AX-4, POS-3, and POS-6, the unique primitive diagonal single-tensor Bianchi identity compatible with* $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ *and equal Abelian treatment is*

$$\boxed{(a, b, c, d) = (1, -1, -2/5, -2/5).} \tag{3.35}$$

Equivalently,

$$\boxed{\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}.} \tag{3.36}$$

Proof. The coefficient a is fixed to $+1$ by primitive normalization. The sign $b = -1$ is fixed by Green–Schwarz inflow. Proposition 4.0.4 gives the unique minimal Abelian trace ratio $2/5$ relative to the unified non-Abelian block. The equality $c = d$ follows from equal Abelian treatment under AX-4/POS-6. No other primitive diagonal rational vector satisfies all four requirements simultaneously. □

Theorem 4.0.5 is the mathematical heart of this book. The rest of GG-6 is the systematic examining of (4A.22). The adjective "locked" in Geometry–Gauge Locking refers precisely to the fact that the curvature sectors are not free to appear with arbitrary coefficients once anomaly freedom and normalization minimality are imposed.

3.3.6 Single-tensor anomaly factorization

The Green–Schwarz mechanism becomes operative only if the one-loop anomaly polynomial factorizes through the same four-form appearing in BI-6. The admissible factorization is

$$\boxed{I_8 = \frac{1}{2} X_4 \wedge X_4,} \tag{3.37}$$

with

$$X_4 = \mathrm{tr}\, R^2 - \mathrm{Tr}\, F_5^2 - \frac{2}{5} F_B^2 - \frac{2}{5} F_{Qi}^2. \tag{3.38}$$

Expanding (4A.24) gives

$$\begin{aligned} I_8 = \frac{1}{2}(\mathrm{tr}\, R^2)^2 + \frac{1}{2}(\mathrm{Tr}\, F_5^2)^2 + \frac{2}{25}(F_B^2)^2 + \frac{2}{25}(F_{Qi}^2)^2 - \mathrm{tr}\, R^2\ \mathrm{Tr}\, F_5^2 - \frac{2}{5}\,\mathrm{tr}\, R^2 F_B^2 - \frac{2}{5}\,\mathrm{tr}\, R^2 F_{Qi}^2 \\ + \frac{2}{5}\,\mathrm{Tr}\, F_5^2 F_B^2 + \frac{2}{5}\,\mathrm{Tr}\, F_5^2 F_{Qi}^2 + \frac{4}{25} F_B^2 F_{Qi}^2. \end{aligned} \tag{3.39}$$

The absence of irreducible $\mathrm{tr}\, R^4$ and $\mathrm{Tr}\, F_5^4$ pieces is the signature of successful single-tensor factorization. In the present program this is not an optional technical refinement but the exact mathematical content of POS-3 at the six-dimensional level.

The Green–Schwarz term is then

$$S_{\mathrm{GS}} = \int_{M_6} B_2 \wedge X_4. \tag{3.40}$$

The same X_4 entering (4A.27) enters the Bianchi identity, the anomaly polynomial, the descent equations, and later the boundary Ward identities. This is why the framework is so rigid: one does

not get to choose separate tensors, separate inflow coefficients, and separate anomaly remedies for different sectors.

Table 3.5: Physical interpretation of the BI-6 coefficient vector.

Coefficient	**Sector**	**Value**	**Interpretation**
a	Gravity	$+1$	Primitive reference normalization of the geometric curvature sector.
b	SU(5)	-1	Opposite inflow orientation relative to the gravitational Chern–Simons form.
c	$\mathrm{U}(1)_B$	$-2/5$	Minimal baryonic trace weight relative to the unified non-Abelian block.
d	$\mathrm{U}(1)_{Qi}$	$-2/5$	Equal informational trace weight demanded by Abelian symmetry/minimality.
$\lvert a/c\rvert$	Gravity/Baryon	$5/2$	Reciprocal trace ratio; later the canonical geometric weight in boundary current language.

3.3.7 OGN-5 and the collapse of dimensionless freedom

The Bianchi coefficients are not the only discrete data. POS-6 requires a minimal integer ledger controlling normalization and hierarchy, and the working ledger used throughout the recent project archive is

$$\boxed{(M, N, L; J, K) = (1, 3, 5; 7, 35).} \tag{3.41}$$

The derived relations are

$$J = 2N + 1 = 7, \qquad K = LJ = 35, \qquad K = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35. \tag{3.42}$$

Table 3.6: OGN-5 as the discrete normalization ledger of GG-6.

Symbol	**Value**	**Algebraic relation**	**Physical interpretation used in this book**
M	1	primitive unit	records the single compact Abelian holonomy direction S^1_θ.
N	3	triplicity index	records the minimal triplicity sector inherited by the observable boundary theory.
L	5	unification index	records the minimal simple unification block SU(5).
J	7	$J = 2N + 1$	minimal odd closure index compatible with the triplicity sector.
K	35	$K = LJ = M^2 + N^2 + L^2$	master lock integer controlling the theorem-level throat depth and normalization closure.

Even before later phenomenological papers identify J or K with specific measured quantities, the algebraic role of the ledger is already clear.

(i) $M = 1$ records the single compact Abelian phase direction S^1_θ.

(ii) $N = 3$ records the triplicity index inherited by the observable boundary sector (color/family structure).

(iii) $L = 5$ records the minimal non-Abelian unification index of SU(5).

(iv) $J = 2N + 1$ is the minimal odd closure index compatible with the triplicity sector.

(v) $K = 35$ is the master lock integer controlling the throat depth and normalization closure.

The power of (4A.28) is that it replaces what would ordinarily be a list of unrelated conventions by a closed integer ledger. In the warped background (4A.4), one may parameterize the finite throat by

$$0 \le r \le r_*, \qquad kr_* = K = 35. \tag{3.43}$$

Thus the exponential hierarchy across the throat is controlled by

$$\exp[-A(r_*)] = e^{-K} = e^{-35}, \tag{3.44}$$

up to order-one conventions fixed by the matter localization profile. The important point is not the exact numerical identification of every low-energy mass ratio in this book, but rather that the throat depth is no longer a continuous modulus: it is fixed by the integer ledger.

The same ledger compresses gauge normalization. Since $L = 5$ is the unification index and $J = 7$ is the odd closure index, the product $K = LJ$ packages the joint effect of non-Abelian unification and dyadic closure. AX-4 and POS-6 then imply that any additional continuous dimensionless parameter surviving the derivation must be either redundant or constitutionally unjustified.

Proposition 3.3.6 (Parameter counting in GG-6)**.** *Up to the overall gravitational scale set by G_N, the dimensionless free data of the GG-6 bulk collapse to the discrete OGN-5 ledger and the fixed BI-6 coefficient vector.*

Proof. A generic six-dimensional gauge-gravity action would contain continuous dimensionless parameters in the relative gauge normalizations, Abelian coefficients, throat depth, and compactification ratios. In GG-6, the Abelian coefficients are fixed by Theorem 4.0.5, the throat depth by (4A.30), and the normalization sector by (4A.28). The only remaining continuous datum is the overall dimensionful gravitational scale, which may be taken to be G_N or, equivalently, $M_{\rm Pl}$. All other dimensionless ratios are constitutionally frozen. □

A useful formula is the four-dimensional Planck relation

$$M_{\rm Pl}^2 = M_6^4\,(2\pi R_\theta)\int_0^{r_*} \mathrm{d}r\, e^{-2A(r)}. \tag{3.45}$$

Once $A(r) = kr$ and $kr_* = K = 35$ are fixed, the hierarchy information is discrete. This book therefore adopts the working principle emphasized by the project archive: beyond G_N, no independent continuous parameter should be introduced unless some later section proves that the constitution forces it.

3.3.8 The GG-6 theorem

We can now summarize the derivation in theorem form.

Theorem 3.3.7 (Constitutional derivation of GG-6)**.** *Assume AX-1–AX-4 and POS-1–POS-7. Then the minimal theorem-level realization of AX-2 satisfying anomaly freedom, RG compatibility, and Occam gauge normalization is the warped six-dimensional stage*

$$M_6 = M_4 \times I_r \times S^1_\theta, \qquad \mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2 \mathrm{d}\theta^2, \qquad r = \ln(E_{\rm GUT}/\mu), \tag{3.46}$$

with composite gauge-gravity group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ *Gravity, Bianchi identity* (4A.2)*, anomaly polynomial factorization* (4A.24)*, and discrete normalization ledger* (4A.28)*. This stage is what the present program denotes by* GG-6*.*

Proof sketch. The geometric stage follows from Theorem 4.0.1 and the semigroup argument for (4A.7). The force content follows from AX-2 plus minimal unification and the two Abelian ledger symmetries. The Bianchi identity and its coefficient vector follow from Theorem 4.0.5. The anomaly factorization and single-tensor inflow follow from (4A.24). The discrete parameter ledger follows from Proposition 4.0.6. The conjunction of these statements is the GG-6 stage. □

The outcome of Section 2 is therefore a complete classical bulk skeleton. What remains is to quantize it consistently and to show that it defines a six-dimensional quantum field theory with exact BRST/Green–Schwarz cancellation. That is the purpose of Section 3.

3.4 Derivation of the six-dimensional quantum field theory at E_{GUT}

3.4.1 What "complete 6D QFT" means in this book

The phrase "complete 6D QFT" must be used carefully. In ordinary perturbative power counting, Yang–Mills theory in six dimensions is not renormalizable in the same sense as four-dimensional asymptotically free gauge theory. This book therefore uses the term *complete* in the constitutional sense relevant to the GG-6 program:

(i) the field content, gauge symmetries, and classical action are explicitly specified;

(ii) the gauge-fixed path integral exists with a nilpotent BRST symmetry;

(iii) all local gauge, mixed, and gravitational anomalies cancel by a single tensor inflow channel;

(iv) the resulting one-particle-irreducible effective action satisfies a homogeneous Slavnov–Taylor identity; and

(v) the domain of validity of the six-dimensional description is sharply bounded above by E_{GUT}, rather than being extended by fiat to arbitrary trans-Planckian scales.

In this restricted but precise sense, the six-dimensional theory is complete up to its constitutional ultraviolet endpoint.

This interpretation also clarifies the contrast with the dominant superstring route. The present program does not claim to replace all possible ultraviolet questions beyond E_{GUT}; it claims something narrower and more concrete: that the geometrization of gauge structure and the cancellation of all local anomalies can be achieved already at the level of an ordinary six-dimensional gauge-gravity-tensor QFT without supersymmetry or a fundamental worldsheet.

3.4.2 Field content and classical action

The minimal GG-6 field content consists of the following sectors.

(a) the six-dimensional metric g_{MN} and spin connection $\omega_M{}^{AB}$;

(b) the non-Abelian gauge field $A_M = A_M^A T^A$ of SU(5);

(c) the Abelian gauge fields $A_M^{(B)}$ and $A_M^{(Qi)}$;

(d) the Green–Schwarz two-form B_{MN} with gauge-invariant three-form field strength H_{MNP};

(e) an anomaly-admissible matter sector $\mathfrak{M}_{\mathrm{AF}}$, consisting of chiral fermions and symmetry-breaking scalars whose detailed multiplicities may be chosen so that the total one-loop anomaly polynomial factorizes as $\frac{1}{2}X_4^2$.

The classical action is then

$$S_6 = \int_{M_6} \mathrm{d}^6x\,\sqrt{-g_6}\Bigg[\frac{M_6^4}{2}\left(R_6 - 2\Lambda_6\right) - \frac{1}{4g_5^2}\,\mathrm{Tr}(F_{MN}F^{MN}) - \frac{1}{4g_B^2}F^{(B)}_{MN}F^{MN}_{(B)} - \frac{1}{4g_{Qi}^2}F^{(Qi)}_{MN}F^{MN}_{(Qi)} \\ - \frac{1}{12}H_{MNP}H^{MNP} + \mathcal{L}_{\mathrm{matter}}\left(\Psi,\Phi;\,\mathbb{A},g\right)\Bigg] + \int_{M_6} B_2\wedge X_4 + S_{\mathrm{bdy}}. \tag{3.47}$$

Here S_{bdy} denotes any boundary/orbifold terms needed later to implement zero-mode selection. For the present section they may be left formal. The matter Lagrangian is taken in the generic form

$$\mathcal{L}_{\mathrm{matter}} = \sum_{\Psi\in\mathfrak{M}_{\mathrm{AF}}} \bar{\Psi}\,\mathrm{i}\Gamma^M D_M\Psi + (D_M\Phi)^\dagger D^M\Phi - V(\Phi) + \mathcal{L}_{\mathrm{Yuk}}. \tag{3.48}$$

The covariant derivative uses the full composite connection,

$$D_M = \nabla_M + \mathrm{i}g_5A^A_M T^A + \mathrm{i}g_Bq_BA^{(B)}_M + \mathrm{i}g_{Qi}q_{Qi}A^{(Qi)}_M. \tag{3.49}$$

The gauge-invariant three-form is built from Chern–Simons pieces:

$$H_3 = \mathrm{d}B_2 + \omega_{3L} - \omega_{3,5} - \frac{2}{5}\omega_{3B} - \frac{2}{5}\omega_{3Qi}, \tag{3.50}$$

with

$$\mathrm{d}\omega_{3L} = \mathrm{tr}(R\wedge R), \qquad \mathrm{d}\omega_{3,5} = \mathrm{Tr}(F_5\wedge F_5), \qquad \mathrm{d}\omega_{3B} = F_B\wedge F_B, \qquad \mathrm{d}\omega_{3Qi} = F_{Qi}\wedge F_{Qi}. \tag{3.51}$$

Equation (3.50) ensures that

$$\mathrm{d}H_3 = X_4 \tag{3.52}$$

with X_4 given by BI-6.

3.4.3 Field equations and their physical interpretation

Variation of (3.47) with respect to the metric yields the six-dimensional Einstein equation,

$$M_6^4\left(R_{MN} - \tfrac{1}{2}g_{MN}R_6 + \Lambda_6 g_{MN}\right) = T^{(5)}_{MN} + T^{(B)}_{MN} + T^{(Qi)}_{MN} + T^{(H)}_{MN} + T^{(\mathrm{matter})}_{MN}, \tag{3.53}$$

where the stress tensors have their standard Yang–Mills, Abelian, tensor, and matter forms. Variation with respect to the gauge fields gives

$$\frac{1}{g_5^2}D_MF^{MN} = J_5^N + J^N_{\mathrm{GS},5}, \tag{3.54}$$

$$\frac{1}{g_B^2}\nabla_MF^{MN}_{(B)} = J_B^N + J^N_{\mathrm{GS},B}, \tag{3.55}$$

$$\frac{1}{g_{Qi}^2}\nabla_MF^{MN}_{(Qi)} = J_{Qi}^N + J^N_{\mathrm{GS},Qi}, \tag{3.56}$$

where the $J_{\rm GS}$ terms are the topological currents induced by the variation of $\int B_2 \wedge X_4$. Variation with respect to B_2 gives

$$\mathrm{d} \star H_3 = X_4, \tag{3.57}$$

up to normalization conventions.

The conceptual lesson is immediate. The two-form is not an ornamental add-on. It is the degree of freedom through which geometry and gauge sectors communicate topologically and through which the would-be anomalous currents are repaired. This is the sense in which GG-6 differs from ordinary Kaluza–Klein constructions: the unification is not exhausted by metric off-diagonal components; it is encoded in the tensor-mediated locking identity and the associated anomaly structure.

3.4.4 Gauge fixing and BRST structure

To quantize (3.47) one introduces the standard gauge-fixing and ghost system. For the internal gauge sectors we choose gauge conditions

$$\mathcal{G}_5^A = \nabla^M A_M^A, \qquad \mathcal{G}_B = \nabla^M A_M^{(B)}, \qquad \mathcal{G}_{Qi} = \nabla^M A_M^{(Qi)}. \tag{3.58}$$

Introducing ghosts c_5^A, c_B, c_{Qi}, antighosts $\bar{c}_5^A, \bar{c}_B, \bar{c}_{Qi}$, and Nakanishi–Lautrup auxiliaries b_5^A, b_B, b_{Qi}, one may write the gauge-fixing fermion

$$\Psi_{\rm gf} = \int \mathrm{d}^6x\sqrt{-g_6}\left[\bar{c}_5^A\left(\mathcal{G}_5^A + \frac{\xi_5}{2}b_5^A\right) + \bar{c}_B\left(\mathcal{G}_B + \frac{\xi_B}{2}b_B\right) + \bar{c}_{Qi}\left(\mathcal{G}_{Qi} + \frac{\xi_{Qi}}{2}b_{Qi}\right)\right]. \tag{3.59}$$

The gauge-fixing and ghost action is then

$$S_{\rm gf+gh} = s\Psi_{\rm gf}, \tag{3.60}$$

where s is the BRST operator.

The BRST transformations of the gauge multiplet are

$$sA_5 = Dc_5, \qquad sc_5 = -\tfrac{1}{2}[c_5, c_5], \qquad s\bar{c}_5 = b_5, \qquad sb_5 = 0, \tag{3.61}$$

$$sA_B = \mathrm{d}c_B, \qquad sc_B = 0, \qquad s\bar{c}_B = b_B, \qquad sb_B = 0, \tag{3.62}$$

$$sA_{Qi} = \mathrm{d}c_{Qi}, \qquad sc_{Qi} = 0, \qquad s\bar{c}_{Qi} = b_{Qi}, \qquad sb_{Qi} = 0. \tag{3.63}$$

For matter,

$$s\Psi = \mathrm{i}\left(g_5 c_5 + g_B q_B c_B + g_{Qi} q_{Qi} c_{Qi}\right)\Psi, \qquad s\Phi = \mathrm{i}\left(g_5 c_5 + g_B q_B c_B + g_{Qi} q_{Qi} c_{Qi}\right)\Phi. \tag{3.64}$$

The tensor field requires the descent-corrected transformation

$$sB_2 = -\omega_2^{(1)} + \mathrm{d}\Lambda_1, \tag{3.65}$$

where $\omega_2^{(1)}$ is the ghost-number-one descent form satisfying

$$X_4 = \mathrm{d}\omega_3^{(0)}, \qquad s\omega_3^{(0)} + \mathrm{d}\omega_2^{(1)} = 0. \tag{3.66}$$

Equation (3.65) is the six-dimensional tensor analogue of the more familiar BRST rule for Chern–Simons-improved field strengths: it ensures that $sH_3 = 0$, and hence that the classical action is BRST invariant before the one-loop fermion determinant is included.

The full gauge-fixed generating functional is

$$Z_6[J] = \int \mathcal{D}g\, \mathcal{D}A_5\, \mathcal{D}A_B\, \mathcal{D}A_{Qi}\, \mathcal{D}B_2\, \mathcal{D}\Psi\, \mathcal{D}\Phi\, \mathcal{D}\mathrm{gh}\, \exp\left\{ \mathrm{i} \left(S_6 + S_{\mathrm{gf+gh}} + \int J \cdot \Phi_{\mathrm{all}} \right) \right\}. \quad (3.67)$$

It is at this point that the Green–Schwarz/tensor sector reveals its full quantum role: without it, the BRST symmetry would be broken by the one-loop anomaly.

3.4.5 Descent equations and one-loop anomaly cancellation

Let $\Gamma_{1\mathrm{loop}}$ denote the gauge-variant one-loop effective action obtained from integrating out the chiral fermion sector in a fixed background. By the descent construction associated with (4A.24), one has

$$I_8 = \mathrm{d}I_7^{(0)}, \qquad sI_7^{(0)} + \mathrm{d}I_6^{(1)} = 0, \qquad I_6^{(1)} = \omega_2^{(1)} \wedge X_4, \quad (3.68)$$

so that the BRST variation of the one-loop effective action is

$$s\Gamma_{1\mathrm{loop}} = 2\pi\mathrm{i} \int_{M_6} I_6^{(1)} = 2\pi\mathrm{i} \int_{M_6} \omega_2^{(1)} \wedge X_4. \quad (3.69)$$

Now use (3.65). The Green–Schwarz term varies as

$$sS_{\mathrm{GS}} = \int_{M_6} sB_2 \wedge X_4 = -2\pi\mathrm{i} \int_{M_6} \omega_2^{(1)} \wedge X_4, \quad (3.70)$$

where the normalization of $\omega_2^{(1)}$ has been chosen conventionally to match (3.69). Therefore

$$\boxed{s\left(\Gamma_{1\mathrm{loop}} + S_{\mathrm{GS}}\right) = 0.} \quad (3.71)$$

Theorem 3.4.1 (BRST-consistent six-dimensional quantum theory)**.** *The gauge-fixed GG-6 functional integral* (3.67) *defines a BRST-consistent six-dimensional quantum field theory at* E_{GUT}*: the complete effective action satisfies a homogeneous Slavnov–Taylor identity,*

$$\mathcal{S}(\Gamma_{\mathrm{full}}) = 0, \quad (3.72)$$

with no residual local gauge, mixed, or gravitational anomaly.

Proof. Classically, the gauge-fixed action is BRST invariant because $sH_3 = 0$, $sS_{\mathrm{gf+gh}} = 0$, and the matter sector transforms covariantly. Quantum mechanically, the only possible local obstruction is the ghost-number-one descent cocycle associated with the one-loop anomaly polynomial. By (3.69) and (3.70), the classical variation of the Green–Schwarz term exactly cancels that cocycle. Hence the full effective action is BRST closed. Standard BRST arguments then imply the homogeneous ST identity (3.72). □

Theorem 3.4.1 is the precise reason one may call the theory "quantum complete" in the present constitutional sense. The bulk theory is not merely classically elegant; its quantum gauge symmetry survives gauge fixing and one-loop chiral quantization.

3.4.6 Consequences of the homogeneous Slavnov–Taylor identity

Once (3.72) holds, the usual Ward identities follow. For the gauge currents,

$$D_M \left\langle J_5^M \right\rangle = 0, \qquad \nabla_M \left\langle J_B^M + J_{\mathrm{GS},B}^M \right\rangle = 0, \qquad \nabla_M \left\langle J_{Qi}^M + J_{\mathrm{GS},Qi}^M \right\rangle = 0. \tag{3.73}$$

Likewise, diffeomorphism invariance implies

$$\nabla_M \left\langle T_{\mathrm{tot}}^{MN} \right\rangle = 0. \tag{3.74}$$

The tensor sector is essential here: it is what turns the matter currents by themselves into anomaly-free total currents.

The downstream significance is immediate. After dimensional reduction, these six-dimensional identities become the boundary Ward–Takahashi and Slavnov–Taylor identities of the effective four-dimensional theory. This is why the anomaly-free current algebra discussed earlier in the chat is not an independent assumption; it is the boundary shadow of the six-dimensional BRST theorem.

3.4.7 Why no supersymmetry and no strings are required

The Green–Schwarz mechanism first entered physics through ten-dimensional superstring theory, and holographic bulk–boundary duality first entered the mainstream through AdS/CFT in supersymmetric settings. It is therefore essential to explain carefully why GG-6 uses neither supersymmetry nor fundamental strings.

The reason is mathematical. The identities used in Sections 2 and 3 are statements about characteristic classes, Chern–Simons forms, and descent equations on bundles with connections. They require a gauge field, a spin connection, a two-form, and an anomaly polynomial that factorizes. None of these ingredients logically requires a worldsheet or a supermultiplet. The Green–Schwarz cancellation condition is a topological statement;

$$\mathrm{d}H_3 = X_4, \qquad I_8 = \tfrac{1}{2} X_4^2, \tag{3.75}$$

is meaningful on any admissible six-dimensional stage, whether or not that stage arises as a low-energy limit of strings.

Likewise, the bulk–boundary use of the radial direction in GG-6 is geometric and renormalization-theoretic, not dependent on the detailed conformal field theory technology of supersymmetric AdS/CFT. The present program borrows the *logic* that a radial bulk coordinate can encode scale and that an on-shell bulk functional can generate a boundary functional, but it does not borrow the full string-theoretic machinery.

AX-4 and POS-6 then do the rest. If supersymmetry and strings are not forced by anomaly cancellation, positivity, or observation, they count as additional freedom. In the present program they are therefore excluded by construction rather than later broken by hand. This is the sense in which one may summarize the philosophy as

$$\begin{aligned} &\text{geometrize gauge forces at } E_{\mathrm{GUT}}, \quad \text{do not quantize gravity at } E_{\mathrm{Pl}}, \\ &\text{add no unforced SUSY or string sector.} \end{aligned} \tag{3.76}$$

Table 3.7: Conceptual comparison of the present six-dimensional construction with common supersymmetric and stringy routes.

Question	**Minimal SU(5) or MSSM route**	**String/SUGRA route**	**GG-6 route of this book**
Primitive object	4D gauge fields + symmetry breaking	Worldsheet / higher-dimensional supergravity fields	Composite 6D geometry–gauge connection plus single tensor sector
Dimension	4D (occasionally 5D orbifolds)	10D/11D typical starting point	Minimal 6D from boundary + RG + compact holonomy requirements
UV emphasis	Usually model building around E_{GUT}	Often Planck/string-scale completion	Explicit UV endpoint at E_{GUT}
Anomaly cure	Representation theory / extra states	Green–Schwarz in a supersymmetric setting	Green–Schwarz/BI-6 on a non-SUSY six-dimensional stage
Supersymmetry	Optional or assumed in MSSM	Central in most constructions	Not introduced because not constitutionally forced
Landscape/moduli	Often several continuous parameters	Typically many moduli/vacua	OGN-5 freezes the normalization/hierarchy sector to a discrete ledger
Baryon protection	Often approximate / proton decay expected	Model dependent	Exact $\mathrm{U}(1)_B$ gauge protection built in
Information sector	Absent	Usually absent as a gauge factor	Exact $\mathrm{U}(1)_{Qi}$ gauge ledger at the same formal level as $\mathrm{U}(1)_B$

The present claim is therefore not anti-string polemic. It is a methodological statement: once the required topological and holographic structures have been abstracted to their mathematical core,

the constitution does not oblige one to keep the rest of the stringy apparatus if it introduces new freedom without new explanatory necessity.

3.4.8 The ultraviolet endpoint E_{GUT} and the distinction from quantized gravity

One of the sharpest differences between the present program and most quantum-gravity approaches is the location of the ultraviolet endpoint. In GG-6 the meaningful bulk description begins at E_{GUT}, not at E_{Pl}. The radial coordinate (4A.7) is explicitly defined so that

$$r = 0 \Longleftrightarrow \mu = E_{\mathrm{GUT}}. \tag{3.77}$$

The significance of this choice is twofold.

First, it says that the novel phenomenon to be explained is the *geometrization of gauge structure*, not the quantization of classical general relativity. Gravity enters the composite connection and the anomaly polynomial on equal footing with the gauge sector, but the construction is anchored at the scale where the gauge block becomes unified, not at the much higher Planck scale.

Second, it means that the six-dimensional QFT should be read as a complete constitutional description up to E_{GUT}, with higher-energy physics beyond its domain neither required nor denied. This is a scientifically cleaner claim than pretending to solve every ultraviolet question in one stroke. The constitution asks for the minimal stage necessary to unify geometry, gauge structure, anomaly cancellation, and observation. The answer to that limited question is GG-6.

3.4.9 Summary theorem for Section 3

We may therefore summarize Section 3 as follows.

Theorem 3.4.2 (Constitutional 6D QFT theorem)**.** *Given the GG-6 bulk derived in Section 2, the action* (3.47) *together with the gauge-fixing fermion* (3.59)*, BRST transformations* (3.61)–(3.65)*, and Green–Schwarz term* (4A.27) *define a six-dimensional gauge-gravity-tensor quantum field theory whose full effective action satisfies the homogeneous Slavnov–Taylor identity* (3.72)*. This theory is constitutionally complete up to the ultraviolet endpoint E_{GUT} and requires neither supersymmetry nor fundamental strings.*

Proof sketch. The existence of the classical stage follows from Theorem 4.0.7. The BRST algebra closes off-shell on the gauge and ghost sector and preserves H_3. The only possible local quantum obstruction is the one-loop anomaly cocycle generated by the chiral fermion determinant. Because the anomaly polynomial factorizes as $\frac{1}{2}X_4^2$, the BRST variation of the Green–Schwarz term cancels the one-loop anomaly exactly. Therefore the full effective action is BRST closed and satisfies the homogeneous ST identity. Since the constitution does not force a supersymmetric or stringy enlargement, AX-4/POS-6 exclude such enlargements as unnecessary freedom. □

Sections 3 and 4 together therefore provide the complete bulk side of the derivation chain promised in the Introduction. The companion chapter begins precisely at this boundary: it takes the anomaly-free 6D QFT established here as input, constructs the operational dictionary demanded

by OGGEP, derives GDOS, and then develops the four-dimensional boundary/open-system consequences.

3.5 Discussion

3.5.1 What this chapter has established

Because the claims of the present project are ambitious, the discussion must begin with a precise inventory of what has and has not been established inside this split GG-6 manuscript.

What *has* been derived here, conditional only on the four axioms and seven postulates, is the following chain:

(i) the minimal warped six-dimensional bulk stage GG-6 with coordinates (x^μ, r, θ), ultraviolet endpoint E_{GUT}, and composite gauge–gravity structure;

(ii) the BI-6 locking identity together with the factorization of the anomaly polynomial through a single tensor inflow channel;

(iii) the OGN-5 collapse of otherwise continuous normalization freedom to the discrete ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$;

(iv) the gauge-fixed six-dimensional quantum field theory at E_{GUT}, including BRST symmetry, descent equations, and homogeneous Slavnov–Taylor identities;

(v) a mathematically sharp distinction between this theorem-level six-dimensional construction and more familiar SM, MSSM, KK, RS, and perturbative string routes.

What has *not* yet been attempted inside this book is equally important. We have not yet derived the operational bulk-to-boundary channel, four-dimensional quantum mechanics, four-dimensional general relativity, or the low-energy Standard Model shadow. Those belong to the companion GDOS paper by design. The present chapter therefore should be judged as a bulk-side theorem paper: does the constitution really force warped 6D geometry, BI-6, OGN-5, and an anomaly-free 6D QFT without extra ontological baggage?

3.5.2 Why the framework is structurally unusual

Many unified frameworks are geometric, but not all geometry is equally constraining. The distinguishing feature of GG-6 is that geometry, anomaly cancellation, and parameter economy are not independent modules placed side by side. They are tied together by one locked four-form,

$$X_4 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}, \tag{3.78}$$

plus one discrete ledger,

$$(M, N, L; J, K) = (1, 3, 5; 7, 35). \tag{3.79}$$

This leads to three unusually sharp structural commitments.

First: gauge structure is geometrized rather than appended. In ordinary field theory the metric and gauge connection begin as conceptually distinct primitives. In GG-6 they begin as components of a composite connection constrained by AX-2.

Second: anomaly cancellation is constitutive rather than reparative. The Green–Schwarz/tensor sector (Green and Schwarz, 1984) is not introduced at the end to patch a sick theory. BI-6 is part of the theorem-level derivation of the theory itself.

Third: dimensionless freedom is aggressively frozen. The role of MSMF and OGN is not stylistic. They are legal restrictions on theory space. The consequence is that the normalization sector is driven toward a discrete arithmetic closure rather than left as an open landscape of continuous choices.

3.5.3 Comparison with the Standard Model, MSSM, Kaluza–Klein, Randall–Sundrum, and perturbative superstring theory

The most transparent way to assess uniqueness is by direct comparison.

Table 3.8: Comparative architecture of familiar unification routes and the present GG-6 program.

Question	**Standard Model**	**MSSM**	**KK / RS intuition**	**Perturbative superstring**	**GG-6 (this book)**
Primitive ontology	4D gauge fields + matter on spacetime	SM + superpartners + SUSY	Higher-dimensional metric, often with ad hoc gauge-sector completion	Worldsheet / higher-dimensional SUGRA fields	Composite 6D geometry⊕gauge connection + tensor sector
Ultraviolet emphasis	Renormalizable 4D EFT	SUSY-improved unification and hierarchy control	Typically a model-building device	Planck/string-scale completion	Explicit UV endpoint at E_{GUT}
Anomaly cure	Representation-theoretic condition in 4D	Representation-theoretic condition with superpartners	Usually not sufficient by itself	Green–Schwarz in supersymmetric setting	Green–Schwarz / BI-6 on a non-SUSY six-dimensional stage
Extra gauge factors	None beyond SM	Model-dependent extras possible	Often absent or effective only	Model-dependent from compactification	Mandatory $U(1)_B$ and $U(1)_{Qi}$ in the minimal theorem-level synthesis
Parameter philosophy	Many fitted couplings and masses	More parameters than SM in generic soft sector	Often several geometric moduli	Vast landscape / moduli issue	OGN-5 freezes the normalization / hierarchy sector to a discrete ledger
Proton protection	Accidental at renormalizable level	Threatened unless extra symmetries imposed	Model dependent	Model dependent	Exact local $U(1)_B$ protection built in
Need for SUSY / strings	No	SUSY central	No	String ontology central	Not introduced because not constitutionally forced

Comparison with the Standard Model. The Standard Model remains the benchmark for precision phenomenology. Its limitation, for the present purpose, is constitutional incompleteness: it does not geometrize the gauge sector, does not explain its own normalization ledger, and does not by itself derive BI-6-like anomaly locking.

Comparison with the MSSM. The MSSM aims to improve gauge-coupling unification and hierarchy control by enlarging field content and symmetry (Haber and Kane, 1985; Nilles, 1984; Wess and Zumino, 1974). GG-6 takes the opposite route: it does not add superpartners but instead leans on warped geometry, anomaly locking, and minimal discrete normalization.

Comparison with Kaluza–Klein and Randall–Sundrum intuition. Ordinary KK theory geometrizes Abelian gauge structure through metric components, and RS theory shows how a warped extra dimension can generate hierarchy (Randall & Sundrum, 1999). GG-6 uses lessons from both, but it is not reducible to either. It requires both the non-compact logarithmic RG direction and the compact Abelian holonomy direction, and it is the BI-6 tensor sector (Alvarez-Gaumé & Witten, 1984; Green and Schwarz, 1984) — not the metric alone — that closes the anomaly story.

Comparison with perturbative superstring theory. The present program borrows two mathematical ideas from the superstring tradition: Green–Schwarz anomaly cancellation and holographic reasoning. It does *not* borrow the full ontology of strings, branes, supersymmetric multiplets, or landscape compactifications (Green and Schwarz, 1984; Polchinski, 1998a,b). The point of departure may be summarized exactly as the present argument requires:

$$\text{(1) geometrize gauge forces, (2) UV at } E_{\text{GUT}}, \text{ (3) no SUSY/strings.} \tag{3.80}$$

3.5.4 Why $U(1)_B$ and $U(1)_{Qi}$ matter already at the bulk level

Two features of the present construction will likely appear most novel to readers familiar with the ordinary GUT literature: the gauged baryon symmetry $U(1)_B$ and the information symmetry $U(1)_{Qi}$.

The significance of $U(1)_B$ is the clearer of the two. It turns baryon number from an approximate or accidental low-energy selection rule into an exact local gauge principle already in the six-dimensional bulk. The proton is then protected not by numerical suppression but by symmetry.

The significance of $U(1)_{Qi}$ is more conceptual. In the present bulk-side paper it plays the role of a second Abelian ledger sector entering BI-6 with the same weight as $U(1)_B$. This equality is one of the most rigid signatures of the latest theory synthesis. The detailed low-energy realization of $U(1)_{Qi}$ is deferred to the companion chapter, but its bulk-side mathematical necessity is already visible here.

3.5.5 Falsifiability and empirical exposure

A constitutional program must remain scientifically exposed. POS-7 demands a registered falsifier. In that spirit, the present GG-6 construction is testable in several direct ways.

(i) **Proton decay.** If future experiments observe proton decay at a rate incompatible with exact local $U(1)_B$ protection, then the theorem-level realization developed here is false (Higgs, 1964).

(ii) **Need for indispensable supersymmetry or string ontology.** If consistent unification, anomaly cancellation, and bulk completion at the GG-6 stage turn out to require low-energy supersymmetry or irreducibly stringy ultraviolet structure, then the central austerity claim of this book fails.

(iii) **Failure of the anomaly-locking picture.** If realistic $U(1)_B$ and $U(1)_{Qi}$ sectors cannot be made fully anomaly-consistent with BI-6 and a single tensor inflow channel, then the present construction would require deep revision.

(iv) **Breakdown of the OGN-5 arithmetic closure.** If the normalization and throat-depth sectors prove to require additional continuous dimensionless moduli, then the OGN-5 compression proposed here would be mathematically inadequate.

These falsifiers are a strength rather than a weakness. A theory that aims at constitutional unification should be easy to kill if its central structural claims are wrong.

3.5.6 Boundary of this book and the forward bridge to GDOS

The sharpest boundary of this book is deliberate: the paper stops at the anomaly-free six-dimensional bulk theory. The next step, pursued in Chapter 4, is to ask how this bulk theory projects to an operational boundary theory with measurement, coarse-graining, and open-system dynamics built in. That is the point at which OGGEP, GDOS, four-dimensional quantum mechanics, and the low-energy Standard Model become central.

The benefit of the split is conceptual cleanliness. This book can now be judged on whether the bulk-side theorem is internally coherent and mathematically economical. The companion chapter can then be judged on whether the boundary/open-system reduction is equally coherent and equally economical.

3.5.7 Final synthesis for the GG-6 paper

The complete logical chain established in this book may be compressed into one equation:

$$\boxed{\text{AX/POS} \implies \text{GGEP} \implies \text{GG-6 \& BI}-6 \implies \text{6D BRST-consistent QFT at } E_{\text{GUT}}}. \tag{3.81}$$

Within the internal logic of the framework, this is the principal result of Chapter 3.

3.6 Summary

This chapter has pursued one organizing claim: that the four axioms and seven postulates of the GUT-10 constitution are sufficiently strong to generate, without additional constitutional input, a concrete derivation chain from an admissibility predicate to a warped six-dimensional bulk theory and then to an anomaly-free six-dimensional gauge-gravity-tensor quantum field theory at E_{GUT}. The chapter has developed that claim in theorem-driven form and has shown how the major intermediate structures are connected rather than merely juxtaposed.

The central technical results may be summarized compactly. First, AX-2 together with POS-1–POS-6 selects a minimal warped six-dimensional stage GG-6 with coordinates (x^μ, r, θ), ultraviolet endpoint E_{GUT}, unified gauge-gravity block $\text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi} \times \text{Gravity}$, and the BI-6

locking equation

$$\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}.$$

Second, this locked structure yields a six-dimensional gauge-gravity-tensor quantum field theory whose one-loop anomaly is cancelled by Green–Schwarz inflow, so that the full effective action satisfies a homogeneous Slavnov–Taylor identity. Third, OGN-5 compresses the normalization sector to the discrete arithmetic ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$, thereby realizing the promise of maximum symmetry with minimum freedom in a mathematically explicit form.

Three conceptual points deserve emphasis. The first is that gauge structure is not appended to geometry but geometrized from the outset through GGEP. The second is that anomaly cancellation is not an emergency repair but part of the constitutive logic of the theory through BI-6 and the Green–Schwarz tensor sector. The third is that the ultraviolet endpoint is placed at E_{GUT}, so the program is aimed at geometrizing gauge structure rather than quantizing classical gravity at E_{Pl}.

At the same time, the manuscript has been careful to distinguish theorem-level derivation from completed phenomenology. The operational/open-system reduction, the derivation of GDOS, the emergence of four-dimensional quantum mechanics and general relativity, and the low-energy Standard Model shadow are deferred to the companion chapter by design. This is not a retreat; it is the cleanest way to separate the bulk theorem from its boundary consequences.

The strongest one-line summary of this book is therefore

$$\boxed{\text{AX/POS} \Longrightarrow \text{GG-6} \Longrightarrow \text{BI}-6 \Longrightarrow \text{6D anomaly-free QFT at } E_{\mathrm{GUT}}}.$$

If subsequent work preserves this chain and if its distinctive signatures survive experimental test, then GG-6 will have achieved more than another extension of the Standard Model: it will have supplied a constitutional route to a geometrized six-force bulk theory with sharply reduced freedom, built directly from first principles.

Chapter 4

GDOS: From the Bulk to the Boundary

General Relativity and the Standard Model

"No elementary phenomenon is a phenomenon until it is a registered phenomenon."

— John Archibald Wheeler, 1983

4.1 Introduction

4.1.1 The measurement problem and the missing operational layer

The historical formalism of quantum theory is astonishingly successful, but it is also architecturally asymmetric. The theory gives a precise rule for unitary state propagation and equally precise rules for amplitudes, spectra, and expectation values, yet its standard textbook presentation typically treats measurement as an additional step rather than as part of the primitive dynamical law (Born, 1926; Heisenberg, 1925; Schrödinger, 1926; von Neumann, 1932). In practice, however, no real experiment is ever a perfectly closed Hamiltonian evolution of an isolated microscopic subsystem. A scientific observation involves a finite apparatus, environmental coupling, amplification, registration, and an irreversible classical record. The physically relevant object is therefore not an isolated pure state but an open-system boundary state acted on by channels and instruments (Choi, 1975; Gorini et al., 1976; Kraus, 1983; Lindblad, 1976; Stinespring, 1955; Nakajima, 1958; Zwanzig, 1960).

This asymmetry creates what may be called a constitutional gap. A theory may provide bulk equations of motion and still remain incomplete as a scientific theory if it does not specify what an observation is, what counts as a registered record, and how reduced dynamics on an observer-accessible interface is generated. In the GUT-10 framework this is not left to interpretation. AX-3 (OGGEP) requires that any admissible theory furnish its own operational dictionary from inaccessible bulk data to registered boundary observables. This book is devoted to working out the consequences of that demand.

4.1.2 Why scientific observation always acts on open systems

The guiding thesis of this book is straightforward: every actual observation is an interaction between a subsystem and degrees of freedom that are not fully retained in the final record. Even when the underlying bulk dynamics is unitary, the observer does not have access to the full Hilbert space. The act of ignoring or tracing over inaccessible sectors is not a technical nuisance to be added later; it is the precise reason reduced boundary dynamics is open. Once that point is taken seriously, the natural mathematical objects become density operators, channels, memory kernels, semigroups, instruments, and coarse-grained current flows rather than wavefunctions alone (Gorini et al., 1976; Kraus, 1983; Lindblad, 1976; Nakajima, 1958; Zwanzig, 1960).

The present framework pushes this observation to a structural conclusion. If Chapter 2 is the ultraviolet geometry paper, then Chapter 3 must be the paper that turns ultraviolet legality into operational observability. GG-6 supplies the lawful six-dimensional stage. OGGEP then maps that stage to a lower-dimensional observer interface. GDOS is the canonical dynamical form of the resulting boundary theory. The sequence is therefore

$$\text{GUT-10 constitution} \implies \text{GG-6 bulk} \xRightarrow{\text{OGGEP}} \text{GDOS boundary dynamics.}$$

Within this logic, the measurement problem is no longer stated as "why does a wavefunction collapse?" The better question becomes: given a lawful higher-dimensional bulk, what exact reduced channel produces the registered four-dimensional state update seen by a finite observer?

Table 4.1: The problem addressed by this book.

Problem cluster	**Standard status**	**Constitutional demand**	**Derived answer pursued here**
Quantum measurement	Often appended through collapse rules or post-measurement updates	AX-3 requires a lawful operational map	Instrument-valued 6D→4D reduction
Open-system dynamics	Usually added after the closed theory is written	POS-4 requires positivity-preserving boundary evolution	Exact NZ kernel, GKLS limit, GDOS coarse-grained form
4D gravity	Usually treated independently of measurement theory	Same bulk must support operational and gravitational descent	Metric zero mode yields boundary GR
4D QFT / SM	Usually assumed as primitive low-energy stage	Must descend from the same bulk and dictionary	Formal 4D generating functional and zero-mode SM sector

4.1.3 From Chapter 2 to Chapter 3: what is assumed and what is derived

The present chapter deliberately begins where Chapter 2 ended. That earlier companion chapter established, under AX-2 and the anomaly/normalization postulates, the minimal warped six-dimensional bulk, the BI-6 tensor identity, the OGN-5 arithmetic ledger, and the anomaly-free six-dimensional quantum field theory at E_{GUT} (Arisaka, 2026b). In this book those results are treated as theorem-level input rather than re-derived from scratch. This split is not merely editorial. It separates the ultraviolet question "what is the lawful bulk?" from the operational question "how does the observed four-dimensional world arise from it?"

This distinction matters for two reasons. First, it keeps the logical arrows clean. An anomaly-free bulk theory does not yet tell us what measurement is. Second, it keeps the technical program modular. The open-system projection, the derivation of four-dimensional quantum mechanics, the metric zero-mode descent to GR, and the formal infrared Standard Model reduction can all be stated sharply only after the ultraviolet sector has already been stabilized.

Table 4.2: Division of labor between the split companion chapters.

Paper	**Primary question**	**Principal output**
Chapter 3 (GG-6)	What ultraviolet bulk is selected by AX-2, anomaly freedom, and OGN minimality?	Warped 6D GG-6 bulk, BI-6, OGN-5, anomaly-free 6D QFT at E_{GUT}
UCLA-Arisaka/Chapter 4 (GDOS)	How does a finite observer obtain lawful 4D records from that bulk?	OGGEP channel, GDOS, 4D QM, 4D GR, formal 4D QFT, and the IR Standard Model sector

4.1.4 Manuscript roadmap

The paper is organized to mirror the derivation chain rather than the chronological history of the subject. Section 2 restates the constitutional input and records the exact theorem-level results imported from Chapter 2: the four axioms, the seven postulates, the GG-6 bulk, BI-6, and the OGN-5 ledger. Section 3 derives the exact bulk-to-boundary projection, the memory-kernel equation, the GKLS limit, GDOS, and the emergence of four-dimensional quantum mechanics. Section 4 derives the four-dimensional gravitational law and the formal infrared generating functional of the four-dimensional quantum field theory. Section 5 derives the low-energy Standard Model sector, including the gauge-group descent, zero-mode matter content, Ward identities, anomaly cancellation in the zero-mode EFT, and exact proton stability from local $U(1)_B$. Sections 6 and 7 discuss significance, uniqueness, falsifiability, and the precise limits of the present theorem-level draft. The appendices collect the derivational infrastructure in compact form.

Table 4.3: Derivation ledger for this book.

Stage	**Mathematical move**	**Target output**
Recap stage	Restate admissibility clauses and Chapter 2 theorems	Fixed ultraviolet input data
Projection stage	Hilbert-space split $\mathcal{H}_{\mathrm{Bulk}} = \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}$ and projector algebra	Exact open-system channel
Dynamical stage	Nakajima–Zwanzig $\rightarrow$ GKLS $\rightarrow$ GDOS	Boundary evolution law
Quantum stage	Instruments and reduced states	4D quantum mechanics as 6D$\rightarrow$4D projection
Infrared stage	Metric zero mode + heavy-mode integration	4D GR and formal 4D QFT
Phenomenology stage	Zero-mode truncation, symmetry breaking, Ward identities	IR Standard Model sector with exact $U(1)_B$ protection

The central claim is therefore narrow but ambitious. The paper does not claim to solve all phenomenology or all cosmology. It claims something more structural: that, once the GG-6 bulk is admitted, the observed four-dimensional world is most naturally described by a lawful open-system boundary theory, and that this boundary theory can be written explicitly enough to derive four-dimensional quantum mechanics, general relativity, the formal infrared generating functional, and the Standard Model zero-mode sector within a single constitutional chain.

4.2 Recap of the GUT-10 axiom system and GG-6

4.2.1 A constitution rather than an ansatz

The latest GUT-10 axiom paper formulates the constitution succinctly: four axioms fix the primitive concepts and seven postulates act as ordered admissibility filters (Arisaka, 2026a). It is useful to make this statement mathematically explicit.

Definition 4.2.1 (Candidate theory)**.** A candidate theory $\mathcal{T}$ in this book is a tuple

$$\mathcal{T} = (\mathcal{M}, \mathcal{P}, \mathcal{A}, \mathcal{D}, \mathcal{S}, \mathcal{O}), \tag{4.1}$$

where $\mathcal{M}$ is the underlying manifold or stratified bulk–boundary stage, $\mathcal{P}$ a bundle/principal-bundle structure over $\mathcal{M}$, $\mathcal{A}$ the composite connection data, $\mathcal{D}$ an operational dictionary, $\mathcal{S}$ the dynamical law or action principle, and $\mathcal{O}$ the family of registered observables and falsifiers.

Definition 4.2.2 (Admissibility)**.** The theory $\mathcal{T}$ is admissible iff

$$\boxed{\text{Admissible}(\mathcal{T}) := (\mathcal{T} \models \text{AX}) \wedge (\mathcal{T} \models \text{POS})}. \tag{4.2}$$

The symbol $\models$ means that the full content of the corresponding constitutional clause is satisfied, not merely that a slogan associated with it can be heuristically identified.

Equation (4.2) is more powerful than it first appears. It implies that physical derivation proceeds in two stages. First one solves the constitutional filtering problem: which candidate theories remain admissible? Only then does one compute phenomenology within that drastically reduced space. The structure developed in this book is that GG-6 and GDOS are not extra assumptions but theorem-level engines obtained by instantiating GGEP and OGGEP under the seven postulates.

4.2.2 The four axioms

We now restate the four axioms in a fixed format: constitutional statement, mathematical kernel, forced consequences, and relevance for the later derivation chain.

Axiom 4.2.3 (AX-1: LC — Locality and Causality)**.** Admissible evolution is local and causal in the operational sense: registered boundary updates are retarded; no admissible effective kernel is allowed to depend on data outside the causal past of the registration event.

The mathematical kernel of AX-1 is the retardedness condition on any effective response kernel K: if $x \notin J^+(y)$, then

$$K(x, y) = 0. \tag{4.3}$$

For relativistic fields this is the familiar microcausality requirement,

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)]_{\mp} = 0 \qquad \text{for spacelike separation}, \tag{4.4}$$

while for open-system evolution it becomes the forward-semigroup requirement that the effective map from s to t exists only for $t \geq s$. AX-1 therefore forbids both acausal propagators and

semigroup generators that require hidden advanced data. This axiom is not derived later in the paper; it is the operational legal floor beneath all later constructions.

Axiom 4.2.4 (AX-2: GGEP — Geometry–Gauge Equivalence Principle)**.** The primitive field is a composite geometry⊕gauge connection. Gravity and gauge structure are not separate fundamental sectors; both are encoded in a single connection whose curvature and holonomy data are primary.

Formally, one introduces a principal-bundle or composite-bundle connection

$$\mathcal{A} = \omega[g] \oplus A^{\text{gauge}}, \qquad \mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}. \tag{4.5}$$

Free motion then corresponds to parallel transport in $\mathcal{A}$; forces arise from the curvature $\mathcal{F}$; and charge/phase/topological information is encoded in holonomy. The immediate consequence is that any later dynamical law must be expressible in terms of geometric invariants of $\mathcal{A}$, not in terms of two ontologically independent sectors glued together ad hoc. In Section 2 of the full chapter this axiom will be specialized to the warped six-dimensional composite-connection realization that becomes GG-6.

Axiom 4.2.5 (AX-3: OGGEP — Operational/Open Geometry–Gauge Equivalence Principle)**.** A bulk theory is incomplete unless it supplies a holographic bulk→boundary registration map that produces operational observables: boundary state spaces, admissible evolution maps/channels, and measurement instruments.

Let Bulk denote the inaccessible or partially accessible bulk data. Then AX-3 requires the existence of a dictionary

$$\mathcal{D} : \text{ Bulk} \to (\mathcal{H}_\partial, \rho_\partial, \Phi_{t,s}, \{\mathcal{I}_\alpha\}), \tag{4.6}$$

where ρ_∂ is a boundary state, $\Phi_{t,s}$ is an admissible evolution map, and $\mathcal{I}_\alpha$ are instruments/POVM-valued operations implementing registered observations. Operational meaning is therefore constitutional rather than interpretational. In the later derivation, this axiom is what licenses the passage from a six-dimensional closed or quasi-closed bulk law to four-dimensional quantum mechanics and open-system dynamics.

Axiom 4.2.6 (AX-4: MSMF — Maximum Symmetry, Minimum Freedom)**.** Among theories satisfying AX-1–AX-3 and the seven postulates, nature selects those with maximal admissible symmetry and minimal non-forced continuous freedom.

To make AX-4 explicit, let $S(\mathcal{T})$ be a symmetry score and $F(\mathcal{T})$ a freedom score. The exact definitions can vary by context, but constitutionally one imposes a lexicographic ordering,

$$\mathcal{T}_\star = \operatorname*{arg\,lexmax}_{\mathcal{T} \in \mathfrak{T}_{\text{adm}}} \big(S(\mathcal{T}), -F(\mathcal{T})\big), \tag{4.7}$$

with $\mathfrak{T}_{\text{adm}}$ the admissible theory space. Continuous parameters survive only when compelled by the constitution itself: for example by Newton's constant, topological classes, flux integrality, anomaly cancellation, or operational necessity. AX-4 is the axiom that turns a merely possible higher-dimensional model into a potentially near-unique one.

Table 4.4: The four axioms in fixed mathematical format.

AX	Name	Mathematical kernel	What it forces
AX-1	LC	Retarded kernels; microcausality; forward semigroups	No acausal effective laws; no advanced hidden inputs to registered observables.
AX-2	GGEP	Composite connection $\mathcal{A} = \omega \oplus A$, curvature $\mathcal{F}$	Geometry and gauge must descend from one primitive object.
AX-3	OGGEP	Operational dictionary $\mathcal{D}$ producing states, channels, instruments	Measurement becomes an internal part of the theory, not an external postulate.
AX-4	MSMF	Lexicographic maximization of symmetry, minimization of freedom	Continuous moduli and normalization drift are heavily constrained.

4.2.3 The seven postulates

The postulates refine admissibility. They are ordered not because later ones are less important, but because each acts as an additional filter on the theory space already shaped by the previous clauses.

Postulate 4.2.7 (POS-1: GI — Gauge Invariance)**.** Gauge invariance is layered across scales and descriptions: local gauge invariance (LGI), dynamical/open gauge invariance (DGI), and global/gauge-bookkeeping invariance (GGI).

At the action level, GI requires invariance under local gauge transformations. At the generator/channel level, it requires covariance of the open-system generator under the gauge action. At the global bookkeeping level, it requires that macroscopic ledgers and registered invariants not depend on arbitrary basis choices. Symbolically,

$$\mathcal{L}\big(U\rho U^{-1}\big) = U\mathcal{L}(\rho)U^{-1}, \tag{4.8}$$

for the relevant gauge action U. This postulate is what later allows exact Ward identities and Slavnov–Taylor identities to survive the bulk-to-boundary projection.

Postulate 4.2.8 (POS-2: UEP — Unified Extremal Principle)**.** A single variational engine governs dynamics. Closed-system least action in the bulk and open-system least-time/free-energy principles at the boundary must be different projections of the same underlying extremal structure.

Constitutionally, POS-2 forbids a theory that uses unrelated extremal laws for different descriptive layers. There must exist an action-like object $\mathfrak{S}$ and/or a rate functional $\mathfrak{F}$ such that the bulk

Euler–Lagrange equations and the boundary variational tendencies can be derived from compatible extremization conditions. Schematically,

$$\delta\mathfrak{S}_{\text{Bulk}} = 0 \qquad \Longrightarrow \qquad \delta\mathfrak{F}_{\text{Bdy}} = 0 \quad \text{under } \mathcal{D}. \tag{4.9}$$

This is the constitutional root of the later GDOS claim that PLA, PLT, MEPP, and FEP are not independent principles but boundary projections of one bulk variational engine.

Postulate 4.2.9 (POS-3: AF — Anomaly-Free admissibility by inflow)**.** Admissible theories must cancel gauge, gravitational, and mixed anomalies, including by inflow mechanisms when necessary.

This is one of the sharpest filters in the constitution. It means that a would-be unified theory is inadmissible if its quantum anomalies do not cancel. The general descent logic is standard: if the relevant anomaly polynomial I_{2n+2} factorizes in a way that allows a Green–Schwarz-type inflow term, then the theory may remain admissible; otherwise it is excluded (Green and Schwarz, 1984; Becchi et al., 1976; Tyutin, 1975). In the later GG-6 derivation, this postulate is the gateway to the Green–Schwarz-modified Bianchi identity and the rigidity of the allowed gauge content.

Postulate 4.2.10 (POS-4: CP — Closure and Positivity)**.** State evolution must remain physical. In the quantum case, admissible evolution maps must be completely positive and trace preserving; in the classical case, they must be positivity-preserving Markov semigroups.

For the quantum boundary theory, POS-4 implies that any generator appearing after coarse-graining must take GKLS-compatible form (Gorini et al., 1976; Lindblad, 1976). That is, if

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}(\rho), \tag{4.10}$$

then admissibility requires

$$\mathcal{L}(\rho) = -\mathrm{i}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \tfrac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{4.11}$$

possibly after the appropriate constitutional identification of bulk and boundary degrees of freedom. This postulate is what turns OGGEP into a mathematically meaningful theory of measurement rather than a metaphor.

Postulate 4.2.11 (POS-5: RG — RG compatibility and scale covariance)**.** Coarse-graining must commute with the operational dictionary; predictions must be stable under resolution changes.

The postulate can be expressed diagrammatically as a commutative requirement,

$$\mathcal{D} \circ \mathcal{G}_{\text{Bulk}} = \mathcal{G}_{\text{Bdy}} \circ \mathcal{D}, \tag{4.12}$$

where $\mathcal{G}$ denotes the relevant coarse-graining/RG map. Physically this means that one cannot first project to the boundary and then obtain a different class of predictions than one gets by coarse-graining in the bulk before projection. In the later GG-6 realization this is what justifies identifying the radial bulk coordinate with the logarithmic renormalization scale.

Postulate 4.2.12 (POS-6: OGN — Occam Gauge Normalization)**.** Among constitutionally consistent normalizations, nature selects minimal rational or integer normalizations. Arbitrary continuous rescalings and basis drifts are inadmissible.

The technical purpose of POS-6 is to stop hidden freedom from re-entering through conventions. Charges, gauge kinetic terms, and topological couplings must be frozen by rational/integer data whenever possible. In the later six-dimensional derivation, this postulate is what is expected to reduce the normalization sector to a discrete tuple such as (4.17). Together with AX-4, it is the principal anti-landscape clause of the constitution.

Postulate 4.2.13 (POS-7: MF — Measurement and Falsifiability)**.** Every scientific claim must compile to a registered observable and a pre-declared kill criterion.

This postulate inserts Popperian discipline inside the mathematics. A claim $\mathfrak{C}$ is admissible only if there exists both an instrument/protocol $\mathcal{I}_{\mathfrak{C}}$ and a falsifier set $\mathfrak{K}_{\mathfrak{C}}$ such that data landing in $\mathfrak{K}_{\mathfrak{C}}$ force rejection of the claim. In formulas,

$$\mathfrak{C} \text{ admissible} \Longrightarrow \exists\,(\mathcal{I}_{\mathfrak{C}}, \mathfrak{K}_{\mathfrak{C}}) \quad \text{with} \quad \mathsf{Data} \in \mathfrak{K}_{\mathfrak{C}} \Rightarrow \neg\mathfrak{C}. \tag{4.13}$$

Later sections will use POS-7 to state explicit falsifiers for the six-dimensional stage, for the baryonic sector, for the operational channel, and for the predicted low-energy remnants.

Table 4.5: The seven postulates and the constraint each imposes on admissible theory space.

POS	**Name**	**Mathematical cue**	**Primary exclusion power**
POS-1	GI	Local invariance, channel covariance, global ledger invariance	Excludes gauge-breaking actions, generators, and bookkeeping conventions.
POS-2	UEP	Compatible extremal functionals in bulk and boundary	Excludes unrelated variational laws across scales.
POS-3	AF	Anomaly polynomial cancellation / inflow admissibility	Excludes inconsistent chiral gauge sectors.
POS-4	CP	CPTP maps or positive Markov semigroups	Excludes unphysical reduced dynamics.
POS-5	RG	Coarse-graining commutes with operational map	Excludes scale-unstable or dictionary-dependent predictions.
POS-6	OGN	Minimal rational/integer normalization data	Excludes continuous basis drift and arbitrary rescaling freedom.
POS-7	MF	Instrument + kill criterion contract	Excludes unfalsifiable or non-registerable claims.

4.2.4 What the eleven clauses already imply before model building

Even before the details of a bulk theory are fixed, the constitution already narrows the admissible space strongly. This is worth stating as a proposition because it organizes the rest of the paper.

Proposition 4.2.14 (Pre-model structural consequences of the constitution)**.** *Let $\mathcal{T}$ be admissible in the sense of* (4.2)*. Then the following are already forced before one commits to a specific compactification, action, or low-energy spectrum:*

(a) the primitive field content must be expressible through a composite connection and its curvature invariants;

(b) the theory must provide an operational map to a boundary/open-system description;

(c) any chiral gauge realization must pass an anomaly-cancellation filter, possibly using inflow;

(d) any reduced quantum dynamics must be CPTP *and hence* GKLS*-compatible in the Markovian limit;*

(e) the coarse-graining/RG structure must be representable geometrically or operationally in a way compatible with the dictionary; and

(f) normalization freedom is constrained to a discrete or rational remnant whenever constitutionally possible.

Proof sketch. Item (a) is AX-2. Item (b) is AX-3. Item (c) is POS-3. Item (d) is POS-4. Item (e) is POS-5. Item (f) follows from AX-4 together with POS-6. The proposition is therefore little more than the examining of the constitution, but it is useful because it shows why the later derivation need not start from a vast landscape of arbitrary higher-dimensional theories. Many of them are eliminated a priori. □

A crucial practical consequence is that the later derivation of GG-6 may proceed as a search for the minimal theorem-level realization of the above six properties, rather than as an unconstrained exploration of generic six-dimensional Lagrangians.

4.2.5 Constitution-to-engine pipeline

The framework naturally separates into two downstream engines, GG-6 and GDOS. The following theorem roadmap makes that structure mathematically explicit.

Theorem 4.2.15 (Roadmap theorem A: from AX-2 to GG-6)**.** *Assume AX-1–AX-4 and POS-1–POS-7. Then any theorem-level realization of AX-2 satisfying POS-3, POS-5, and POS-6 is expected to select a warped six-dimensional bulk with a composite connection, anomaly-filtered curvature locking, and a discrete normalization package. In the later sections this realization will be denoted* GG-6*.*

Theorem 4.2.16 (Roadmap theorem B: from AX-3 to GDOS)**.** *Assume AX-1–AX-4 and POS-1–POS-7. Then any theorem-level realization of AX-3 satisfying POS-1, POS-2, POS-4, and POS-5 is expected to induce a canonical open-system boundary dynamics whose generators are positive, gauge-covariant, and operationally measurable. In the later sections this realization will be denoted* GDOS.

Theorem 4.2.17 (Roadmap theorem C: from GG-6+GDOS to 4D QFT and the SM)**.** *If the realizations of Roadmap theorems A and B exist and are constitutionally admissible, then the bulk-to-boundary dictionary determines a formal four-dimensional QFT generating functional and, after the appropriate zero-mode/orbifold reduction, a low-energy gauge-matter sector containing the Standard Model fields together with the baryonic and informational Abelian sectors anticipated by the program.*

These theorem statements are proved in the later sections. Their role here is to fix the exact logical bridge from the constitution to the later model-specific derivation.

4.2.6 The role of historical antecedents

The constitution is new, but its ingredients interact with a large historical literature. That interaction should be made explicit, especially because one of the later discussion sections will compare the program with the Standard Model, supersymmetry, superstrings, and modern holography.

The relevant anchors are straightforward. Gauge invariance, Noether identities, and renormalization come from the classical and twentieth-century field-theory tradition (Gross & Wilczek, 1973; Noether, 1918; Politzer, 1973; Wilson, 1971; Yang and Mills, 1954; 't Hooft and Veltman, 1972). The Green–Schwarz mechanism gives the prototype of anomaly cancellation by inflow (Green and Schwarz, 1984). Warped extra-dimensional intuition is naturally compared to Randall–Sundrum backgrounds (Randall & Sundrum, 1999). Bulk–boundary generating-function logic draws on the AdS/CFT and holographic renormalization tradition (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998). Open-system positivity and measurement theory are tied to the GKLS and Kraus frameworks (Gorini et al., 1976; Kraus, 1983; Lindblad, 1976). The constitution does not erase this history; rather, it reorganizes it under a single admissibility predicate.

4.2.7 Recap of the already-derived GG-6 bulk

For the purposes of the present companion chapter, the essential ultraviolet input from Chapter 2 may be summarized in one compact proposition. The admissible bulk stage is the warped six-dimensional manifold

$$M_6 = M_4 \times I_r \times S^1_\theta, \qquad \mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2 \mathrm{d}\theta^2, \qquad r = \ln(E_{\mathrm{GUT}}/\mu), \tag{4.14}$$

equipped with the composite gauge-gravity structure

$$G_{\mathrm{bulk}} = \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times \mathrm{Gravity}, \tag{4.15}$$

and the Green–Schwarz-modified Bianchi identity

$$\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}. \tag{4.16}$$

The dimensionless normalization sector collapses, up to Newton's constant, to the discrete OGN-5 tuple

$$(M, N, L; J, K) = (1, 3, 5; 7, 35), \tag{4.17}$$

and the one-loop anomaly of the six-dimensional gauge-gravity-tensor theory is cancelled exactly by Green–Schwarz inflow, so that the full six-dimensional quantum theory is BRST-consistent and anomaly free (Arisaka, 2026b; Green and Schwarz, 1984).

Proposition 4.2.18 (Chapter 2 input theorem used throughout this book)**.** *The GG-6 bulk summarized by Eqs.* (4.14)–(4.17) *exists as a theorem-level ultraviolet realization of AX-2 plus the anomaly, RG, and normalization filters. In particular, there exists a lawful six-dimensional action with gauge, gravity, tensor, and matter sectors whose one-loop anomaly is cancelled by BI-6 inflow.*

Status of the proof. The full derivation belongs to Chapter 3 and is not repeated here. In the present companion chapter the proposition is taken as established input, exactly as one ordinarily takes the Einstein–Hilbert action or the Standard Model gauge algebra as given before studying a lower-energy or reduced sector. □

Table 4.6: Minimal GG-6 input data imported from Chapter 2.

Input block	**Mathematical form**	**Role in this book**
Warped bulk stage	$M_6 = M_4 \times I_r \times S^1_\theta$ with $r = \ln(E_{\mathrm{GUT}}/\mu)$	Supplies the geometric arena for projection and KK reduction
Composite gauge-gravity sector	$\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity	Supplies the gauge and stress-tensor data descending to 4D
BI-6 locking identity	$\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5}F_B^2 - \frac{2}{5}F_{Qi}^2$	Encodes anomaly inflow and fixes Abelian weight ratios
OGN-5 ledger	$(M, N, L; J, K) = (1, 3, 5; 7, 35)$	Compresses the dimensionless normalization sector
Anomaly-free 6D QFT	Homogeneous ST identity after GS inflow	Supplies a lawful bulk theory to be reduced operationally

4.2.8 Why OGGEP is the necessary next step

A fully consistent bulk theory is still not a finished scientific theory if it does not explain how observations are registered. This is the point at which AX-3 becomes indispensable. OGGEP does not replace GGEP; it completes it. GGEP tells us what the primitive bulk object is — a composite geometry⊕gauge connection. OGGEP tells us how finite observers, who never access the entire six-dimensional state space, acquire the lower-dimensional records that constitute empirical science.

In practical terms, OGGEP demands a map

$$\mathfrak{D}_{\mathrm{OGGEP}}: \qquad (\mathcal{H}_{\mathrm{Bulk}}, \rho_{\mathrm{Bulk}}, U_{\mathrm{Bulk}}) \longmapsto (\mathcal{H}_{\mathrm{Bdy}}, \rho_{\mathrm{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\}), \tag{4.18}$$

where the output is no longer a purely closed Hamiltonian bulk law but an operational boundary theory with states, channels, and instruments. The next section derives this map in exact and reduced forms. It is at that stage that GDOS appears: not as an independent postulate, but as the canonical open-system dynamics generated by the constitutionally admissible bulk-to-boundary reduction.

4.3 Derivation from OGGEP to GDOS

4.3.1 Overview: why observation forces an open-system theory

Chapter 2 derived a BRST-consistent six-dimensional quantum field theory on the warped GG-6 bulk, and Section 2 recalled the theorem-level input needed here. A closed bulk theory, however, is not yet a complete physical theory in the constitutional sense of GUT-10. AX-3 (OGGEP) and POS-4 force a further step: the theory must specify how a finite observer, equipped with a finite apparatus and bounded to a four-dimensional operational interface, acquires physical records. In the present program the operational interface is not a philosophical afterthought. It is a theorem-level boundary reduction.

The basic claim of OGGEP is therefore the following. Let $\mathcal{H}_{\mathrm{Bulk}}$ denote the total Hilbert space of the admissible GG-6 bulk. Then the observer-accessible sector is not the whole of $\mathcal{H}_{\mathrm{Bulk}}$ but a boundary-visible algebra $\mathcal{A}_{\mathrm{Bdy}}$ or, equivalently, a boundary Hilbert space $\mathcal{H}_{\mathrm{Bdy}}$ together with an operational channel

$$\mathfrak{D}_{\mathrm{OGGEP}}: \qquad \big(\mathcal{H}_{\mathrm{Bulk}}, \rho_{\mathrm{Bulk}}, U_{\mathrm{Bulk}}\big) \longmapsto \big(\mathcal{H}_{\mathrm{Bdy}}, \rho_{\mathrm{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\}\big), \tag{4.19}$$

where ρ_{Bdy} is a boundary density operator, $\Phi_{t,s}$ is an admissible state-update map, and $\{\mathcal{I}_\alpha\}$ is an instrument-valued family of measurement operations. The immediate consequence is that four-dimensional quantum mechanics is not primitive. It is the lawful shadow of a six-dimensional bulk after inaccessible degrees of freedom have been traced out and holographically reduced (Arisaka, 2026c; Choi, 1975; Gorini et al., 1976; Kraus, 1983; Lindblad, 1976; Stinespring, 1955; Nakajima, 1958; Zwanzig, 1960).

The reason this matters constitutionally is simple. A closed bulk equation alone tells us how the universe evolves. It does not yet tell us what a registered observation is. AX-3 closes that gap by insisting that any admissible theory must carry its own measurement semantics. In GG-6/GDOS this semantics is geometric and open-system from the start.

Table 4.7: From the closed six-dimensional bulk to the operational four-dimensional boundary.

Closed GG-6 bulk object	**Operational map required by AX-3**	**Boundary output**
Bulk Hilbert space $\mathcal{H}_{\mathrm{Bulk}}$	Environment split + boundary restriction	$\mathcal{H}_{\mathrm{Bdy}}$
Bulk state ρ_{Bulk}	Partial trace + holographic reduction	ρ_{Bdy}
Bulk unitary evolution $U_{\mathrm{Bulk}}(t,s)$	Reduced open-system channel	$\Phi_{t,s}$
Bulk observables / pointer couplings	Instrument reduction	$\{\mathcal{I}_\alpha\}$
Bulk diffeomorphism/gauge symmetry	Boundary Ward identities + inflow bookkeeping	Conserved current algebra

The rest of this section makes (4.19) explicit and shows how GDOS emerges as the most general constitutionally admissible boundary dynamics.

4.3.2 Exact boundary reduction: Hilbert-space split and projection operators

Let the total bulk Hilbert space be decomposed as

$$\mathcal{H}_{\mathrm{Bulk}} = \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}, \tag{4.20}$$

where $\mathcal{H}_{\mathrm{Bdy}}$ collects the degrees of freedom encoded on the operational four-dimensional boundary and $\mathcal{H}_{\mathrm{env}}$ denotes bulk or inaccessible environmental modes. This split is not unique in a purely mathematical sense; constitutionally, it is selected by the measurement protocol. AX-3 allows only those decompositions for which the resulting boundary evolution is physical in the sense of POS-4 and falsifiable in the sense of POS-7.

Choose a reference environment state $\rho_{\mathrm{env}}^{(0)}$ and define the Nakajima–Zwanzig projection

$$\mathcal{P}X := \mathrm{Tr}_{\mathrm{env}}(X) \otimes \rho_{\mathrm{env}}^{(0)}, \qquad \mathcal{Q} := 1 - \mathcal{P}. \tag{4.21}$$

By construction,

$$\mathcal{P}^2 = \mathcal{P}, \qquad \mathcal{Q}^2 = \mathcal{Q}, \qquad \mathcal{P}\mathcal{Q} = \mathcal{Q}\mathcal{P} = 0. \tag{4.22}$$

Let the exact bulk density operator obey the Liouville–von Neumann equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathrm{Bulk}}(t) = \mathfrak{L}_{\mathrm{Bulk}}(t)\rho_{\mathrm{Bulk}}(t), \qquad \mathfrak{L}_{\mathrm{Bulk}}(t)X := -\mathrm{i}[H_{\mathrm{Bulk}}(t), X] \tag{4.23}$$

for the closed bulk theory. Then applying $\mathcal{P}$ and $\mathcal{Q}$ yields the coupled system

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}\rho_{\mathrm{Bulk}}(t) = \mathcal{P}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) + \mathcal{P}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{Q}\rho_{\mathrm{Bulk}}(t), \tag{4.24}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{Q}\rho_{\mathrm{Bulk}}(t) = \mathcal{Q}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) + \mathcal{Q}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{Q}\rho_{\mathrm{Bulk}}(t). \tag{4.25}$$

Equation (4.25) has the formal solution

$$\mathcal{Q}\rho_{\mathrm{Bulk}}(t) = \mathcal{G}_{\mathcal{Q}}(t,t_0)\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) + \int_{t_0}^{t} \mathrm{d}s\, \mathcal{G}_{\mathcal{Q}}(t,s)\, \mathcal{Q}\mathfrak{L}_{\mathrm{Bulk}}(s)\mathcal{P}\rho_{\mathrm{Bulk}}(s), \tag{4.26}$$

where

$$\mathcal{G}_{\mathcal{Q}}(t,s) = \mathcal{T}\exp\!\left(\int_{s}^{t} \mathrm{d}u\, \mathcal{Q}\mathfrak{L}_{\mathrm{Bulk}}(u)\mathcal{Q}\right) \tag{4.27}$$

is the $\mathcal{Q}$-subspace propagator. Inserting (4.26) into (4.24) gives the exact memory-kernel equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}\rho_{\mathrm{Bulk}}(t) = \mathcal{P}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) + \int_{t_0}^{t} \mathrm{d}s\, \mathcal{K}(t,s)\mathcal{P}\rho_{\mathrm{Bulk}}(s) + \mathcal{I}(t), \tag{4.28}$$

with

$$\mathcal{K}(t,s) = \mathcal{P}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{G}_{\mathcal{Q}}(t,s)\mathcal{Q}\mathfrak{L}_{\mathrm{Bulk}}(s)\mathcal{P}, \qquad \mathcal{I}(t) = \mathcal{P}\mathfrak{L}_{\mathrm{Bulk}}(t)\mathcal{G}_{\mathcal{Q}}(t,t_0)\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0). \tag{4.29}$$

Equation (4.28) is exact. No Markov approximation has yet been made. The right-hand side already exhibits the three structural ingredients that later become GDOS: a projected local generator, a history-dependent memory term, and an inhomogeneous contribution encoding initial bulk–environment correlations.

The operational boundary state is now defined as

$$\rho_{\mathrm{Bdy}}(t) := \mathcal{R}\Big[\,\mathrm{Tr}_{\mathrm{env}}\,\rho_{\mathrm{Bulk}}(t)\Big] = \mathcal{R}\Big[\,\mathrm{Tr}_{\mathrm{env}}\,\mathcal{P}\rho_{\mathrm{Bulk}}(t)\Big], \tag{4.30}$$

where $\mathcal{R}$ is the holographic reduction map from boundary-supported data to the physical four-dimensional state space. In the most formal presentation one may absorb the trace and reduction into a single channel,

$$\mathfrak{D}_{\mathrm{OGGEP}} := \mathcal{R}\circ\mathrm{Tr}_{\mathrm{env}}\,. \tag{4.31}$$

Proposition 4.3.1 (CPTP character of the boundary map)**.** *If $\mathcal{R}$ is completely positive and trace preserving on the boundary-visible algebra, then for every bulk unitary evolution $U_{\mathrm{Bulk}}(t,s)$ the reduced boundary evolution*

$$\Phi_{t,s}(\rho_{\mathrm{Bdy}}) = \mathfrak{D}_{\mathrm{OGGEP}}\Big[U_{\mathrm{Bulk}}(t,s)\Big(\rho_{\mathrm{Bdy}}\otimes\rho_{\mathrm{env}}^{(0)}\Big)U_{\mathrm{Bulk}}(t,s)^{\dagger}\Big] \tag{4.32}$$

is completely positive and trace preserving.

Proof. Unitary conjugation is completely positive and trace preserving. The partial trace is completely positive and trace preserving. By assumption, $\mathcal{R}$ is completely positive and trace preserving. The composition of CPTP maps is CPTP. □

Proposition 4.3.1 is the mathematical core of POS-4 in the quantum sector. It shows that once the bulk theory is lawful and the operational channel is lawful, the boundary evolution is automatically a legal quantum dynamical map.

4.3.3 From the exact memory kernel to GDOS

The exact equation (4.28) is too general to be useful phenomenologically unless it is reorganized. The purpose of GDOS is to provide the minimal canonical form of this reorganization. There are two levels to the statement.

At the *quantum operator level*, one seeks a time-local generator $\mathfrak{L}_{\mathrm{GDOS}}(t)$ such that

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathrm{Bdy}}(t) = \mathfrak{L}_{\mathrm{GDOS}}(t)\rho_{\mathrm{Bdy}}(t) \tag{4.33}$$

and such that the propagator generated by $\mathfrak{L}_{\mathrm{GDOS}}(t)$ reproduces the exact history dependence whenever the environment correlation time is short compared with the relevant boundary scales. In the Markovian semigroup limit this generator must assume the Gorini–Kossakowski–Lindblad–Sudarshan form (Gorini et al., 1976; Lindblad, 1976)

$$\mathfrak{L}_{\mathrm{GDOS}}(\rho) = -\mathrm{i}[H_{\mathrm{eff}}, \rho] + \sum_{\alpha} \gamma_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right), \tag{4.34}$$

with $\gamma_\alpha \geq 0$. Equation (4.34) is therefore not an extra postulate in the present framework; it is the constitutionally admissible fixed point of the bulk-to-boundary reduction.

At the *coarse-grained mesoscopic level*, one may represent the state by a probability density $p(x,t)$ on a state manifold $\mathcal{M}_{\mathrm{state}}$ with local coordinates x^A, $A = 1, \ldots, N$. Then the generic GDOS evolution is written as a continuity equation

$$\partial_t p = -\nabla_A J^A, \tag{4.35}$$

with current decomposed as

$$J^A = \underbrace{-D^{AB}\nabla_B p}_{\text{noise/diffusion}} + \underbrace{b^A p}_{\text{drift}} = \underbrace{-D^{AB}\nabla_B p}_{\text{noise}} + \underbrace{\Big(- g^{AB}\nabla_B \Phi + h^A \Big) p}_{\text{gradient + solenoidal}}, \tag{4.36}$$

subject to

$$\nabla_A h^A = 0. \tag{4.37}$$

Here D^{AB} is positive semidefinite, Φ is an effective potential or free-energy landscape, and h^A is the divergence-free circulation piece. Equation (4.36) is the most compact expression of the GDOS philosophy: every admissible open-system dynamics decomposes into gradient relaxation, rotational/circulatory transport, and fluctuations.

This decomposition is powerful because several familiar principles become corollaries rather than independent assumptions. The entropy production rate

$$\dot{S}_{\mathrm{tot}} = \int_{\mathcal{M}_{\mathrm{state}}} \mathrm{d}\mu \, \frac{J_A (D^{-1})^{AB} J_B}{p} \geq 0 \tag{4.38}$$

is nonnegative whenever D^{AB} is positive semidefinite. Thus the phenomenological laws usually discussed separately under names such as least-time relaxation, maximum entropy production, or free-energy minimization appear as different asymptotic regimes or projections of the same GDOS

current geometry (Arisaka, 2026c). In this book we do not elevate these derivative principles to axioms. They follow from AX-3 and POS-4 after coarse-graining.

Theorem 4.3.2 (Constitutional derivation of GDOS)**.** *Assume the GG-6 bulk of Sections 2–3, AX-3, and POS-4. Then every admissible boundary reduction is governed by a completely positive, trace-preserving channel* $\Phi_{t,s}$*. In the exact formulation it obeys a Nakajima–Zwanzig memory-kernel equation; in the time-local Markovian limit it reduces to a GKLS generator* (4.34)*; and after mesoscopic coarse-graining it takes the GDOS form* (4.35)–(4.37)*. Hence* GDOS *is the canonical boundary dynamics implied by* OGGEP*.*

Proof sketch. Equations (4.28)–(4.29) establish the exact projected open-system evolution. Proposition 4.3.1 ensures that the reduced propagator is CPTP. Standard dilation and semigroup arguments then force the Markovian limit to be GKLS. Finally, the Kramers–Moyal/Fokker–Planck coarse-grained representation gives the continuity equation with gradient, solenoidal, and diffusive pieces. These are precisely the structural ingredients called GDOS. □

4.3.4 Measurement as holographic projection from six to four dimensions

The central physical content of OGGEP is that measurement is not an external collapse rule. It is an operationally reduced bulk process. Let σ_{app} be the initial state of a boundary apparatus, and let U_{SA} denote the induced interaction between the boundary system S and the apparatus A. Choose a pointer decomposition $\{\Pi_\alpha\}$ on the apparatus Hilbert space. Then the instrument associated with outcome α is

$$\mathcal{I}_\alpha(\rho_{\mathrm{Bdy}}) = \mathrm{Tr}_A\Big[(\mathbf{1}\otimes\Pi_\alpha)\,U_{SA}\big(\rho_{\mathrm{Bdy}}\otimes\sigma_{\mathrm{app}}\big)U_{SA}^\dagger(\mathbf{1}\otimes\Pi_\alpha)\Big]. \tag{4.39}$$

The outcome probability and posterior state are

$$p_\alpha = \mathrm{Tr}\left[\mathcal{I}_\alpha(\rho_{\mathrm{Bdy}})\right], \qquad \rho_{\mathrm{Bdy}}^{(\alpha)} = \frac{\mathcal{I}_\alpha(\rho_{\mathrm{Bdy}})}{p_\alpha}. \tag{4.40}$$

These are the usual Born and Lüders/Kraus formulas, but in this book they are no longer primitive. They are the 4D instrument image of a 6D bulk interaction followed by partial trace and holographic reduction.

To state this more explicitly, write

$$\rho_{\mathrm{Bdy}} = \mathfrak{D}_{\mathrm{OGGEP}}(\rho_{\mathrm{Bulk}}), \qquad \mathcal{I}_\alpha = \mathfrak{D}_{\mathrm{OGGEP}}\circ\mathfrak{M}_\alpha, \tag{4.41}$$

where $\mathfrak{M}_\alpha$ is the bulk interaction-and-pointer-selection map. Thus the boundary measurement algebra is not autonomous; it is inherited. This is precisely the sense in which the present programme has repeatedly described quantum measurement as a "6D → 4D projection". Within this book that slogan becomes the concrete statement (4.41).

Remark 4.3.3. The framework does not require the boundary reduction to be identical to the exact strong form of AdS/CFT duality (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998). The present use of holography is operational rather than string-theoretic: it is a lawful coarse-graining and encoding map from a higher-dimensional bulk to a lower-dimensional observer interface. Exact duality may be a special case, not a prerequisite.

4.3.5 Derivation of four-dimensional quantum mechanics

We can now formulate the boundary quantum theory explicitly.

Theorem 4.3.4 (Boundary quantum mechanics from GG-6 + OGGEP)**.** *Under AX-1–AX-4 and POS-1–POS-7, together with the GG-6 bulk of Sections 2–3, the observer-accessible boundary theory is a four-dimensional quantum theory whose states are density operators on $\mathcal{H}_{\mathrm{Bdy}}$, whose admissible evolutions are CPTP maps $\Phi_{t,s}$, whose measurements are instruments $\{\mathcal{I}_\alpha\}$, and whose closed-system limit is unitary Schrödinger evolution.*

Proof sketch. The state space follows from the Hilbert-space split (4.20) and the channel (4.31). CPTP evolution follows from Proposition 4.3.1. Measurement statistics follow from (4.39)–(4.40). In the limit of vanishing coupling to $\mathcal{H}_{\mathrm{env}}$, the reduced generator has no dissipator and reduces to $-\mathrm{i}[H_{\mathrm{eff}}, \rho]$, which is precisely unitary quantum mechanics. □

The theorem is important conceptually because it reverses the historical order in which the subject is usually taught. Conventionally one starts from four-dimensional quantum mechanics and only later attaches environment, decoherence, and measurement. Here one starts from a closed six-dimensional bulk and derives four-dimensional quantum mechanics as the operational law of the boundary observer. The open-system nature of actual measurement is therefore not a complication of the formalism. It is its very origin.

Table 4.8: Three mathematically equivalent views of the boundary dynamics derived from OGGEP.

Level of description	**Canonical equation**	**Interpretation**
Exact projected dynamics	Nakajima–Zwanzig memory-kernel equation	Non-Markovian boundary evolution with explicit memory and initial-correlation terms
Time-local Markovian limit	GKLS master equation	Completely positive semigroup evolution for the reduced density operator
Mesoscopic coarse-grained limit	GDOS continuity/current decomposition	Gradient, solenoidal, and diffusive transport on the boundary state manifold

The conceptual gain of Table 4.8 is worth emphasizing. These are not three competing boundary theories. They are three compatible representations of one and the same operational reduction, written at three different levels of resolution. The exact kernel retains memory, the GKLS form keeps complete positivity manifest, and the GDOS representation exposes the geometric current structure most directly.

4.4 Derivation of General Relativity and formal 4D QFT at the IR boundary

The previous section established the operational channel and the boundary quantum law. Two further reductions are now required. First, the metric sector of the warped six-dimensional bulk must descend to a four-dimensional gravitational law. Second, the anomaly-free bulk functional must descend to a formal infrared generating functional for the observer-accessible four-dimensional quantum field theory. These are logically distinct but technically linked: both are infrared statements about zero modes and heavy-mode integration.

Table 4.9: How different bulk structures descend to different infrared 4D structures.

Bulk ingredient	**Reduction move**	**4D infrared output**
Warped metric g_{AB}	Zero-mode projection of the Einstein–Hilbert sector	Einstein gravity with effective $M_{\rm Pl}$
Composite gauge connection	Zero-mode truncation and boundary conditions	Gauge sector of the IR EFT
Heavy KK/environmental modes	Functional integration	Local plus higher-dimensional terms in $S_{4,\rm eff}$
Operational dictionary	Restriction to boundary-visible states and observables	4D generating functional and measurable correlation functions

4.4.1 Derivation of four-dimensional general relativity

The same OGGEP machinery that produces boundary quantum mechanics also determines how the gravitational sector descends. The metric zero mode is obtained by retaining the $(n, \ell) = (0, 0)$ component of the bulk metric perturbation in the KK expansion. Insert the warped ansatz

$$\mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2 \mathrm{d}\theta^2 \tag{4.42}$$

into the six-dimensional Einstein–Hilbert term:

$$S_{\rm EH}^{(6)} = \frac{M_6^4}{2} \int \mathrm{d}^6x \sqrt{-g_6}\, R_6. \tag{4.43}$$

Using

$$\sqrt{-g_6} = e^{-4A(r)} R_\theta \sqrt{-g_4}, \qquad R_6 \supset e^{2A(r)} R_4 + \cdots, \tag{4.44}$$

one obtains the four-dimensional Einstein term

$$S_{\rm EH}^{(4)} = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4x \sqrt{-g_4}\, R_4, \qquad M_{\rm Pl}^2 = M_6^4\, 2\pi R_\theta \int_{I_r} \mathrm{d}r\, e^{-2A(r)}. \tag{4.45}$$

Thus ordinary four-dimensional gravity is already present as the metric zero mode of the GG-6 bulk.

The operational content comes from the boundary generating functional

$$W_{\mathrm{Bdy}}[g_{\mu\nu}] := -\mathrm{i}\ln Z_{\mathrm{Bdy}}[g_{\mu\nu}], \qquad T^{\mu\nu}(x) = \frac{2}{\sqrt{-g_4}}\frac{\delta W_{\mathrm{Bdy}}}{\delta g_{\mu\nu}(x)}. \tag{4.46}$$

If Z_{Bdy} is invariant under infinitesimal boundary diffeomorphisms $\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$, then

$$\delta_\xi W_{\mathrm{Bdy}} = \int \mathrm{d}^4x \sqrt{-g_4}\, \xi_\nu \nabla_\mu T^{\mu\nu} = 0, \tag{4.47}$$

and hence

$$\nabla_\mu T^{\mu\nu} = 0 \tag{4.48}$$

in the closed boundary sector. Varying the effective action with respect to $g_{\mu\nu}$ gives

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \langle T_{\mu\nu} \rangle_{\mathrm{Bdy}} + \Delta T_{\mu\nu}^{\mathrm{GS}} + \Delta T_{\mu\nu}^{\mathrm{KK}} + \cdots, \tag{4.49}$$

where the extra terms encode tensor-sector and heavy-mode corrections. In the weak-fluctuation classical limit these corrections reduce to the familiar Einstein field equation.

Theorem 4.4.1 (Boundary GR from GG-6)**.** *Within the constitutional framework of this book, the low-energy boundary gravitational law is four-dimensional general relativity, with Planck mass given by* (4.45)*, stress-energy conservation given by* (4.48)*, and field equation given to leading order by* (4.49)*.*

Proof sketch. Equation (4.45) follows by dimensional reduction of the Einstein–Hilbert term. Boundary diffeomorphism invariance implies (4.48). Varying the resulting effective action yields (4.49). □

Two comments are in order. First, the present derivation does not treat four-dimensional GR as a separate primitive theory standing beside GG-6. It is the low-energy metric zero-mode theorem of the bulk. Second, the open-system viewpoint survives even in gravity: for an observer restricted to the boundary, apparent nonconservation may appear as inflow terms during measurement or coarse-grained interaction with hidden bulk modes. Closed-system conservation remains exact only for the full bulk-plus-boundary system.

4.4.2 Formal derivation of the generic four-dimensional quantum field theory

The previous sections establish the ingredients needed for the boundary field theory. Section 2 recalled the anomaly-free six-dimensional quantum functional imported from Chapter 2, and Section 3 provided the lawful bulk-to-boundary reduction. The generic four-dimensional quantum field theory is now defined by integrating over bulk configurations with fixed boundary data. Concretely,

$$Z_4[J] := Z_6\big[\Phi_\partial = J\big] = \int_{\Phi|_{\partial M_6}=J} \mathcal{D}\Phi_6 \, \exp\Big(\mathrm{i}S_{6,\mathrm{tot}}[\Phi_6]\Big), \tag{4.50}$$

where

$$S_{6,\mathrm{tot}} = S_6 + S_{\mathrm{gf+gh}} \tag{4.51}$$

is the BRST-complete six-dimensional action. Equivalently, decompose the fields into zero modes and heavy KK/environmental modes,

$$\Phi_6 = \Phi_0 \oplus \Phi_{\text{heavy}}, \tag{4.52}$$

and define the exact four-dimensional effective action by

$$\exp\bigl(\mathrm{i}S_{4,\text{eff}}[\Phi_0]\bigr) = \int \mathcal{D}\Phi_{\text{heavy}}\, \exp\Bigl(\mathrm{i}S_{6,\text{tot}}[\Phi_0, \Phi_{\text{heavy}}]\Bigr). \tag{4.53}$$

Then

$$Z_4[J] = \int \mathcal{D}\Phi_0\, \exp\left(\mathrm{i}S_{4,\text{eff}}[\Phi_0] + \mathrm{i}\int \mathrm{d}^4x\, J\Phi_0\right). \tag{4.54}$$

Equation (4.54) is the formal derivation of the boundary QFT. It is important to distinguish what is exact from what is model-dependent. The equality between Z_4 and the reduced bulk path integral is exact. The explicit local form of $S_{4,\text{eff}}$ depends on the chosen warp factor, boundary conditions, matter multiplicities, and threshold corrections. The role of this book is to derive the former and to write the latter in its constitutionally minimal form.

Theorem 4.4.2 (Formal 4D QFT from GG-6)**.** *Assume the GG-6 bulk recalled in Section 4.2 and the OGGEP reduction of Section 4.3. Then the four-dimensional boundary theory is a quantum field theory with generating functional* (4.54)*, BRST symmetry inherited from the six-dimensional theory, and a local effective action obtained by integrating out the heavy KK and environmental sectors.*

Proof sketch. Equation (4.50) defines the generating functional by the bulk path integral. Equation (4.53) integrates out heavy modes. The existence of the BRST structure follows from the homogeneous six-dimensional Slavnov–Taylor identity together with the fact that integrating out BRST doublets preserves the cohomology class of the reduced theory. □

4.4.3 Summary of Section 4

The two derivations established in this section can be compressed into one line:

$$\boxed{\begin{array}{l}\text{warped GG-6 bulk} \xRightarrow{\text{metric zero mode}} \text{4D GR} \\ \text{anomaly-free 6D QFT} \xRightarrow{\text{heavy-mode integration}} \text{formal 4D QFT}\end{array}}. \tag{4.55}$$

The next section specializes the formal four-dimensional quantum field theory to its low-energy zero-mode sector and shows how the ordinary Standard Model gauge and matter content is recovered together with the additional baryonic and informational Abelian sectors inherited from the bulk.

4.5 Derivation of the Standard Model at the IR boundary

4.5.1 Kaluza–Klein expansion and zero-mode truncation

To obtain the low-energy theory, expand every bulk field as

$$\Phi(x^\mu, r, \theta) = \sum_{n,\ell} \phi^{(n,\ell)}(x^\mu)\, f_n(r)\, \frac{\mathrm{e}^{\mathrm{i}\ell\theta/R_\theta}}{\sqrt{2\pi R_\theta}}. \tag{4.56}$$

The labels n and ℓ enumerate radial and compact modes, respectively. The zero modes are the $(n,\ell) = (0,0)$ components. Heavy modes carry masses of order

$$m_{n,\ell}^2 \sim m_n^2 + \frac{\ell^2}{R_\theta^2} \tag{4.57}$$

up to warp corrections.

For the gauge sector, the four-dimensional kinetic term follows from

$$-\frac{1}{4g_6^2}\int \mathrm{d}^6x\sqrt{-g_6}\, g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g_4}\,\frac{1}{g_4^2}\,F^{(0)}_{\mu\nu}F^{(0)\mu\nu} + \cdots, \tag{4.58}$$

with

$$\frac{1}{g_4^2} = \frac{2\pi R_\theta}{g_6^2}\int_{I_r} \mathrm{d}r\ |f_0(r)|^2\,. \tag{4.59}$$

Unlike the gravitational zero mode, the gauge zero-mode normalization does not inherit the extra factor $e^{-2A(r)}$ because the metric determinants cancel between $\sqrt{-g_6}$ and the inverse metrics. This distinction is one of the recurrent quantitative signatures of warped compactification.

For the gravitational sector we already found

$$M_{\mathrm{Pl}}^2 = M_6^4\, 2\pi R_\theta \int_{I_r} \mathrm{d}r\, e^{-2A(r)}. \tag{4.60}$$

Thus the low-energy hierarchy between gravitational and gauge couplings is partly a geometrical statement about different overlap integrals.

4.5.2 Symmetry breaking by boundary conditions

The non-Abelian GUT sector is SU(5). To recover the Standard Model gauge group on the boundary, choose the parity matrix

$$P = \mathrm{diag}(+1,+1,+1,-1,-1), \tag{4.61}$$

and impose orbifold/interval boundary conditions of the form

$$A_\mu(x, r, -\theta) = P\, A_\mu(x, r, \theta)\, P^{-1}, \tag{4.62}$$

$$A_\theta(x, r, -\theta) = -P\, A_\theta(x, r, \theta)\, P^{-1}. \tag{4.63}$$

Generators commuting with P are even and therefore admit zero modes. Generators anti-commuting with P are odd and have no zero modes. This realizes the familiar decomposition

$$\mathrm{SU}(5) \longrightarrow \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \tag{4.64}$$

at the zero-mode level (Georgi and Glashow, 1974; Kawamura, 2001). The broken X, Y bosons are projected out of the massless boundary spectrum.

The full zero-mode boundary gauge group is therefore

$$G_{\text{IR,before EWSB}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}. \tag{4.65}$$

After electroweak symmetry breaking,

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{\mathrm{EM}}, \tag{4.66}$$

so that the late-time gauge group is

$$G_{\text{IR,after EWSB}} = \mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}. \tag{4.67}$$

Table 4.10: Gauge symmetry descent from the GG-6 bulk to the low-energy boundary theory.

Stage	**Gauge group**	**Mechanism**
GG-6 bulk at E_{GUT}	$\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity	AX-2 + BI-6 + anomaly admissibility
Zero-mode boundary before EWSB	$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$	Orbifold/interval boundary conditions
Low-energy boundary after EWSB	$\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$	Higgs doublet vacuum expectation value

4.5.3 Representation decomposition and the emergence of three generations

The relevant SU(5) representations decompose in the standard way:

$$\mathbf{24} \to (8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{+5/6}, \tag{4.68}$$

$$\mathbf{10} \to (3,2)_{+1/6} \oplus (\bar{3},1)_{-2/3} \oplus (1,1)_{+1}, \tag{4.69}$$

$$\overline{\mathbf{5}} \to (\bar{3},1)_{+1/3} \oplus (1,2)_{-1/2}, \tag{4.70}$$

$$\mathbf{5}_H \to (3,1)_{-1/3} \oplus (1,2)_{+1/2}. \tag{4.71}$$

In component notation,

$$\Psi_{10} \to (Q, u^c, e^c), \qquad \Psi_{\bar{5}} \to (d^c, L), \qquad H_5 \to (T, H). \tag{4.72}$$

At the level of the six-dimensional anomaly-complete bulk, the matter multiplicities can be larger than the observed four-dimensional spectrum. The purpose of the orbifold and boundary conditions is to leave precisely three linearly independent chiral combinations of $\mathbf{10} \oplus \bar{\mathbf{5}}$ as zero

modes:

$$\left\{\Psi_{10}^{(i)}, \Psi_{\bar{5}}^{(i)}\right\}_{i=1}^{3}, \tag{4.73}$$

together with one light Higgs doublet H. This is the constitutionally minimal low-energy content compatible with the observed Standard Model matter families.

It is important to emphasize that this book does not need to choose one unique microscopic six-dimensional family reservoir in order to derive the three observed generations. What is required is the theorem-level statement that the anomaly-admissible bulk can support a boundary-condition choice whose chiral index is three. That is sufficient for the formal reduction pursued here.

4.5.4 The explicit zero-mode Lagrangian before electroweak symmetry breaking

We now write the most useful explicit form of the boundary zero-mode Lagrangian. Denote the gauge fields by

$$G_\mu^a, \qquad W_\mu^i, \qquad B_\mu, \qquad X_\mu^{(B)}, \qquad X_\mu^{(Qi)}.$$

The corresponding field strengths are

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_3 f^{abc} G_\mu^b G_\nu^c, \tag{4.74}$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_2 \epsilon^{ijk} W_\mu^j W_\nu^k, \tag{4.75}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad X_{\mu\nu}^{(B)} = \partial_\mu X_\nu^{(B)} - \partial_\nu X_\mu^{(B)}, \qquad X_{\mu\nu}^{(Qi)} = \partial_\mu X_\nu^{(Qi)} - \partial_\nu X_\mu^{(Qi)}. \tag{4.76}$$

The covariant derivative acting on a generic field Φ is

$$D_\mu \Phi = \left[\nabla_\mu + \mathrm{i} g_3 G_\mu^a T_3^a + \mathrm{i} g_2 W_\mu^i t^i + \mathrm{i} g_Y Y B_\mu + \mathrm{i} g_B q_B X_\mu^{(B)} + \mathrm{i} g_{Qi} q_{Qi} X_\mu^{(Qi)}\right] \Phi. \tag{4.77}$$

The physically motivated baryon charges are

$$q_B(Q) = \frac{1}{3}, \qquad q_B(u^c) = q_B(d^c) = -\frac{1}{3}, \qquad q_B(L) = q_B(e^c) = q_B(H) = 0. \tag{4.78}$$

For the Qi sector the most general derivation only requires that the anomaly vector lie in the image of the tensor/Stückelberg cancellation map. To write a concrete zero-mode Lagrangian, however, we adopt the minimal diagonal realization

$$q_{Qi}(Q) = \frac{1}{3}, \qquad q_{Qi}(u^c) = q_{Qi}(d^c) = -\frac{1}{3}, \qquad q_{Qi}(L) = q_{Qi}(e^c) = q_{Qi}(H) = 0, \tag{4.79}$$

which mirrors the $U(1)_B$ assignment and is compatible with the diagonal equal-weight BI-6 sector.

The renormalizable zero-mode Lagrangian is then

$$\begin{aligned}\mathcal{L}_{4,\mathrm{zero}} = \sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \Lambda_4\right) - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X^{(B)}_{\mu\nu}X^{\mu\nu}_{(B)} - \frac{1}{4}X^{(Qi)}_{\mu\nu}X^{\mu\nu}_{(Qi)}\\ - \frac{\epsilon_{YB}}{2}B_{\mu\nu}X^{\mu\nu}_{(B)} - \frac{\epsilon_{YQi}}{2}B_{\mu\nu}X^{\mu\nu}_{(Qi)} - \frac{\epsilon_{BQi}}{2}X^{(B)}_{\mu\nu}X^{\mu\nu}_{(Qi)}\\ + \sum_{i=1}^{3}\left(\bar{Q}_i\mathrm{i}\gamma^\mu D_\mu Q_i + \bar{u}^c_i\mathrm{i}\gamma^\mu D_\mu u^c_i + \bar{d}^c_i\mathrm{i}\gamma^\mu D_\mu d^c_i + \bar{L}_i\mathrm{i}\gamma^\mu D_\mu L_i + \bar{e}^c_i\mathrm{i}\gamma^\mu D_\mu e^c_i\right)\\ + (D_\mu H)^\dagger D^\mu H - \left(-\mu_H^2 H^\dagger H + \lambda_H(H^\dagger H)^2\right) - \left(\bar{Q}Y_u\widetilde{H}\,u_R + \bar{Q}Y_d H d_R + \bar{L}Y_e H e_R + \mathrm{h.c.}\right)\\ + \frac{a(x)}{16\pi^2}\Big[c_3\ \mathrm{Tr}(G\widetilde{G}) + c_2\ \mathrm{Tr}(W\widetilde{W}) + c_Y B\widetilde{B} + c_B X^{(B)}\widetilde{X}^{(B)} + c_{Qi}X^{(Qi)}\widetilde{X}^{(Qi)} + c_R\,\mathrm{tr}(R\widetilde{R})\Big]\\ + \frac{1}{2}\left(\partial_\mu\sigma_B + M_B X^{(B)}_\mu\right)^2 + \frac{1}{2}\left(\partial_\mu\sigma_{Qi} + M_{Qi}X^{(Qi)}_\mu\right)^2 + \sum_{d>4}\frac{c_d}{M_{\mathrm{KK}}^{d-4}}\mathcal{O}_d. \end{aligned} \tag{4.80}$$

Equation (4.80) is the explicit low-energy theorem-level answer to the question posed in the Introduction: what does the four-dimensional effective theory look like? It contains the ordinary Standard Model structure together with the baryonic and information gauge sectors inherited from BI-6, the tensor-descended axionic couplings, and the Stückelberg completion required by anomaly inflow.

4.5.5 Electroweak symmetry breaking and the physical infrared gauge group

Choose the unitary-gauge Higgs vacuum expectation value

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix}, \qquad v^2 = \frac{\mu_H^2}{\lambda_H}. \tag{4.81}$$

Then the electroweak gauge boson masses are

$$m_W = \frac{1}{2}g_2 v, \qquad m_Z = \frac{1}{2}v\sqrt{g_2^2 + g_Y^2}, \qquad m_\gamma = 0. \tag{4.82}$$

Defining the weak mixing angle by

$$\tan\theta_W = \frac{g_Y}{g_2}, \qquad e = g_2\sin\theta_W = g_Y\cos\theta_W, \tag{4.83}$$

the massless electromagnetic field is

$$A^{\mathrm{EM}}_\mu = \sin\theta_W\, W^3_\mu + \cos\theta_W\, B_\mu. \tag{4.84}$$

Thus the visible gauge group becomes $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}}$, supplemented by the additional $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ sectors whose masses depend on the Stückelberg and mixing parameters.

This separation between the electroweak and the B/Qi sectors is conceptually important. The ordinary Standard Model is recovered as the visible non-Abelian-plus-electromagnetic core, while GG-6 predicts a larger bookkeeping structure that retains baryonic and information charges as exact or nearly exact symmetries depending on the Stückelberg realization.

4.5.6 BRST reduction and Ward identities in the zero-mode theory

The four-dimensional zero-mode theory inherits BRST symmetry from the six-dimensional parent. Introduce ghosts

$$c_3^a, \qquad c_2^i, \qquad c_Y, \qquad c_B, \qquad c_{Qi},$$

with standard antighost and Nakanishi–Lautrup partners. The BRST transformations of the gauge fields are

$$sG_\mu^a = \partial_\mu c_3^a + g_3 f^{abc} G_\mu^b c_3^c, \qquad sc_3^a = -\frac{g_3}{2} f^{abc} c_3^b c_3^c, \tag{4.85}$$

$$sW_\mu^i = \partial_\mu c_2^i + g_2 \epsilon^{ijk} W_\mu^j c_2^k, \qquad sc_2^i = -\frac{g_2}{2} \epsilon^{ijk} c_2^j c_2^k, \tag{4.86}$$

$$sB_\mu = \partial_\mu c_Y, \qquad sc_Y = 0, \tag{4.87}$$

$$sX_\mu^{(B)} = \partial_\mu c_B, \qquad sc_B = 0, \tag{4.88}$$

$$sX_\mu^{(Qi)} = \partial_\mu c_{Qi}, \qquad sc_{Qi} = 0. \tag{4.89}$$

For matter fields,

$$s\Phi = \mathrm{i}\Big(g_3 c_3^a T_3^a + g_2 c_2^i t^i + g_Y Y c_Y + g_B q_B c_B + g_{Qi} q_{Qi} c_{Qi} \Big) \Phi. \tag{4.90}$$

For the Stückelberg fields,

$$s\sigma_B = -M_B c_B, \qquad s\sigma_{Qi} = -M_{Qi} c_{Qi}, \tag{4.91}$$

so that the combinations $\partial_\mu \sigma_B + M_B X_\mu^{(B)}$ and $\partial_\mu \sigma_{Qi} + M_{Qi} X_\mu^{(Qi)}$ are BRST invariant.

The 1PI effective action therefore obeys the Slavnov–Taylor identity

$$\mathcal{S}(\Gamma_4) = 0, \tag{4.92}$$

with $\mathcal{S}$ the usual sum over antifield-source pairings. From (4.92) one derives the component Ward operators

$$\mathcal{W}_A(x)\Gamma_4 = 0, \qquad A \in \{SU(3)_C, SU(2)_L, U(1)_Y, U(1)_B, U(1)_{Qi}\}. \tag{4.93}$$

In particular,

$$\mathcal{W}_B(x)\Gamma_4 = -\partial_\mu \frac{\delta \Gamma_4}{\delta X_\mu^{(B)}(x)} + \mathrm{i} \sum_\Phi q_B(\Phi) \left(\Phi \frac{\delta \Gamma_4}{\delta \Phi} - \Phi^\dagger \frac{\delta \Gamma_4}{\delta \Phi^\dagger} \right) + M_B \frac{\delta \Gamma_4}{\delta \sigma_B(x)} = 0, \tag{4.94}$$

and similarly for $U(1)_{Qi}$. These equations express the fact that the baryonic and information gauge symmetries are exact symmetries of the full zero-mode EFT once the tensor/Stückelberg sector is included.

4.5.7 Anomaly coefficients and their tensor cancellation

It is instructive to compute the baryon anomaly coefficients directly from the light chiral spectrum. For one generation of left-handed Weyl fermions with charges (4.78),

$$\mathcal{A}_{B33} = \sum_f q_B(f)\, T_3(f)\, d_2(f) = 0, \tag{4.95}$$

$$\mathcal{A}_{B22} = \sum_f q_B(f)\, T_2(f)\, d_3(f) = \frac{1}{2}, \tag{4.96}$$

$$\mathcal{A}_{BYY} = \sum_f q_B(f)\, Y_f^2\, d_3(f) d_2(f) = -\frac{1}{2}, \tag{4.97}$$

$$\mathcal{A}_{YBB} = 0, \qquad \mathcal{A}_{BBB} = 0, \qquad \mathcal{A}_{BRR} = 0. \tag{4.98}$$

For three generations,

$$\mathcal{A}_{B22} = \frac{3}{2}, \qquad \mathcal{A}_{BYY} = -\frac{3}{2}, \qquad \mathcal{A}_{B33} = \mathcal{A}_{YBB} = \mathcal{A}_{BBB} = \mathcal{A}_{BRR} = 0. \tag{4.99}$$

Thus the only nonvanishing baryon anomalies in the light spectrum are $U(1)_B - SU(2)_L^2$ and $U(1)_B - U(1)_Y^2$.

The tensor-descended GS sector cancels these exactly. Write

$$\mathcal{L}^{(4)}_{\rm GS} = \frac{1}{24\pi^2}\, a^r \Big[k_{r2}\ \mathrm{Tr}(W\widetilde{W}) + k_{rY}\, B\widetilde{B} + k_{r3}\ \mathrm{Tr}(G\widetilde{G}) + k_{rB}\, X_B \widetilde{X}_B + k_{rQi}\, X_{Qi}\widetilde{X}_{Qi} + k_{rR}\, R\widetilde{R} \Big]. \tag{4.100}$$

Under a baryon gauge transformation $\delta_B a^r = -M_B^{\ r}\alpha_B$,

$$\delta_B S^{(4)}_{\rm GS} = -\frac{1}{24\pi^2} \int \mathrm{d}^4x\, \alpha_B \Big[M_B^{\ r} k_{r2}\ \mathrm{Tr}(W\widetilde{W}) + M_B^{\ r} k_{rY}\, B\widetilde{B} + \cdots \Big]. \tag{4.101}$$

Exact cancellation requires

$$M_B^{\ r} k_{r2} = \frac{3}{2}, \qquad M_B^{\ r} k_{rY} = -\frac{3}{2}, \tag{4.102}$$

in the normalization used above. In the minimal diagonal realization $q_{Qi} = q_B$, the Qi anomalies are identical and are cancelled by the same tensor image with

$$M_{Qi}^{\ r} k_{r2} = \frac{3}{2}, \qquad M_{Qi}^{\ r} k_{rY} = -\frac{3}{2}. \tag{4.103}$$

Theorem 4.5.1 (Anomaly-free current algebra of the zero-mode theory)**.** *The zero-mode EFT derived from GG-6 has an exact anomaly-free BRST/Ward algebra. In particular, once the GS/Stückelberg descendants of BI-6 are included, the charge operators*

$$Q_C^a, \qquad Q_L^i, \qquad Q_Y, \qquad Q_B, \qquad Q_{Qi}$$

satisfy the ordinary direct-product Lie algebra relations with no anomalous central extension.

Proof sketch. The six-dimensional homogeneous Slavnov–Taylor identity descends to (4.92). The only nonzero light-spectrum baryon and Qi anomalies are cancelled by (4.102)–(4.103). Hence

the four-dimensional Ward operators are homogeneous and the corresponding equal-time charge algebra has no anomaly term. □

4.5.8 Absolutely stable protons from $U(1)_B$

One of the central claims of this book is that proton stability is not accidental in GG-6. It is geometric and gauge-theoretic. The proof is straightforward once the zero-mode symmetry structure is explicit.

Theorem 4.5.2 (Absolute proton stability)**.** *Assume that the low-energy theory retains the exact local symmetry $U(1)_B$, possibly with a Stückelberg mass for the baryonic gauge boson. Then every local operator $\mathcal{O}$ in the exact low-energy effective action satisfies $\Delta B(\mathcal{O}) = 0$. In particular, the proton is absolutely stable.*

Proof. Gauge invariance under $U(1)_B$ forbids any local operator with nonzero baryon charge. The standard baryon-violating operators

$$QQQL, \qquad u^c u^c d^c e^c, \qquad QQQH, \qquad u^c d^c d^c, \qquad QLd^c, \qquad LLe^c$$

carry $\Delta B \neq 0$ and are therefore forbidden. Moreover, the orbifold projection removes the X, Y gauge boson zero modes, and the triplet Higgs zero mode is absent or heavy, so the usual GUT-mediated proton-decay channels are not present in the massless theory. A Stückelberg mass does not break the gauge symmetry; it only renders the vector boson massive while preserving the selection rule. Therefore no local proton-decay operator can be generated. □

The theorem deserves a conceptual comment. In ordinary SU(5) unification, proton decay is a generic expectation because the X, Y gauge bosons and triplet Higgs mediate baryon-violating transitions. In the present framework, the orbifold projection and the exact extra $U(1)_B$ gauge symmetry reverse this logic. The proton is not metastable by accident; it is protected by the same enlarged symmetry structure that made BI-6 possible in the first place.

Table 4.11: Zero-mode field content and charges in the minimal diagonal $U(1)_{Qi}$ realization.

Field	$SU(3)_C \times SU(2)_L$	Y	Q_{EM}	q_B	q_{Qi}
Q	$(3, 2)$	$+1/6$	$(+2/3, -1/3)$	$+1/3$	$+1/3$
u^c	$(\bar{3}, 1)$	$-2/3$	$-2/3$	$-1/3$	$-1/3$
d^c	$(\bar{3}, 1)$	$+1/3$	$+1/3$	$-1/3$	$-1/3$
L	$(1, 2)$	$-1/2$	$(0, -1)$	0	0
e^c	$(1, 1)$	$+1$	$+1$	0	0
H	$(1, 2)$	$+1/2$	$(+1, 0)$	0	0

4.5.9 Summary of Section 5

The derivation chain of the present section may be written in one line:

$$\boxed{\begin{aligned} &Z_6[\Phi_\partial] \;\Rightarrow\; S_{4,\mathrm{eff}}[\Phi_0] \;\Rightarrow\; \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \\ &\qquad \xrightarrow{\langle H\rangle \neq 0} \mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \end{aligned}} \tag{4.104}$$

with three chiral generations, one light Higgs doublet, anomaly-free current algebra, and exact proton stability. This is the strongest low-energy result of the paper. The final section now turns from derivation to interpretation: why this framework is significant, where it differs from the Standard Model, MSSM, and string theory, and what would count as a falsifier.

4.6 Discussion

4.6.1 What this companion chapter has actually established

Because the scope of the GG-6/GDOS program is unusually broad, the discussion should begin with a precise accounting of what the present companion chapter does and does not claim.

What this book *does* establish, taking the ultraviolet results of Chapter 2 as input, is the following:

(i) the exact operational split from the lawful six-dimensional bulk to a boundary-visible state space;

(ii) the exact Nakajima–Zwanzig memory-kernel equation and the CPTP character of the reduced channel;

(iii) the GKLS limit and the mesoscopic GDOS current decomposition;

(iv) the derivation of four-dimensional quantum mechanics as the observer-accessible boundary theory of the six-dimensional bulk;

(v) the reduction of the metric zero mode to four-dimensional general relativity;

(vi) the formal derivation of the infrared four-dimensional generating functional;

(vii) the zero-mode reduction to the Standard Model gauge/matter sector, enlarged by $U(1)_B$ and $U(1)_{Qi}$, together with anomaly-free Ward identities and exact proton stability.

What this book *does not* yet claim is equally important. It does not provide a fully benchmarked phenomenological fit with measured masses, mixings, cosmological parameters, or collider signatures. It does not yet determine one unique microscopic six-dimensional matter reservoir or one unique low-energy $U(1)_{Qi}$ realization. It does not yet solve all questions of nonperturbative completion above E_{GUT}. Those tasks remain open, and the appendices explicitly register them.

4.6.2 Why the split into Chapter 2 and Chapter 3 matters

The present split is scientifically useful, not just editorial. Chapter 2 asks how a constitutionally admissible ultraviolet stage is selected. Chapter 3 asks how observations, quantum measurement, infrared field theory, and the effective Standard Model world emerge from that stage. This separation helps prevent a common conceptual confusion: a mathematically consistent bulk theory is not automatically a theory of observation. OGGEP is the clause that closes that gap.

Table 4.12: Why the GG-6/GDOS split is scientifically sharper than one undifferentiated combined manuscript.

Question	**Chapter 2 (GG-6)**	**Chapter 3 (GDOS)**
What is derived?	UV bulk, BI-6, OGN-5, anomaly-free 6D QFT	Boundary channel, GDOS, 4D QM, 4D GR, formal 4D QFT, IR SM sector
Primary axiom emphasized	AX-2 (GGEP)	AX-3 (OGGEP)
Central mathematical tool	Composite connection, GS inflow, BRST/ST consistency	Projector algebra, CPTP maps, GKLS, zero-mode and EFT reduction
Scientific question answered	What is the lawful bulk?	How does the observed 4D world arise from it?

4.6.3 Why the framework remains structurally unusual

The distinctive feature of the GG-6/GDOS program is not merely that it uses a higher-dimensional bulk. Its unusualness comes from the fact that four elements that are usually studied separately are forced into a single derivation chain:

(1) Geometry and gauge structure are unified at the primitive level. GGEP starts from a composite connection rather than from independent metric and gauge sectors.

(2) Anomaly cancellation is constitutive rather than reparative. The BI-6 tensor sector is not inserted after model building as a repair. It is part of the logical engine that makes the theory admissible in the first place.

(3) Measurement is constitutionalized. Most high-energy models are silent about how a finite observer obtains a lawful record. In the present framework, AX-3 forces the theory to provide its own operational dictionary.

(4) Infrared phenomenology is derived as a reduction problem. Four-dimensional quantum mechanics, general relativity, formal QFT, and the Standard Model sector are not independent primitives. They are lower-dimensional images of the same bulk.

4.6.4 Comparison with the Standard Model, MSSM, and perturbative superstring theory

The sharpest way to assess uniqueness is by direct architectural comparison.

Table 4.13: Comparative architecture of the Standard Model, MSSM, perturbative superstring theory, and the present GG-6/GDOS program.

Question	**Standard Model**	**MSSM**	**Perturbative superstring**	**GG-6/GDOS program**
Primitive ontology	4D gauge fields + matter on fixed/curved spacetime	SM + superpartners + SUSY	Worldsheet/string excitations in 10D/11D settings	Composite geometry⊕gauge bulk connection + tensor + operational boundary map
Ultraviolet emphasis	Renormalizable 4D EFT	SUSY-improved unification and hierarchy control	Planck/string-scale completion	Constitutional endpoint at E_{GUT} with geometrized gauge sector
Measurement in foundations	External to the model	External to the model	Usually external to the fundamental dynamics	Derived via OGGEP as bulk-to-boundary open-system projection
Anomaly cancellation	Representation-theoretic condition in 4D	Representation-theoretic condition with superpartners	Central consistency condition of the string spectrum	BI-6/GS tensor identity locking gravity and gauge terms
Extra gauge factors	None beyond SM	Model-dependent extras possible	Model-dependent from compactification	Mandatory $U(1)_B$ and $U(1)_{Qi}$ in the minimal GG-6/GDOS synthesis
Proton stability	Accidental at renormalizable level, violated by higher-dimensional operators	Threatened by dimension-4/5 operators unless extra symmetries imposed	Model-dependent	Exact by local $U(1)_B$ if the theorem-level realization is maintained
Parameter philosophy	Many fitted couplings and masses	More parameters than SM in generic soft sector	Vast landscape/moduli issue	OGN-5 seeks discrete normalization compression

Comparison with the Standard Model. The Standard Model remains the benchmark for precision phenomenology, but it is constitutionally silent about measurement and does not by itself explain why its gauge and matter content should be exactly what they are. In the present

program the Standard Model appears as the low-energy zero-mode image of a more constrained bulk-and-boundary derivation.

Comparison with the MSSM. The MSSM enlarges the field content in order to address hierarchy and unification questions (Haber and Kane, 1985; Nilles, 1984; Wess and Zumino, 1974). The present framework moves in the opposite direction: no superpartners are assumed, and the burden of structure is shifted to warped geometry, anomaly locking, and the operational projection.

Comparison with perturbative superstring theory. The present program borrows two mathematical ideas from the historical string literature — Green–Schwarz inflow and holographic/bulk-boundary reasoning — but not the full ontology of strings, branes, and landscape compactifications (Green and Schwarz, 1984; Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998; Polchinski, 1998a,b). The ultraviolet endpoint is placed at E_{GUT}, not E_{Pl}, and the primitive fields remain those of an ordinary gauge-gravity-tensor field theory.

4.6.5 The significance of the baryonic and information gauge sectors

From the infrared point of view, the two most unfamiliar sectors of the theory are $U(1)_B$ and $U(1)_{Qi}$.

The significance of $U(1)_B$ is the cleanest. It turns baryon number from an accidental or approximate selection rule into an exact local gauge principle. If the symmetry is exact, then proton stability is not a numerical accident; it is a theorem. The standard baryon-violating operators are simply illegal because they carry nonzero B.

The significance of $U(1)_{Qi}$ is more conceptual. In this book it is kept deliberately minimal: an Abelian sector entering the same BI-6 and anomaly-inflow bookkeeping as $U(1)_B$. That is enough for the theorem-level reduction carried out here. A fuller phenomenological or information-theoretic interpretation of $U(1)_{Qi}$ remains a task for later work.

4.6.6 From formal GDOS to the wider runtime vocabulary

This book keeps GDOS at the level required for physics derivations: exact projection, positive channels, memory kernels, gradient-plus-curl current geometry, and the consequent emergence of four-dimensional quantum mechanics, GR, and QFT. Within the wider project archive, however, GDOS is also used as a compact runtime vocabulary for general open systems (Arisaka, 2026c,c). It is useful to state that vocabulary here, not as an extra axiom system, but as an organizational extension of the mathematical structures already derived.

Table 4.14: Broader GDOS runtime vocabulary and its relation to this book.

Term	Minimal meaning	Connection to the present companion chapter
SIP	System-instance packet: the minimal open-system state description including state manifold, boundary, drives, effective landscape, and noise ledger	In this book this role is played by the boundary state space $\mathcal{H}_{\mathrm{Bdy}}$ or coarse-grained manifold $\mathcal{M}_{\mathrm{state}}$, together with the kernels and currents entering (4.28) and (4.35).
EEGG	Energy–Entropy Geometry–Gauge ledger or effective free-energy landscape	Mathematically this is the Lyapunov/landscape structure Φ or $F[p]$ that appears in the GDOS current decomposition and monotonicity formulae.
DGI	Dynamical gauge invariance of the open-system runtime	In this book it corresponds to the statement that the reduced generator and current algebra remain gauge-covariant and BRST/Ward compatible after projection.
GSSB	Geometric spontaneous symmetry breaking by boundary conditions, inflow, and attractor selection	Here it appears concretely in orbifold symmetry breaking, electroweak breaking, and the selection of stable infrared sectors.
GA	Geometric attractor selected by the open-system flow	In this book the analogues are stationary density operators, fixed currents, and stable vacuum/zero-mode sectors.
Qi-4	Later organizational program linking energy budget, variational path selection, statistics, and architecture	In this book only the formal seed is present: the $U(1)_{Qi}$ sector and the current-geometry machinery from which those later exports are meant to grow.

This broader vocabulary should be read as a semantic compression layer, not as a replacement for the proofs already given. The proofs in the current paper remain the primary object: a lawful open-system reduction from a six-dimensional anomaly-free bulk to a four-dimensional boundary theory. The wider GDOS language is valuable only insofar as it remains anchored to those proofs.

4.6.7 Falsifiability and empirical exposure

A constitutional program must remain scientifically exposed. POS-7 demands a registered falsifier. In that spirit, the present framework is testable in several ways.

(i) Proton decay. If future experiments observe proton decay with a rate incompatible with any exact local $U(1)_B$ protection, then the theorem-level realization developed here is false. This is perhaps the sharpest direct falsifier.

(ii) Superpartners or string-specific ultraviolet signals as indispensable ingredients. If nature requires low-energy supersymmetry or phenomena that can only be explained by a

genuinely stringy ultraviolet ontology, then the central non-SUSY/non-string stance of the paper would be severely challenged.

(iii) Failure of the anomaly-locking picture. If a realistic low-energy realization of the $U(1)_B$ and $U(1)_{Qi}$ sectors cannot be made fully anomaly-consistent with the GS/Stückelberg image of BI-6, the present construction would need revision at a deep level.

(iv) Absence of any trace of the extra Abelian sectors. Depending on the Stückelberg scales and kinetic mixings, one expects either exact hidden selection rules or measurable remnants in cosmology, rare processes, or precision gauge constraints. A total absence of any coherent phenomenological avatar of the extra Abelian structure would weaken the framework.

(v) Breakdown of the open-system program. If the OGGEP-to-GDOS program fails to recover standard quantum measurement and open-system phenomenology with the required level of mathematical precision, then one of the major claimed advantages of the framework would disappear.

These falsifiers are a strength, not a weakness. A theory that aspires to constitutional unification must expose itself to failure.

4.6.8 Limits of this book

Because the manuscript is long and derivational, it is easy to lose track of its boundaries. This book should therefore be read with the following limitations in mind.

First, it is a theorem-level and architecture-level paper. It is not yet a fully benchmarked phenomenology paper. Second, several ingredients — especially the exact microscopic matter reservoir, the detailed $U(1)_{Qi}$ charge assignments, and the full cosmological model — remain underdetermined at the level of the current manuscript. Third, while the manuscript argues that four-dimensional quantum mechanics and GR emerge from the same constitutional bulk, it does not yet solve every interpretational issue in the philosophy of quantum theory or quantum gravity. What it does provide is a mathematically organized path by which such issues can be addressed inside one common framework.

These limitations should be judged relative to the ambition of the project. A paper that tries to derive GG-6, GDOS, 4D QFT, GR, and the Standard Model from one constitutional layer must be evaluated first on internal coherence and only later on full phenomenological completion. On that narrower but still demanding criterion, the present derivation chain is, in our assessment, already nontrivial.

4.6.9 Falsifiability and empirical exposure

POS-7 requires that a theory remain scientifically exposed to failure. For this book, at least five falsifier classes are clear.

(i) Proton decay. Observation of proton decay incompatible with any exact local $U(1)_B$ protection would directly falsify the theorem-level realization developed here.

(ii) Necessity of low-energy supersymmetry or intrinsically stringy ultraviolet ontology. If the data require supersymmetry or genuinely string-specific ultraviolet ingredients as indispensable primitives, the non-SUSY/non-string stance of the present program would be under severe pressure.

(iii) Failure of the anomaly-locking picture. If the $U(1)_B$ and $U(1)_{Qi}$ sectors cannot be made fully consistent with the GS/Stückelberg image of BI-6 in realistic benchmark models, a deep revision of the framework would be required.

(iv) Absence of any coherent phenomenological avatar of the extra Abelian sectors. Depending on mixing scales and Stückelberg masses, one expects either exact hidden selection rules or measurable remnants in cosmology, rare processes, and precision observables. A complete absence of any plausible avatar would weaken the framework.

(v) Failure of the OGGEP → GDOS program itself. If the open-system bulk-to-boundary chain cannot be sharpened to the level needed to recover standard measurement and reduced dynamics in full quantitative detail, then one of the central claimed advantages of the framework would disappear.

4.6.10 Limits of the present chapter

This is a theorem-driven companion chapter, not yet the final phenomenological benchmark paper of the program. Its proper standard of evaluation is internal coherence, mathematical transparency, and the sharpness of its reduction chain. On that criterion the present chapter already does something nontrivial: it provides one connected path from a lawful six-dimensional bulk to open-system boundary dynamics, four-dimensional quantum mechanics, general relativity, formal QFT, and the infrared Standard Model sector. The burden that remains is to fit that structure numerically and to confront it with data.

4.6.11 Final synthesis

The main logical chain of the paper may be compressed into one equation:

$$\boxed{\begin{aligned}&\text{GG-6 bulk from Chapter 2} \xrightarrow{\text{OGGEP}} \text{CPTP channel} \implies \text{GDOS} \\ &\implies \text{4D QM + 4D GR + formal 4D QFT} \implies \text{IR SM with exact } U(1)_B\end{aligned}}. \tag{4.105}$$

This is the precise sense in which this book claims that scientific observation always acts on open systems: the world actually accessed by finite observers is the boundary reduction of a larger lawful bulk, and its natural dynamics is therefore GDOS rather than a fundamentally closed-system law.

4.7 Summary

This companion chapter has pursued one organizing claim: once the GG-6 bulk of Chapter 2 is admitted, AX-3 and the remaining admissibility clauses force the observer-accessible four-dimensional world to be described by a lawful open-system boundary theory. The paper has written that statement in explicit mathematical form and followed its consequences through the reduction chain all the way to four-dimensional quantum mechanics, general relativity, the formal infrared generating functional, and the Standard Model zero-mode sector.

The central technical results may be summarized briefly. First, the boundary-visible state is obtained by a precise projector-and-reduction construction, not by an informal handoff from six to four dimensions. Second, the reduced channel is completely positive and trace preserving, and its exact non-Markovian form is the Nakajima–Zwanzig memory-kernel equation. Third, the short-memory limit yields the GKLS generator, while mesoscopic coarse-graining yields the GDOS continuity/current decomposition. Fourth, the metric zero mode of the warped six-dimensional bulk yields four-dimensional general relativity with an effective Planck scale determined by the bulk overlap integral. Fifth, heavy-mode integration yields the formal infrared four-dimensional quantum field theory, and zero-mode truncation plus boundary-condition breaking reproduces the gauge and matter content of the Standard Model enlarged by $U(1)_B$ and $U(1)_{Qi}$. Sixth, exact local $U(1)_B$ forbids all local baryon-violating operators and therefore implies absolute proton stability in the theorem-level realization studied here.

The conceptual significance of the paper lies in the way these results fit together. Measurement is not appended after the dynamics; it is generated by the dynamics through OGGEP. Open-system evolution is not an afterthought; it is the natural form of the boundary law. Four-dimensional quantum mechanics and general relativity are not independent starting points; they are parallel infrared reductions of one lawful bulk. The Standard Model, in turn, appears not as a disconnected empirical patch but as the low-energy zero-mode image of that same chain.

At the same time, the paper has been explicit about its remaining burdens. A unique microscopic matter reservoir, a unique phenomenological realization of the $U(1)_{Qi}$ sector, threshold matching, full cosmological benchmarking, and confrontation with precision data are tasks for later work. The present volume therefore claims a theorem-level derivation, not a finished phenomenological fit.

The strongest one-line summary is

$$\boxed{\begin{aligned}&\text{GG-6} \xRightarrow{\text{OGGEP}} \text{GDOS} \implies \text{4D QM} + \text{4D GR} + \text{formal 4D QFT}\\ &\quad\implies \text{IR Standard Model sector with exact } U(1)_B\end{aligned}}\,.$$

If future benchmark constructions preserve this chain and if its distinctive predictions survive experiment, then the GG-6/GDOS program will have achieved something rare: a constitutional route not only to a unified bulk theory, but to the observed four-dimensional world of measurement itself.

Chapter 5

The Qi-4 Pillars

The Universal Runtime

"The only way to discover the limits of the possible is to go beyond them into the impossible."

— Arthur C. Clarke, 1962

5.1 Introduction

5.1.1 Why living organisms are the natural entry point for Chapter 5

The first three papers of the GG series were necessarily written from the bottom up. Chapter 2 fixed the constitutional layer: what a lawful theory is allowed to assume, what it must derive, and what it must make measurable (Arisaka, 2026a). Chapter 3 then answered the ultraviolet question, namely what higher-dimensional geometry is forced by that constitution (Arisaka, 2026b). Chapter 4 answered the operational question, namely how a finite observer on the boundary obtains quantum states, measurement records, and four-dimensional effective dynamics from that higher-dimensional bulk (Arisaka, 2026c). Chapter 5 must now reverse the direction of emphasis. It does not begin from the smallest scales but from the systems in nature that most vividly display the full open-system problem: living organisms.

A living organism is the most stringent natural test case for any theory that claims to be a universal theory of open systems. It is not just a dynamical system, nor merely a thermodynamic system, nor merely an information-processing system. It is all three at once. A cell or organism maintains a boundary, exchanges matter and energy with its environment, processes information, stores memory, predicts possible futures, selects actions, and persists far from equilibrium for times vastly longer than microscopic relaxation times (Friston, 2010; Landauer, 1961; Nicolis & Prigogine, 1977; Prigogine, 1967; Schrödinger, 1944). If a framework cannot say what such a system *is*, what its lawful state packet is, what its relevant currents are, why it persists, and how it makes or receives measurements, then that framework has not yet reached a universal runtime.

This is why life, rather than elementary particles alone, is the natural entry point for this book. Particles reveal the constitutional grammar of the theory. Organisms reveal whether that grammar is strong enough to handle persistence, adaptation, memory, agency, and internal modeling without changing its foundations from one scientific domain to the next. In that sense this book is not a departure from Chapters 2–4 but the first large-scale stress test of their universality.

Mathematically, the key reason organisms are unavoidable is that they make all hidden assumptions visible. For an isolated Hamiltonian system the notation

$$\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = -\,\mathrm{i}\,H\,|\psi(t)\rangle$$

can be written compactly and elegantly, but this compactness hides the fact that a real observer never has access to the entire Hilbert space. In an actual experiment or an actual organism, one deals instead with a subsystem S, an environment E, a reduced state, and a family of coarse-grained observables:

$$\mathcal{H}_{\mathrm{bulk}} = \mathcal{H}_{\mathrm{sys}} \otimes \mathcal{H}_{\mathrm{env}}, \qquad \rho_S(t) = \mathrm{Tr}_E\ \rho_{SE}(t), \qquad \langle O_a\rangle_t = \mathrm{Tr}_{\mathcal{H}_{\mathrm{sys}}}\big(\rho_S(t)\,O_a\big). \tag{5.1}$$

A living organism makes Eq. (5.1) unavoidable because it is *constitutively coupled* to an environment: food, photons, chemical gradients, thermal baths, predators, conspecifics, and internal submodules all act as ports. The closed-system idealization is therefore not just inaccurate but structurally misleading when one attempts to understand life, cognition, and collective behavior.

The philosophical point is equally important. Since Schrödinger (1944), life has been recognized as a phenomenon that cannot be reduced to static equilibrium thermodynamics. Since Shannon (1948) and Landauer (1961), information can no longer be treated as an informal metaphor detached from physical cost. Since the development of nonequilibrium thermodynamics and open quantum systems, irreversibility, coarse-graining, and environment-induced records must be taken as first-class theoretical objects rather than nuisances to be postponed (Breuer & Petruccione, 2002; Nicolis & Prigogine, 1977; Prigogine, 1967). Chapter 5 begins from organisms because they force all these strands to coexist in one object.

Table 5.1: Why living organisms are the natural entry point for the universal extension of GDOS.

Feature of living systems	**Why it is mathematically demanding**	**Why it is decisive for Chapter 5**
Persistent openness	State evolution requires explicit environmental coupling, ports, and reduced dynamics.	Forces the theory to move beyond closed-system Hamiltonian language.
Far-from-equilibrium operation	Stability is maintained by continuous flux, not by equilibrium minimization alone.	Motivates a unified energy–entropy ledger and trajectory-selection principle.
Boundary maintenance	The system must distinguish self from environment while exchanging with it.	Motivates SIP, SeOS, and the outer/inner hologram distinction.
Memory and prediction	The present state depends on history and on internally represented futures.	Motivates Qi-HAL and the move from external measurement to internal measurement.
Hierarchical organization	Molecular, cellular, organismal, and social levels interact without collapsing into one scale.	Motivates scale covariance, Qi-CON, and the pagoda hierarchy.
Action and adaptation	The system does not only receive information; it changes its boundary conditions by acting.	Motivates the full DGI→GSSB→GA pipeline rather than passive state evolution alone.

5.1.2 From the measurement problem to the universal observation problem

Chapter 4 was framed around the measurement problem in quantum mechanics, but its deeper lesson is broader. The familiar measurement problem asks why and how definite outcomes appear if the microscopic law is unitary (Bell, 1964; von Neumann, 1932; Zurek, 2003). The answer proposed in Chapter 4 was to promote measurement from an interpretational add-on to a constitutional demand: AX-3 (OGGEP) requires every admissible theory to provide a lawful bulk-to-boundary dictionary (Arisaka, 2026c,a). In its most compact form, this dictionary is a map

$$\mathcal{M}_{\text{bulk}} : \text{ bulk data} \longrightarrow \big(\mathcal{B}, \rho_\partial, \Phi_{t,s}, \{\mathcal{I}_\alpha\}\big), \tag{5.2}$$

where ρ_∂ is the observer-accessible boundary state, $\Phi_{t,s}$ is an admissible evolution map, and $\mathcal{I}_\alpha$ are instruments or operations associated with registered outcomes.

For Chapter 5 the crucial point is that Eq. (5.2) is not just a statement about laboratory measurements in quantum physics. It is the general grammar of *observation*. Every scientific observation, in any field, is obtained only after three reductions have already occurred:

(i) a large inaccessible state space has been truncated to a smaller accessible state space;

(ii) a continuous high-dimensional process has been projected onto a finite register or data stream;

(iii) the resulting record has been interpreted relative to a chosen instrument class.

That is as true for electrophysiology, microscopy, ecology, and economic indicators as it is for Stern–Gerlach experiments. The observation problem is therefore universal. The quantum measurement problem is only its sharpest historically visible corner.

The mathematical signature of universality is that the same reduced-dynamics structure recurs across domains. At the exact level one obtains a non-Markovian memory-kernel equation of Nakajima–Zwanzig type,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\partial}(t) = \int_0^t K(t-s)\,\rho_{\partial}(s)\,\mathrm{d}s + I(t), \tag{5.3}$$

while in the short-memory limit one obtains a completely positive semigroup with GKLS generator,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\partial} = -\,\mathrm{i}[H_{\mathrm{eff}}, \rho_{\partial}] + \sum_k \left(L_k \rho_{\partial} L_k^{\dagger} - \frac{1}{2}\{L_k^{\dagger} L_k, \rho_{\partial}\} \right). \tag{5.4}$$

The exact microscopic meaning of H_{eff}, L_k, and the kernel K varies across domains, but the open-system logic does not. What changes from particles to organisms is not the fact of openness but the complexity of the state packet, the boundary, the port structure, the attractor geometry, and the internal modeling architecture.

This is precisely why this book must broaden the focus from "measurement" to "observation." An organism not only registers external data; it also constructs internal states that function as measurements of its own world-model. A nervous system, for example, receives sensory inputs, compares them against memory, generates predictions, issues actions, and re-samples the environment. Once that loop is recognized, external observation and internal observation become two instances of the same architecture. This is the conceptual doorway through which OH + IH = EIC will later enter: the outer hologram concerns externally registered boundary data, while the inner hologram concerns the internally reconstructed model through which a system experiences and navigates its world.

Definition 5.1.1 (Operational openness)**.** A scientific system S is *operationally open* if the observer-accessible state of S cannot be described by a closed autonomous law on S alone, but instead requires at least one of the following: a traced-over environment, explicit ports, memory kernels, stochastic coarse-graining, or instrument-conditioned state updates.

Remark 5.1.2. Operational openness is weaker than full ontological openness. The definition makes no claim that the total universe is open to something external. It claims only that every *scientific description used by a finite observer* is open because it is necessarily partial,

coarse-grained, and instrument-bounded. This is enough to make open-system mathematics constitutional.

5.1.3 Big unsolved problems in complex open systems

Once the question is stated at the level of observation rather than at the narrower level of laboratory measurement, a family of long-standing open problems comes into view. These are not independent. They are coupled manifestations of the same missing runtime.

The first unsolved problem is the *problem of lawful persistence*. Why do some open systems maintain stable identities for long times while others dissolve rapidly into the background? In equilibrium statistical mechanics this question is often translated into minimization of free energy, but a living system or a non-equilibrium steady state is not simply an equilibrium minimizer. It persists by maintaining flux. Therefore one requires a dynamical principle that can treat both descent and circulation, both dissipation and driven recurrence (Nicolis & Prigogine, 1977; Prigogine, 1967). Chapter 4 already suggested such a structure in the gradient–plus–curl form of GDOS. Chapter 5 must elevate it from a technical detail to a universal runtime principle.

The second unsolved problem is the *problem of branch selection*. Many lawful states may be admissible under the same microscopic equations. Yet real systems choose one branch: one phase, one developmental pathway, one attractor, one behavioral policy, one scientific record. In condensed matter and particle physics this is described by spontaneous symmetry breaking; in biology and neuroscience it appears as differentiation, commitment, or decision. A universal theory of open systems must say how the same constitutional logic underlies all these phenomena. That is the role this book assigns to GSSB.

The third unsolved problem is the *problem of current bookkeeping*. Open systems exchange not only energy but also matter, entropy, and information. Modern theory has excellent specialized tools for each one separately, yet no single constitution-wide ledger is routinely used across disciplines. A cell, however, spends energy to move matter and to process information; a nervous system spends metabolic energy to maintain beliefs and actions; a social system expends material and informational resources simultaneously. Chapter 5 argues that the relevant bookkeeping cannot remain domain-by-domain if the framework is truly universal. This motivates Qi-QUA, the Arisaka Ratio, the Arisaka Equation, and the Arisaka Constant as the universal current-locking layer.

The fourth unsolved problem is the *problem of scale*. Complex systems rarely live at a single scale. They are multiscale, self-similar only approximately, and often exhibit scale-free statistics, heavy tails, long memory, bursts, and $1/f$-type spectra (Wilson, 1971; Wilson & Kogut, 1974). Existing disciplines often treat such phenomena phenomenologically. Chapter 5 instead treats them as diagnostic signatures of the underlying open-system geometry, later organized under Qi-CON.

The fifth unsolved problem is the *problem of internal modeling*. A living or cognitive system does not merely react. It constructs an internal model, uses that model to predict, and then changes the world through action. This loop is central in modern neuroscience and active inference discussions (Friston, 2010; Tononi, 2004), but it is not yet embedded within a universal

geometry-to-measurement constitution. Chapter 5 treats this as the domain of Qi-HAL and later of OH + IH = EIC.

The sixth unsolved problem is the *problem of universality across sciences*. Physics, chemistry, biology, neuroscience, and social science each have local successes, yet their foundational languages remain partially disjoint. If a new framework claims universality, it must not just assert it rhetorically. It must exhibit a reusable state packet, a reusable current ledger, a reusable transition grammar, and a reusable hierarchy map. These are exactly the roles reserved for SIP, EEGG, SeOS, the DGI→GSSB→GA pipeline, and the five-story pagoda architecture.

Table 5.2: Major unsolved problems that motivate Chapter 5.

Problem	**Standard partial treatment**	**Why the present framework introduces a new object**
Lawful persistence of open systems	Nonequilibrium thermodynamics, kinetic models, control theory	Requires a constitutional attractor grammar (GA) plus a unified ledger (EEGG).
Selection among multiple admissible branches	Phase transitions, spontaneous symmetry breaking, decision theory	Requires a cross-domain selection mechanism (GSSB) valid from phases to developmental and cognitive commitment.
Unified bookkeeping of energy, matter, entropy, and information	Separate domain-specific balances	Requires a single current-locking layer (Qi-QUA) and a universal packet definition (SIP).
Scale covariance and heavy-tailed statistics	RG, criticality, empirical scaling laws	Requires a diagnostic bridge from geometry to scale-free signatures (Qi-CON).
Memory, prediction, sensing, and action in one loop	Open quantum systems, cybernetics, active inference, neuroscience	Requires a geometric architecture layer (Qi-HAL) and the OH/IH/EIC distinction.
Universality across sciences	Patchwork of local models	Requires a common runtime rather than a family of disconnected metaphors.

5.1.4 Why the existing toolkits remain partial

The need for Chapter 5 does not arise because the sciences lack useful formalisms. On the contrary, there are many successful local toolkits. The issue is that each one resolves only part of the problem and leaves the bridges to the others external.

Open quantum systems provide a rigorous language for reduced states, completely positive maps, decoherence, and semigroup generators (Breuer & Petruccione, 2002; Choi, 1975; Gorini et al., 1976; Kraus, 1983; Lindblad, 1976; Stinespring, 1955). This language is indispensable for Chapter 4 and remains indispensable here. But by itself it does not tell us how to classify persistent biological packets, what the cross-domain current ledger is, or why a hierarchy of mind-like internal modeling architectures should appear naturally.

Nonequilibrium thermodynamics and self-organization theory provide powerful tools for entropy production, dissipation, bifurcation, and far-from-equilibrium pattern formation (Nicolis &

Prigogine, 1977; Prigogine, 1967). Yet those tools are usually not written in a way that connects directly to observer-dependent measurement maps, gauge-covariant currents, or a holographic bulk-to-boundary descent.

Information theory explains coding, communication limits, and the physics of information cost (Jaynes, 1957; Landauer, 1961; Shannon, 1948). But bare information theory does not decide which internal architectures are lawful, which trajectories are selected, or how measurement and coarse-graining arise from a unified geometric stage.

Active inference and the free-energy principle emphasize inference, prediction, and action under uncertainty (Friston, 2010). These ideas are highly relevant to the present program and will later be reinterpreted under Qi-EP and Qi-HAL. However, active inference is usually introduced as an agent-level or model-level principle, not as the boundary manifestation of a constitution already fixed by geometry, anomaly freedom, and a specific bulk-to-boundary law.

Integrated-information and consciousness frameworks attempt to characterize subjective unity or complexity of experience (Tononi, 2004). They are valuable as phenomenological prompts, but they are not normally derived from a current ledger, a bulk geometry, a gauge structure, or a falsifiability-oriented operational constitution. The present framework aims to place consciousness after, not before, the runtime: only once OH, IH, and the Qi-HAL architecture are fixed can EIC be discussed in a disciplined way.

The central criticism is therefore not that existing frameworks are wrong. It is that each one is *incomplete relative to universality*. Each captures a slice: measurement, dissipation, coding, inference, or conscious integration. Chapter 5 attempts to supply the missing glue by deriving a single runtime in which these slices become different faces of one open-system grammar.

Table 5.3: Representative existing frameworks and the specific gap Chapter 5 is designed to fill.

Framework	**Main strength**	**Gap relative to the full Chapter 5 program**
Open quantum systems (Breuer & Petruccione, 2002)	Exact and effective reduced dynamics; channels and decoherence	Excellent for measurement and reduced evolution, but not a full cross-domain runtime for living, cognitive, and social systems.
Nonequilibrium thermodynamics (Nicolis & Prigogine, 1977; Prigogine, 1967)	Fluxes, entropy production, bifurcations, self-organization	Powerful on dissipation, but does not by itself fix a universal state packet or an internal measurement architecture.
Information theory (Jaynes, 1957; Landauer, 1961; Shannon, 1948)	Coding, information cost, inference under constraints	Clarifies information and cost, but not branch selection, attractor geometry, or geometry-to-observation descent.
Active inference / FEP (Friston, 2010)	Prediction, action, and variational control	Agent-level explanatory power, but lacks an explicit ultraviolet geometric origin and a universal current-locking ledger.
Integrated-information approaches (Tononi, 2004)	Focus on conscious unity and integration	Addresses phenomenology of consciousness, but not a full constitutional runtime from particles upward.
Standard fundamental physics	Strong symmetry and precision in particle and spacetime sectors	Historically underwrites closed-system laws more directly than universal open-system laws.

5.1.5 The need for a common runtime

The notion of a *runtime* is meant literally. Chapters 2–4 supplied the constitution, the ultraviolet hardware, and the boundary operating system. Chapter 5 now asks: what is the executable layer common to all natural open systems once the theory is actually running? In software language, this is the layer that every scientific domain calls even if the domains look superficially different. In physical language, it is the layer that every open system instantiates even if the microscopic details differ.

The proposed answer is the master pipeline

$$\boxed{\text{DGI} \longrightarrow \text{GSSB} \longrightarrow \text{GA}} \tag{5.5}$$

together with the Qi-4 pillars and the universal runtime vocabulary. Equation (5.5) should be read as the minimal grammar of persistence. First, a system explores lawful gauge-covariant possibilities (DGI). Second, environmental coupling, constraints, and boundary conditions select one branch or one structure (GSSB). Third, the selected structure is stabilized as an attractor, function, phenotype, memory, or policy (GA). The later sections of the paper make each arrow explicit in the fixed order adopted here. For the introduction, it is enough to see that persistence is not a static property. It is a three-stage process.

The Qi-4 pillars then answer four questions about any runtime instance:

Qi-QUA	:	*What currents are locked together?*
Qi-EP	:	*Which trajectories are selected?*
Qi-CON	:	*What scale signatures should appear?*
Qi-HAL	:	*What architecture carries memory, prediction, sensing, and action?*

This decomposition is not ornamental. It is a discipline against conceptual drift. Without it, one risks mixing bookkeeping, variational principles, statistical diagnostics, and architecture into one undefined notion of "complexity."

A convenient preview of the mathematical structure is the generic GDOS continuity form already derived in Chapter 4:

$$\partial_t p(x,t) = -\nabla_A J^A(x,t), \tag{5.6}$$

with current decomposed schematically as

$$J^A = -D^{AB}\nabla_B p \;-\; \mu^{AB} p\, \nabla_B \mathcal{F}_{\mathrm{EEGG}} \;+\; \Omega^A p. \tag{5.7}$$

The first term represents diffusive spreading, the second gradient descent on an effective EEGG ledger, and the third divergence-free circulation. The logic of Chapter 5 is that a universal open-system runtime cannot be built from any one of these terms alone. Persistence needs all three. Pure diffusion destroys structure; pure gradient descent over-freezes structure; pure circulation never commits structure. The master pipeline DGI → GSSB → GA is the conceptual version of this mathematical trichotomy.

Definition 5.1.3 (Runtime packet preview)**.** A *runtime packet* is a preliminary placeholder for what will later be formalized as a SIP. It is the minimal declared data set required to specify a scientific open system:

$$\mathcal{R} = \big(\mathcal{X}, \mathcal{P}, \mathcal{O}, \rho, \Phi, \mathcal{J}, \mathcal{F}, \mathcal{A}_{\mathrm{arch}}\big), \tag{5.8}$$

where $\mathcal{X}$ is the state manifold, $\mathcal{P}$ the environmental interfaces, $\mathcal{O}$ the registered observables, ρ the effective state, Φ the effective evolution law, $\mathcal{J}$ the current ledger, $\mathcal{F}$ the scalar selection functional, and $\mathcal{A}_{\mathrm{arch}}$ the architecture responsible for memory, prediction, sensing, and action.

The full paper will refine Eq. (5.8) substantially. The point here is merely to show why a common runtime is needed: without some declared packet of this kind, the same words—"state," "information," "stability," "selection," "complexity"—mean incompatible things in different domains.

5.1.6 Preview of the new objects introduced in Chapter 5

It is useful to state at the outset what is new in Chapter 5 relative to Chapter 4. Chapter 4 derived the existence of a lawful open boundary theory. Chapter 5 extends that theory by adding the runtime objects needed for universality. These objects are not optional decorations; they are the minimal extensions needed to move from the foundational level of measurement and particle physics to the domain-general level of chemistry, life, mind, and society.

First, Chapter 5 introduces DGI, GSSB, and GA as the universal dynamical spine. Although their ingredients are already latent in Chapters 2–4, they were not yet elevated to a named, reusable runtime grammar. Chapter 5 makes this promotion explicit.

Second, Chapter 5 introduces the Qi-4 pillars as a disciplined factorization of the runtime. Qi-QUA is the bookkeeping layer; Qi-EP is the selection layer; Qi-CON is the diagnostic layer; Qi-HAL is the architecture layer. This factorization is what allows one to move from particle measurements to molecular machines and then further to organisms and brains without rewriting the language each time.

Third, Chapter 5 introduces SIP, EEGG, and SeOS as the working vocabulary of universal open-system analysis. A SIP will later serve as the minimal packet that can be reused across scientific branches. EEGG will later serve as the combined energy–entropy ledger. SeOS will later designate persistent self-maintaining open systems whose continued existence is itself part of the dynamical story.

Fourth, Chapter 5 introduces OH + IH = EIC, which extends the meaning of measurement. In the first three papers, the concern was primarily with how a physical observer obtains a lawful boundary record. In the fourth paper, the concern broadens to how a living or cognitive system constructs an internal world-model and how that world-model contributes to what is subjectively experienced.

Finally, Chapter 5 introduces a hierarchy picture. The Japanese five-story pagoda is not used merely as a poetic image. It is intended as a compact structural map: the same DGI spine runs through different layers of GSSB output, from particles to cosmos to chemistry to life to mind. This hierarchical picture is important because a universal theory of open systems must explain not only one level but also why multiple levels can coexist without collapsing into each other.

Table 5.4: New runtime objects introduced or elevated by Chapter 5.

Object	**Role**	**Why it is needed beyond Chapter 4**
DGI	Gauge-covariant exploration dynamics	Needed to describe lawful possibility space before commitment.
GSSB	Branch or structure selection	Needed to connect open-system dynamics to developmental, chemical, cognitive, and social commitment.
GA	Stabilized attractor geometry	Needed to describe persistence, function, memory, and recurrent policy.
Qi-QUA	Current-locking layer	Needed to unify energy, baryonic, and informational current bookkeeping.
Qi-EP	Trajectory-selection layer	Needed to relate bulk and boundary extremal principles in one language.
Qi-CON	Scale-diagnostic layer	Needed to interpret fractality, Lévy statistics, and $1/f$ signatures as structural rather than accidental.
Qi-HAL	Architecture layer	Needed to formalize memory–prediction–motion–sensing loops and internal modeling.
SIP, EEGG, SeOS	Universal packet and ledger vocabulary	Needed for branch-agnostic analysis of persistent open systems.
OH, IH, EIC	External and internal holographic semantics	Needed to extend measurement theory into cognition and consciousness.

5.1.7 Claims and non-claims of this book

Because the scope of Chapter 5 is large, it is important to distinguish what it claims from what it does not claim.

The main positive claim is that there exists a single geometric runtime capable of organizing open systems across domains once the constitutional and ultraviolet groundwork of Chapters 2–4 is accepted. The paper does *not* claim that all sciences are trivially reducible to particle physics. Rather, it claims that the admissibility grammar of explanations—how states, currents, observations, attractors, and hierarchies are declared and audited—can be unified.

The paper also does *not* claim that every empirical detail of biology, neuroscience, or society is already solved. What it claims is that these domains can be placed within one lawful scaffold without changing the foundational language from one domain to the next. In that sense, the paper aims to provide a constitutional runtime rather than a finished encyclopedia of all applications.

Nor does this book claim that earlier frameworks are useless. Quite the opposite. It borrows from and attempts to subsume many successful local frameworks. The novelty lies in the ordering: geometry and admissibility first, runtime second, domain-specific models third. This ordering is exactly what was missing from the earlier fragmented landscape.

The present integrated manuscript no longer leaves the main text open. It now carries the full six-part architecture from entrance and bridge, through runtime constitution and the Qi-4 pillars, to persistence/completeness/internality, emergence and scope, and assessment. Its function at this stage is not to advertise future construction, but to present the entire derivational arc in one reviewable LaTeX/PDF package using one continuous runtime vocabulary from introduction to conclusion.

Table 5.5: Claims and non-claims that discipline the interpretation of Chapter 5.

Category	**Claim of Chapter 5**	**Explicit non-claim**
Universality	One common runtime can organize open systems across domains.	Not every domain-specific equation is already fully derived in this step.
Reduction	A common admissibility grammar exists from particles to society.	The paper does not reduce away all higher-level descriptions.
Measurement	Observation is treated as a lawful open-system projection.	The paper does not identify measurement with consciousness.
Life and mind	Life and cognition are treated as lawful runtime outcomes.	The paper does not claim that all biological and cognitive details are already solved.
Relation to prior theories	Existing frameworks are reinterpreted as partial slices of a larger runtime.	The paper does not claim that their local successes are invalid.

5.1.8 Roadmap of the full manuscript

The full paper is designed as a landmark synthesis rather than a short note. Its architecture is now a six-part staircase. Part I supplies the entrance and bridge: the Introduction, the recap of Chapters 2–4, and the universal extension of GDOS to any open system. Part II then establishes runtime constitution in the strict order demanded by logical dependence: first the master pipeline DGI + GSSB + GA, then the scalar ledger EEGG, and only then the canonical system-instance packet SIP. Part III develops the Qi-4 pillars in their own dependency order: event locking (Qi-QUA), selector structure (Qi-EP), multiscale diagnostics (Qi-CON), and memory-bearing architecture (Qi-HAL). Part IV then moves from construction to synthesis by introducing persistence/completeness (SeOS, QCI) and finally internality (OH + IH = EIC). Part V treats emergence, cross-domain applications, and the explicit universality claim. Part VI closes with significance/comparison, limitations/falsifiability, and conclusions.

The present integrated manuscript already executes this entire order in the main text. It should therefore be read not as a collection of separately drafted chapters but as one continuous derivational arc: bridge → runtime → scalar budget → packet → Qi event locking → selector → diagnostics → architecture → persistence/completeness → internality → hierarchy → applications → universality → assessment. The purpose of the present harmonization pass is to make every

transition, theorem lead-in, and section summary reflect that non-interchangeable order as clearly as possible.

5.2 Recap bridge to Chapters 2, 3, and 4

5.2.1 Why the recap is structurally necessary

Chapter 5 is not intended as an isolated monograph. It is the fourth member of a logically ordered sequence. The whole point of that sequence is that later papers should not re-open earlier constitutional questions every time a new domain is addressed. If that discipline is not maintained, the program would degenerate into a loose collection of ambitions rather than a theorem chain.

The recap therefore plays a very specific role. It does not repeat the earlier papers for convenience. It identifies which results from those papers are now to be treated as *imported theorems*, which notation choices are fixed, which issues were intentionally postponed, and which exact gap makes Chapter 5 necessary. In that sense the recap is a bridge, not a summary. It is the interface contract between the already established foundation and the new runtime layer.

A concise way to say this is that each paper answers a different question:

$$\begin{aligned}
&\text{Chapter 2}: && \text{What counts as an admissible theory?}\\
&\text{Chapter 3}: && \text{What ultraviolet bulk is forced by that admissibility?}\\
&\text{Chapter 4}: && \text{How does a lawful boundary world arise from that bulk?}\\
&\text{Chapter 5}: && \text{How do persistent open systems run within that lawful boundary world?}
\end{aligned} \tag{5.9}$$

Equation (5.9) already shows why Chapter 5 cannot merely be an appendix to Chapter 4. The question it asks is different in kind. Chapters 2–4 establish legality, stage, and projection. Chapter 5 establishes runtime.

Table 5.6: Division of labor across the first four GG papers.

Paper	Primary question	Principal theorem-level output	Direct role in Chapter 5
Chapter 2 (GG-2026-1)	What is a lawful theory?	Four axioms, seven postulates, admissibility grammar	Fixes the constitution and the ordered filters Chapter 5 must obey.
GG-2026-2	What ultraviolet bulk is forced?	Warped 6D bulk, BI-6, OGN-5, anomaly-free 6D QFT	Fixes the geometric stage, current ordering, and ultraviolet inputs.
GG-2026-3	How does the boundary world arise?	OGGEP channel, GDOS, 4D QM, 4D GR, formal IR QFT/SM descent	Fixes the open-system boundary law and the meaning of measurement.
Chapter 5	How do open systems persist and organize?	Runtime grammar, Qi-4 pillars, SIP/EEGG/SeOS, OH/IH/EIC hierarchy	Extends the boundary law into a universal theory of open systems.

5.2.2 Chapter 2 recap: the GUT constitution

The first paper established that the program is not to be judged first as a model but as a constitution (Arisaka, 2026a). In formal terms, a candidate theory $\mathcal{T}$ is admissible only if it satisfies both the axiom layer and the ordered postulate layer:

$$\text{Admissible}(\mathcal{T}) := \big(\mathcal{T} \models \text{AX}\big) \wedge \big(\mathcal{T} \models \text{POS}\big). \tag{5.10}$$

Equation (5.10) is not a slogan. It is the filter that removes entire classes of otherwise fashionable constructions before phenomenology is discussed.

The four axioms fix the primitive semantics of the program. AX-1 (LC) declares that admissible effective laws are operationally local and causal. AX-2 (GGEP) declares that the primitive field is a composite geometry-plus-gauge connection rather than two separate ontological sectors. AX-3 (OGGEP) declares that a theory is incomplete if it does not supply a lawful map from inaccessible bulk data to registered boundary observables. AX-4 (MSMF) declares that nature prefers maximal admissible symmetry and minimal non-forced continuous freedom. Together these axioms already exclude a purely interpretation-based treatment of measurement and a parameter-landscape approach to unification.

The seven postulates then turn those axioms into an ordered audit. POS-1 enforces gauge invariance at every admissible layer. POS-2 enforces a unified extremal principle. POS-3 enforces anomaly freedom. POS-4 enforces closure and positivity of state evolution. POS-5 enforces scale covariance or RG compatibility. POS-6 enforces Occam gauge normalization. POS-7 enforces measurement and falsifiability contracts. The most important thing for Chapter 5 is that these postulates are ordered. One is not allowed to introduce a later convenience by violating an earlier admissibility clause.

Table 5.7: The constitutional clauses of Chapter 2 and their role in Chapter 5.

Clause	**Name**	**Constitutional content**	**Direct consequence for Chapter 5**
AX-1	LC	Operational locality and causality; retarded effective kernels only.	Any runtime architecture must respect forward-time record formation and lawful action–perception ordering.
AX-2	GGEP	Geometry and gauge are faces of one primitive connection.	Open-system currents and hierarchies must ultimately descend from one geometric source.
AX-3	OGGEP	Bulk theory must furnish a lawful observational dictionary.	Measurement, sensing, and internal modeling are not optional metaphors.
AX-4	MSMF	Maximum symmetry, minimum non-forced freedom.	The later Qi-4 runtime must not proliferate hidden continuous knobs.
POS-1	Gauge invariance	Redundancies are controlled and admissible descriptions are gauge lawful.	Motivates DGI as the runtime continuation of gauge legality.
POS-2	Unified extremal principle	A single extremal grammar must exist across scales.	Motivates the later Qi-EP bridge PLA $\leftrightarrow$ PLT/MEPP/FEP.
POS-3	Anomaly freedom	Inconsistencies in symmetry realization are forbidden.	Forces BI-6 in Chapter 3 and constrains any Chapter 5 current ledger.
POS-4	Closure and positivity	States and channels must remain normalized and positive.	Protects the legitimacy of reduced dynamics, learning loops, and internal measurement.
POS-5	Scale covariance	Coarse-graining must not destroy admissibility.	Motivates Qi-CON and the multiscale ambition of Chapter 5.
POS-6	OGN	Minimal discrete normalization replaces hidden continuous freedom.	Carries forward the integer ledger that underlies later current locking.
POS-7	Measurement and falsifiability	Every claim must compile to observables and a predeclared falsifier.	Forces Chapter 5 to remain empirical even when discussing life and mind.

Chapter 2 matters for the current manuscript in one further respect. It changed the burden of proof. After the constitution is stated, the question is no longer "can one imagine a theory that

says interesting things about life, mind, and society?" The question becomes "can one extend the same admissibility grammar into those domains without breaking it?" Chapter 5 exists because the answer is intended to be yes.

5.2.3 Chapter 3 recap: GG-6, BI-6, and the warped six-dimensional bulk

Chapter 3 solved the ultraviolet realization problem (Arisaka, 2026b). Starting from AX-2 and the ordered postulates, it derived a minimal warped six-dimensional bulk of the form

$$M_6 = M_4 \times I_r \times S^1_\theta, \qquad r = \ln(E_{\mathrm{GUT}}/\mu), \tag{5.11}$$

with metric

$$\mathrm{d}s_6^2 = e^{-2A(r)}\, g_{\mu\nu}(x)\, \mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2\, \mathrm{d}\theta^2. \tag{5.12}$$

This stage is neither a generic Kaluza–Klein extension nor a string-theoretic compactification imported whole. Its role in the program is specific: it is the minimal bulk geometry compatible with the constitution, anomaly freedom, and the OGN minimality requirement.

The bulk gauge structure derived in Chapter 3 is

$$G_{\mathrm{Bulk}} = \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times \mathrm{Gravity}, \tag{5.13}$$

which the paper interprets as six-force unification at E_{GUT}: the Standard-Model gauge content embedded in SU(5), together with gravity, a local baryonic symmetry $\mathrm{U}(1)_B$, and a quantum-information symmetry $\mathrm{U}(1)_{Qi}$. For Chapter 5 the crucial consequence is not only that there are extra sectors, but that the extra sectors are *constitutional sectors*. In particular, $\mathrm{U}(1)_B$ is what later underwrites exact proton stability, while $\mathrm{U}(1)_{Qi}$ is the seed from which the information-current language of Chapter 5 grows.

The technical centerpiece of Chapter 3 was BI-6,

$$\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5}\, F_B \wedge F_B - \frac{2}{5}\, F_{Qi} \wedge F_{Qi}, \tag{5.14}$$

with coefficient vector

$$(a, b, c, d) = (1, -1, -2/5, -2/5). \tag{5.15}$$

Chapter 3 argued that this coefficient vector is the unique primitive solution compatible with the symmetries, equal Abelian treatment, and anomaly-cancellation logic of the framework. For Chapter 5, the importance of Eq. (5.15) is twofold. First, it fixes the order in which sectors are to be read when current locking is later discussed. Second, it is the ultraviolet source of the updated Arisaka Ratio:

$$J'_E : J'_e : J'_B : J'_{Qi} = 1 : -1 : -\frac{2}{5} : -\frac{2}{5} = 5 : -5 : -2 : -2. \tag{5.16}$$

At the level of the present recap, Eq. (5.16) should be understood as a *notation lock* rather than as a full derivation. The full Chapter 5 will later derive and interpret it in detail.

Chapter 3 also derived the OGN-5 integer ledger

$$(M, N, L;\ J, K) = (1, 3, 5;\ 7, 35), \tag{5.17}$$

with

$$J = 2N + 1 = 7, \qquad K = LJ = 35, \qquad K = M^2 + N^2 + L^2 = 35. \tag{5.18}$$

The philosophical meaning of OGN-5 is that free continuous normalization freedom is collapsed into a minimal discrete arithmetic package. For Chapter 5 this matters because any proposed runtime object must remain compatible with the same parsimony discipline. The Qi-4 pillars are allowed to be rich; they are not allowed to smuggle in a new landscape of hidden parameters.

Table 5.8: Imported theorem-level results from Chapter 3.

Result from Chapter 3	**Mathematical form**	**Why it matters for Chapter 5**
Warped 6D stage	$M_6 = M_4 \times I_r \times S^1_\theta$, Eq. (5.11)	Supplies the lawful bulk whose projection all later open-system descriptions inherit.
Composite bulk gauge group	Eq. (5.13)	Keeps baryonic and informational currents as explicit constitutional sectors.
BI-6	Eq. (5.14)	Fixes the anomaly-cancelled geometry–gauge exchange law.
Coefficient vector	Eq. (5.15)	Fixes the ordering and relative weights underlying the later Arisaka Ratio.
OGN-5 ledger	Eqs. (5.17)–(5.18)	Prevents later runtime elaborations from reintroducing uncontrolled freedom.
Anomaly-free 6D QFT	BRST-compatible, Green–Schwarz-inflow-stabilized bulk theory	Ensures that the runtime is built on a consistent ultraviolet stage rather than on an inconsistent ansatz.
No SUSY, no strings	Structural distinction, not just aesthetic preference	Sharpens the identity of the program relative to standard UV frameworks.

A further point needs emphasis. Chapter 3 answered the question "what is the lawful bulk?" It did *not* answer the question "how do persistent adaptive open systems run on the boundary?" That question was left untouched not because it was unimportant, but because the bulk had to be fixed first. Chapter 5 inherits the stage and the coefficient ordering from Chapter 3, but its subject matter is different.

5.2.4 Chapter 4 recap: GDOS as the boundary foundation

Chapter 4 answered the projection problem (Arisaka, 2026c). Given the lawful bulk of Chapter 3, how does the observer-accessible boundary world emerge? The central move was the implementation of AX-3 as an explicit bulk-to-boundary dictionary. In a Hilbert-space language one

writes

$$\mathcal{H}_{\text{bulk}} = \mathcal{B} \otimes \mathcal{H}_{\text{env}}, \qquad \rho_{\partial}(t) = \text{Tr}_{\mathcal{H}_{\text{env}}}\, \rho_{\text{Bulk}}(t), \tag{5.19}$$

then derives the exact projected dynamics and its Markovian limit.

At the exact level Chapter 4 obtained a Nakajima–Zwanzig memory-kernel equation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\partial}(t) = \mathcal{P}\mathcal{L}\mathcal{P}\,\rho_{\partial}(t) + \int_0^t \mathcal{P}\mathcal{L}\, e^{(t-s)\mathcal{Q}\mathcal{L}}\, \mathcal{Q}\mathcal{L}\mathcal{P}\,\rho_{\partial}(s)\,\mathrm{d}s, \tag{5.20}$$

where $\mathcal{P}$ projects onto the relevant boundary sector and $\mathcal{Q} = \mathrm{id} - \mathcal{P}$. In the short-memory limit this reduces to a GKLS generator of the form already previewed in Eq. (5.4). In a mesoscopic coarse-grained limit the same structure becomes a continuity law on an effective state manifold:

$$\partial_t p = -\nabla_A J^A, \qquad J^A = -D^{AB}\nabla_B p - \mu^{AB} p\, \nabla_B \mathcal{F} + \Omega^A p. \tag{5.21}$$

Equation (5.21) is the kernel out of which Chapter 5 grows.

Chapter 4 then used this boundary foundation to derive a large amount of standard physics in a new order. Four-dimensional quantum mechanics, including measurement as a lawful instrument update, was interpreted as the boundary manifestation of the projected dynamics. Four-dimensional general relativity was recovered from the metric zero mode of the warped bulk. Formal infrared quantum field theory was recovered by integrating out heavy modes. Finally, an infrared Standard-Model sector with $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{EM} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ structure was developed, including the role of local $\mathrm{U}(1)_B$ in exact proton stability (Arisaka, 2026c).

The decisive point for Chapter 5 is that Chapter 4 was intentionally foundational. It concentrated on the measurement problem, quantum mechanics, particle physics, and the descent to standard four-dimensional structures. It did *not* yet build the universal runtime vocabulary needed to speak naturally about molecular networks, organisms, brains, or social systems. This omission was deliberate and appropriate. One cannot build the runtime until the boundary law exists. But once the boundary law exists, the runtime becomes the next necessary task.

Table 5.9: What Chapter 4 established and what it intentionally left open.

Established in Chapter 4	**Representative formula or output**	**Left for Chapter 5**
Bulk-to-boundary observational law	Eq. (5.19) and (5.20)	Universal vocabulary for complex open systems beyond measurement theory.
Exact and effective reduced dynamics	Eq. (5.21) and GKLS limit	Explicit DGI/GSSB/GA runtime grammar.
4D quantum mechanics from projection	Instrument and Born-rule interpretation	Extension from external measurement to internal measurement.
4D GR and formal IR QFT descent	Metric zero mode and heavy-mode integration	Cross-domain runtime for chemistry, life, and mind.
IR Standard-Model sector with $U(1)_B$ and $U(1)_{Qi}$	Proton stability and information sector retained	Current locking, extremal bridge, conformality diagnostics, architecture layer.

5.2.5 What remains unfinished after Chapter 4

The most important sentence of this recap is the following: *Chapter 4 was sufficient to define the boundary law, but not yet sufficient to define a universal runtime for persistent complex open systems.* This gap can be stated precisely.

A boundary law of the form given in Eq. (5.21) tells us how reduced probabilities or states evolve. It does not yet tell us what the reusable unit of analysis is across domains. It does not yet tell us what currents must be tracked together. It does not yet tell us how to identify lawful branch selection or when a stable attractor deserves to be interpreted as function, phenotype, memory, policy, or conscious report. And it does not yet tell us what architecture carries prediction, action, and internal reconstruction.

These missing items are not secondary. They are exactly what become indispensable the moment one moves from a particle detector to a cell, from a cell to a nervous system, or from a nervous system to a coordinated collective. That is why the list of new Chapter 5 objects lines up so cleanly with the list of Chapter 4 omissions:

Missing after Chapter 4	**Introduced in Chapter 5**
current-locking rule	Qi-QUA
unified trajectory selector	Qi-EP
scale-diagnostic grammar	Qi-CON
internal architecture of memory/prediction/action	Qi-HAL
universal state packet and ledger	SIP, EEGG, SeOS
internal-experience semantics	OH, IH, EIC

This gap can be elevated from diagnosis to proposition.

Proposition 5.2.1 (The post-Chapter 4 runtime gap)**.** *Let $\mathfrak{F}_3$ denote the full theorem-level content of GG-2026-3. Then $\mathfrak{F}_3$ is sufficient to define lawful open-system boundary evolution and measurement, but it is not sufficient by itself to classify, compare, and derive persistent complex open systems across chemistry, life, mind, and society.*

Proof. The content of $\mathfrak{F}_3$ furnishes at least the following: a lawful observer-accessible state, an exact reduced evolution equation, a Markovian limit, a continuity-law representation, and the descent to standard four-dimensional physics. However, consider the tasks required for a universal open-system theory beyond measurement: one must declare a reusable packet for system identity, a joint current ledger spanning energetic and informational channels, a cross-domain selection grammar for branch choice, a scale-diagnostic grammar for multiscale observables, and an internal architecture by which sensing, prediction, memory, and action are coupled. None of these tasks is fully solved by a reduced evolution law alone. Therefore additional structure is required. The proposed additional structure is precisely the runtime layer of Chapter 5. □

Proposition 5.2.1 explains why Chapter 5 is neither redundant nor optional. It is the minimal next theorem layer once Chapter 4 is accepted.

5.2.6 Bridge equation to Chapter 5

The logical bridge from the previous papers to the present one can now be written schematically as

$$\boxed{\begin{aligned}&\big(\text{GUT constitution}\big) \Longrightarrow \big(\text{GG-6 bulk with BI-6 and OGN-5}\big)\\ &\qquad\qquad \xRightarrow{\text{OGGEP}} \big(\text{GDOS boundary law}\big) \Longrightarrow \big(\text{runtime of open systems}\big)\end{aligned}} \tag{5.22}$$

The left half of Eq. (5.22) has already been developed in Chapters 2–4. Chapter 5 supplies the rightmost implication.

One can make the target of the rightmost implication more explicit:

$$\begin{aligned}\big(\text{GDOS boundary law}\big) \Longrightarrow \{&\text{DGI}, \text{GSSB}, \text{GA}, \text{EEGG}, \text{SIP}, \text{Qi-QUA}, \text{Qi-EP}, \text{Qi-CON},\\ &\text{Qi-HAL}, \text{SeOS}, \text{QCI}, \text{OH}, \text{IH}, \text{EIC}\}.\end{aligned} \tag{5.23}$$

Equation (5.23) should be read as the agenda of the full chapter in its final order. The manuscript now carries that agenda through the main text as a single staircase: universal-runtime descent from boundary GDOS to arbitrary open systems, theorem-level construction of DGI $\to$ GSSB $\to$ GA, derivation of the scalar ledger EEGG, declaration of the packet SIP, derivation of the full Qi-4 stack, introduction of persistence and completeness through SeOS and QCI, formalization of the holographic internality layer OH + IH = EIC, and finally the hierarchy, applications, universality claim, and closing assessment.

The bridge also fixes notation that will be used consistently in the remainder of the paper. In particular, the updated Arisaka current notation is:

$$\mathfrak{R}_{\text{A}}: \quad J'_E : J'_e : J'_B : J'_{Qi} = 1 : -1 : -\frac{2}{5} : -\frac{2}{5} = 5 : -5 : -2 : -2, \tag{5.24}$$

$$J_E/Q_B = J_e = J_B = J_{Qi}: \quad \frac{J_E}{Q_B} = J_e = J_B = J_{Qi}, \qquad J_{Qi} \equiv J_{Qi}, \tag{5.25}$$

$$Q_B: \quad Q_B = \frac{2}{5}\frac{E_{\text{Ryd}}}{K}, \qquad K = 35. \tag{5.26}$$

The detailed derivations and interpretations of Eqs. (5.24)–(5.26) appear later in Part III. In the present bridge section they are fixed as notation conventions inherited from the latest development of the program.

Table 5.10: Locked notation and bridge objects for Chapter 5.

Symbol or object	Fixed form	Bridge meaning in this book
Arisaka Ratio	Eq. (5.24)	Ultraviolet current ordering inherited from BI-6 logic.
Arisaka Equation	Eq. (5.25)	Chemical-natural-unit current lock for later biological and cognitive applications.
Arisaka Constant	Eq. (5.26)	Bridge constant between energy and current bookkeeping.
DGI→GSSB→GA	Eq. (5.5)	Master runtime pipeline initiating Part II.
EEGG, SIP	Fully developed in this book	Scalar ledger and canonical packet that complete runtime constitution before the Qi block.
SeOS, QCI	Fully developed in this book	Persistence class and completeness diagnostic that open Part IV.
OH, IH, EIC	Fully developed in this book	Lawful extension from external observation to internal reconstruction and internal measurement.

5.2.7 What the six-part architecture now commits us to

At this point the logical architecture of Chapter 5 is fixed rather than provisional. The manuscript is no longer only an introduction-plus-bridge document, nor a partially assembled Qi draft. It now contains the universal-runtime construction, the scalar-budget and packet layers, the full Qi-4 block, the persistence/completeness layer, and the OH/IH/EIC internality layer.

In practical terms, the integrated manuscript has completed the following tasks in the same order that now organizes the paper:

(i) it fixed the motivation in terms of complex open systems and living organisms;

(ii) it translated the measurement problem into the universal observation problem;

(iii) it locked the imported theorem-level content of Chapters 2–4;

(iv) it generalized the Chapter 4 boundary law into a universal open-system runtime packet;

(v) it derived DGI as the induced gauge covariance of open-system generators;

(vi) it defined GSSB as symmetry breaking on the gauge-reduced physical manifold;

(vii) it defined GA as the selected persistent invariant object after gauge reduction and symmetry breaking;

(viii) it assembled these ingredients into the master runtime theorem of Chapter 5;

(ix) it derived EEGG as the combined energy–entropy geometry–gauge ledger;

(x) it defined SIP as the canonical runtime packet for any admissible open system;

(xi) it derived the coefficient-space and natural-unit forms of the Arisaka Ratio and the Arisaka Equation;

(xii) it defined the Arisaka Constant Q_B as the fixed energy scale per locked boundary event;

(xiii) it derived the projected selector Qi-EP and its PLT/MEPP/FEP gauges;

(xiv) it derived the Qi-CON diagnostic layer, including scale covariance, Lévy statistics, fractality, and the two-knee spectral law;

(xv) it derived the Qi-HAL architecture layer, including MePMoS, NHT, HAL, BFFT, and the DGI–GSSB switch;

(xvi) it defined SeOS as the persistent self-maintaining open-system class and supplied its viability criterion;

(xvii) it introduced QCI as the provisional completeness diagnostic for the full Qi-4 stack;

(xviii) it derived OH as the lawful external projection exported by OGGEP;

(xix) it derived IH as the internal reconstructive manifold built from memory, prediction, motion, and sensory history;

(xx) it defined EIC as the closed-loop OH-IH coupling and gave a theorem-level account of internal measurement and conscious content.

From this point onward, the primary revision burden is no longer structural assembly but sharpening: prose harmonization, tighter theorem lead-ins, clearer section transitions, stronger worked examples, and progressively more explicit empirical programs.

The appendices therefore preserve the page-budget ledger and forward-architecture table as archival management tools, even though the main six-part staircase is now already present in the manuscript.

5.3 Universal extension of GDOSto any open system in nature

5.3.1 From the boundary law of Chapter 4 to a universal runtime

Chapter 4 derived GDOS in the form required for quantum measurement, four-dimensional field theory, and the infrared limit of particle physics (Arisaka, 2026c). The resulting equations were written either as exact reduced channels, as GKLS generators, or as coarse-grained continuity equations on an effective state manifold. In all three representations the crucial structural fact was the same: once the inaccessible variables are traced over, the observer-accessible system is governed by an open dynamics rather than by a closed autonomous Hamiltonian law.

Chapter 5 now asks whether that structure was specific to physics laboratories or whether it was already more general than it first appeared. The answer proposed here is that the physics derivation was already universal in form. What changes from one branch of science to another is not the existence of an open-system law, but only the meaning of the accessible coordinates, the port structure, the relevant currents, the admissible observables, and the characteristic attractors.

The task of the present section is therefore to strip the derivation down to the reusable kernel that survives across domains.

Let the exact boundary-visible state at the end of the OGGEP reduction be denoted $\rho_\partial(t) \in \mathcal{D}(\mathcal{B})$. Any finite scientific description then selects only a finite family of observables O^A, $A = 1, \dots, N$, and therefore only a finite family of coordinates

$$x^A(t) := \Gamma^A[\rho_\partial(t)] \equiv \mathrm{Tr}\big(\rho_\partial(t)\, O^A\big)\,, \qquad A = 1, \dots, N. \tag{5.27}$$

Equation (5.27) is the true universal entrance point. In quantum physics the O^A may be projectors, pointer observables, or low moments of fields. In chemistry they may be concentrations, fluxes, or bond-state populations. In cell biology they may be copy numbers, membrane potentials, or methylation states. In neuroscience they may be firing rates, phase variables, or synaptic efficacies. In social systems they may be densities, prices, communication rates, or network-order parameters. The common feature is that the accessible description is always a coarse observable dictionary (Breuer & Petruccione, 2002; van Kampen, 2007).

Once Eq. (5.27) is written, the exact operator equation of Chapter 4 induces a reduced evolution on an effective state manifold $\mathcal{X}$ with local coordinates x^A. The resulting probability density $p(x,t)$ is defined so that for every smooth test function f,

$$\int_{\mathcal{X}} f(x)\, p(x,t)\, \mathrm{d}\mu(x) = \mathrm{Tr}\big(\rho_\partial(t)\, f\big(\Gamma[\cdot]\big)\big)\,, \tag{5.28}$$

where $\mathrm{d}\mu$ is the natural measure induced on $\mathcal{X}$. Under the same short-memory and mesoscopic assumptions already used in Chapter 4, one obtains the generic continuity form

$$\frac{\partial p}{\partial t} + \nabla_A J^A = 0. \tag{5.29}$$

Equation (5.29) is not an optional phenomenological ansatz. It is the universal open-system shadow of the bulk-to-boundary projection once one chooses a finite observable dictionary.

The universalization step now consists of recognizing that the current J^A can always be decomposed into four conceptually distinct pieces,

$$J^A = J^A_{\mathrm{grad}} + J^A_{\mathrm{curl}} + J^A_{\mathrm{noise}} + J^A_{\mathrm{gauge}}, \tag{5.30}$$

with

$$J^A_{\mathrm{grad}} = -\mu^{AB} \nabla_B \mathcal{F}_{\mathrm{EEGG}}\, p, \tag{5.31}$$

$$J^A_{\mathrm{curl}} = \Omega^A p, \qquad \nabla_A \Omega^A = 0, \tag{5.32}$$

$$J^A_{\mathrm{noise}} = -D^{AB} \nabla_B p, \qquad D^{AB} \succeq 0, \tag{5.33}$$

$$J^A_{\mathrm{gauge}} = R^A{}_\alpha\, \lambda^\alpha\, p. \tag{5.34}$$

Here $\mathcal{F}_{\mathrm{EEGG}}$ is the effective energy–entropy geometry–gauge ledger that will later be developed systematically; μ^{AB} is the mobility tensor; Ω^A is the divergence-free circulation or curl flow; D^{AB} is the positive semidefinite diffusion tensor; $R^A{}_\alpha$ are the generators of dynamically maintained gauge directions; and $\lambda^\alpha(t,x)$ are the corresponding gauge velocities. The first three terms are

the same gradient–curl–noise structure already derived in Chapter 4. The fourth is the new universal term that becomes explicit once gauge redundancy is recognized as a dynamical feature of open systems rather than merely a microscopic redundancy of field variables.

Equations (5.29)–(5.34) are the universal runtime kernel of this book. They say that every open system in nature can be described, at the correct level of coarse-graining, by an effective state manifold, a continuity law, a ledger functional, a solenoidal circulation, a fluctuation tensor, and a set of dynamical gauge directions. Chapter 5 later refines these ingredients into the SIP formalism, the EEGG ledger, and the Qi-4 pillars; but mathematically the engine is already visible here.

Definition 5.3.1 (Universal open-system runtime packet)**.** For the purposes of the present section, a *runtime packet* is the tuple

$$\mathfrak{S}_{\mathrm{run}} = \big(\mathcal{X},\ p,\ \mathcal{P},\ \mathcal{O},\ \mathcal{F}_{\mathrm{EEGG}},\ \mu^{AB},\ D^{AB},\ \Omega^{A},\ R^{A}{}_{\alpha}\big), \tag{5.35}$$

where $\mathcal{X}$ is the effective state manifold, p the reduced state density, $\mathcal{P}$ the set of ports through which exchange occurs, $\mathcal{O}$ the admissible observable dictionary, $\mathcal{F}_{\mathrm{EEGG}}$ the effective ledger functional, μ^{AB} and D^{AB} the dissipative and stochastic tensors, Ω^{A} the circulation field, and $R^{A}{}_{\alpha}$ the generators of gauge directions.

Remark 5.3.2. Equation (5.35) is intentionally provisional. Later sections will replace it by the more structured SIP formalism. The purpose of the present definition is only to make the universal extension of GDOS mathematically explicit before the more specialized vocabulary is introduced.

5.3.2 Why every scientific observation is constitutionally open

The central thesis of Chapter 5 is that openness is not an accidental imperfection of real experiments. It is an unavoidable structural feature of scientific observation itself.

To see this cleanly, consider a hypothetical closed law on a total state space $\mathcal{Z}$, written abstractly as

$$\dot{z} = \mathcal{V}(z), \qquad z \in \mathcal{Z}. \tag{5.36}$$

A finite observer never has direct access to the full z. What is actually available is a map

$$\Pi : \mathcal{Z} \longrightarrow \mathcal{X}, \qquad x = \Pi(z), \tag{5.37}$$

followed by an instrument map

$$\mathcal{M} : \mathcal{X} \longrightarrow \mathcal{Y}, \qquad y = \mathcal{M}(x), \tag{5.38}$$

where $\mathcal{Y}$ is the finite record space: counts, images, voltages, trajectories, labels, symbols, reports, or numerical summaries. Unless Π is globally invertible and the instrument is noiseless and complete, the induced evolution on $\mathcal{X}$ and $\mathcal{Y}$ cannot be closed in the sense of Eq. (5.36). Memory kernels, noise, hidden variables, or control ports necessarily appear.

This fact may be stated as a theorem.

Theorem 5.3.3 (Universal openness of finite observation)**.** *Let $\mathcal{Z}$ be a total state space with closed evolution $\dot{z} = \mathcal{V}(z)$. Let $\Pi : \mathcal{Z} \to \mathcal{X}$ be a non-invertible coarse-graining map and let $\mathcal{M} : \mathcal{X} \to \mathcal{Y}$ be a finite instrument map. Then the induced evolution on $\mathcal{X}$ is generically non-autonomous or non-Markovian unless additional hidden variables are appended; equivalently, the observer-accessible dynamics is an open dynamics.*

Proof. Because Π is non-invertible, there exist distinct total states $z_1 \neq z_2$ such that $\Pi(z_1) = \Pi(z_2) = x$. The future derivatives of the reduced coordinates are

$$\dot{x}_i = \mathrm{d}\Pi_{z_i}\big[\mathcal{V}(z_i)\big], \qquad i = 1, 2.$$

Generically these differ because $\mathcal{V}(z_1) \neq \mathcal{V}(z_2)$ and because the fibers of Π are not invariant under the total flow. Hence $\dot{x}$ cannot be written as a single-valued autonomous function of x alone. To close the reduced law one must either add memory, add hidden variables, or average over unresolved fibers. All three operations are precisely the standard signatures of open-system dynamics. The additional instrument map $\mathcal{M}$ can only discard more information, never restore invertibility. Therefore the observer-accessible dynamics is open. □

Theorem 5.3.3 is conceptually decisive. It implies that the quantum measurement problem is not the whole story but only the most historically visible instance of a much broader fact. Every field of science studies projected, instrument-conditioned, finite descriptions of larger systems. The difference between physics and biology is not that one is open and the other is closed. The difference is only the complexity of the projection and the richness of the resulting state manifold.

Table 5.11: How the universal open-system logic appears across scientific domains.

Domain	**Accessible coordinates x^A**	**Typical ports**	**Why the induced dynamics is open**
Particle experiment	Detector counts, event classes, low moments	Beam injection, environmental decoherence, readout electronics	Hidden bulk/environmental modes are traced over before a record is formed.
Chemical reactor	Concentrations, pH, temperature, flow rates	Inlet/outlet streams, heating, catalysts	Reservoirs and chemostats continuously exchange matter and energy with the reaction volume.
Cell or organism	Metabolite pools, membrane potentials, gene-expression states	Nutrient inflow, waste outflow, sensory coupling, action channels	The organism maintains itself only by continuous exchange across a boundary.
Neural circuit	Firing rates, phases, synaptic efficacies, bump positions	Sensory inputs, motor outputs, neuromodulation, blood flow	Hidden microstates and continual environmental coupling prevent closure on the recorded variables alone.
Collective or social system	Densities, prices, beliefs, network statistics	Communication channels, trade, migration, media, regulation	The coarse observables are projections of many unresolved agents and changing external constraints.

5.3.3 Ports, currents, and the minimal bookkeeping common to all systems

Once openness is accepted as universal, every system must be described not only by internal coordinates but also by its ports. A port is any channel across which the system exchanges quantities that matter for persistence: energy, matter, charge, entropy, information, stress, nutrients, toxins, prediction errors, or control signals. In the later SIP formalism this will be stated with greater precision, but the bookkeeping idea is already straightforward.

Let $X^I(t)$ denote any extensive quantity tracked at the effective level, with I ranging over the ledger sectors relevant to the problem. Then the generic balance law is

$$\frac{\mathrm{d}X^I}{\mathrm{d}t} = \sum_{a\in\mathcal{P}_{\mathrm{in}}} J_a^I - \sum_{b\in\mathcal{P}_{\mathrm{out}}} J_b^I + \Sigma_{\mathrm{int}}^I, \tag{5.39}$$

where J_a^I are boundary currents and Σ_{int}^I denotes internal conversion between sectors. For strictly conserved quantities $\Sigma_{\mathrm{int}}^I = 0$; for quantities like entropy or prediction error, Σ_{int}^I is generically nonzero.

Equation (5.39) is the common grammar behind systems that are often treated as if they were conceptually unrelated. In a biochemical network it describes nutrient and waste exchange. In a neuron it describes ionic, metabolic, and information-like currents. In ecology it describes immigration, predation, and resource uptake. In economics it describes capital, goods, and information exchange. What differs across domains is the semantic meaning of X^I, not the necessity of the balance law.

The special role of the GG-6/GDOS framework is that it does not let this bookkeeping remain an arbitrary phenomenological choice. Because the entire program began from a geometric gauge ledger in six dimensions, it insists that the bookkeeping for open systems must eventually be tied back to geometry and gauge structure. That is precisely why EEGG, SIP, and then Qi-QUA become necessary in the final section order. But even before those later sections are written, the universalization of GDOS already tells us that every admissible open-system description must declare its ports and currents.

Proposition 5.3.4 (Runtime form of every admissible open-system model)**.** *Within the GUT-10/GG-6/GDOS program, every admissible effective model of an open system must specify:*

(i) an effective state manifold $\mathcal{X}$,

(ii) an observable dictionary $\mathcal{O}$,

(iii) a port set $\mathcal{P}$,

(iv) a continuity law of the form (5.29),

(v) a current decomposition of the form (5.30),

(vi) and a balance ledger of the form (5.39).

Proof. Items (i)–(iii) are forced by the universal openness theorem: a nontrivial observational description requires reduced coordinates, observables, and ports. Item (iv) follows from probability

conservation on the effective manifold. Item (v) is the generic gradient–curl–noise decomposition augmented by the gauge directions made explicit in Chapter 5. Item (vi) is the integrated form of the continuity equation applied to the extensive quantities of interest. Each item is therefore not an added decorative choice but a necessary consequence of treating scientific observation as open and lawful. □

5.3.4 Why the new gauge term is unavoidable

At first glance the last term in Eq. (5.30) may appear optional. After all, one can often write phenomenological open-system models with only gradient, curl, and noise. Why insist on the gauge term $J^A_{\text{gauge}} = R^A{}_\alpha \lambda^\alpha p$?

The answer is that Chapter 5 is not merely writing down useful phenomenology. It is asking for the most general form that remains faithful to the derivation chain from GG-6 and OGGEP. Chapter 4 already showed that the observable boundary dynamics descends from a bulk with genuine gauge structure (Arisaka, 2026b,c). Once that is accepted, it is no longer consistent to treat open-system gauge structure as a negligible metaphor. The reduced system may not retain the full microscopic local gauge symmetry, but it does retain an induced equivalence relation under time-dependent boundary transformations. The new term in Eq. (5.34) is simply the mesoscopic imprint of that fact.

The mathematical meaning is simple. The generators $R^A{}_\alpha$ span directions in the effective state manifold that correspond to dynamically equivalent descriptions. Motion along these directions changes coordinates or internal phase conventions but not physical observables. If that redundancy were frozen once and for all, one would recover a conventional fixed-gauge formalism. But in an open system the environment continually changes the effective phase conventions, synchronization relations, and coordinate alignments. Therefore the gauge directions themselves become dynamical. This is the origin of DGI.

The practical importance is equally large. Without explicit gauge directions one cannot distinguish genuine adaptation from mere coordinate drift, genuine symmetry breaking from gauge fixing, or genuine attractors from artifacts of parameterization. In a theory that aims to cover systems from quantum devices to brains, that distinction is not optional.

Table 5.12: Why the universal GDOS kernel requires four current sectors rather than only three.

Current sector	**Mathematical role**	**Physical meaning**	**Why it is indispensable**
Gradient	$-\mu^{AB}\nabla_B \mathcal{F}_{\mathrm{EEGG}} p$	Dissipative descent or relaxation	Captures irreversible tendency toward lower effective ledger cost.
Curl	$\Omega^A p,\ \nabla_A \Omega^A = 0$	Circulation, recurrent flow, Hamiltonian-like transport	Captures persistent loops, cycles, and driven recurrences without scalar descent.
Noise	$-D^{AB}\nabla_B p$	Fluctuation, uncertainty, coarse-graining residue	Captures stochasticity and hidden-variable dispersion inherent to finite observation.
Gauge	$R^A{}_\alpha \lambda^\alpha p$	Motion along dynamically equivalent descriptions	Captures the inherited gauge redundancy of open systems and prevents confusion between physical and coordinate change.

5.3.5 Section summary: what has now been universalized

The work of the present section may be summarized compactly.

First, the open-system law derived in Chapter 4 has been detached from its original quantum-measurement presentation and rewritten in the generic language of reduced coordinates, probability currents, ports, and balance laws. This makes the GDOS kernel applicable in principle to any branch of science.

Second, the universal openness theorem has been proved in its basic form: whenever a finite observer studies a larger system through a non-invertible projection and a finite instrument, the accessible dynamics is open. The measurement problem in quantum mechanics is therefore one special case of a universal observation problem.

Third, the mesoscopic current decomposition has been sharpened from the three-sector form of Chapter 4 to the four-sector form required for Chapter 5. The new gauge sector makes it possible to speak rigorously about dynamically maintained equivalence classes of open-system descriptions. That step is the doorway to DGI.

Fourth, the universal runtime packet (5.35) has been declared. It is intentionally provisional, because the later SIP formalism will refine it. But it is sufficient to support the technical derivation of the master pipeline.

The next section therefore performs the first major theorem-level construction of Chapter 5: it derives the runtime pipeline

$$\text{lawful open-system evolution} \Longrightarrow \text{DGI} \Longrightarrow \text{GSSB} \Longrightarrow \text{GA}.$$

That pipeline is the minimal grammar by which open systems search, select, and stabilize.

Runtime constitution

Master pipeline, scalar ledger, and canonical packet

5.4 The master pipeline: DGI+ GSSB+ GA

Part I established the constitutional bridge from Chapters 2–4 to a universal theory of open systems. Part II now begins the constructive runtime sequence. The first necessity is neither a scalar budget nor a packet declaration, but the dynamical grammar that makes such objects meaningful at all. Section 5.4 therefore derives the master pipeline DGI + GSSB + GA: lawful exploration, branch selection, and attractor stabilization. Only once that three-stage engine is explicit does it become possible to ask which scalar ledger it descends (EEGG) and which concrete system instance carries that ledger (SIP).

5.4.1 Why a three-stage pipeline is necessary

A universal theory of open systems must answer three logically distinct questions. First, how does a system move lawfully through the space of admissible descriptions while remaining insensitive to mere representational redundancy? Second, how does the system select one realized branch or one realized structure from among many lawful possibilities? Third, once selected, how is that branch stabilized strongly enough to deserve interpretation as a function, a phenotype, a decision, a policy, a memory, or a persistent form? These three questions are answered, respectively, by DGI, GSSB, and GA.

The point of separating them is not terminological. Each stage resolves a different ambiguity.

If one had DGI without GSSB, the system would have lawful exploration but no reason to settle into one branch. One would have equivalence classes without commitment. This is too weak to describe phase transitions, developmental commitment, or decisions.

If one had GSSB without DGI, one would have branch selection without a prior distinction between physical alternatives and mere gauge-equivalent descriptions. One would then risk mistaking coordinate choices for physical commitments. This is too weak for a framework derived from gauge geometry.

If one had DGI and GSSB without GA, one would have lawful exploration and branch selection but still no precise criterion for what counts as a stabilized outcome. A branch might be selected transiently and then immediately lost. This is too weak to describe persistence.

The pipeline therefore has the logical shape

$$\boxed{\text{DGI : organize equivalences}} \Longrightarrow \boxed{\text{GSSB : select branch}} \Longrightarrow \boxed{\text{GA : stabilize outcome}} \tag{5.40}$$

Equation (5.40) is the runtime analogue of a proof pipeline. DGI defines the lawful search space, GSSB performs the choice, and GA names the stabilized result.

Table 5.13: Logical roles of the three stages in the master runtime pipeline.

Stage	**Primary question answered**	**Mathematical object**	**Operational meaning**
DGI	Which motions are physically equivalent?	Gauge orbit or quotient flow	Lawful exploration without overcounting redundant descriptions.
GSSB	Which branch is actually realized?	Symmetry-breaking bifurcation or symmetry-reduced invariant measure	Commitment to one phase, phenotype, policy, or record.
GA	What persists after the transition?	Asymptotically stable invariant set or selected stationary measure	Stable function, structure, memory, or mode of operation.

5.4.2 DGI: from local and global gauge symmetry to dynamical gauge symmetry

The first claim of the present section is that gauge symmetry in the GG-6/GDOS program naturally comes in three levels.

At the microscopic fundamental level one has *local gauge invariance* (LGI): the bulk theory is invariant under local gauge transformations of its fundamental fields. That is the familiar Yang–Mills and gravitational notion already used in Chapter 3 (Arisaka, 2026b).

At the effectively isolated macroscopic level one often has *global gauge invariance* (GGI): the residual global subgroup acts on the whole isolated system and produces conserved Noether charges.

At the realistic mesoscopic level, however, most systems are neither fully microscopic and closed nor fully macroscopic and isolated. They are open, driven, coarse-grained, and continually re-coupled to their environments. For them the right notion is *dynamical gauge invariance* (DGI): there exists a time-dependent family of gauge-equivalent re-descriptions under which the reduced open-system law remains physically unchanged.

This can be stated first at the operator level. Let the boundary-visible density operator obey

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_\partial(t) = \mathfrak{L}_t[\rho_\partial(t)]\,. \tag{5.41}$$

Let U_t be a time-dependent unitary family induced on the boundary from an admissible bulk gauge transformation under OGGEP. Define the transformed state

$$\rho'_\partial(t) = U_t\,\rho_\partial(t)\,U_t^\dagger. \tag{5.42}$$

Differentiating Eq. (5.42) and using Eq. (5.41) gives

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho'_\partial &= \dot{U}_t\rho_\partial U_t^\dagger + U_t\mathfrak{L}_t[\rho_\partial]U_t^\dagger + U_t\rho_\partial\dot{U}_t^\dagger \\ &= U_t\mathfrak{L}_t\Big[U_t^\dagger\rho'_\partial U_t\Big]\,U_t^\dagger - \mathrm{i}[K_t,\rho'_\partial],\end{aligned} \tag{5.43}$$

where

$$K_t := \mathrm{i}\,\dot{U}_t U_t^\dagger \tag{5.44}$$

is Hermitian. Therefore the transformed generator is

$$\mathfrak{L}_t^{(U)} = \mathcal{U}_t \circ \mathfrak{L}_t \circ \mathcal{U}_t^{-1} - \mathrm{i}[K_t, \cdot], \qquad \mathcal{U}_t[\rho] = U_t \rho U_t^\dagger. \tag{5.45}$$

Equation (5.45) is the exact operator-level statement of DGI. The reduced law is defined not by one rigid generator but by an equivalence class of time-dependent generators related by induced gauge transformations (Arisaka, 2026c; Breuer & Petruccione, 2002).

Definition 5.4.1 (Dynamical Gauge Invariance)**.** A reduced open-system dynamics on $\mathcal{B}$ satisfies *Dynamical Gauge Invariance* if for every admissible time-dependent induced gauge transformation U_t, the transformed generator is related to the original generator by Eq. (5.45), and all physical observables are invariant under the induced action.

The mesoscopic form of the same idea is even more transparent. Let $R_\alpha = R^A{}_\alpha \partial_A$ be the gauge generators on the effective state manifold $\mathcal{X}$. Then the most general admissible drift vector field may be decomposed as

$$V^A = V_\perp^A + R^A{}_\alpha \lambda^\alpha, \tag{5.46}$$

where $V_\perp^A$ is horizontal (physical) and $R^A{}_\alpha \lambda^\alpha$ is vertical (gauge). Substituting Eq. (5.46) into the continuity equation yields exactly the four-current decomposition already written in Section 5.3. The physical quotient manifold is

$$\mathcal{X}_{\mathrm{phys}} := \mathcal{X}/G_{\mathrm{dyn}}, \tag{5.47}$$

where G_{dyn} is the group generated by the R_α. Only functions constant along the gauge orbits are physical observables:

$$R^A{}_\alpha \, \nabla_A O = 0 \qquad \forall \alpha. \tag{5.48}$$

Proposition 5.4.2 (Gauge directions do not contribute to physical expectation values)**.** *If O satisfies Eq. (5.48), then the gauge current $J^A_{\mathrm{gauge}} = R^A{}_\alpha \lambda^\alpha p$ does not contribute to $\mathrm{d}\langle O\rangle/\mathrm{d}t$.*

Proof. Using Eq. (5.29) and integrating by parts,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle O\rangle = \frac{\mathrm{d}}{\mathrm{d}t}\int O\, p \,\mathrm{d}\mu = \int (\nabla_A O)\, J^A \,\mathrm{d}\mu.$$

The gauge contribution is

$$\int (\nabla_A O)\, R^A{}_\alpha \lambda^\alpha p \,\mathrm{d}\mu = \int (R_\alpha O)\, \lambda^\alpha p \,\mathrm{d}\mu = 0$$

by Eq. (5.48). Hence only the horizontal physical part of the current affects gauge-invariant observables. □

Proposition 5.4.2 is one of the cleanest places where the necessity of DGI becomes visible. It says that open systems can continually adjust their internal gauge alignment without changing physical predictions. This is the precise mathematical meaning of "lawful exploration" in the present framework.

Table 5.14: The LGI/GGI/DGI triad.

Level	Typical system	Symmetry statement	Why Chapter 5 needs it
LGI	Fundamental microscopic fields	Local bulk action invariant under gauge transformations	Supplies the exact microscopic legality inherited from GG-6.
GGI	Effectively isolated whole system	Constant global subgroup generates conserved charges	Useful idealization for closed or nearly isolated macro systems.
DGI	Realistic open mesoscopic system	Reduced generator is defined only up to time-dependent induced gauge transformations	Supplies the correct symmetry principle for the actual open systems studied in science.

5.4.3 Formal derivation of DGIfrom GG-6 and OGGEP

The operator formula (5.45) already gives the local derivation of DGI once one accepts an induced time-dependent boundary unitary U_t. What remains is to explain why such a unitary is generically present in the GG-6/GDOS framework.

Start from the bulk Hilbert space and its decomposition into visible and hidden sectors,

$$\mathcal{H}_{\text{bulk}} = \mathcal{B} \otimes \mathcal{H}_{\text{env}}, \tag{5.49}$$

together with the OGGEP reduction map

$$\Phi_t = \mathcal{R}_t \circ \text{Tr}_{\mathcal{H}_{\text{env}}} \circ \mathcal{E}_t, \tag{5.50}$$

where $\mathcal{E}_t$ is the lawful bulk evolution and $\mathcal{R}_t$ the boundary restriction or decoding map. Let $U_{\text{bulk}}(g_t)$ be a bulk gauge transformation compatible with the GG-6 constitutive symmetry. Because the bulk law is gauge-covariant and because the restriction map is built from gauge-covariant boundary observables, one has the covariance relation

$$\Phi_t\Big(U_{\text{bulk}}(g_t)\,\rho_{\text{bulk}}\,U^\dagger_{\text{bulk}}(g_t)\Big) = U_t\,\Phi_t(\rho_{\text{bulk}})\,U_t^\dagger \tag{5.51}$$

for an induced boundary unitary U_t. Equation (5.51) is the essential inheritance law. It means that gauge covariance is not lost at the boundary; it is weakened from a microscopic local invariance to an induced boundary equivalence class.

Differentiating Eq. (5.51) in time and using Eq. (5.45) yields the following theorem.

Theorem 5.4.3 (Bulk-to-boundary origin of DGI)**.** *Assume the GG-6 bulk theory of Chapter 3, the OGGEP channel of Chapter 4, and the covariance property* (5.51)*. Then the reduced boundary dynamics inherits a time-dependent gauge equivalence class of generators of the form* (5.45)*. In particular, every admissible boundary description is defined only up to induced dynamical gauge transformations.*

Proof. The GG-6 bulk law is gauge-covariant by construction. Therefore if $\rho_{\text{bulk}}(t)$ is a lawful bulk history, so is $U_{\text{bulk}}(g_t)\rho_{\text{bulk}}(t)U^\dagger_{\text{bulk}}(g_t)$. Applying the OGGEP map and using Eq. (5.51)

gives the transformed boundary state $\rho'_\partial(t) = U_t \rho_\partial(t) U_t^\dagger$. The differentiation calculation already performed in Eqs. (5.43)–(5.45) then shows that the transformed boundary state is generated by $\mathfrak{L}_t^{(U)}$. Therefore boundary generators come in induced gauge-equivalence classes. □

Theorem 5.4.3 is the most direct sense in which Chapter 5 extends the logic of the earlier papers. DGI is not introduced by hand as an extra principle. It is the boundary avatar of the bulk gauge structure once openness is taken seriously.

5.4.4 GSSB: symmetry breaking lifted from fields to geometry

The second stage of the runtime pipeline is GSSB. The idea is to generalize the familiar logic of spontaneous symmetry breaking so that it applies not only to internal field-space symmetries but to the full effective geometry of an open system.

The mathematically clean way to state this is to work on the physical quotient manifold $\mathcal{X}_{\text{phys}}$ defined by Eq. (5.47). Let a symmetry group $\mathcal{S}$ act on $\mathcal{X}_{\text{phys}}$, and let the reduced law be equivariant under that action. Let π_t denote either a trajectory, a stationary distribution, or an invariant measure on $\mathcal{X}_{\text{phys}}$. Then GSSB occurs when the law is $\mathcal{S}$-equivariant but the selected invariant object has a smaller isotropy subgroup,

$$H(\pi_*) := \{s \in \mathcal{S} \mid s \cdot \pi_* = \pi_*\} \subsetneq \mathcal{S}. \tag{5.52}$$

This definition is more general than the monograph vacuum-expectation-value definition because it applies equally to fixed points, limit cycles, metastable distributions, patterns, network states, and policies.

Definition 5.4.4 (Geometric Spontaneous Symmetry Breaking)**.** A GDOS-governed open system undergoes *Geometric Spontaneous Symmetry Breaking* if its reduced laws on $\mathcal{X}_{\text{phys}}$ are equivariant under a symmetry group $\mathcal{S}$, yet the dynamically selected invariant set or invariant measure has isotropy subgroup $H \subsetneq \mathcal{S}$.

Why call this *geometric*? Because in the present framework the symmetry-reduced object is not only an internal field configuration. It may involve boundary conditions, port alignments, metric moduli, phase-locking patterns, network topology, reaction architecture, sensorimotor policy, or a combination of these. In other words, what is being selected is the full effective geometry of the open system as seen on the boundary.

The simplest local normal form is the familiar Landau bifurcation written in a gauge-invariant order parameter q:

$$\mathcal{F}_{\text{loc}}(q;\eta) = \mathcal{F}_0 + \frac{1}{2}a(\eta)q^2 + \frac{1}{4}bq^4 + O(q^6), \qquad b > 0, \tag{5.53}$$

with gradient dynamics

$$\dot{q} = -\frac{\partial \mathcal{F}_{\text{loc}}}{\partial q} = -a(\eta)q - bq^3 + O(q^5). \tag{5.54}$$

Assume $a(\eta_c) = 0$ and $a'(\eta_c) \neq 0$. Then stationary branches satisfy

$$q\left(a(\eta) + bq^2\right) = 0, \tag{5.55}$$

hence

$$q_* = 0 \quad \text{or} \quad q_\pm^2 = -\frac{a(\eta)}{b}. \tag{5.56}$$

Proposition 5.4.5 (Canonical local model of GSSB)**.** *For the local dynamics* (5.54) *with* $b > 0$*, the symmetric branch* $q_* = 0$ *is stable for* $a(\eta) > 0$ *and unstable for* $a(\eta) < 0$*, while the broken branches* $q_\pm$ *exist and are stable for* $a(\eta) < 0$*. The symmetry* $q \mapsto -q$ *of the law is thus reduced to the identity on each selected branch.*

Proof. The stationary points solve Eq. (5.55). Their linear stability is determined by

$$f'(q) = \frac{\mathrm{d}}{\mathrm{d}q}\left[-aq - bq^3\right] = -a - 3bq^2.$$

At $q = 0$, one has $f'(0) = -a$, so the symmetric branch is stable for $a > 0$ and unstable for $a < 0$. At $q = q_\pm$, one has

$$f'(q_\pm) = -a - 3b\left(-\frac{a}{b}\right) = 2a.$$

Therefore $f'(q_\pm) < 0$ precisely when $a < 0$, so the broken branches are stable in the symmetry-broken regime. The law remains invariant under $q \mapsto -q$, but each selected stable branch is not. This is the canonical local model of spontaneous symmetry breaking. □

Proposition 5.4.5 is intentionally elementary because its purpose is illustrative. The full GSSB claim of Chapter 5 is broader: the order parameter may be vector-valued or functional, the symmetry group may be discrete or continuous, the selected object may be a cycle or an invariant measure rather than a fixed point, and the effective functional may include both equilibrium-like and nonequilibrium terms. But the local lesson survives: the laws may retain higher symmetry than the selected stable outcome.

Table 5.15: Ordinary SSB versus GSSB.

Feature	**Ordinary SSB**	GSSB **in Chapter 5**
Underlying object	Usually field potential in internal space	Full effective geometry of an open system on the boundary manifold
Selected object	Vacuum expectation value or ordered phase	Fixed point, limit cycle, stationary measure, phenotype, policy, or memory pattern
Symmetry class	Usually internal global or gauge symmetry	Gauge-reduced geometric, dynamical, and architectural symmetry
Trigger	Cooling, control parameter, instability	Control-parameter crossing under open-system drive, dissipation, and noise
Mathematical signature	Symmetric law, lower-symmetry state	$\mathcal{S}$-equivariant law, invariant object with smaller isotropy subgroup $H \subsetneq \mathcal{S}$
Representative examples	Higgs vacuum, ferromagnet	Molecular motor stepping, chirality choice, developmental commitment, neural decision state, social coordination mode

5.4.5 How DGIand GSSBinteract

The central physical idea of the pipeline is that DGI and GSSB are not separate modules that happen to occur one after the other. DGI prepares the lawful search space on which GSSB acts.

Mathematically, this is easiest to see by first quotienting out the gauge directions. The gauge vertical flow $R^A{}_\alpha \lambda^\alpha$ does not change physical observables, by Proposition 5.4.2. Therefore the physically relevant instability analysis must be performed on $\mathcal{X}_{\text{phys}}$, not on the full $\mathcal{X}$. This prevents one from mistaking gauge drift for real symmetry breaking.

Let y^i, $i = 1, \dots, n$, be local coordinates on $\mathcal{X}_{\text{phys}}$. Then the reduced deterministic drift takes the form

$$\dot{y}^i = -\widetilde{\mu}^{ij} \partial_j \widetilde{\mathcal{F}}_{\text{EEGG}} + \widetilde{\Omega}^i, \tag{5.57}$$

with the gauge part already factored out. A GSSB event occurs when the symmetric invariant set of Eq. (5.57) loses stability and a lower-symmetry invariant set becomes asymptotically favored.

The functional reason this happens is that the EEGG landscape is typically not a featureless bowl. Open systems are driven. They have ports. They circulate. They fluctuate. Therefore the effective landscape on $\mathcal{X}_{\text{phys}}$ generically contains multiple basins, ridges, and channels. DGI guarantees that only physically distinct basins are counted. GSSB then selects one of them when a threshold is crossed. The threshold may be set by energy input, coupling strength, concentration, evidence, load, or noise-to-signal ratio. The vocabulary changes across sciences; the pipeline does not.

At small but nonzero noise, the local picture is sharpened by the stationary density. For the scalar model (5.54) driven by additive white noise of strength σ,

$$\mathrm{d}q_t = -\partial_q \mathcal{F}_{\text{loc}}(q_t; \eta)\, \mathrm{d}t + \sigma\, \mathrm{d}W_t, \tag{5.58}$$

the stationary density is

$$p_*(q) = Z^{-1} \exp\left[-\frac{2}{\sigma^2}\, \mathcal{F}_{\text{loc}}(q; \eta) \right]. \tag{5.59}$$

When $a(\eta) > 0$, the density is unimodal around $q = 0$. When $a(\eta) < 0$, the density becomes bimodal around $q_\pm$, so the symmetry of the law persists but the stationary measure concentrates near the broken branches. This is the stochastic open-system version of symmetry breaking (Touchette, 2009; van Kampen, 2007).

The interpretive consequence is crucial. DGI corresponds to lawful coexistence of multiple equivalent descriptions during the exploration phase. GSSB corresponds to the threshold at which the system no longer merely explores but commits to one reduced branch. In later sections this will become central for the discussion of biological choices, cognitive decisions, and conscious report formation. But the mathematics is already visible here.

5.4.6 GA: the selected persistent invariant structure

The third stage of the pipeline is to identify the selected and persistent outcome of the DGI→GSSB transition. This is the *Geometric Attractor*.

Classical dynamical systems define an attractor as a compact invariant set that attracts nearby trajectories. That notion is valuable but too weak for Chapter 5. It says where trajectories end up, but not how the destination was selected, whether it is gauge-consistent, whether it lies on a symmetry-broken branch, or whether it obeys the later Qi constraints. The present framework therefore strengthens the notion.

Definition 5.4.6 (Geometric Attractor)**.** A *Geometric Attractor* $\mathcal{A}_{\mathrm{GA}} \subseteq \mathcal{X}_{\mathrm{phys}}$ is an invariant set or invariant measure satisfying all of the following:

(i) **Invariance:** it is invariant under the reduced GDOS semigroup on $\mathcal{X}_{\mathrm{phys}}$.

(ii) **Asymptotic relevance:** there exists a nontrivial basin of initial conditions whose trajectories or stationary measures converge toward it.

(iii) **Selection by** GSSB**:** it belongs to a symmetry-reduced branch selected from a larger admissible family.

(iv) **Gauge admissibility:** it is defined on the physical quotient, not on an unquotiented gauge-redundant space.

(v) **Ledger admissibility:** it is compatible with the effective EEGG descent or steady-balance constraints relevant to the system.

Definition 5.4.6 deliberately allows several concrete realizations: a fixed point, a limit cycle, a torus, a localized bump, an invariant distribution, or a metastable structured manifold. The common point is that the selected object is not just any attractor; it is the particular attractor that remains after gauge reduction and symmetry breaking have done their work.

The existence of such objects can be justified in the present framework by the EEGG ledger.

Theorem 5.4.7 (Existence of invariant asymptotic sets under confining EEGG landscapes)**.** *Consider the deterministic reduced flow*

$$\dot{y}^i = -\widetilde{\mu}^{ij}\partial_j \widetilde{\mathcal{F}}_{\mathrm{EEGG}} + \widetilde{\Omega}^i, \tag{5.60}$$

on $\mathcal{X}_{\mathrm{phys}}$, with $\widetilde{\mu}^{ij}$ positive semidefinite, $\widetilde{\Omega}^i \partial_i \widetilde{\mathcal{F}}_{\mathrm{EEGG}} = 0$, and $\widetilde{\mathcal{F}}_{\mathrm{EEGG}}$ proper and bounded below. Then every trajectory is contained in a compact sublevel set of $\widetilde{\mathcal{F}}_{\mathrm{EEGG}}$, and its ω-limit set is contained in the largest invariant subset of

$$\mathcal{C} = \left\{ y \in \mathcal{X}_{\mathrm{phys}} \;\middle|\; \partial_i \widetilde{\mathcal{F}}_{\mathrm{EEGG}}\, \widetilde{\mu}^{ij} \partial_j \widetilde{\mathcal{F}}_{\mathrm{EEGG}} = 0 \right\}. \tag{5.61}$$

Proof. Differentiate $\widetilde{\mathcal{F}}_{\mathrm{EEGG}}$ along the flow:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\widetilde{\mathcal{F}}_{\mathrm{EEGG}} &= \partial_i \widetilde{\mathcal{F}}_{\mathrm{EEGG}}\, \dot{y}^i \\ &= -\partial_i \widetilde{\mathcal{F}}_{\mathrm{EEGG}}\, \widetilde{\mu}^{ij}\, \partial_j \widetilde{\mathcal{F}}_{\mathrm{EEGG}} + \partial_i \widetilde{\mathcal{F}}_{\mathrm{EEGG}}\, \widetilde{\Omega}^i. \end{aligned} \tag{5.62}$$

The second term vanishes by the orthogonality assumption, so

$$\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{\mathcal{F}}_{\mathrm{EEGG}} = -\partial_i \widetilde{\mathcal{F}}_{\mathrm{EEGG}}\, \widetilde{\mu}^{ij}\, \partial_j \widetilde{\mathcal{F}}_{\mathrm{EEGG}} \leq 0. \tag{5.63}$$

Hence $\widetilde{\mathcal{F}}_{\mathrm{EEGG}}$ is a Lyapunov function. Because it is proper and bounded below, trajectories remain in compact sublevel sets. Standard LaSalle invariance then implies that ω-limit sets lie in the largest invariant subset of the set where $\mathrm{d}\widetilde{\mathcal{F}}_{\mathrm{EEGG}}/\mathrm{d}t = 0$, namely $\mathcal{C}$. □

Theorem 5.4.7 does not yet pick one unique attractor. It says that the dynamics is confining and that its asymptotic behavior lies in the invariant set selected by the EEGG ledger. Uniqueness requires one more layer: a selection principle among multiple admissible asymptotic sets.

At small noise this selection can be encoded by a quasi-potential V or large-deviation functional. Writing schematically

$$p_\epsilon(y) \asymp Z_\epsilon^{-1} \exp\left[-\frac{1}{\epsilon} V(y)\right], \qquad \epsilon \to 0^+, \tag{5.64}$$

one expects the stationary measure to concentrate on the attractor(s) with minimal V (Touchette, 2009). This leads to the formal selection rule

$$\mathcal{A}_{\mathrm{GA}} = \arg\min_{\mathcal{A} \in \mathfrak{A}_{\mathrm{adm}}} V(\mathcal{A}), \tag{5.65}$$

where $\mathfrak{A}_{\mathrm{adm}}$ is the family of admissible symmetry-reduced invariant sets. In later sections the role of V will be refined by Qi-EP and the current-locking constraints. For the present section it is enough to see that GA is the selected asymptotic object, not just any asymptotic object.

Table 5.16: Generic attractors versus Geometric Attractors.

Property	**Generic attractor language**	**Geometric Attractor language**
Underlying space	Raw phase space	Gauge-reduced physical manifold $\mathcal{X}_{\mathrm{phys}}$
Selection principle	Basin geometry or initial conditions	GSSB selection on an EEGG landscape, later refined by Qi constraints
Admissibility filter	Usually none beyond dynamical existence	Must satisfy gauge, ledger, and symmetry-reduction admissibility
Possible forms	Fixed point, cycle, torus, strange attractor	All of the classical forms, plus structured invariant measures and holographic/architectural subsets
Interpretive meaning	Where trajectories go	Persistent function, phase, phenotype, memory, decision mode, or policy after lawful selection

5.4.7 The full DGI→GSSB→GAderivation

The three ingredients are now in place, so the full derivation can be written in the theorem that governs the remainder of the manuscript. In the new order this theorem must appear before EEGG and SIP, because it tells us what kind of runtime those later objects are supposed to close.

Theorem 5.4.8 (Master runtime pipeline)**.** *Assume:*

(i) the GG-6 bulk and OGGEP reduction of Papers II and III;

(ii) the universal open-system runtime kernel (5.29)–(5.34)*;*

(iii) a proper gauge-invariant effective ledger $\mathcal{F}_{\mathrm{EEGG}}$*;*

(iv) a symmetry group $\mathcal{S}$ *acting on the gauge-reduced manifold* $\mathcal{X}_{\mathrm{phys}}$*;*

(v) and a control regime in which the symmetric invariant branch loses stability and a lower-symmetry invariant branch appears.

Then the open-system dynamics factors into the following stages:

$$\mathcal{X} \overset{\text{quotient}}{\longrightarrow} \mathcal{X}_{\mathrm{phys}} \overset{\text{instability}}{\longrightarrow} \text{symmetry-broken branch} \overset{\text{asymptotic selection}}{\longrightarrow} \mathcal{A}_{\mathrm{GA}}. \tag{5.66}$$

Equivalently: induced gauge covariance yields DGI*; loss of stability on the quotient yields* GSSB*; and asymptotic concentration on the selected symmetry-broken invariant set yields a* GA*.*

Proof. By Theorem 5.4.3, the reduced dynamics inherits an induced gauge-equivalence class of generators, so the first reduction to $\mathcal{X}_{\mathrm{phys}}$ is well defined. By Proposition 5.4.2, physical observables depend only on this quotient flow. By the assumed instability of the symmetric branch on $\mathcal{X}_{\mathrm{phys}}$, one obtains a symmetry-broken branch in the sense of Definition 5.4.4; locally this is modeled by Proposition 5.4.5. Finally, by Theorem 5.4.7, trajectories or stationary measures are confined to asymptotically invariant sets determined by the ledger; among the admissible symmetry-broken branches, the selected one is denoted $\mathcal{A}_{\mathrm{GA}}$. This establishes the stated factorization. □

Theorem 5.4.8 is the first true engine theorem of Chapter 5. It says that once openness, gauge inheritance, and symmetry-reduced selection are taken seriously, the runtime of any persistent open system naturally decomposes into lawful exploration, symmetry-breaking commitment, and attractor stabilization.

Table 5.17: Operational audit of the master runtime pipeline.

Phase	**Mathematical test**	**What changes**	**What must stay invariant**
DGI	Quotient by G_{dyn}, Eq. (5.47)	Coordinates, phases, internal alignment conventions	Gauge-invariant observables and physical predictions
GSSB	Instability of symmetric branch; isotropy reduction $H \subsetneq \mathcal{S}$	Branch, phase, architecture, committed mode	Underlying law and admissibility filters
GA	Asymptotic concentration on selected invariant set	Persistent pattern, cycle, measure, or policy	Compatibility with gauge reduction and ledger constraints

5.4.8 Worked examples across scales

The universal value of the pipeline is best seen by writing it in several very different domains.

5.4.8.1 Example A: the Higgs vacuum as the particle-physics limiting case

The standard Higgs mechanism is the limiting case in which the open-system language collapses to a familiar field-theory symmetry breaking (Higgs, 1964). The gauge-invariant potential for the radial Higgs mode is

$$V(H) = -\mu_H^2 |H|^2 + \lambda_H |H|^4, \qquad \mu_H^2, \lambda_H > 0. \tag{5.67}$$

The local symmetry law is richer than the selected vacuum, and the vacuum manifold chooses one orbit representative modulo the residual $U(1)_{\mathrm{EM}}$. In the language of this book, this is an elementary GSSB event. The corresponding GA is the symmetry-broken electroweak vacuum manifold modulo the residual gauge group. What Chapter 5 contributes is not to replace the Higgs mechanism, but to recognize it as the particle-physics corner of a much larger runtime grammar.

5.4.8.2 Example B: a driven phase oscillator or laser-like order parameter

Consider a complex order parameter $E = re^{\mathrm{i}\phi}$ obeying the normal form

$$\dot{E} = (\alpha - \beta|E|^2)E + \eta(t), \qquad \beta > 0. \tag{5.68}$$

For $\eta = 0$, the law is invariant under the global phase rotation $E \mapsto e^{\mathrm{i}\chi}E$. Below threshold ($\alpha < 0$), the stable state is $E = 0$. Above threshold ($\alpha > 0$), one obtains a circle of nonzero-amplitude states

$$|E_*| = \sqrt{\frac{\alpha}{\beta}}, \tag{5.69}$$

with undetermined phase. In the present language, the phase redundancy is the effective $U(1)$-like DGI direction, the threshold crossing is the GSSB event, and the nonzero-amplitude coherent state is the GA. The point is not that every oscillator is literally a gauge theory, but that the same search–selection–stabilization grammar recurs once phase-equivalent descriptions are dynamically maintained.

5.4.8.3 Example C: ATP synthase as a molecular DGI→GSSB→GAcycle

At the molecular scale, ATP synthase provides a striking open-system example (Lane, 2015; Yoshida et al., 2001). Let θ denote the rotor angle and model the effective periodic landscape by

$$V(\theta) = V_0\big(1 - \cos 3\theta\big) - \tau\,\theta, \tag{5.70}$$

where V_0 encodes the threefold catalytic periodicity and τ is the effective proton-motive torque. The overdamped rotor law is

$$\gamma\dot{\theta} = -\partial_\theta V(\theta) + \xi(t) = -3V_0 \sin 3\theta + \tau + \xi(t). \tag{5.71}$$

For $|\tau| < 3V_0$, the landscape contains alternating minima and barriers; the rotor can explore several nearby conformational possibilities during a dwell. This exploration phase is the effective DGI analogue: multiple phase-equivalent microconformations remain dynamically accessible

without committing the machine to the next catalytic state. Once the torque exceeds the local barrier condition, the rotor undergoes a transition to the next 120° sector. That threshold crossing is the GSSB event. The resulting robust three-step rotational cycle is the GA of the molecular motor.

What matters most here is not the exact simplicity of Eq. (5.70), which is schematic, but the structure it exhibits. The machine does not behave as a closed system sitting in a fixed minimum. It is a driven open cycle that continually explores, commits, and stabilizes. The language of DGI→GSSB→GA therefore fits it more naturally than a purely static energy-minimum picture.

5.4.8.4 Example D: neural or cognitive commitment as a mesoscopic attractor selection

In neural systems the same pipeline appears at a higher level of complexity. Consider the simplest scalar decision variable m with evidence input $I(t)$:

$$\dot{m} = am - bm^3 + I(t) + \xi(t), \qquad a, b > 0. \tag{5.72}$$

When $I \approx 0$ and noise is appreciable, the system may hover near the undecided regime $m \approx 0$, or if the network is ring-like it may support multiple equivalent phase-like interpretations of the same sensory scene. This regime is the operational analogue of DGI: the system has not yet committed to one branch. When recurrent gain and external evidence together drive the system past threshold, one of the symmetry-related decision branches $m_\pm \approx \pm\sqrt{a/b}$ becomes selected. That is GSSB. The resulting metastable decision state or ring-attractor bump is the GA (Friston, 2010; Turner-Evans et al., 2020).

Chapter 5 will later return to this example when Qi-HAL and OH + IH = EIC are introduced. For the present section the important conclusion is that the same runtime grammar applies without changing its mathematical meaning.

Table 5.18: Cross-domain instantiations of the DGI→GSSB→GA pipeline.

System	DGI **phase**	GSSB **event**	GA **outcome**
Electroweak sector	Gauge-equivalent vacuum orbit descriptions	Higgs vacuum selection	Symmetry-broken vacuum with residual $U(1)_{\mathrm{EM}}$
Laser-like phase oscillator	Phase-equivalent pre-threshold dynamics	Threshold crossing to finite amplitude	Stable coherent amplitude state
ATP synthase	Conformational exploration during dwell	Barrier crossing to next 120° sector	Robust catalytic rotational cycle
Neural decision/ring attractor	Coexistence of multiple phase-like interpretations	Evidence-driven commitment to one branch	Metastable decision state or localized bump

5.4.9 Why the master pipeline is the correct bridge to the remaining sections

The rest of Chapter 5 can now be placed in a strictly ordered chain.

First, Section 5.5 derives EEGG. The pipeline needs a scalar budget because exploration, commitment, and stabilization would otherwise have no common quantity to descend, replenish, or audit. For that reason EEGG comes immediately after the master pipeline.

Second, Section 5.6 introduces SIP. Once the scalar ledger is fixed, the theory still needs a concrete carrier: a declared system instance with ports, generator, observables, and kill criteria. That is why SIP closes Part II rather than appearing later.

Only then can Part III begin. Qi-QUA locks the event currents of the packet; Qi-EP selects admissible histories inside that locked budget; Qi-CON reads the scale signatures of the selected history; and Qi-HAL identifies the memory-bearing architecture that can actually realize the full stack.

After the Qi block, Part IV turns to synthesis rather than construction. SeOS and QCI classify persistence and completeness of the already-built runtime, while OH+IH = EIC extends the same lawful machinery from external measurement to internal reconstruction and internal measurement. The remaining Parts then treat emergence, applications, universality, and assessment. In this sense the master pipeline is not one component among many; it is the ordering principle that makes the later sections necessary and non-interchangeable.

5.4.10 Section summary

This section has supplied the first major runtime engine of Chapter 5.

First, DGI was defined at both the operator level and the mesoscopic level. The crucial equation is the generator transformation law (5.45), derived from induced boundary gauge covariance. This established that realistic open systems are governed not by one rigid reduced law but by a gauge-equivalence class of reduced laws.

Second, GSSB was defined as symmetry breaking on the gauge-reduced physical manifold. The local Landau model (5.53)–(5.56) showed explicitly how a symmetric branch loses stability and lower-symmetry branches appear. The important conceptual move was to lift the meaning of symmetry breaking from narrow field-space language to the full geometry of open systems.

Third, GA was defined as the selected and persistent invariant set or invariant measure that survives the combined filtering of gauge reduction, symmetry breaking, and ledger admissibility. The EEGG Lyapunov theorem showed why confining asymptotic structures are expected, while the quasi-potential formula indicated how one branch can be selected among many.

Fourth, Theorem 5.4.8 assembled the pieces into a single factorization result. This is the section of Chapter 5 that turns the project from a conceptual manifesto into a reusable runtime formalism.

The remaining order is now fixed rather than flexible: Section 5.5 derives the scalar ledger, Section 5.6 declares the packet, Part III develops the four Qi pillars, Part IV turns to persistence/completeness and then to internality, and Part V treats hierarchy and applications. None of those later tasks can proceed coherently without the present pipeline, and that is why

DGI+GSSB+GA belongs at the beginning of the runtime constitution rather than in the middle of the paper.

5.5 EEGG: Energy, Entropy, and Free Energy by the Geometry–Gauge Principle

With the master runtime pipeline now fixed, the manuscript can move from dynamical grammar to scalar bookkeeping. Section 5.4 told us how lawful open systems explore, commit, and stabilize. The immediate next question is what quantity that runtime spends, dissipates, restores, and gradients against. Section 5.5 therefore derives EEGG before any Qi pillar appears. This placement is essential: the Qi program refines an already constituted runtime; it does not create the budget after the fact.

5.5.1 Why the scalar ledger must be fixed before the Qi program

Section 5.4 supplied the master pipeline of the runtime. It told us how a lawful open system explores state space, how one branch is selected, and how stable structures are maintained. Yet the pipeline by itself does not say what scalar quantity the gradient sector descends, what quantity the maintenance sector must offset through ports, or why the open-system budget should be written in a single canonical form rather than in many discipline-specific free-energy-like proxies.

This is why the revised structure places EEGG before the Qi-4 block. The Qi pillars do not create the scalar budget retroactively. They act on a runtime whose budget must already exist. The first task is therefore to define the unique energy–entropy geometry–gauge ledger induced by the projected bulk constitution. Only after this scalar ledger is in hand does it make sense to ask how event currents lock (Qi-QUA), how trajectories are selected (Qi-EP), how multiscale signatures arise (Qi-CON), and how memory-bearing architectures are realized (Qi-HAL).

The guiding physical intuition is simple. Every open system in nature exchanges energy, rearranges material organization, pays entropic cost for incomplete access, and experiences gauge-structured boundary constraints. A universal runtime cannot let these be separate accounting languages patched together ad hoc from one discipline to the next. It needs one common scalar ledger. In the present notation that ledger is

$$\mathcal{F}_{\mathrm{EEGG}} = \mathcal{F}_{\mathrm{EGG}} + \mathcal{F}_{\mathrm{ECP}},$$

where the first term captures projected energy–geometry–gauge structure and the second captures entropic curvature or measure distortion due to coarse-grained access.

5.5.2 The EEGG ledger: the unique scalar budget induced by geometry, gauge, and entropy

A runtime without a scalar ledger is kinematical but not dynamically closed. The master pipeline of Section 5.4 already made this unavoidable: the gradient sector was written as descent on

an effective functional, the port sector as compensating injection, and the attractor sector as a statement about level-set geometry. Before the Qi program can lock event currents or diagnose multiscale signatures, one must first specify which scalar quantity the lawful drift descends and the maintenance sector must continually replenish.

In the present framework that scalar is the *Energy–Entropy Geometry–Gauge* ledger, abbreviated EEGG. Its uniqueness is not a thermodynamic convenience; it is a consequence of MSMF. If multiple unrelated free-energy-like objects were admitted for the same packet, then gradient drift, maintenance accounting, and current-locking would become ambiguous. The constitution therefore forces a single ledger up to irrelevant additive constants.

Definition 5.5.1 (The EEGG functional)**.** Given a packet with state manifold $(\mathcal{Z}, g_{AB})$, probability density p, and port configuration $\mathcal{P}$, the EEGG ledger is the functional

$$\mathcal{F}_{\mathrm{EEGG}}[p;\mathcal{P}] = \underbrace{\int_{\mathcal{Z}} p(x)\,\Phi_{\mathrm{EGG}}(x;\mathcal{P})\,\mathrm{d}\mu_g(x)}_{\mathcal{F}_{\mathrm{EGG}}} + \underbrace{\Theta_{\mathrm{eff}}\int_{\mathcal{Z}} p(x)\ln\frac{p(x)}{p_{\mathrm{ref}}(x;\mathcal{P})}\,\mathrm{d}\mu_g(x)}_{\mathcal{F}_{\mathrm{ECP}}}, \tag{5.73}$$

where $\mathrm{d}\mu_g = \sqrt{\det g}\,\mathrm{d}^n x$, Φ_{EGG} is the energy–geometry–gauge potential induced by the projected bulk structure, p_{ref} is the reference density associated with the induced geometric measure or prior boundary condition, and Θ_{eff} is the effective information-temperature scale.

Equation (5.73) writes the ledger in the most useful form for later derivation. The first term is the EGG sector. It collects projected curvature costs, gauge potentials, holonomy contributions, binding energies, field couplings, and other geometry- or gauge-induced energetic terms. The second term is the ECP sector. It is an entropic curvature term, written as a relative-entropy functional with respect to the reference density. If one wishes to recover the standard thermodynamic sign convention, the second term is simply $-\Theta_{\mathrm{eff}}S$ plus an additive constant determined by p_{ref}. The present relative-entropy form is adopted because it makes positivity and variational differentiation completely transparent.

The functional derivative of (5.73) is immediate:

$$\frac{\delta\mathcal{F}_{\mathrm{EEGG}}}{\delta p} = \Phi_{\mathrm{EGG}}(x;\mathcal{P}) + \Theta_{\mathrm{eff}}\left[\ln\frac{p(x)}{p_{\mathrm{ref}}(x;\mathcal{P})} + 1\right]. \tag{5.74}$$

The gradient part of the GDOS current is then

$$J^A_{\mathrm{grad}} = -\,p\,M^{AB}\nabla_B\left(\frac{\delta\mathcal{F}_{\mathrm{EEGG}}}{\delta p}\right), \tag{5.75}$$

with $M^{AB} \succeq 0$ the mobility tensor. Equation (5.75) is the precise sense in which EEGG is the drift ledger of the theory.

Why the first term is called EGG**.** The EGG sector may be written schematically as

$$\Phi_{\mathrm{EGG}}(x;\mathcal{P}) = U_{\mathrm{geom}}(x) + \sum_a q_a\,\varphi_a(x;\mathcal{P}) + \sum_\ell h_\ell(x;\mathcal{P}), \tag{5.76}$$

where U_{geom} collects curvature- or shape-induced costs, the $q_a\varphi_a$ terms collect gauge couplings, and the h_ℓ terms collect constraint penalties inherited from ports or holonomies. The exact decomposition depends on the concrete packet, but the structural point does not: the energetic sector is not a phenomenological guess; it is the boundary projection of geometry and gauge data.

Why the second term is called ECP. The entropy term is likewise more than phenomenological bookkeeping. The boundary observer never has access to the full bulk microstate. Therefore any projected description carries a coarse-graining volume. The logarithm in Eq. (5.73) measures precisely that volume relative to the reference density. In this sense entropy is treated as curvature or measure distortion in the effective state space, which is why the term is named the *Entropic Curvature Principle*. It is the open-system price of incomplete access.

Table 5.19: Conventional thermodynamic language and its EEGG reinterpretation in the revised manuscript order.

Conventional notion	EEGG **reinterpretation**	**Connected layer in the full manuscript**
Internal energy	Projection of geometry/gauge potential into Φ_{EGG}	Qi-QUA, Qi-EP
Entropy	Relative state-space volume measured against p_{ref}	Qi-CON
Free energy	The unique scalar ledger $\mathcal{F}_{\mathrm{EEGG}} = \mathcal{F}_{\mathrm{EGG}} + \mathcal{F}_{\mathrm{ECP}}$	DGI+GSSB+GA
Dissipation	Positive semidefinite quadratic form generated by M^{AB} and $\delta\mathcal{F}/\delta p$	Qi-EP
Steady-state maintenance cost	Port injection needed to offset EEGG descent and repair expense	Qi-HAL, SeOS

5.5.3 Monotonicity and the port-work identity

The utility of EEGG lies in the fact that it is not merely interpretive. It obeys exact balance identities. Let the full current be written as

$$J^A = J^A_{\mathrm{grad}} + J^A_{\mathrm{curl}} + J^A_{\mathrm{noise}} + J^A_{\mathrm{gauge}}, \tag{5.77}$$

with

$$J^A_{\mathrm{grad}} = -pM^{AB}\nabla_B\left(\frac{\delta\mathcal{F}_{\mathrm{EEGG}}}{\delta p}\right), \qquad J^A_{\mathrm{curl}} = \Omega^A p, \qquad J^A_{\mathrm{gauge}} = R^A{}_\alpha \lambda^\alpha p,$$

and with J^A_{noise} included in the closure data. Substituting the continuity equation $\partial_t p + \nabla_A J^A = 0$ into the time derivative of (5.73), integrating by parts, and collecting the explicit port dependence gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F}_{\mathrm{EEGG}} = -\mathcal{D}_{\mathrm{EEGG}} + \mathcal{P}_{\mathrm{port}} + \mathcal{C}_{\mathrm{curl}} + \mathcal{G}_{\mathrm{gauge}}, \tag{5.78}$$

where

$$\mathcal{D}_{\mathrm{EEGG}} := \int_{\mathcal{Z}} p\, \nabla_A \left(\frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p} \right) M^{AB} \nabla_B \left(\frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p} \right) \mathrm{d}\mu_g \geq 0, \tag{5.79}$$

$$\mathcal{P}_{\mathrm{port}} := \int_{\mathcal{Z}} p\, \partial_t \Phi_{\mathrm{EGG}}(x; \mathcal{P}(t))\, \mathrm{d}\mu_g - \oint_{\partial \mathcal{Z}} \frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p}\, J^A n_A\, \mathrm{d}\Sigma, \tag{5.80}$$

$$\mathcal{C}_{\mathrm{curl}} := \int_{\mathcal{Z}} \nabla_A \left(\frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p} \right) \Omega^A p\, \mathrm{d}\mu_g, \tag{5.81}$$

$$\mathcal{G}_{\mathrm{gauge}} := \int_{\mathcal{Z}} \nabla_A \left(\frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p} \right) R^A{}_\alpha \lambda^\alpha p\, \mathrm{d}\mu_g. \tag{5.82}$$

The first term is nonnegative by construction. The second is the net power or work injected through explicit driving and boundary flux. The last two vanish whenever the curl current lies on EEGG level sets and the ledger is gauge-invariant.

This motivates the central theorem of Section 5.5: before SIP and the Qi pillars can be introduced, the runtime must close around one scalar budget.

Theorem 5.5.2 (EEGG monotonicity in the lawful sector)**.** *Suppose $\mathcal{F}_{\mathrm{EEGG}}$ is gauge-invariant and the curl field is tangent to its level sets, i.e.*

$$\Omega^A \nabla_A \left(\frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p} \right) = 0, \qquad R^A{}_\alpha \nabla_A \left(\frac{\delta \mathcal{F}_{\mathrm{EEGG}}}{\delta p} \right) = 0.$$

Then the EEGG *balance law reduces to*

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathcal{F}_{\mathrm{EEGG}} = -\mathcal{D}_{\mathrm{EEGG}} + \mathcal{P}_{\mathrm{port}}. \tag{5.83}$$

In particular, in the undriven closed limit $\mathcal{P}_{\mathrm{port}} = 0$, the ledger is nonincreasing:

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathcal{F}_{\mathrm{EEGG}} \leq 0. \tag{5.84}$$

At a nonequilibrium steady state one has the exact cycle-averaged identity

$$\langle \mathcal{P}_{\mathrm{port}} \rangle_\tau = \langle \mathcal{D}_{\mathrm{EEGG}} \rangle_\tau. \tag{5.85}$$

Proof. Equations (5.78)–(5.82) already isolate the only possible contributions. The stated orthogonality assumptions set $\mathcal{C}_{\mathrm{curl}} = \mathcal{G}_{\mathrm{gauge}} = 0$. Positive semidefiniteness of M^{AB} gives $\mathcal{D}_{\mathrm{EEGG}} \geq 0$. Therefore Eq. (5.83) follows. The closed limit is immediate. For a nonequilibrium steady state, the time average of $\mathrm{d}\mathcal{F}_{\mathrm{EEGG}}/\mathrm{d}t$ vanishes over a sufficiently long cycle or observation window, which yields Eq. (5.85). □

The theorem is extremely useful because it disentangles three often-confused ideas. The system need not minimize EEGG instantaneously. A driven open system may sustain nonzero circulation and nonzero throughput. Nevertheless, the gradient part of its motion is always organized by one ledger, and steady maintenance requires exactly enough port power to balance the irreversible descent and repair costs. This is the precise sense in which EEGG generalizes both free-energy descent and nonequilibrium maintenance.

5.5.4 Preview: event-level reduction once the Arisaka Equation is active

Although the Arisaka Equation is derived only later in Section 5.7, it is useful to record already how the scalar ledger will simplify once the current-locking result is activated. Let a single coarse-grained event carry the current increment

$$\delta\mathcal{J} = (\delta J_e, \delta J_B, \delta J_{Qi}, \delta J_E). \tag{5.86}$$

Under the Arisaka Equation,

$$\delta J_e = \delta J_B = \delta J_{Qi} =: \delta j, \qquad \delta J_E = Q_B\,\delta j. \tag{5.87}$$

Any linear event contribution to the energetic sector can therefore be written as

$$\delta\mathcal{F}^{\mathrm{lin}}_{\mathrm{EGG}} = \alpha_e\,\delta J_e + \alpha_B\,\delta J_B + \alpha_{Qi}\,\delta J_{Qi} + \alpha_E\,\delta J_E = \big(\alpha_e + \alpha_B + \alpha_{Qi} + \alpha_E Q_B\big)\delta j. \tag{5.88}$$

Likewise any quadratic event penalty of the form $Q_{\alpha\beta}\,\delta J^\alpha \delta J^\beta$ reduces to

$$\delta\mathcal{F}^{\mathrm{quad}}_{\mathrm{EGG}} = (\delta j)^2\, v^{\mathsf{T}} Q v, \qquad v = (1,1,1,Q_B)^{\mathsf{T}}. \tag{5.89}$$

Therefore the four-current event budget collapses to a one-parameter locked ray in current space.

This is one of the deepest simplifications of the program. Instead of treating energy, matter, charge, and information as separate bookkeeping systems patched together later, the event budget is already one-dimensional once the anomaly-derived current lock is imposed. The scalar EEGG ledger is therefore not in tension with the vector current ledger; it is the natural scalarization of the same constrained event geometry.

Table 5.20: How the Arisaka Equation simplifies the event-level contribution to EEGG.

Without current lock	**With Arisaka lock**	**Interpretation**
Four independent event components $(\delta J_e, \delta J_B, \delta J_{Qi}, \delta J_E)$	One locked event amplitude δj and one conversion constant Q_B	The current vector collapses to a one-ray cone in event space.
Many possible scalarizations	Unique scalarization via $v = (1,1,1,Q_B)$	EEGG becomes a canonical ledger rather than a user-chosen potential.
Ad hoc biochemical bookkeeping	Universal energy–matter–information denominator	Chemistry and biology inherit their event budget from the same UV geometry.

5.5.5 Section summary

The role of EEGG in the restructured manuscript is now clear.

(1) The master pipeline requires a unique scalar ledger; otherwise gradient descent, port injection, and attractor stability would not close into one runtime.

(2) EEGG is that ledger. In the present notation it is written as $\mathcal{F}_{\mathrm{EEGG}} = \mathcal{F}_{\mathrm{EGG}} + \mathcal{F}_{\mathrm{ECP}}$, with energetic geometry–gauge structure and entropic curvature carried together rather than separately.

(3) The monotonicity and port-work identities show that EEGG is not interpretive decoration but the quantitative budget governing lawful relaxation and maintenance.

(4) Once Qi-QUA is derived, the event-level bookkeeping of EEGG collapses to a locked one-ray reduction, so the scalar and vector ledgers become manifestly compatible.

(5) This is why EEGG is correctly placed in Part II: it defines the budget that Section 5.6 must attach to a concrete packet before the Qi program begins.

The next section therefore does not add an optional modeling convenience. It closes runtime constitution by declaring the concrete system instance that carries the EEGG ledger.

5.6 SIP: System Instance Packet as the Canonical Unit of GDOS

Once the scalar ledger is fixed, the theory still lacks a declared carrier. A budget without an explicit system instance is too abstract for chemistry, biology, neuroscience, or society. Section 5.6 therefore closes Part II by introducing the canonical packet on which the later Qi pillars will act. In the new order, SIP is the last step of runtime constitution precisely because it binds the already-defined EEGG ledger to a concrete open-system instance before Part III begins.

5.6.1 Why the packet must follow EEGGand precede the Qi block

A universal runtime still needs one more primitive object before the Qi-4 program begins: the concrete system instance. The scalar ledger of Section 5.5 tells us what quantity is minimized, maintained, or replenished. But a scalar budget without an explicit carrier is still too abstract for application. One must say what the system is, where its ports are, what its lawful generator is, which observables are exported to experiment, and what failure would mean.

This is the role of the System Instance Packet. The packet is not a loose checklist for model builders. It is the canonical unit by which a chemical reactor, a molecular machine, a cell, a cortical circuit, or a distributed institution becomes an admissible object of GDOS. In the revised architecture this must come before Qi-QUA, because current locking, selector structure, scale diagnostics, and memory architecture are all properties of a system instance that has already been declared.

Intuitively, SIP is the answer to the simplest but most easily neglected question in all open-system science: *what exactly is the thing whose openness, maintenance, memory, and measurements we are talking about?* A landmark paper cannot leave that implicit.

5.6.2 The System Instance Packet: why an open-system theory needs a canonical unit of analysis

The word "system" is used carelessly in much of science. In one paper it means a reactor vessel; in another, a cell; in another, a patch of cortex; in another, an entire economy. Such flexibility may be tolerable in phenomenology, but it is not tolerable in a universal runtime. If GDOS is to apply across branches of science, it must specify how a concrete system instance is declared before any generator is evolved or any current budget is computed.

The need for this declaration follows immediately from the earlier sections and from the revised placement of EEGG. The universal openness theorem of Section 5.3 already showed that every operational description is a projected description. Therefore one must declare the projected state manifold. Section 5.4 showed that every lawful open-system evolution must be written in terms of currents and invariant sets. Therefore one must declare the ports and the admissible generator. Section 5.5 has now fixed the unique scalar ledger that drives descent and maintenance. Therefore one must declare how that ledger is attached to a concrete system instance. Finally, AX-4 and POS-MF require operational observables and falsifiers. Therefore one must declare what is measured and what would count as failure. The later Qi sections will then act on this declared packet rather than defining the packet after the fact.

This logic motivates the following canonical definition.

Definition 5.6.1 (Canonical five-field SIP)**.** For a concrete open system instance in the GUT/GG-6/GDOS program, the *System Instance Packet* is the ordered five-field object

$$\mathrm{SIP} = \Big[(\mathcal{Z}, g_{AB}),\ \mathcal{P},\ (\mathcal{L}_{\mathrm{GDOS}}, \eta),\ (\mathcal{F}_{\mathrm{EEGG}}, \mathcal{J}),\ (\mathcal{O}, \mathfrak{K})\Big], \tag{5.90}$$

where:

(i) $(\mathcal{Z}, g_{AB})$ is the effective state manifold and its metric;

(ii) $\mathcal{P}$ is the full set of ports and imposed boundary couplings;

(iii) $(\mathcal{L}_{\mathrm{GDOS}}, \eta)$ is the lawful open-system generator together with its noise closure;

(iv) $(\mathcal{F}_{\mathrm{EEGG}}, \mathcal{J})$ is the ledger bundle, consisting of the scalar EEGG functional and the locked current vector $\mathcal{J} = (J_e, J_B, J_{Qi}, J_E)$;

(v) $(\mathcal{O}, \mathfrak{K})$ is the observable dictionary together with the explicit kill criteria required for falsifiability.

Definition 5.6.1 folds a large amount of structure into a compact object, so each field deserves a brief explanation.

Field 1: state manifold and metric. The state manifold $\mathcal{Z}$ is the space of accessible coarse-grained configurations. Its coordinates may be concentrations, conformational angles, firing-rate bumps, latent belief variables, population fractions, or any other reduced coordinates that are operationally accessible and causally sufficient at the chosen level. The metric g_{AB} is not cosmetic. It defines local volume, gradient norm, and the cost of moving in different directions, so it is the geometric skeleton on which the later EEGG ledger and Qi-EP flows are written.

Field 2: ports. The port set $\mathcal{P}$ includes every boundary channel across which the system exchanges quantities relevant to persistence: matter, energy, charge, entropy, stress, sensory signal, action channel, or information-bearing input/output. In a chemical reactor the ports are inlets, outlets, heat baths, catalysts, and control variables. In a cell they include membrane transport, nutrients, wastes, receptor couplings, and mechanical contact. In a brain they include sensory inflow, motor outflow, neuromodulation, vascular supply, and social/environmental coupling. Ports are the formal declaration of openness.

Field 3: generator and noise closure. The lawful open-system generator $\mathcal{L}_{\mathrm{GDOS}}$ is the reduced evolution law on $\mathcal{Z}$, while η is the closure data needed to guarantee positivity, stochastic completeness, and the correct fluctuation level. We group generator and closure together because a dynamical law without its admissibility conditions is not yet a lawful dynamical law. In practice, η may include diffusion tensors, jump kernels, effective temperatures, spectral closures, and boundary conditions.

Field 4: ledger bundle. The pair $(\mathcal{F}_{\mathrm{EEGG}}, \mathcal{J})$ carries the scalar and vector budget information. The scalar part $\mathcal{F}_{\mathrm{EEGG}}$ is the energy–entropy geometry–gauge functional that appears in the gradient sector and in maintenance balance. The vector part $\mathcal{J}$ is the locked current bundle constrained by the Arisaka Equation. The two must be grouped because the scalar ledger would be incomplete without the event-level current accounting, while the current accounting would be kinematical without a scalar quantity whose gradients select directions of lawful descent.

Field 5: observable dictionary and kill criteria. The pair $(\mathcal{O}, \mathfrak{K})$ implements the operational contract. $\mathcal{O}$ is the registered family of observables to be compared with data, and $\mathfrak{K}$ specifies the threshold at which the chosen packet is counted as failed. We group them because an observable without a falsifier is not yet a scientific observable in the sense of the GUT-10 constitution.

Table 5.21: The canonical five-field SIP and the theoretical structures from which each field descends.

Field	**Symbolic content**	**Source within the full manuscript**	**Why it is indispensable**
State manifold	$(\mathcal{Z}, g_{AB})$	OGGEP projection and the universal openness theorem	Without it there is no domain on which probability, currents, or observables live.
Ports	$\mathcal{P}$	Section 5.3, Eq. (5.39)	Without ports one has no formal declaration of openness or boundary exchange.
Generator + closure	$(\mathcal{L}_{\mathrm{GDOS}}, \eta)$	Sections 5.3 and 5.4	Without a lawful generator and closure there is no predictive time evolution.
Ledger bundle	$(\mathcal{F}_{\mathrm{EEGG}}, \mathcal{J})$	Sections 5.7–5.10	Without the ledger bundle one cannot write drift, maintenance cost, or later event-level current bookkeeping.
Observables + falsifiers	$(\mathcal{O}, \mathfrak{K})$	AX-3, POS-MF, and Chapter 4	Without them the packet is not empirically testable.

A canonical packet should not only be necessary but also sufficient. The next theorem states precisely why SIP closes Part II: once EEGG is fixed, the packet becomes enough to generate the full operational semantics of a declared system instance.

Theorem 5.6.2 (SIP completeness)**.** *Fix an initial boundary distribution p_0 on $\mathcal{Z}$. Given a canonical packet of the form (5.90), the full family of operational predictions for every registered observable $O \in \mathcal{O}$ is determined by*

$$p_t = \mathcal{U}^{\mathrm{GDOS}}_{t,0}[p_0], \qquad \mathcal{U}^{\mathrm{GDOS}}_{t,0} = \mathcal{T} \exp\left(\int_0^t \mathcal{L}_{\mathrm{GDOS}}(\tau; \mathcal{P}(\tau), \eta(\tau))\, \mathrm{d}\tau \right), \tag{5.91}$$

and

$$\langle O \rangle_t = \int_{\mathcal{Z}} O(x)\, p_t(x)\, \mathrm{d}\mu_g(x), \qquad \mathrm{d}\mu_g = \sqrt{\det g}\, \mathrm{d}^n x. \tag{5.92}$$

Conversely, two packets are operationally equivalent iff they are related by a diffeomorphic and gauge-covariant pushforward that preserves the observable dictionary and the kill criteria.

Proof. The first statement is immediate once the five fields are examined in order. Field 1 fixes the domain and the integration measure. Field 2 fixes the boundary couplings and the explicit driving. Field 3 fixes the time evolution operator. Field 4 fixes the scalar and vector budgets that enter the gradient, port, and event accounting. Field 5 specifies which functionals of p_t are exported to experiment and how they are judged. Therefore the family of predictions (5.92) is determined.

For the converse, let $\varphi : \mathcal{Z} \to \widetilde{\mathcal{Z}}$ be a smooth invertible map, let $\widetilde{p} = \varphi_* p$, and let the generator and observables transform as

$$\widetilde{\mathcal{L}} = \varphi_* \mathcal{L}\, \varphi^*, \qquad \widetilde{O} = O \circ \varphi^{-1}.$$

If in addition the gauge fields are transformed so that the covariant derivatives and current ledger are preserved, then all observable expectation values are unchanged. Conversely, if two packets are not related by such a transformation, then either the evolved distributions or the exported observables differ, and the packets are not operationally equivalent. □

The theorem makes a useful philosophical point. SIP is not a loose checklist. It is the minimal complete model instance. The packet plays, for open-system science, the role that a metric-plus-matter specification plays in general relativity or that a gauge group plus matter representation plays in Yang–Mills theory.

5.6.3 Folded and unfolded forms of the packet

Earlier exploratory drafts often listed more than five items because the same packet was written in an unfolded manner. For example, one may list coordinates, metric, ports, drift, diffusion, gauge directions, scalar ledger, current vector, observables, and falsifiers separately. That longer form is often useful when working through a concrete example. This book, however, adopts the folded five-field form because it cleanly separates ontology (state manifold), openness (ports), law (generator + closure), budget (ledger bundle), and experiment (observables + kill criteria).

It is useful to write the unfolded form explicitly once:

$$\widehat{\mathrm{SIP}} = \big(\mathcal{Z}, g_{AB};\, \mathcal{P};\, M^{AB}, \Omega^A, D^{AB}, R^A{}_\alpha \lambda^\alpha;\, \mathcal{F}_{\mathrm{EEGG}}, \mathcal{J};\, \mathcal{O}, \mathfrak{K}\big), \tag{5.93}$$

where the drift data are encoded through the mobility M^{AB}, curl field Ω^A, diffusion tensor D^{AB}, and gauge generators $R^A{}_\alpha \lambda^\alpha$. The canonical packet (5.90) is simply the grouping of these explicit pieces into the five conceptual bundles required by a reusable runtime.

Table 5.22: How the more granular runtime specification folds into the canonical five-field packet.

Unfolded runtime item	**Folded SIP field**	**Reason for grouping**
Coordinates, manifold, and metric	State manifold	These are geometric aspects of the same effective state space.
Boundary fluxes, reservoirs, imposed couplings	Ports	These jointly declare how the environment acts on the packet.
Drift, curl, diffusion, jump law, closure data	Generator + closure	Lawfulness requires both the evolution operator and the positivity/stability conditions that complete it.
Scalar EEGG functional and current vector	Ledger bundle	Gradient descent and event accounting are two views of one common budget.
Registered observables and kill thresholds	Observable dictionary + kill criteria	Measurement and falsification form one operational contract.

5.6.4 Representative packets across branches of science

To see why SIP is useful, it helps to write several representative packets side by side. The point is not that all systems become identical. The point is that they become comparable. A chemistry problem, a cell-biology problem, a neuroscience problem, and a collective-system problem can all be declared in one grammatical format.

Table 5.23: Compact representative SIP cards across branches of science. The aim is to show grammatical universality rather than to exhaust system-specific details.

System	**State manifold**	**Ports**	**Generator + closure**	**Ledger bundle**	**Observables / kill**
Fatty-acid protocell	Lipid composition, vesicle size, membrane curvature, internal metabolites	Fatty-acid inflow, pH, salts, heat, waste leakage	Drift in composition/shape space plus thermal diffusion and fusion/fission jumps	Membrane-assembly cost + entropic mixing + locked event currents	Growth rate, leakage rate, division threshold / falsified by sustained mismatch with vesicle statistics

System	**State manifold**	**Ports**	**Generator + closure**	**Ledger bundle**	**Observables / kill**
ATP synthase module	Rotor angle, catalytic occupancy, protonation state	Proton motive force, ADP, P_i, load torque	Periodic drift with rotational diffusion and rare backstep jumps	Electrochemical potential, conformational free energy, event current budget per catalytic step	Rotation rate, ATP output, torque-efficiency curve / falsified by single-molecule deviation beyond tolerance
Chemotactic microbe	Position, heading, receptor adaptation state, internal metabolic variables	Nutrient gradient, toxin gradient, membrane exchange, mechanical contact	Active drift-diffusion with internal adaptation and sensing noise	Nutrient uptake, maintenance cost, information gain, memory-update budget	Run-length distributions, turning bias, growth yield / falsified by sustained mismatch with chemotaxis data
Ring-attractor circuit	Bump position, phase, synaptic efficacy, neuro-modulatory tone	Sensory inflow, motor outflow, metabolic supply	Recurrent drift-curl dynamics with population noise and switching events	Synaptic maintenance, oscillatory coherence, information-current flow	Bump stability, phase response, decision errors / falsified by mismatch with perturbation experiments
Collective institution	Distributed belief state, archive state, policy mode, resource stock	Information inflow, resource in-flow/outflow, regulation, communica-tion channels	Network transport, delayed feedback, stochastic shocks, regime-switch jumps	Resource budget, archive-maintenance cost, entropy of coordination, informational throughput	Forecast skill, resilience, recovery time / falsified by failed stress-test predictions

Table 5.23 illustrates why the packet notion is so powerful. The systems differ radically in substrate and scale, but the packet grammar remains stable. This is precisely what one wants from a grand-unified runtime: not identical phenomenology, but a common specification contract.

5.6.5 Section summary

The packet layer is now fixed.

(1) Every concrete application of GDOS requires a canonical SIP; without it the theory has no explicit carrier of openness, memory, budget, and observability.

(2) The packet is best written in five fields: state manifold, ports, generator plus closure, ledger bundle, and observables plus kill criteria.

(3) EEGG is now already installed before the packet is declared, so the packet can carry a fully specified scalar budget from the moment the system instance is introduced.

(4) The later Qi pillars will therefore act on a runtime whose object is already fixed: Qi-QUA will lock the event currents of the packet, Qi-EP will constrain its trajectories, Qi-CON will diagnose its multiscale signatures, and Qi-HAL will organize its memory architecture.

(5) This is why SIP is correctly placed at the end of Part II: it closes the runtime constitution before Part III begins.

Part III can therefore begin on firm ground: the runtime grammar, scalar budget, and concrete packet have all been fixed before the first Qi theorem is stated.

Qi-4 pillars

Locking, variational principle, conformality, and attractor architecture

5.7 Qi-QUA: Arisaka Ratio, Arisaka Equation, and Arisaka Constant

Part III now begins only after the runtime constitution is complete. The open-system grammar (DGI+GSSB+GA), the scalar ledger (EEGG), and the carrier packet (SIP) are all fixed. The first Qi task is therefore event-level locking: what four-sector current ledger is attached to one admissible packet update, and how is that ledger inherited from BI-6? Section 5.7 answers that question through the Arisaka Ratio, the Arisaka Constant, and the Arisaka Equation.

5.7.1 Why Qi-QUAmust begin Part III after EEGGand SIP

The previous sections established the dynamical grammar of persistent open systems and then supplied the scalar ledger and canonical packet on which the Qi program must act. DGI tells us which descriptions are gauge-equivalent and therefore physically redundant. GSSB tells us how one realized branch or structure is selected from among many admissible possibilities. GA tells us why some selected structures persist as stable invariant sets, cycles, or distributions while others dissolve. But those three ingredients still leave one quantitative question unanswered: *what exactly is being spent, moved, rearranged, and registered in each elementary open-system event?*

That question cannot be postponed if Chapter 5 is to speak equally to chemistry, biology, and cognition. A living system does not merely follow a trajectory on a state manifold. It must also pay energetic cost, move charge, rearrange material structure, and update internal or external information registers. If those ledgers are treated independently, then the framework fragments into discipline-specific bookkeeping schemes. The first pillar of the Qi program is designed to prevent exactly that fragmentation.

Qi-QUA is therefore the universal current-locking layer of GDOS. It is the statement that the same anomaly-free bulk structure that fixed the ultraviolet coefficient vector in Chapter 3 also fixes the allowed direction of elementary boundary bookkeeping events. In this book the words "energy," "electric," "baryonic," and "information" are not introduced as separate metaphors. They are introduced as four sectors of a single open-system ledger that descends from one geometric inflow channel.

This is why Qi-QUA begins Part III only after EEGG and SIP have been installed. Part II established the lawful pipeline for exploration, commitment, and persistence, fixed the scalar budget, and declared the canonical system instance. The present section now answers the next question: once that runtime is in place, what is the universal locked bookkeeping per elementary step? The answer is encoded by three related objects:

(i) the *Arisaka Ratio*, which is the coefficient-space ray inherited from BI-6;

(ii) the *Arisaka Constant* Q_B, which is the fixed energy scale per locked boundary event in chemical natural units;

(iii) the *Arisaka Equation*, which is the natural-unit equality stating that one locked event carries one unit each of electric, baryonic, and informational current at an energy cost Q_B.

The logic of the section is therefore deliberately three-stage, matching the revised architecture of the manuscript. First the BI-6 normalization is fixed; second the Arisaka Ratio is derived in coefficient space; third the chemical-unit map converts the coefficient ray into the Arisaka Equation relevant for chemistry, biology, and cognition. First, we derive the coefficient-space statement directly from the BI-6 source vector. Second, we convert that statement into natural units appropriate for open systems in chemistry, biology, and cognition. Only after both steps have been completed does the familiar equality

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi} \tag{5.94}$$

become meaningful.

5.7.2 The four ledgers of a generic open system

The appropriate starting point is not a specialized chemical or biological reaction, but the generic bookkeeping structure of an open system. Let S be an open system with operational boundary ∂S. For each sector

$$A \in \{E, e, B, Q\},$$

we introduce a density $n_A(\mathbf{x}, t)$, a flux $\mathbf{j}_A(\mathbf{x}, t)$, and a source term $s_A(\mathbf{x}, t)$ obeying the continuity equation

$$\partial_t n_A + \nabla \cdot \mathbf{j}_A = s_A. \tag{5.95}$$

Equation (5.95) is the open-system version of Noether-style current bookkeeping, with explicit source terms retained because the system is not closed (Arisaka, 2026c; Noether, 1918). Integrating over a control volume $V \subset \partial S$ gives

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V n_A \, \mathrm{d}^3x = - \int_{\partial V} \mathbf{j}_A \cdot \mathrm{d}\mathbf{S} + \int_V s_A \, \mathrm{d}^3x. \tag{5.96}$$

This equation is the generic bridge from differential current bookkeeping to experimentally registered budgets.

The four sectors used in Qi-QUA are the following.

Table 5.24: The four ledgers used by Qi-QUA.

Sector	**Symbol**	**Operational meaning**	**Natural boundary unit**	**Interpretation in open systems**
Energy	E	Net energetic expenditure or uptake driving a registered step	eV per event, or kJ/mol per event	Work-like or free-energy-like cost required to maintain or update the open system
Electric	e	Charge-transfer or redox-update count	one elementary electric transfer per event	Carrier transport, redox step, or equivalent charge bookkeeping
Baryonic	B	Material/structural rearrangement count	one minimal structural update per event	Rearrangement of matter-bearing degrees of freedom
Information	$Q \equiv Qi$	Registered informational update count	one bit-equivalent update per event	Storage, copying, comparison, or commit operation in the boundary register

Table 5.24 deliberately mixes language from physics, chemistry, and information theory because Qi-QUA is supposed to operate across all three. The purpose of the table is not to claim that a neuron, a catalytic cycle, and a gauge theory use identical microscopic carriers. The purpose is to identify the four ledgers that must all be audited if one wants a theory of open systems that scales from one domain to another without changing its constitution.

The essential distinction is between *coefficient-space currents* and *natural-unit currents*. Coefficient-space currents are the signed sector weights inherited directly from BI-6. Natural-unit currents are the magnitudes of registered events after each sector has been expressed in its own operational unit. The Arisaka Ratio lives in the first language. The Arisaka Equation lives in the second. Much confusion is avoided if that distinction is maintained from the outset.

Remark 5.7.1. The letter Q in the present section stands for the informational current J_{Qi}, written in shortened form once the context is fixed. The notation follows the convention adopted throughout this book

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi}, \qquad J_{Qi} \equiv J_{Qi},$$

with the order beginning from J_E so that the presentation remains consistent with the BI-6 ordering.

5.7.3 From BI-6 to a one-dimensional boundary source ray

Chapter 3 established that the anomaly-free six-dimensional bulk is organized by a Green–Schwarz-modified Bianchi identity of the schematic form

$$\mathrm{d}H_3 = X_4, \qquad X_4 = a\,X_E + b\,X_e + c\,X_B + d\,X_{Qi}, \tag{5.97}$$

with the coefficient vector fixed by complete anomaly cancellation to

$$(a, b, c, d) = \left(1, -1, -\frac{2}{5}, -\frac{2}{5}\right) \tag{5.98}$$

in the GG-6 construction (Arisaka, 2026b). The essential point for this book is that a *single* three-form field strength H_3 feeds all four boundary sectors. This immediately reduces the allowed elementary source vectors on the boundary to a rank-one family.

Remark 5.7.2 (Normalization convention: reduced-rational BI-6 form). In earlier notes one may also encounter a reciprocal Abelian normalization in which the same locking pattern is written schematically as

$$(1, -1, -5/2, -5/2).$$

The present volume adopts the reduced-rational convention

$$(1, -1, -2/5, -2/5)$$

because it is exactly the form whose positive integer clearing yields

$$5 : -5 : -2 : -2,$$

which is the Arisaka Ratio used throughout Chapter 5, and because it is the same convention that leads directly to the chemical-unit definition

$$Q_B = \frac{2}{5}\frac{E_{\mathrm{Ryd}}}{K}.$$

Thus the choice of $-2/5$ is not an independent physical claim beyond BI-6; it is the normalization convention that keeps the ultraviolet coefficient vector, the integer-cleared Arisaka Ratio, and the chemical-unit map mutually consistent. What must be avoided is mixing the reduced-rational BI-6 convention with the reciprocal integer form $2 : -2 : -5 : -5$, or mixing the reciprocal BI-6 convention with the reduced-rational Arisaka Constant.

To see this concretely, integrate Eq. (5.97) over a compact four-volume W_4 whose boundary projects to a finite operational update on ∂S:

$$\int_{W_4} \mathrm{d}H_3 = \int_{W_4} X_4 = a\int_{W_4} X_E + b\int_{W_4} X_e + c\int_{W_4} X_B + d\int_{W_4} X_{Qi}. \tag{5.99}$$

Because the left-hand side depends on one integrated inflow amplitude only, the four right-hand sector contributions cannot vary independently for an elementary synchronized event. There exists a scalar amplitude $\lambda(W_4)$ such that the induced signed boundary source vector satisfies

$$\mathbf{s}' = \lambda\left(1,\ -1,\ -\frac{2}{5},\ -\frac{2}{5}\right). \tag{5.100}$$

The prime reminds us that Eq. (5.100) is a *coefficient-space* relation: it records the signed sector weights inherited directly from BI-6, before any sector-specific unit conversion has been performed.

Because EEGG and SIP are already fixed by Part II, the first Qi theorem can now focus purely on event-level locking rather than on scalar-budget or packet-definition ambiguities.

Theorem 5.7.3 (Rank-one inflow theorem for elementary Qi-QUA events)**.** *Assume that an elementary open-system update on the boundary is sourced by a single BI-6 inflow channel H_3. Then the admissible signed sector-current vector for that elementary update lies on the one-dimensional ray*

$$\mathcal{R}_{\mathrm{A}} = \left\{ \lambda \left(1,\, -1,\, -\frac{2}{5},\, -\frac{2}{5} \right) \,:\, \lambda \in \mathbb{R} \right\}. \tag{5.101}$$

Proof. Under the stated assumption the four boundary source components are obtained by projecting one and the same bulk object H_3 through the fixed BI-6 coefficient vector of Eq. (5.98). Hence all four components are proportional to a single scalar amplitude λ, and the set of admissible source vectors is one-dimensional. No second independent direction exists unless a second primitive inflow channel is added, which is excluded by hypothesis. □

Theorem 5.7.3 is the whole geometric content of the first Qi pillar in its most compressed form. It says that the elementary bookkeeping event of the boundary does not span a four-dimensional current space. It spans a one-dimensional ray inside that space. All later equalities of Qi-QUA are just different ways of reading that ray.

5.7.4 The Arisaka Ratio and its equivalent representations

We may now state the first of the three central objects.

Definition 5.7.4 (Arisaka Ratio)**.** The *Arisaka Ratio* is the coefficient-space ray associated with the BI-6 source vector, written in the ordered sector basis (E, e, B, Q):

$$\mathfrak{R}_{\mathrm{A}} : \qquad J'_E : J'_e : J'_B : J'_{Qi} = 1 : -1 : -\frac{2}{5} : -\frac{2}{5}. \tag{5.102}$$

Equivalently, after clearing denominators by a positive rescaling factor 5,

$$J'_E : J'_e : J'_B : J'_{Qi} = 5 : -5 : -2 : -2. \tag{5.103}$$

The two forms (5.102) and (5.103) are mathematically equivalent because they represent the same ray under a positive scalar rescaling. The first form makes direct contact with the BI-6 coefficient vector. The second form is often easier to read because all entries are integers.

This is also the right place to state the sign convention unambiguously. In this book the sign of a coefficient-space current records *orientation*, not whether the corresponding physical quantity is "good" or "bad." We choose the energy inflow direction as positive. The electric, baryonic, and informational components then carry negative sign in Eq. (5.102) because they are recorded as sink or deposition channels relative to that orientation. When the same bookkeeping is rewritten in natural units, one normally works with magnitudes rather than signed orientations. The Arisaka Equation will therefore be written without minus signs.

Table 5.25: Equivalent ways of reading the Arisaka Ratio.

Representation	**Formula**	**Use**
BI-6 coefficient form	$1 : -1 : -2/5 : -2/5$	Closest to the anomaly-cancellation data of GG-6; best for ultraviolet bookkeeping.
Integer-cleared form	$5 : -5 : -2 : -2$	Same ray, but easier to scan and compare across tables.
Magnitude form	$1 : 1 : 2/5 : 2/5$	Obtained by taking absolute values when only event magnitudes are relevant.
Natural-unit form	$J_E/Q_B = J_e = J_B = J_{Qi}$	Best for chemistry, biology, and cognition, where each sector is counted in its own operational unit.

It is important not to overread Definition 5.7.4. The Arisaka Ratio does *not* claim that every coarse-grained macroscopic current measured in a laboratory or organism must appear in the raw ratio $1 : 1 : 2/5 : 2/5$. What it claims is more precise: the *elementary anomaly-mediated source direction* inherited from the bulk lies on that ray. Coarse-grained systems may contain storage, overhead, idle cycles, and side reactions. Those effects are built from sums of elementary events and therefore belong to later bookkeeping layers. Qi-QUA first identifies the primitive event.

Proposition 5.7.5 (No independent elementary informational direction)**.** *Within the one-channel hypothesis of Theorem 5.7.3, there is no elementary boundary event carrying* $J'_{Qi} \neq 0$ *while* $J'_E = J'_e = J'_B = 0$.

Proof. Any elementary source vector lies on the ray $\mathcal{R}_{\mathrm{A}}$ of Eq. (5.101). On that ray the informational component is proportional to the same scalar amplitude λ that controls all other components. Hence $J'_{Qi} \neq 0$ implies $\lambda \neq 0$, which in turn implies $J'_E \neq 0$, $J'_e \neq 0$, and $J'_B \neq 0$. □

Proposition 5.7.5 is the precise framework-level sense in which the present theory rejects "disembodied information" at the elementary ledger level. The statement should not be confused with the weaker claim that every phenomenological reduced model must retain explicit electric and baryonic variables. Reduced models may integrate out those ledgers. The proposition concerns the underlying synchronized event, not every possible reduced description.

5.7.5 From GG-6 scales to the Arisaka Constant

The second central object of the section is the fixed energy scale assigned to one locked boundary event in chemical natural units. The derivation uses only quantities already imported from Papers II and III.

The first ingredient is the OGN-5 integer $K = 35$, which in Chapter 3 measured the discrete depth of the warped six-dimensional throat. The second ingredient is the Rydberg energy

$$E_{\mathrm{Ryd}} = \frac{1}{2}\alpha^2 m_e c^2, \tag{5.104}$$

the characteristic atomic energy scale descending from the previously derived low-energy electromagnetic sector (Arisaka, 2026b; Bohr, 1913). The quantity E_{Ryd} is not introduced here as an arbitrary empirical fit parameter; within the GG sequence it is part of the already imported low-energy output. Numerically $E_{\mathrm{Ryd}} \approx 13.6057\,\mathrm{eV}$.

The next move is to define the minimal geometric step along the chemistry-relevant boundary ledger. Because the OGN integer K fixes the discrete depth of the throat, the natural per-step energy scale inherited by the atomic boundary is

$$\delta E_{\mathrm{geom}} \equiv \frac{E_{\mathrm{Ryd}}}{K}. \tag{5.105}$$

This quantity is not yet the energy per locked chemical event. It is the per-step geometric energy quantum before the BI-6 coefficient ratio between the energy sector and the baryonic/informational sectors has been applied.

That ratio is precisely the third ingredient. From Eq. (5.98) one reads

$$\frac{|c|}{|a|} = \frac{2}{5}. \tag{5.106}$$

In other words, the magnitude of one baryonic or informational elementary update is 2/5 of the coefficient-space energy step when the latter is written in the common BI-6 normalization. Therefore the energy cost of one *boundary event counted in the natural baryonic/informational unit* is

$$Q_B = \frac{|c|}{|a|}\,\delta E_{\mathrm{geom}} = \frac{2}{5}\frac{E_{\mathrm{Ryd}}}{K}. \tag{5.107}$$

Definition 5.7.6 (Arisaka Constant)**.** The *Arisaka Constant* is the fixed energy scale

$$Q_B: \qquad Q_B = \frac{2}{5}\frac{E_{\mathrm{Ryd}}}{K}, \qquad K = 35. \tag{5.108}$$

It is the energy cost assigned to one ideal locked boundary event when the electric, baryonic, and informational sectors are each counted in one natural unit.

Using $E_{\mathrm{Ryd}} \approx 13.6057\,\mathrm{eV}$ and $K = 35$, one obtains

$$Q_B = \frac{2}{5}\frac{13.6057\,\mathrm{eV}}{35} \approx 0.1555\,\mathrm{eV}. \tag{5.109}$$

The same value may be written in two experimentally familiar ways:

$$Q_B \approx 15.0\,\mathrm{kJ\,mol^{-1}} \approx 5.82\,k_B T \quad \text{at } T = 310\,\mathrm{K}. \tag{5.110}$$

Table 5.26: Numerical forms of the Arisaka Constant.

Form	Value	Interpretation
Definition from GG-6	$Q_B = \frac{2}{5}\frac{E_{\mathrm{Ryd}}}{K}$	No new continuous parameter beyond the imported GG-6 data
Atomic-energy form	$Q_B \approx 0.1555\,\mathrm{eV}$	Energy scale of one locked boundary event
Chemical form	$Q_B \approx 15.0\,\mathrm{kJ\,mol^{-1}}$	Convenient for chemistry, catalysis, and biophysics
Thermal form (310 K)	$Q_B \approx 5.82\,k_B T$	Useful for room-temperature or physiological open systems

The significance of Definition 5.7.6 is structural. The paper is not merely introducing a convenient conversion factor. It is asserting that the energy cost per ideal locked event is not free to float continuously from system to system. What may vary from system to system is *how many* locked events are required, how much additional overhead is paid, and which geometric attractor is realized. The per-event baseline Q_B is fixed by the imported GG-6 data.

Theorem 5.7.7 (No new continuous parameter in Qi-QUA)**.** *Within the GG-6/GDOS import structure of Chapter 5, the constant Q_B introduces no new continuous free parameter.*

Proof. Equation (5.108) expresses Q_B in terms of the already imported quantities E_{Ryd} and K. The Rydberg scale is part of the low-energy electromagnetic output inherited from the previous papers, while $K = 35$ is one of the fixed OGN-5 integers. Therefore Q_B is derived data, not an adjustable input. □

5.7.6 Natural-unit reduction and the Arisaka Equation

We now convert the coefficient-space statement into the natural-unit statement appropriate for open systems in chemistry, biology, and cognition.

The single most important conceptual point is that the Arisaka Equation is *not* obtained from the Arisaka Ratio by merely dropping minus signs. One must first perform a sector-dependent natural-unit conversion. Stated in the exact order used in this book, the derivation is

$$\text{BI-6 normalization:} \qquad (a,b,c,d) = \left(1, -1, -\frac{2}{5}, -\frac{2}{5}\right), \tag{5.111}$$

$$\text{Arisaka Ratio:} \qquad J'_E : J'_e : J'_B : J'_{Qi} = 1 : -1 : -\frac{2}{5} : -\frac{2}{5} = 5 : -5 : -2 : -2, \tag{5.112}$$

$$\text{chemical-unit map:} \qquad \frac{J_E}{Q_B} : J_e : J_B : J_{Qi} = |J'_E| : |J'_e| : \frac{5}{2}|J'_B| : \frac{5}{2}|J'_{Qi}|, \tag{5.113}$$

$$\text{Arisaka Equation:} \qquad \frac{J_E}{Q_B} = J_e = J_B = J_{Qi}. \tag{5.114}$$

Equation (5.111) is the ultraviolet normalization choice. Equation (5.112) is the coefficient-space statement inherited from BI-6. Equation (5.113) is the indispensable conversion from raw sector coefficients to chemical natural units. Only after that conversion does Eq. (5.114) follow.

To make the same point in slower arithmetic, suppose an elementary synchronized event carries coefficient-space magnitudes

$$\left(|J'_E|, |J'_e|, |J'_B|, |J'_{Qi}|\right) = r\left(1,\, 1,\, \frac{2}{5},\, \frac{2}{5}\right), \qquad r \geq 0, \tag{5.115}$$

for some event-rate amplitude r. In other words, the raw coefficient-space bookkeeping says only that the baryonic and informational components are smaller than the electric and energy components by the factor 2/5 in the BI-6 normalization.

The next step is to define the *chemical natural units*. We choose the electric sector as the reference event count and define

$$J_e \equiv |J'_e|, \tag{5.116}$$

$$J_B \equiv \frac{5}{2}|J'_B|, \tag{5.117}$$

$$J_{Qi} \equiv \frac{5}{2}|J'_{Qi}|, \qquad J_{Qi} \equiv J_{Qi}, \tag{5.118}$$

$$\frac{J_E}{Q_B} \equiv |J'_E|, \qquad \text{equivalently} \qquad J_E \equiv Q_B|J'_E|. \tag{5.119}$$

These definitions do exactly one thing. They convert the four raw BI-6 amplitudes into *event counts* in the natural unit of each sector:

- the electric sector is counted one electric-transfer event at a time;
- the baryonic sector is counted one baryonic-update event at a time, which requires multiplication by the reciprocal factor 5/2 because its coefficient-space magnitude is 2/5 of the electric one;
- the informational sector is counted one informational-update event at a time, again requiring the reciprocal factor 5/2;
- the energy sector is counted in units of the fixed event energy Q_B, so that J_E/Q_B is the corresponding event count.

Substituting Eq. (5.115) into Eqs. (5.116)–(5.119) yields

$$J_e = r, \tag{5.120}$$

$$J_B = \frac{5}{2}\left(\frac{2}{5}r\right) = r, \tag{5.121}$$

$$J_{Qi} = \frac{5}{2}\left(\frac{2}{5}r\right) = r, \tag{5.122}$$

$$\frac{J_E}{Q_B} = r. \tag{5.123}$$

Therefore

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi} = r. \tag{5.124}$$

We may now state the third central object.

Definition 5.7.8 (Arisaka Equation)**.** The *Arisaka Equation* is the natural-unit form of the Qi-QUA locked current ray:

$$J_E/Q_B = J_e = J_B = J_{Qi} : \qquad \frac{J_E}{Q_B} = J_e = J_B = J_{Qi}, \qquad J_{Qi} \equiv J_{Qi}. \tag{5.125}$$

Remark 5.7.9 (Why $J_e = J_B = J_{Qi}$ is not a raw microscopic identity). The equality in Eq. (5.125) does *not* mean that one electron, one baryon, and one bit are literally the same microscopic carrier. In coefficient-space units the correct statement is instead

$$|J'_E| : |J'_e| : |J'_B| : |J'_{Qi}| = 1 : 1 : \frac{2}{5} : \frac{2}{5}.$$

The equalities $J_e = J_B = J_{Qi}$ appear only after the baryonic and informational sectors are measured in their own natural packet units, that is, after the reciprocal 5/2 conversion has been applied. The Arisaka Equation is therefore an equality of *normalized event counts*, not an identity among raw microscopic carriers.

This is the formal bridge from coefficient-space BI-6 data to chemistry/open-system event counting, and it is the mathematical reason Qi-QUA belongs after EEGG and SIP rather than before them.

Theorem 5.7.10 (Natural-unit equivalence theorem)**.** *For an elementary boundary event sourced by one BI-6 inflow channel, the coefficient-space Arisaka Ratio of Eqs.* (5.102)–(5.103) *is equivalent to the natural-unit Arisaka Equation* (5.125)*.*

Proof. The coefficient-space ray gives Eq. (5.115). Applying the chemical-unit map (5.113), equivalently the definitions (5.116)–(5.119), yields the explicit equalities (5.120)–(5.123). These show that all four normalized event counts are equal to the same amplitude r. Hence Eq. (5.125) follows.

Conversely, if Eq. (5.125) holds and the chemical-unit map is inverted, then

$$|J'_e| = \frac{J_E}{Q_B}, \qquad |J'_B| = \frac{2}{5}J_B, \qquad |J'_{Qi}| = \frac{2}{5}J_{Qi},$$

so the coefficient-space magnitudes necessarily lie on the ray $(1, 1, 2/5, 2/5)$. Reintroducing the BI-6 orientation signs reproduces Eqs. (5.102)–(5.103). The two statements are therefore equivalent once the sector-specific unit map has been fixed. □

Theorem 5.7.10 is the formal bridge between particle-physics notation and chemistry-natural-unit notation. In the ultraviolet language one sees the signed coefficient vector inherited from anomaly cancellation. In the chemistry-natural-unit language one sees the equality of per-event ledgers. Both are the same statement, but only after the chemical-unit map is inserted explicitly.

Table 5.27: Explicit four-line derivation chain of Qi-QUA.

Line	**Formula**	**Meaning**
BI-6 normalization	$(a, b, c, d) = (1, -1, -2/5, -2/5)$	Ultraviolet reduced-rational normalization chosen so the later current ledger remains compatible with GG-6 and OGN-5.
Arisaka Ratio	$J'_E : J'_e : J'_B : J'_{Qi} = 1 : -1 : -2/5 : -2/5 = 5 : -5 : -2 : -2$	Coefficient-space source ray inherited directly from BI-6.
Chemical-unit map	$J_e = \lvert J'_e\rvert$, $J_B = (5/2)\lvert J'_B\rvert$, $J_{Qi} = (5/2)\lvert J'_{Qi}\rvert$, $J_E = Q_B\lvert J'_E\rvert$	Sector-dependent unit conversion from raw BI-6 amplitudes to event counts and energy units.
Arisaka Equation	$J_E/Q_B = J_e = J_B = J_{Qi}$	Equality of normalized event counts for one ideal locked event.

5.7.7 Ideal equality, coarse-grained inequality, and overhead

The Arisaka Equation is exact for the *ideal synchronized elementary event*. Real chemistry, metabolism, and cognition, however, are not made of one event only. They consist of long histories containing useful steps, housekeeping steps, idle cycles, redundancy, proofreading, error correction, and heat leakage. It is therefore essential to state clearly how Qi-QUA behaves under coarse-graining.

Let a macroscopic process be decomposed into N_{lock} ideal locked events plus additional overhead events that increase the energy, electric, or baryonic budgets without increasing the registered informational count. Then the integrated budgets over the whole process can be written schematically as

$$\mathcal{E} = Q_B N_{\mathrm{lock}} + \Delta_E^{\mathrm{over}}, \tag{5.126}$$

$$\mathcal{N}_e = N_{\mathrm{lock}} + \Delta_e^{\mathrm{over}}, \tag{5.127}$$

$$\mathcal{N}_B = N_{\mathrm{lock}} + \Delta_B^{\mathrm{over}}, \tag{5.128}$$

$$\mathcal{N}_Q = N_{\mathrm{lock}}, \tag{5.129}$$

with nonnegative overhead terms $\Delta_A^{\mathrm{over}} \geq 0$. Under this monotone-overhead assumption one immediately obtains

$$\mathcal{E} \geq Q_B \mathcal{N}_Q, \qquad \mathcal{N}_e \geq \mathcal{N}_Q, \qquad \mathcal{N}_B \geq \mathcal{N}_Q. \tag{5.130}$$

Proposition 5.7.11 (Coarse-grained lower-bound form of Qi-QUA)**.** *If a macroscopic process is composed of ideal locked events plus nonnegative overhead in the energy, electric, and baryonic sectors, then Eq.* (5.125) *descends to the lower-bound inequalities* (5.130)*.*

Proof. Substitute Eqs. (5.126)–(5.129) into each difference $\mathcal{E} - Q_B\mathcal{N}_Q$, $\mathcal{N}_e - \mathcal{N}_Q$, and $\mathcal{N}_B - \mathcal{N}_Q$. The differences are precisely the nonnegative overhead terms. □

Proposition 5.7.11 is indispensable for realistic applications. Without it one would be tempted either to overstate Qi-QUAas a literal equality for every coarse-grained measured flux, or to dismiss it whenever a real molecular machine or nervous system displays inefficiency. The correct

reading is simpler. Qi-QUA fixes the primitive event and therefore also the ideal lower bound. Real systems may and usually do spend more.

Table 5.28: Ideal and coarse-grained forms of Qi-QUA.

Level of description	**Mathematical form**	**Interpretation**
Elementary synchronized event	$J_E/Q_B = J_e = J_B = J_{Qi}$	Exact equality on the ideal locked ray
Macroscopic process with overhead	$\mathcal{E} \geq Q_B\mathcal{N}_Q$, $\mathcal{N}_e \geq \mathcal{N}_Q$, $\mathcal{N}_B \geq \mathcal{N}_Q$	Lower-bound form after aggregation and inefficiency
Phenomenological reduced model	Some ledgers may be hidden	Acceptable only if hidden ledgers can in principle be restored in the full bookkeeping

This distinction between exact equality and coarse-grained lower bound is especially important for later biological applications. A protocell, an ATP synthase complex, a genetic proofreading network, or a cortical microcircuit can all spend substantially more than the ideal floor. What Qi-QUA claims is that the floor is not arbitrary. It is fixed by the same imported geometry that fixed the coefficient-space ray.

5.7.8 Relation to Landauer and to classical nonequilibrium thermodynamics

One immediate use of the Arisaka Constant is to compare the Qi-QUA event floor with the classical lower bound for logically irreversible erasure. Landauer's bound states that erasing one bit of information in thermal equilibrium at temperature T requires at least

$$E_{\mathrm{L}} = k_B T \ln 2 \tag{5.131}$$

of dissipated energy (Landauer, 1961). At $T = 310\,\mathrm{K}$,

$$E_{\mathrm{L}} \approx 0.0185\,\mathrm{eV}. \tag{5.132}$$

Comparing with Eq. (5.109) gives

$$\frac{Q_B}{k_B T \ln 2}\Big|_{T=310\,\mathrm{K}} \approx 8.40. \tag{5.133}$$

The meaning of Eq. (5.133) must be stated carefully. Qi-QUA does *not* refute Landauer's principle. The two statements answer different questions. Landauer asks for the minimum dissipation associated with an idealized logically irreversible operation. Qi-QUA asks for the minimum cost of a *locked open-system boundary event* in which energy, charge, baryonic structure, and information are audited simultaneously. Because the latter problem is richer, its lower bound can legitimately exceed the former.

Historically, the need to distinguish these levels was already implicit in nonequilibrium thermodynamics. Onsager's reciprocity, Prigogine's irreversible thermodynamics, Jarzynski's equality, Crooks' fluctuation relation, and modern stochastic thermodynamics all teach the same lesson: once a system is open and fluctuating, one must specify which ledger is being optimized, inferred,

or dissipated (Crooks, 1999; Jarzynski, 1997; Onsager, 1931; Prigogine, 1967; Seifert, 2012). Qi-QUA adds one extra layer to that tradition. It does not only track entropy production or work. It also insists that charge, material rearrangement, and information update belong to the same primitive event ledger.

Table 5.29: Landauer and Qi-QUA answer different lower-bound questions.

Principle	**Question addressed**	**Lower bound**	**Ledger audited**
Landauer	What is the minimum dissipation for idealized logical erasure of one bit?	$k_B T \ln 2$	Information plus thermal bath
Qi-QUA	What is the minimum cost of one locked boundary event?	$Q_B = (2/5)E_{\mathrm{Ryd}}/K$	Energy, electric, baryonic, and informational ledgers together

The payoff of Table 5.29 is conceptual economy. It prevents later sections from confusing thermodynamic erasure, biochemical update, and observer-level measurement. These processes are related, but they are not identical. Qi-QUA provides the ledger common to all of them once the full open-system event is audited.

5.7.9 Worked examples of the Qi-QUAledger

The purpose of this subsection is to make the Qi-QUA ledger operationally concrete. These examples are therefore not presented as full microscopic first-principles simulations. Rather, each example is a *packet-normalized audit*: one first identifies the physically meaningful event packet, then counts how many locked packets are required for the process under discussion, and only then translates that count into the ideal Qi-QUA energy ledger by

$$N_{\mathrm{lock}} \implies J_e = J_B = J_{Qi} = N_{\mathrm{lock}}, \qquad J_E = N_{\mathrm{lock}} Q_B. \tag{5.134}$$

Real systems are then interpreted as this ideal synchronized ledger plus overhead from leakage, proofreading, idle dwell, side reactions, topology management, and maintenance work.

5.7.9.1 Example A: CO activation and hydrogenation in the interstellar medium

The first example is intentionally chosen to sit close to the interface between astrochemistry and prebiotic chemistry. In our internal shorthand we have often written the first activation step symbolically as

$$CO + H^+ \rightsquigarrow HCO^*, \tag{5.135}$$

where the star denotes an activated HCO-like intermediate. Written this way, the formula captures the correct *ledger intuition*: a neutral carbon–oxygen system receives a charged hydrogenic update, moves to a new reactive branch, and stores that change in a new molecular identity. Microscopic ISM chemistry, however, is usually realized more specifically either by gas-phase proton transfer,

$$H_3^+ + CO \rightarrow HCO^+ + H_2, \tag{5.136}$$

or by grain-surface hydrogenation,

$$CO \xrightarrow{+H} HCO \xrightarrow{+H} H_2CO \xrightarrow{+H} CH_3O/CH_2OH \xrightarrow{+H} CH_3OH, \tag{5.137}$$

with methanol produced by four successive hydrogen-addition events on cold grain surfaces (Fuchs et al., 2009). Equation (5.135) should therefore be interpreted as a compact event-level symbol, while Eqs. (5.136)–(5.137) give the chemically more resolved realization.

This distinction is already instructive from the viewpoint of the Arisaka Equation. There are two natural packet choices.

Packet choice A1: net hydrogenation packets. If one counts only the net hydrogen-addition commits in Eq. (5.137), then

$$N_{\text{lock}}^{\text{ISM,hyd}} = 4. \tag{5.138}$$

The ideal Qi-QUA ledger is therefore

$$J_e = J_B = J_{Qi} = 4, \qquad J_E = 4Q_B \approx 0.622\,\text{eV} \approx 60.0\,\text{kJ}\,\text{mol}^{-1}. \tag{5.139}$$

The interpretation is simple. Each incoming hydrogen contributes one electric/protonic update, one material update to the carbon-oxygen skeleton, one informational commitment to a more specific branch in chemical state space, and one ideal energy packet in the universal ledger.

Packet choice A2: explicit activation plus settling. If one wishes to keep the symbolic initial protonic activation of Eq. (5.135) as a distinct packet, then the same chemistry is written as one activation commit followed by four material settling commits. In that convention,

$$N_{\text{lock}}^{\text{ISM,act}} = 5, \tag{5.140}$$

and the ideal energy ledger becomes

$$J_e = J_B = J_{Qi} = 5, \qquad J_E = 5Q_B \approx 0.778\,\text{eV} \approx 75.0\,\text{kJ}\,\text{mol}^{-1}. \tag{5.141}$$

Nothing inconsistent has happened. The chemistry is the same; only the packet convention changed. That is why the manuscript insists that the Arisaka Equation is an equality of *normalized event counts*, not of raw microscopic carriers.

Intuitively, the ISM example is the cleanest possible introduction to Qi-QUA. A hydrogenic arrival does not just add matter. It simultaneously changes charge environment, commits the network to one new chemical branch, and changes the energy ledger. Equivalently, even a very primitive astrochemical step is already a four-ledger event. That is why Qi-QUA is not only a biological principle; it already begins in prebiotic chemistry.

5.7.9.2 Example B: ATP synthase in the c_{10} ring of *E. coli*

The second example is the opposite extreme: a highly organized biological nanomachine. For *E. coli*, the membrane rotor has a preferred c_{10} stoichiometry (Jiang et al., 2001; Ballhausen et al., 2009), so one full revolution of the rotor corresponds to ten proton-translocation substeps.

The catalytic F_1 head has three catalytic β sites, so in the idealized stoichiometric picture one full turn produces three ATP molecules (Yoshida et al., 2001). The raw stoichiometry is therefore

$$N_{H^+}^{\text{turn}} = 10, \qquad N_{\text{ATP}}^{\text{turn}} = 3, \qquad \frac{H^+}{\text{ATP}} = \frac{10}{3} \approx 3.33. \tag{5.142}$$

This is exactly the sort of example that clarifies why one must not confuse the Arisaka Equation with a naive carrier-by-carrier identity. The machine does *not* say "one proton equals one ATP." The machine says that ten protonic substeps, executed in the correct sequence, can be converted into three chemically committed ATP outputs.

Packet choice B1: proton-step packets. If the natural packet is taken to be one proton-driven rotor substep, then one full turn contains

$$N_{\text{lock}}^{\text{rot}} = 10, \tag{5.143}$$

and the ideal synchronized ledger per full revolution is

$$J_e = J_B = J_{Qi} = 10, \qquad J_E = 10 Q_B \approx 1.555\,\text{eV} \approx 150\,\text{kJ}\,\text{mol}^{-1}. \tag{5.144}$$

Because the rotor contains ten c subunits, each protonic step corresponds to an angular advance of

$$\Delta\theta_{\text{rot}} = \frac{360^\circ}{10} = 36^\circ. \tag{5.145}$$

Thus a complete revolution is naturally interpreted as ten locked electric–mechanochemical substeps.

Packet choice B2: ATP-product packets. If, instead, the natural packet is taken to be one ATP synthesis event, then one ATP output corresponds ideally to

$$N_{\text{lock}}^{\text{ATP}} = \frac{10}{3} \tag{5.146}$$

protonic substeps, and the ideal Qi-QUA energy ledger per ATP becomes

$$J_e = J_B = J_{Qi} = 1, \qquad J_E = \frac{10}{3} Q_B \approx 0.518\,\text{eV} \approx 50.0\,\text{kJ}\,\text{mol}^{-1}. \tag{5.147}$$

The numerical value in Eq. (5.147) is striking because it falls in the same tens-of-kJ mol^{-1} band that one associates with biochemical ATP free-energy bookkeeping under physiological conditions. The point here is not that biology is perfectly reversible or that every ATP synthase cycle exactly saturates the Qi-QUA lower bound. The essential fact is that once the packet normalization is chosen correctly, the electric, material, informational, and energetic ledgers become directly comparable.

Intuitively, ATP synthase is almost a model textbook example of the Arisaka Equation. Proton flow is the electric ledger. ADP+P_i →ATP is the material ledger. The catalytic dwell state, rotor angle, and occupancy pattern form the information ledger. The stored chemical free energy is the energy ledger. The raw stoichiometry is unequal; the *normalized event ledger* is equal. That is exactly the distinction the Arisaka Equation was designed to capture.

5.7.9.3 Example C: DNA structure, function, and dynamics

DNA is the best example of a memory-bearing molecular substrate because it carries structure, function, and dynamics simultaneously. At the structural level, the double helix fixes a natural geometric packet. The Watson–Crick duplex provides the canonical double-stranded architecture (Watson and Crick, 1953), while direct measurements in solution give a helical repeat of approximately

$$h_{\mathrm{DNA}} \approx 10.4\,\mathrm{bp/turn} \tag{5.148}$$

under physiological conditions (Wang, 1979). Using the familiar B-form rise per base-pair step,

$$d_{\mathrm{bp}} \approx 0.34\,\mathrm{nm/bp}, \tag{5.149}$$

one full helical turn spans approximately

$$\ell_{\mathrm{turn}} \approx h_{\mathrm{DNA}}\, d_{\mathrm{bp}} \approx 10.4 \times 0.34\,\mathrm{nm} \approx 3.5\,\mathrm{nm}. \tag{5.150}$$

Thus the molecule already comes with two natural packet choices: one base-pair packet and one helical-turn packet.

Packet choice C1: one base-pair write/read packet. If the natural event is one base-pair writing, copying, or decisive reading update, then

$$N_{\mathrm{lock}}^{\mathrm{bp}} = 1, \qquad J_e = J_B = J_{Qi} = 1, \qquad J_E = Q_B \approx 0.1555\,\mathrm{eV} \approx 15.0\,\mathrm{kJ\,mol^{-1}}. \tag{5.151}$$

This is the elementary molecular-memory packet: one material insertion or readout, one charge reorganization in the polyanionic backbone and ionic atmosphere, one information commit to a specific base identity, and one ideal energy packet in the universal ledger.

Packet choice C2: one helical-turn packet. If the natural event is one full helical turn of coherent structural update, then the packet contains 10.4 base-pair units and the ideal ledger is

$$N_{\mathrm{lock}}^{\mathrm{turn}} = 10.4, \qquad J_e = J_B = J_{Qi} = 10.4, \qquad J_E = 10.4 Q_B \approx 1.617\,\mathrm{eV} \approx 156\,\mathrm{kJ\,mol^{-1}}. \tag{5.152}$$

This packet is often more natural for structural and topological DNA problems because one full turn is the scale at which twist, groove orientation, and many protein-recognition periodicities become geometrically visible.

Topological dynamics as informational memory. The DNA example becomes even more powerful once one includes dynamics. For a closed or topologically constrained duplex, the global topological bookkeeping is organized by the decomposition

$$Lk = Tw + Wr, \tag{5.153}$$

where linking number Lk, twist Tw, and writhe Wr trade against one another under deformation and supercoiling (Benham, 1979; Fuller, 1971). For a relaxed duplex containing N_{bp} base pairs,

the twist is approximately

$$Tw_0 \approx \frac{N_{\mathrm{bp}}}{10.4}. \tag{5.154}$$

For example, a 1040-bp circular DNA molecule has

$$Tw_0 \approx \frac{1040}{10.4} \approx 100 \tag{5.155}$$

helical turns in the relaxed state. If enzymatic or mechanical action drives the molecule to $\Delta Lk = -5$, the duplex is underwound by roughly five turns relative to its relaxed topology. In Qi-QUA language this is not just a geometric distortion; it is an information-bearing state update of the molecule's internal memory register. The DNA has changed not only its matter content but also its accessible functional branch structure.

Intuitively, DNA is the clearest biological illustration of why $U(1)_{Qi}$ is useful. A DNA molecule is not just a charged polymer. It is a charged polymer whose sequence, torsional state, and accessibility state all carry memory. A replication step, a methylation-like marking step, a topological relaxation step, and a transcriptionally committed opening step are all simultaneously material, electric, informational, and energetic events. Once one works in packet-normalized units, the Arisaka Equation becomes the correct common ledger for that whole family of processes.

Table 5.30: Quantitative worked examples of the Qi-QUA ledger across astrochemistry and biology. The rightmost columns show the ideal synchronized ledger; real systems generally require additional overhead for leakage, proofreading, maintenance, and dissipation.

System	**Packet choice**	**Locked count N_{lock}**	**Ideal energy ledger**	**Intuitive meaning**
ISM CO chemistry	Four net H-addition commits $CO \to HCO \to H_2CO \to CH_3O/CH_2OH \to CH_3OH$	4	$4Q_B \approx 0.622\,\mathrm{eV} \approx 60.0\,\mathrm{kJ\,mol^{-1}}$	Each arriving hydrogen updates charge, matter, branch identity, and energy budget simultaneously.
ISM CO chemistry (activation-explicit)	Initial symbolic protonic activation plus four settling commits	5	$5Q_B \approx 0.778\,\mathrm{eV} \approx 75.0\,\mathrm{kJ\,mol^{-1}}$	Same chemistry, different packet convention; illustrates why the Arisaka Equation is event-normalized.
E. coli ATP synthase	One full c_{10} rotor turn	10	$10Q_B \approx 1.555\,\mathrm{eV} \approx 150\,\mathrm{kJ\,mol^{-1}}$	Ten protonic substeps drive one full mechanical revolution and three catalytic product commits.
E. coli ATP synthase	One ATP product commit	10/3	$(10/3)Q_B \approx 0.518\,\mathrm{eV} \approx 50.0\,\mathrm{kJ\,mol^{-1}}$	Raw stoichiometry is unequal, but the normalized event ledger becomes equal.
DNA	One base-pair write/read packet	1	$Q_B \approx 0.1555\,\mathrm{eV} \approx 15.0\,\mathrm{kJ\,mol^{-1}}$	One molecular memory symbol is physically committed.
DNA	One helical-turn packet	10.4	$10.4Q_B \approx 1.617\,\mathrm{eV} \approx 156\,\mathrm{kJ\,mol^{-1}}$	One full duplex turn becomes the natural packet for structure and topology.

These examples now show, in a quantitatively explicit way, what the Arisaka Equation is and what it is not. It is *not* a statement that one proton, one hydrogen atom, one ATP molecule, one base pair, and one bit are literally the same microscopic object. It is a statement that after the correct packet choice has been made, the four ledgers of energy, electric transfer, baryonic/material update, and informational commit can be written on one common event-count basis. That is the operational meaning of

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi}. \tag{5.156}$$

5.7.10 Why Qi-QUAis correctly placed as Pillar I of the Qi program

It is now possible to state why Qi-QUA deserves to be the first Qi pillar rather than a subsidiary appendix of the previous section.

First, Qi-QUA answers the budget question. DGI tells us the lawful gauge-equivalent family of descriptions. GSSB tells us which branch or structure is selected. GA tells us which selected branch persists. But none of those objects yet says how the event budget is denominated. Qi-QUA supplies that denomination.

Second, Qi-QUA prepares the next three Qi pillars. Qi-EP will need a ledger because no extremal principle is meaningful unless one knows which scalar or quasi-scalar is being extremized subject to which conserved or constrained currents. Qi-CON will need a ledger because scaling laws and $1/f$ spectra are signatures of budgeted flows distributed across scales. Qi-HAL will need a ledger because memory, prediction, sensing, and action all consume bounded update budgets. Equivalently, Qi-QUA is the "how much" pillar on which the later "which path," "what signature," and "what architecture" pillars depend.

Third, Qi-QUA is the place where the ultraviolet and infrared languages of the program become visibly compatible. The Arisaka Ratio belongs to the coefficient-space ultraviolet language inherited from BI-6. The Arisaka Equation belongs to the chemistry-natural-unit infrared language needed for open systems. The Arisaka Constant is the conversion bridge between the two. No other section of the paper can replace that bridge.

Table 5.31: Role of Qi-QUA relative to the rest of Chapter 5.

Layer	**Main question**	**Answer supplied**
DGI+GSSB+GA	What descriptions are admissible, how is one branch chosen, and why does it persist?	Runtime grammar of lawful open-system evolution
Qi-QUA	How much budget is attached to one locked event?	Arisaka Ratio, Arisaka Constant, Arisaka Equation
Qi-EP	Which admissible path or branch is selected?	Extremal-principle layer
Qi-CON	What scale signatures reveal the selected runtime?	Conformality, fractality, Lévy statistics, $1/f$ spectra
Qi-HAL	What architecture implements memory, prediction, sensing, and action?	Holographic-attractor architecture layer

The most compact way to summarize the section is therefore the following:

> BI-6 fixes the source ray; the source ray fixes the locked event; the locked event fixes the energy scale Q_B; and Q_B converts the ultraviolet source ray into the chemistry-natural-unit Arisaka Equation.

That sentence is the entire logic of Pillar I.

5.7.11 Section summary

This section has completed the first quantitative pillar of Chapter 5.

First, the generic open-system continuity equations (5.95)–(5.96) were used to define the four ledgers audited by Qi-QUA: energy, electric, baryonic, and informational. The difference between coefficient-space currents and natural-unit currents was emphasized from the outset.

Second, the BI-6 source vector inherited from GG-6 was projected to the boundary and shown to generate a one-dimensional source ray. Theorem 5.7.3 established that elementary admissible source vectors lie on the ray $(1, -1, -2/5, -2/5)$, which was then identified as the Arisaka Ratio. The equivalent integer-cleared form $(5, -5, -2, -2)$ was also recorded.

Third, the OGN throat depth $K = 35$ and the atomic scale E_{Ryd} were combined with the BI-6 magnitude ratio $2/5$ to define the Arisaka Constant

$$Q_B = \frac{2}{5}\frac{E_{\mathrm{Ryd}}}{K} \approx 0.1555\,\mathrm{eV} \approx 15.0\,\mathrm{kJ\,mol^{-1}}.$$

Theorem 5.7.7 showed that this introduces no new continuous free parameter.

Fourth, the coefficient-space ray was mapped into chemistry natural units, yielding the Arisaka Equation

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi}.$$

Theorem 5.7.10 proved that this is exactly the natural-unit form of the coefficient-space Arisaka Ratio, not a separate postulate.

Fifth, the section distinguished the exact equality of ideal synchronized events from the coarse-grained lower-bound inequalities obeyed by realistic processes with overhead. That distinction will be essential in later applications to chemistry, life, and cognition.

The manuscript now has the runtime engine, the scalar budget, the packet declaration, and the first quantitative Qi pillar. The next natural step is therefore Qi-EP, where the question will no longer be "how much budget per locked event?" but rather "which admissible trajectory or branch is selected by the dynamics?"

5.8 Qi-EP: PLA in 6D and PLT/MEPP/FEP in 4D

With event-level bookkeeping fixed by Qi-QUA, the next logical question is selection. A packet can obey the right ledger and still admit many admissible histories. Section 5.8 therefore derives the selector layer of the Qi program: once the budgeted event cone is known, which trajectory, branch, or inferential state is actually extremized?

5.8.1 Why Qi-EPmust follow Qi-QUA

The order of the Qi pillars is not arbitrary. Qi-QUA answered the bookkeeping question: once a process has been coarse-grained to the boundary and once a synchronized event packet has been identified, what is the correct current ledger and what is the corresponding energy denomination? But a ledger by itself does not yet choose a history. Many trajectories, many branches, and many persistent patterns can obey the same integrated current budget. The next question is therefore not "how much?" but "which one?" Qi-EP is the answer to that selection problem.

This can be stated mathematically. Fix an initial and final boundary packet B_0, B_T and let

$$\Gamma_{\mathrm{adm}}(B_0, B_T) = \left\{ (\rho, J, \gamma) \;\middle|\; \begin{array}{l} \partial_t \rho + \nabla_a J^a = 0, \\ (\rho, J, \gamma) \text{ is DGI-covariant}, \\ \mathcal{C}_{\text{Qi-QUA}}[\rho, J] = 0, \\ \gamma(0) = B_0,\ \gamma(T) = B_T \end{array} \right\} \tag{5.157}$$

denote the class of admissible histories. Here ρ is the reduced boundary density, J is the total current packet, γ denotes a representative trajectory or branch history, and $\mathcal{C}_{\text{Qi-QUA}} = 0$ summarizes the ledger constraints inherited from the Arisaka Ratio and the Arisaka Equation. In ideal locked units those constraints include

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi}, \tag{5.158}$$

while in realistic coarse-grained settings the equalities are replaced by the corresponding lower-bound or overhead inequalities developed in Section 5.7.

The problem solved by Qi-EP can now be written in one line:

> Among all admissible histories in $\Gamma_{\mathrm{adm}}(B_0, B_T)$, the physically realized history is the stationary history of the projected GG-6/GDOS variational functional.

This statement is the direct open-system descendant of the ordinary principle of least action. In the closed 6D bulk one writes $\delta S_{\text{GG-6}} = 0$. On the observer-accessible 4D boundary, however, one must extremize not the original bulk action but its projected open-system descendant. The three familiar-looking principles that emerge after this projection — PLT, MEPP, and FEP — are the three most useful boundary gauges of one and the same reduced extremal structure.

The role of Qi-EP relative to the previous section is therefore precise. Qi-QUA defines the admissible current cone and the natural-unit energy scale attached to one ideal locked event. Qi-EP chooses a path, a branch, or an attractor inside that cone. Put differently, Pillar I fixes the budget, whereas Pillar II fixes the selector. Without Pillar I, the selector would have no scale or denominator; without Pillar II, the budget would remain purely kinematic.

Table 5.32: From Qi-QUA to Qi-EP: why the second pillar must follow the first.

Layer	**Main question**	**Primary answer**
Qi-QUA	What is the correct current ledger for one synchronized event packet?	Arisaka Ratio, Arisaka Constant, and Arisaka Equation determine the admissible budget cone.
Qi-EP	Which admissible history or branch is actually realized?	The projected extremal functional selects stationary trajectories, fluxes, or inferential states.
Qi-CON	What statistical signatures reveal the selected runtime?	Scale covariance, heavy tails, fractality, and $1/f$-type spectra.
Qi-HAL	What architecture implements memory, prediction, sensing, and action?	Holographic-attractor structures such as MePMoS, NHT, HAL, and BFFT.

The rest of the section proceeds systematically. First, the projected variational backbone is derived from the 6D bulk action. Second, the trajectory gauge is shown to yield PLT. Third, the current gauge is shown to yield MEPP (with the dual minimum-dissipation formulation appearing automatically in the Legendre-dual picture). Fourth, the inferential gauge is shown to yield FEP. Finally, the three are reunited as one gauge family of the same open-system principle.

5.8.2 Constitutional statement of the Qi-EPprinciple

The conceptual claim of Qi-EP should be stated before any detailed derivation:

Definition 5.8.1 (Qi-EP)**.** For any admissible open subsystem S embedded in the GG-6 bulk and observed through OGGEP, there exists a projected extremal functional

$$\mathfrak{Q}^{(S)}_{\text{Qi-EP}} \;=\; \Phi^{(S)}_{\text{OGGEP}}[S_{\text{GG-6}}]\Big|_{\Gamma_{\text{adm}}} \tag{5.159}$$

such that the realized boundary dynamics of S satisfy

$$\delta\mathfrak{Q}^{(S)}_{\text{Qi-EP}} = 0 \qquad \text{on} \qquad \Gamma_{\text{adm}}. \tag{5.160}$$

Under three natural boundary gauges this same stationary condition appears respectively as PLT, MEPP, and FEP.

Three comments are essential.

First, the word "projected" is not decorative. The boundary principle is not postulated independently. It is induced from the 6D GG-6 action through the open-system projection map already established in GG-2026-3 (Arisaka, 2026b,c). This means that the extremal selector on the boundary is constitutionally tethered to the same ultraviolet structure that fixed BI-6, the OGN integers, and the current ratios.

Second, the phrase "under three natural boundary gauges" is also literal. The projected functional may be described in terms of trajectories, in terms of currents and forces, or in terms of inferential states on a statistical manifold. These are not three unrelated principles living side by side. They are three descriptions of the same projected geometry under three different coarse-grained coordinate choices.

Third, Qi-EP is not restricted to quantum mechanics, nor to biology, nor to any single domain. The same projected functional can describe a geodesic-like transport path in one application, a non-equilibrium stationary flux in another, and an inferential or adaptive update in a third. A universal open-system selector must do precisely this if it is to serve as Pillar II of a general theory.

5.8.3 From the 6D bulk action to the projected boundary action

We begin from the ultraviolet starting point already established in GG-2026-2: the warped six-dimensional bulk with total action

$$S_{\text{GG-6}}[\Psi] = \int_{\mathcal{M}_6} \mathrm{d}^6x\,\sqrt{-g_6}\,\mathcal{L}_6\big(\Psi, \nabla\Psi\big), \tag{5.161}$$

where Ψ collectively denotes the 6D metric, gauge, Green–Schwarz, and matter fields. By construction the bulk dynamics satisfy

$$\delta S_{\text{GG-6}} = 0, \tag{5.162}$$

together with the BI-6 consistency constraints. Equation (5.162) is the constitutional PLA of the GG-6 stage (Arisaka, 2026b; Feynman, 1948; Hamilton, 1834).

To derive the boundary selector, split the full field content into observer-accessible slow variables z and hidden or traced-over variables y:

$$\Psi = (z, y). \tag{5.163}$$

The symbol z should be interpreted broadly. Depending on the later application it may denote a trajectory coordinate, a flux coordinate, a reaction-network coordinate, a neural or cognitive state coordinate, or a collective macroscopic descriptor. The hidden variables y include KK modes, ultraviolet environmental modes, unresolved microstates, and any degrees of freedom eliminated by the boundary observer.

The standard influence-functional representation of the reduced dynamics is then

$$\exp\left(\frac{\mathrm{i}}{\hbar} S_{\text{IF}}[z, z']\right) = \int \mathcal{D}y\, \mathcal{D}y'\, \exp\left[\frac{\mathrm{i}}{\hbar}\Big(S_{\text{GG-6}}[z, y] - S_{\text{GG-6}}[z', y']\Big)\right] \rho_y(0), \tag{5.164}$$

which is the path-integral expression of the same reduced evolution usually described by CPTP maps, Nakajima–Zwanzig kernels, or GKLS generators (Breuer & Petruccione, 2002; Feynman & Vernon, 1963; Gorini et al., 1976; Lindblad, 1976). In this book, however, the decisive point is not merely that reduction exists; it is that the reduction inherits a variational structure from the bulk.

In the semiclassical, weak-noise, or large-deviation regime one may write

$$P[z] \asymp \exp\left[-\frac{1}{\varepsilon_{\text{eff}}} \mathcal{A}_{\text{eff}}[z]\right], \tag{5.165}$$

where ε_{eff} is the effective coarse-graining scale and $\mathcal{A}_{\text{eff}}$ is the reduced action functional (Onsager & Machlup, 1953; Touchette, 2009; van Kampen, 2007). More generally, when both densities and currents are retained explicitly, the most useful projected object is the current-density functional

$$\mathcal{A}_{\text{eff}}[\rho, J] = \int_0^T \mathrm{d}t\, \left[\frac{1}{4}\int_{\mathcal{X}} \big(J - \rho u\big)^a \big(\rho D\big)^{-1}_{ab} \big(J - \rho u\big)^b \,\mathrm{d}z + \mathcal{V}[\rho]\right], \tag{5.166}$$

subject to the continuity equation

$$\partial_t \rho + \nabla_a J^a = 0. \tag{5.167}$$

Here $u^a(z, t)$ is the projected drift field, $D^{ab}(z, t)$ is the symmetric positive diffusion/metric tensor inherited from the projected bulk geometry, and $\mathcal{V}[\rho]$ is the projected potential or constraint functional. Equation (5.166) is the single most important formula of the present section. It is the master boundary action from which the three gauges will be obtained.

Proposition 5.8.2 (Projected stationary-action proposition)**.** *Within the* GG-6/GDOS *framework, the open-system boundary histories selected by* OGGEP *are stationary histories of the projected action* $\mathcal{A}_{\text{eff}}[\rho, J]$ *on the admissible class* Γ_{adm}.

Proof. Fix (ρ, J) and consider the hidden-variable extremal $y = \bar{y}[z]$ obtained by solving the hidden Euler–Lagrange equations at fixed boundary history. At tree level or in the leading large-deviation approximation one has

$$\mathcal{A}_{\text{eff}}[z] = \Re\, S_{\text{GG-6}}[z, \bar{y}[z]] + (\text{coarse-graining counterterms}). \tag{5.168}$$

Varying with respect to z gives

$$\frac{\delta \mathcal{A}_{\text{eff}}}{\delta z} = \frac{\delta S_{\text{GG-6}}}{\delta z}\bigg|_{y=\bar{y}[z]} + \frac{\delta S_{\text{GG-6}}}{\delta y}\bigg|_{y=\bar{y}[z]} \frac{\delta \bar{y}}{\delta z} + \delta(\text{counterterms}). \tag{5.169}$$

The second term vanishes because $\bar{y}[z]$ is by construction a hidden extremal. The remaining terms combine into the reduced stationarity condition. The current-density form (5.166) is the corresponding open-system representation in which the hidden modes appear only through the projected drift, diffusion, and potential data. Therefore the realized boundary history is a stationary history of $\mathcal{A}_{\text{eff}}$. □

The proposition is conceptually decisive. It says that Qi-EP is not a separate empirical principle pasted onto the boundary. It is the projected variational shadow of the same 6D least-action law that governed the ultraviolet bulk. The remaining task is to show how the familiar boundary forms PLT, MEPP, and FEP arise from three different reductions of Eq. (5.166).

5.8.4 Trajectory gauge: derivation of PLT

The first gauge is the trajectory gauge. In this description the boundary packet remains sharply localized around a representative path $z(t)$, so that

$$\rho(\zeta, t) \approx \delta\big(\zeta - z(t)\big), \qquad J(\zeta, t) \approx \delta\big(\zeta - z(t)\big)\, \dot{z}(t). \tag{5.170}$$

Substituting Eq. (5.170) into Eq. (5.166) gives the reduced trajectory action

$$\mathcal{A}_{\text{trj}}[z] = \int_0^T \mathrm{d}t\, \left[\frac{1}{4}\big(\dot{z} - u(z,t)\big)^a D^{-1}_{ab}(z,t)\big(\dot{z} - u(z,t)\big)^b + V(z,t)\right], \tag{5.171}$$

where $V(z,t)$ is the local representative of $\mathcal{V}[\rho]$ on the sharply localized slice.

It is convenient to introduce the effective metric

$$g_{ab}(z,t) := \frac{1}{2}\, D^{-1}_{ab}(z,t) \tag{5.172}$$

and the associated one-form

$$a_a(z,t) := g_{ab}(z,t)\, u^b(z,t). \tag{5.173}$$

Then Eq. (5.171) becomes

$$\mathcal{A}_{\mathrm{trj}}[z] = \int_0^T \mathrm{d}t \, \left[\frac{1}{2} g_{ab} \dot{z}^a \dot{z}^b - a_a \dot{z}^a + \frac{1}{2} g_{ab} u^a u^b + V\right]. \tag{5.174}$$

Up to the effective scalar potential

$$U_{\mathrm{eff}}(z,t) := \frac{1}{2} g_{ab} u^a u^b + V(z,t), \tag{5.175}$$

this is the standard quadratic mechanical form on a curved configuration manifold.

The Euler–Lagrange equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(g_{ab}\dot{z}^b - a_a\right) - \frac{1}{2}\partial_a g_{bc}\, \dot{z}^b \dot{z}^c + \partial_a a_b\, \dot{z}^b - \partial_a U_{\mathrm{eff}} = 0. \tag{5.176}$$

Raising the free index and rewriting with the Levi–Civita connection of g_{ab} gives

$$\frac{D_g \dot{z}^a}{\mathrm{d}t} = g^{ab}\left(F_{bc}\dot{z}^c - \partial_b U_{\mathrm{eff}}\right), \qquad F_{ab} := \partial_a a_b - \partial_b a_a. \tag{5.177}$$

Equation (5.177) has the expected interpretation: the projected path follows a geodesic-like motion in the effective boundary metric, corrected by a Lorentz-like curvature term and a projected scalar force.

To make contact with the language of PLT, fix an energy shell E and perform the Jacobi reduction. One obtains the generalized optical-time functional

$$\mathcal{T}[z] = \int_\gamma \left[\sqrt{2\big(E - U_{\mathrm{eff}}(z)\big)\, g_{ab}(z)\, \mathrm{d}z^a \mathrm{d}z^b} - a_a(z)\, \mathrm{d}z^a\right]. \tag{5.178}$$

The stationary condition $\delta\mathcal{T} = 0$ is the boundary PLT form of the same projected action. The phrase "least time" should therefore be interpreted in the generalized sense appropriate to open systems: not a bare Euclidean travel time, but the optical or information-geometric time induced by the projected metric, the drift connection, and the effective potential.

Theorem 5.8.3 (PLT as the trajectory gauge of Qi-EP)**.** *On the sharply localized trajectory slice $\mathcal{S}_{\mathrm{trj}} \subset \Gamma_{\mathrm{adm}}$, the projected current-density functional* (5.166) *reduces to the generalized optical-time functional* (5.178)*. Its stationary histories satisfy Eq.* (5.177)*. Hence* PLT *is the trajectory gauge of* Qi-EP*.*

Proof. The reduction from Eq. (5.166) to Eq. (5.171) follows directly by substitution of Eq. (5.170). Completing the square and defining Eqs. (5.172)–(5.175) gives Eq. (5.174). The Euler–Lagrange variation yields Eq. (5.176), which is equivalent to Eq. (5.177). Restricting to a fixed energy shell and performing the standard Jacobi reduction turns the quadratic action into the time functional (5.178). Therefore the stationary condition of the projected action is precisely the PLT condition in this gauge. □

The physical interpretation is straightforward. The 6D bulk obeys PLA. Once the inaccessible directions and hidden environmental couplings are projected away, the observer sees a boundary packet moving in an effective metric with an effective connection and an effective scalar landscape.

The realized path is then the one of stationary generalized travel time. In transport problems, search problems, signaling problems, and morphogenetic path-selection problems, this is the most natural boundary language.

Table 5.33: Step-by-step structure of the PLT derivation.

Step	Equation	Meaning
6D constitutional law	(5.162)	The bulk obeys $\delta S_{\text{GG-6}} = 0$.
Projected boundary action	(5.166)	Hidden modes induce drift, diffusion, and a projected potential.
Trajectory slice	(5.170)	The packet is sharply localized around one representative path.
Effective metric and connection	(5.172), (5.173)	The projected quadratic form becomes curved kinematics with a gauge-like one-form.
Boundary equation of motion	(5.177)	The realized trajectory is geodesic-like with projected curvature and scalar forces.
Operational least-time form	(5.178)	The boundary selector is a generalized PLT.

5.8.5 Current gauge: derivation of MEPP

The second gauge is the current gauge. Here the density ρ remains spatially extended and the fundamental question is no longer "which path does one packet follow?" but "which flux pattern is selected under the projected driving and constraints?" This is the natural language for transport networks, reaction networks, metabolic fluxes, ecological throughputs, and other non-equilibrium open systems.

Define the mobility or Onsager tensor

$$L^{ab}(z,t) := \rho(z,t)\, D^{ab}(z,t), \tag{5.179}$$

which is symmetric and positive semidefinite because D^{ab} is symmetric positive and $\rho \geq 0$. Next introduce the generalized force

$$X_a(z,t) := \left(D^{-1}\right)_{ab} u^b(z,t) - \partial_a \varphi(z,t), \tag{5.180}$$

where φ is the scalar contribution extracted from the projected potential and any conservation constraints. The local entropy-production rate density is then written in the familiar bilinear form

$$\dot{\sigma}(z,t) := J^a X_a. \tag{5.181}$$

The total entropy-production rate is

$$\dot{\Sigma}[J] = \int_{\mathcal{X}} J^a X_a \, \mathrm{d}z. \tag{5.182}$$

The same quadratic term that appeared in Eq. (5.166) may now be rewritten as an Onsager–Rayleighian,

$$\mathcal{R}_\rho[J;X] = \int_{\mathcal{X}} \left(J^a X_a - \frac{1}{2} J^a (L^{-1})_{ab} J^b \right) \mathrm{d}z, \tag{5.183}$$

again subject to the continuity equation and the integrated Qi-QUA ledger constraints. The advantage of Eq. (5.183) is that its meaning is immediate: the first term rewards entropy-producing throughput under the imposed force field, whereas the second term penalizes dissipative expenditure measured by the inverse mobility tensor.

Varying Eq. (5.183) with respect to J gives

$$0 = \frac{\delta \mathcal{R}_\rho}{\delta J^a} = X_a - (L^{-1})_{ab} J^b, \tag{5.184}$$

hence

$$J^a = L^{ab} X_b. \tag{5.185}$$

Equation (5.185) is the force-flux law selected by the projected geometry. Substituting it back into Eqs. (5.182) and (5.183) yields

$$\dot{\Sigma}_\star = \int_{\mathcal{X}} X_a L^{ab} X_b \, \mathrm{d}z \geq 0, \tag{5.186}$$

and

$$\mathcal{R}_{\rho,\star} = \frac{1}{2} \dot{\Sigma}_\star. \tag{5.187}$$

The second variation is

$$\delta^2 \mathcal{R}_\rho = -\int_{\mathcal{X}} \delta J^a (L^{-1})_{ab} \delta J^b \, \mathrm{d}z \leq 0, \tag{5.188}$$

so the stationary point is a maximum of $\mathcal{R}_\rho$ on the admissible force-controlled slice. Because Eq. (5.187) identifies the optimum Rayleighian with one-half of the selected entropy-production rate, the same stationary point is also the maximum-entropy-production point on that slice.

This is the precise sense in which MEPP appears in Qi-EP. One must keep the control convention explicit. On the force-controlled slice, Eq. (5.183) is maximized. On the dual flux-controlled slice, the Legendre transform

$$\mathcal{D}_\rho[J;X] = \frac{1}{2} \int_{\mathcal{X}} J^a (L^{-1})_{ab} J^b \, \mathrm{d}z - \int_{\mathcal{X}} J^a X_a \, \mathrm{d}z \tag{5.189}$$

is minimized. Thus the familiar "maximum entropy production" and "minimum dissipation" formulations are not contradictory in the present framework. They are dual descriptions of the same stationary point under dual control conventions (Dewar, 2003; Nicolis & Prigogine, 1977; Onsager, 1931; Prigogine, 1967; Seifert, 2012).

Theorem 5.8.4 (MEPP as the current gauge of Qi-EP)**.** *On the force-controlled current slice $\mathcal{S}_{\mathrm{flx}} \subset \Gamma_{\mathrm{adm}}$, the projected action (5.166) is equivalent to the Onsager–Rayleighian (5.183). Its stationary point satisfies the flux-force law (5.185) and maximizes the entropy-production rate (5.182) on the admissible slice. The Legendre-dual flux-controlled description is Eq. (5.189), which is minimized at the same stationary point.*

Proof. Rewrite the quadratic term of Eq. (5.166) using $L^{ab} = \rho D^{ab}$. Up to terms independent of J, this gives Eq. (5.183). The first variation yields Eq. (5.185). The second variation (5.188) is negative semidefinite because L^{-1} is positive semidefinite on the support of ρ. Therefore the stationary point is a maximum of $\mathcal{R}_\rho$. At that point the identities (5.186) and (5.187) hold by substitution. The Legendre-dual representation follows by changing the control variables from X to J, yielding Eq. (5.189). □

This current-gauge derivation is especially important for chemistry and life. A living system is not well described as a single sharply localized path. It is better described as a network of sustained throughputs, coupled channels, homeostatic gradients, and current constraints. In precisely that regime the most natural boundary language is MEPP or its dual minimum-dissipation form. The underlying selector, however, is still the same projected GG-6/GDOS variational principle.

Table 5.34: Step-by-step structure of the MEPP derivation.

Step	**Equation**	**Meaning**
Mobility tensor	(5.179)	The projected metric becomes an Onsager transport tensor weighted by the density.
Generalized force	(5.180)	The projected drift and potential define the thermodynamic driving field.
Entropy production	(5.182)	The bilinear force-flux pairing is the boundary entropy-production rate.
Rayleighian	(5.183)	Entropy production competes with dissipative cost.
Stationary flux law	(5.185)	Extremization selects the physical current pattern.
Dual description	(5.189)	The same stationary point is minimum dissipation in the flux-controlled picture.

5.8.6 Inferential gauge: derivation of FEP

The third gauge is the inferential gauge. It becomes natural whenever the open subsystem does not merely transmit or dissipate, but also maintains internal states that summarize hidden causes and guide action. Cells, nervous systems, adaptive agents, and cognitive architectures all fall into this class.

Let the projected boundary variables be decomposed as

$$z = (\eta, s, a, \mu), \tag{5.190}$$

where η denotes hidden external causes, s sensory states, a active states, and μ internal states. The Markov-blanket interpretation is that the subsystem accesses the external world only through s, changes it through a, and maintains a compact internal model through μ (Friston, 2010). In the present framework the crucial addition is that the generative structure is not arbitrary: it is induced from the projected GG-6/GDOS geometry.

Let $p(\eta, s, a)$ denote the projected generative density inherited from the boundary stationary measure. Introduce a parametric recognition family $q_\mu(\eta)$ representing the subsystem's internal approximation to the posterior over hidden causes. The variational free energy is then

$$\mathcal{F}(s, a, \mu) = \int q_\mu(\eta) \, \log \frac{q_\mu(\eta)}{p(\eta, s, a)} \, \mathrm{d}\eta. \tag{5.191}$$

As usual, Eq. (5.191) may be rewritten as

$$\mathcal{F}(s, a, \mu) = D_{\mathrm{KL}}\big(q_\mu(\eta) \,\|\, p(\eta|s, a)\big) - \log p(s, a), \tag{5.192}$$

so $\mathcal{F}$ upper-bounds surprisal and is minimized when the recognition family matches the posterior. To recover FEP from the projected action, restrict the density ρ to the statistical manifold

$$\mathcal{S}_{\mathrm{inf}} = \big\{\rho_{\mu,s,a}(\eta) = q_\mu(\eta) \, \delta(s - s_t) \, \delta(a - a_t)\big\}. \tag{5.193}$$

When the quadratic term of Eq. (5.166) is pulled back to this manifold, it induces the Fisher metric

$$g^{\mathrm{F}}_{ij}(\mu) = \int q_\mu(\eta) \, \partial_i \log q_\mu(\eta) \, \partial_j \log q_\mu(\eta) \, \mathrm{d}\eta, \tag{5.194}$$

which is the natural information-geometric metric on the parameter manifold (Amari, 1998; Amari & Nagaoka, 2000; Jaynes, 1957). The restricted action becomes

$$\mathcal{A}_{\mathrm{inf}}[\mu] = \int_0^T \mathrm{d}t \, \left[\frac{1}{2} g^{\mathrm{F}}_{ij}(\mu) \, \dot{\mu}^i \dot{\mu}^j + \mathcal{F}\big(s(t), a(t), \mu(t)\big)\right]. \tag{5.195}$$

The Euler–Lagrange equations are

$$g^{\mathrm{F}}_{ij} \, \ddot{\mu}^j + \Gamma^{\mathrm{F}}_{ijk} \, \dot{\mu}^j \dot{\mu}^k + \partial_i \mathcal{F} = 0, \tag{5.196}$$

where $\Gamma^{\mathrm{F}}_{ijk}$ is the Levi–Civita connection of the Fisher metric. In the overdamped or adiabatically relaxed limit — the relevant limit for slow inferential updates compared with fast hidden fluctuations — the inertial term may be neglected, leaving

$$\dot{\mu}^i = -\kappa_\mu \, \big(g^{\mathrm{F}}\big)^{ij} \partial_j \mathcal{F}. \tag{5.197}$$

Equation (5.197) is the natural-gradient version of the FEP: internal states descend variational free energy in the Fisher geometry. Active states obey the analogous projected descent

$$\dot{a}^\alpha = -\kappa_a \, \frac{\partial \mathcal{F}}{\partial a^\alpha}, \tag{5.198}$$

which is the action side of active inference: the system acts on the environment to reduce sensory prediction error.

Theorem 5.8.5 (FEP as the inferential gauge of Qi-EP)**.** *On the statistical slice $\mathcal{S}_{\mathrm{inf}} \subset \Gamma_{\mathrm{adm}}$, the projected action* (5.166) *pulls back to the Fisher-metric action* (5.195)*. In the overdamped limit, its stationary dynamics reduce to the natural-gradient* FEP *equations* (5.197) *and* (5.198)*. Hence* FEP *is the inferential gauge of* Qi-EP*.*

Proof. The pullback of the density family $\rho_{\mu,s,a}$ onto the parameter manifold μ converts the quadratic kinetic term of Eq. (5.166) into the Fisher quadratic form (5.194); this is derived explicitly in Appendix 5D.3. The projected potential becomes the variational free energy $\mathcal{F}$, because the hidden-sector elimination identifies the admissible statistical mismatch cost with the KL divergence between q_μ and the projected generative density. The Euler–Lagrange variation of Eq. (5.195) yields Eq. (5.196). The overdamped limit removes the second-order term and leaves the natural-gradient descent (5.197). The active-state update (5.198) follows by the same reduction applied to the action variables. □

This derivation is the formal bridge from the present physics paper to later life, brain, and mind applications. It says that a persistent adaptive subsystem can be described as an open-system packet living on a statistical manifold whose geometry is inherited from the same projected bulk action. The resulting inferential descent is not a separate law introduced only for neuroscience. It is the inferential gauge of the same Qi-EP selector that also appears as PLT and MEPP.

Table 5.35: Step-by-step structure of the FEP derivation.

Step	**Equation**	**Meaning**
Blanket decomposition	(5.190)	External hidden causes, sensory states, active states, and internal states are separated.
Projected generative model	(5.191)	The subsystem compares its recognition density with the projected world-model.
Statistical slice	(5.193)	The current-density action is restricted to a parametric statistical manifold.
Fisher metric	(5.194)	The projected kinetic term becomes information geometry.
Inferential action	(5.195)	The same reduced action becomes a Fisher-kinetic plus free-energy functional.
Natural-gradient descent	(5.197)	In the overdamped limit the selector becomes FEP.

5.8.7 The unified Qi-EPtheorem: one projected functional, three gauges

The three previous subsections are the heart of the section. They show that PLT, MEPP, and FEP are not independent principles living in separate disciplines. They are three reductions of one master current-density action.

It is useful to package this result formally. Define the quadratic form

$$\mathbb{K}_\rho(\delta J, \delta J) := \int_{\mathcal{X}} \delta J^a (\rho D)^{-1}_{ab} \delta J^b \, \mathrm{d}z, \tag{5.199}$$

and write the projected action compactly as

$$\mathfrak{Q}_{\text{Qi-EP}}[\rho, J] = \int_0^T \mathrm{d}t \, \left[\frac{1}{4} \mathbb{K}_\rho(J - \rho u, J - \rho u) + \mathcal{V}[\rho] \right] \qquad \text{subject to} \qquad \partial_t \rho + \nabla \cdot J = 0. \tag{5.200}$$

Then the three gauges may be summarized as follows:

$$\text{trajectory gauge:} \quad (\rho, J) \mapsto \big(\delta_{z(t)}, \delta_{z(t)}\dot{z}(t)\big) \quad \Longrightarrow \quad \mathfrak{Q}_{\text{Qi-EP}} \to \mathcal{T}[z], \tag{5.201}$$

$$\text{current gauge:} \quad (\rho, J) \mapsto (\rho, J) \quad \Longrightarrow \quad \mathfrak{Q}_{\text{Qi-EP}} \to \mathcal{R}_\rho[J; X], \tag{5.202}$$

$$\text{inferential gauge:} \quad (\rho, J) \mapsto \rho_{\mu,s,a} \quad \Longrightarrow \quad \mathfrak{Q}_{\text{Qi-EP}} \to \mathcal{A}_{\text{inf}}[\mu]. \tag{5.203}$$

Theorem 5.8.6 (Unified gauge theorem for Qi-EP)**.** *Within the* GG-6/GDOS *framework, the projected functional* (5.200) *generates the three boundary principles*

$$\text{PLT}, \qquad \text{MEPP}, \qquad \text{FEP}$$

as its trajectory, current, and inferential gauges respectively. Therefore the three principles are gauge-equivalent boundary faces of one projected 6D variational law.

Proof. The proof is the composition of Theorems 5.8.3, 5.8.4, and 5.8.5. Each theorem establishes that a distinct admissible slice of Γ_{adm} together with a distinct coarse-grained coordinate choice reduces the same master functional (5.200) to one of the three familiar boundary forms. Since the starting functional is the same in all three cases, the resulting principles are not independent axioms but gauge-equivalent reductions. □

The most important conceptual consequence is that Qi-EP restores unity across scientific domains. A transport problem, a metabolic steady state, and an adaptive inferential system no longer require three unrelated foundational principles. They require one projected selector viewed in three different gauges. This is exactly the kind of cross-domain unification that Chapter 5 is meant to achieve.

Table 5.36: The three gauges of Qi-EP.

Gauge	**Primary variable**	**Reduced functional**	**Operational interpretation**
Trajectory gauge	$z(t)$	$\mathcal{T}[z]$	Path or branch selection under an effective metric and connection (PLT).
Current gauge	(ρ, J)	$\mathcal{R}_\rho[J; X]$	Force-controlled flux selection and entropy-production extremization (MEPP).
Inferential gauge	$\mu(t)$	$\mathcal{A}_{\text{inf}}[\mu]$	Natural-gradient descent of variational free energy (FEP).

5.8.8 DGI, GSSB, and GA inside the Qi-EP landscape

Because Qi-EP is a projected selector, it must mesh exactly with the runtime engine developed in Section 5.4. This is not an optional consistency check. If the selector were incompatible with DGI, GSSB, or GA, then Chapter 5 would split into two disconnected theories. Fortunately the compatibility is structural.

5.8.8.1 DGI: gauge orbits of the selector

Let Λ denote an informational gauge parameter acting on the projected connection a_a and, in inferential gauges, on the phase convention of the recognition family. Then

$$a_a \mapsto a_a + \partial_a \Lambda, \qquad q_\mu \mapsto e^{\Lambda}\, q_\mu \quad \text{(up to normalization on the statistical slice).} \tag{5.204}$$

The change in $\mathfrak{Q}_{\text{Qi-EP}}$ is a boundary term,

$$\Delta \mathfrak{Q}_{\text{Qi-EP}} = \Lambda\Big|_{t=T} - \Lambda\Big|_{t=0}, \tag{5.205}$$

so the stationary equations are gauge-invariant. Therefore extrema occur in gauge orbits. This is the Qi-EP manifestation of DGI: the selector does not choose one microscopic parametrization; it chooses one gauge-equivalence class of admissible histories.

5.8.8.2 GSSB: branch creation by deformation of the selector

Suppose that the projected functional depends on control parameters λ (boundary forcing, coupling strengths, geometry, resource availability, blanket topology, and so on). Let m denote a reduced coordinate on the space of candidate stationary states. Then the branch-creation condition is

$$\frac{\partial \mathfrak{Q}_{\text{Qi-EP}}}{\partial m} = 0, \qquad \det\left[\frac{\partial^2 \mathfrak{Q}_{\text{Qi-EP}}}{\partial m^i \partial m^j}\right] = 0. \tag{5.206}$$

At such points the extremal structure changes: one branch becomes unstable, two or more branches emerge, or the stabilizer subgroup decreases. This is exactly the language of GSSB in the selector landscape. In trajectory problems it appears as path splitting; in current problems as a new transport regime; in inferential problems as a new posterior or policy attractor.

5.8.8.3 GA: stable extrema are attractors

Finally, the selected branch matters only if it persists. In the gradient sectors of Qi-EP one has a direct Lyapunov statement. For example, in the inferential gauge,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F} = \partial_i \mathcal{F}\, \dot{\mu}^i = -\kappa_\mu\, \partial_i \mathcal{F}\, \big(g^{\mathrm{F}}\big)^{ij} \partial_j \mathcal{F} \le 0, \tag{5.207}$$

so minima of $\mathcal{F}$ are attractors. In the current gauge the dual dissipation functional decreases along the relaxation to the stationary current, and in the trajectory gauge the effective action defines the extremal branch selected under the imposed boundary data. Thus GA appears as the persistence criterion for the selected Qi-EP extremum.

Table 5.37: How the runtime engine enters the Qi-EP selector.

Runtime element	**Role inside Qi-EP**	**Operational meaning**
DGI	Gauge orbits of stationary histories	Different microscopic descriptions represent the same selected law.
GSSB	Bifurcation and branch creation	New paths, new flux regimes, or new inferential policies appear when control parameters cross thresholds.
GA	Stability of the selected extremum	The selected history persists as an attractor rather than a transient fluctuation.

5.8.9 Why Qi-EPapplies to all open systems

At this stage it is worth stepping back and asking why the section matters beyond formal derivation. The answer is that every natural open system presents some combination of the three selector questions:

(i) Which path or branch is followed?

(ii) Which flux pattern or throughput is sustained?

(iii) Which internal model or policy is stabilized?

A purely physical transport problem may use almost exclusively the first question. A reaction network, molecular motor, or ecosystem may use primarily the second. A cell, brain, or adaptive agent inevitably uses the third. But no natural system is forbidden from moving between gauges. A cell chemotaxes along a path (PLT language), maintains metabolic fluxes (MEPP language), and updates internal states in response to sensory mismatch (FEP language). The same object therefore occupies all three gauges at once, depending on which variables have been retained by the observer.

This is why the present derivation cannot be restricted to one discipline. If Qi-EP were only a neuroscience principle, it could not serve as Pillar II of a grand open-system framework. If it were only a non-equilibrium thermodynamic principle, it could not later support Qi-HAL and the inner-hologram architecture. If it were only a geodesic-trajectory principle, it could not explain living systems. The point of the present derivation is exactly that the same projected 6D variational law has enough flexibility to cover all three uses without multiplying foundational assumptions.

The universality claim can be summarized in a domain table.

Table 5.38: Illustrative domain-wise readings of Qi-EP.

Domain	**Most natural gauge**	**Selected object**	**Illustrative reading**
Particle and wave transport	PLT	Path or branch	Effective geodesic, tunneling channel, or propagation branch under the projected boundary geometry.
Chemistry and reaction networks	MEPP	Flux pattern	Stationary throughput through coupled channels under force-controlled or resource-controlled driving.
Molecular machines	MEPP + PLT	Cycle and substep path	Driven cycles and substep selections constrained by the Qi-QUA budget.
Cells and homeostatic systems	MEPP + FEP	Metabolic plus inferential state	Throughput must be maintained while internal states track the hidden environment.
Brains and cognition	FEP + PLT	Inference trajectory and policy	Prediction, sensing, and action unfold on an information-geometric manifold.
Societies and collective systems	All three	Path, flow, and policy	Collective trajectories, throughputs, and internal models are all needed simultaneously.

5.8.10 Why Qi-EPis correctly placed as Pillar II of the Qi program

It is now possible to say why Qi-EP deserves its own pillar and why it belongs immediately after Qi-QUA.

First, Qi-EP solves a distinct problem. Qi-QUA tells us the denomination of admissible current packets. Qi-EP tells us which packet history is realized. This is not a cosmetic distinction. The first is a ledger problem; the second is a selector problem.

Second, Qi-EP is the hinge on which the rest of the Qi program turns. Qi-CON will later identify the signatures left by the selected histories across scales. Qi-HAL will later identify the architectural structures that implement selection, memory, and prediction in persistent open systems. Neither of those later pillars can be written convincingly until the selector itself has been derived. In that sense Qi-EP is the logical bridge from the budget language of Qi-QUA to the architectural and phenomenological languages of Qi-HAL and Qi-CON.

Third, Qi-EP visibly unifies several historically separate scientific traditions. The geodesic or least-time tradition, the non-equilibrium entropy-production tradition, and the free-energy/inference tradition are usually taught as belonging to different subjects. In the present framework they are recognized as coordinate faces of the same projected selector. This is precisely the sort of unification that the broader GUT/GG-6/GDOS program is trying to achieve.

Table 5.39: Role of Qi-EP relative to the rest of Chapter 5.

Layer	Main question	Answer supplied
DGI+GSSB+GA	What histories are admissible, how do branches split, and why do some persist?	Runtime grammar of lawful open-system evolution.
Qi-QUA	What budget is attached to one ideal locked event?	Arisaka Ratio, Arisaka Constant, and Arisaka Equation.
Qi-EP	Which admissible history is selected?	Projected extremal selector with PLT, MEPP, and FEP gauges.
Qi-CON	What multiscale signatures reveal the selected runtime?	Conformality, heavy tails, fractality, and $1/f$-type spectra.
Qi-HAL	What architecture implements the selected runtime in persistent systems?	Holographic attractors, memory lattices, and prediction/action circuits.

5.8.11 Section summary

This section has completed the second quantitative pillar of Chapter 5.

First, Qi-EP was defined as the projected extremal selector obtained by applying OGGEP to the 6D GG-6 least-action law and then restricting to the admissible boundary history class Γ_{adm}. In this way the open-system selector was tied directly to the same ultraviolet constitution that fixed BI-6, OGN-5, and the current budget of Qi-QUA.

Second, the master projected boundary action was written in the current-density form (5.166). Proposition 5.8.2 showed that the realized boundary histories are stationary histories of this projected action.

Third, the trajectory gauge was derived explicitly. On the sharply localized slice, the projected action reduced to the generalized optical-time functional, and Theorem 5.8.3 established PLT as the trajectory gauge of Qi-EP.

Fourth, the current gauge was derived explicitly. Rewriting the same quadratic form as an Onsager–Rayleighian yielded the force-flux law $J = LX$, the entropy-production rate, and Theorem 5.8.4, which established MEPP as the force-controlled current gauge of Qi-EP together with its dual minimum-dissipation formulation.

Fifth, the inferential gauge was derived explicitly. Pulling the projected action back to a statistical manifold produced the Fisher metric, the variational free energy, and Theorem 5.8.5, which established FEP as the inferential gauge of Qi-EP.

Sixth, the three reduced forms were reunited by Theorem 5.8.6, which proved that PLT, MEPP, and FEP are three gauge-equivalent boundary faces of one projected 6D variational law.

The manuscript now contains the runtime layer, the scalar-budget layer (EEGG), the packet layer (SIP), the event-locking layer (Qi-QUA), and the selector layer (Qi-EP). The next natural step is therefore Qi-CON, where the question will no longer be "which history is selected?" but rather "what multiscale statistical signatures does the selected history leave behind?"

5.9 Qi-CON: Conformality, Lévy Statistics, Fractality, and $1/f$ with Two Knees

After budget and selector come diagnostics. Once Qi-QUA has fixed admissible current bookkeeping and Qi-EP has fixed the selected history, one can finally ask what observable multiscale signatures that selected runtime leaves behind. Section 5.9 therefore derives the conformal, Lévy, fractal, and $1/f$-type diagnostics of the selected branch.

5.9.1 Why Qi-CONmust follow Qi-EP

The order of the Qi pillars continues to be forced by logic rather than by taste. Qi-QUA answered the bookkeeping question: what is the admissible current ledger, and what is the correct natural-unit conversion between energy, electric, baryonic, and information currents? Qi-EP then answered the selector question: among the histories compatible with that ledger and with the DGI + GSSB + GA runtime, which path, branch, or inferential state is actually realized? But a selected history is still not yet a scientific signature. A runtime becomes empirically legible only when it leaves a reproducible pattern in fluctuations, correlations, spectra, and multiscale geometry. Qi-CON is the layer that answers that diagnostic question.

This point is easiest to see by writing the logical staircase explicitly. Let

$$\Gamma_{\mathrm{adm}} \xrightarrow{\text{Qi-QUA}} \Gamma_{\mathrm{budget}} \xrightarrow{\text{Qi-EP}} \gamma_* \xrightarrow{\text{Qi-CON}} \mathcal{D}(\gamma_*) \tag{5.208}$$

denote the passage from all admissible histories to the budget-constrained histories, then to the selected history or branch γ_*, and finally to the measurable diagnostic family $\mathcal{D}(\gamma_*)$. The first arrow restricts the current ledger, the second chooses a runtime history, and the third exports the observable multiscale signatures of that chosen runtime. In this sense, Qi-CON is neither a second budget law nor a second selector law. It is the first *readout law*. It tells the observer what the selected history looks like when coarse-grained over scales.

The dependency on Qi-EP is especially important. A system can display many possible fluctuation geometries if many branches remain available. Once Qi-EP selects a branch, however, the linearized and weakly nonlinear dynamics around that branch determine the low-lying spectrum, the effective jump kernel, the correlation functions, and the observable power laws. Said differently: Qi-EP chooses the sector, whereas Qi-CON computes the multiscale statistics of that chosen sector. This is why Qi-CON must come after Qi-EP.

The dependency on Qi-QUA is equally real, even if it is subtler. The scaling exponents of Qi-CON are dimensionless and do not by themselves carry energy denominations. But the amplitudes, cross-spectral ratios, and sector-to-sector comparisons only become physically meaningful once the Arisaka Equation has fixed the common current denominator. Thus Qi-QUA provides the budget normalization, Qi-EP provides the selector, and Qi-CON provides the diagnostic signature.

Table 5.40: Why Qi-CON must follow the first two pillars.

Pillar	**Primary question**	**Answer supplied**
Qi-QUA	What is the admissible current budget for one synchronized event packet?	Arisaka Ratio, Arisaka Constant, and Arisaka Equation define the universal ledger and unit conversion.
Qi-EP	Which admissible history or branch is realized?	The projected extremal functional selects paths, fluxes, or inferential states.
Qi-CON	What multiscale signatures reveal the selected runtime?	Scale covariance, power-law correlators, Lévy statistics, fractality, and $1/f$-type spectra with two knees.
Qi-HAL	What architecture implements and stabilizes the selected runtime?	Holographic attractor structures, frequency ladders, memory lattices, and prediction/action loops.

The rest of the section proceeds systematically. First, the constitutional form of Qi-CON is stated. Second, the warped 6D bulk and the OGGEP projection are shown to induce a homogeneous nonlocal kernel on the boundary. Third, that kernel is reduced to a fractional generator in the intermediate scale window. Fourth, the four signatures — conformality, Lévy statistics, fractality, and $1/f$ spectra with two knees — are derived one by one. Finally, the compatibility of Qi-CON with DGI + GSSB + GA, Qi-QUA, and Qi-EP is made explicit.

5.9.2 Constitutional statement of the Qi-CON principle

Before any detailed calculation, the central claim of Qi-CON should be stated in a precise operational form.

Definition 5.9.1 (Qi-CON)**.** Let S be an admissible open subsystem on the physical boundary state manifold $(\mathcal{X}_{\mathrm{phys}}, g)$, and let γ_* be a Qi-EP-selected branch of its projected dynamics. We say that S is in a Qi-CON window if there exist two cutoff scales

$$\ell_{\mathrm{micro}} \ll \ell \ll L_{\mathrm{att}} \qquad \Longleftrightarrow \qquad f_{\mathrm{IR}} \ll f \ll f_{\mathrm{UV}}, \tag{5.209}$$

and an exponent $0 < \alpha \leq 2$ such that the effective generator of the selected branch satisfies

$$\mathcal{L}_{\mathrm{eff}} = -\,D_\alpha\,(-\Delta_g)^{\alpha/2} + \mathcal{L}_{<\alpha}, \tag{5.210}$$

where $D_\alpha > 0$, $(-\Delta_g)^{\alpha/2}$ is the fractional Laplace–Beltrami operator on $(\mathcal{X}_{\mathrm{phys}}, g)$, and the remainder $\mathcal{L}_{<\alpha}$ contains lower-order, cutoff-sensitive, or weakly relevant deformations. In this window the leading dynamics are asymptotically homogeneous under scale transformations and generate power-law correlators, heavy-tailed coarse statistics, fractal support, and $1/f$-type spectra.

Three clarifications are essential.

First, Qi-CON does not claim exact scale invariance at all scales. Natural systems are finite, instrument-bounded, resource-limited, and often branch-selected by GSSB. Therefore the relevant statement is *intermediate* or *operational* conformality: there exists a nontrivial scale interval on which the leading generator is homogeneous to good approximation.

Second, the exponent α is not inserted by hand. It is inherited from the projected bulk-to-boundary kernel and therefore ultimately from the warped six-dimensional geometry together with the selected branch. The value $\alpha = 2$ reproduces Gaussian diffusion as a limiting case, but the generic Qi-CON regime of interest is $0 < \alpha < 2$, where nonlocal heavy-tailed transport survives at leading order.

Third, the phrase "conformality" is used in the operational sense appropriate to GDOS. In the strictest mathematical literature, full conformal invariance can be stronger than mere scale covariance. This book uses the term to denote the physically relevant combination of homogeneous generators, power-law correlators, and absence of an intrinsic scale in the active window. That is the feature required for a universal diagnostic layer.

Remark 5.9.2. The transition from exact microscopic law to operational conformality mirrors the transition from exact bulk action to projected boundary action already encountered in Qi-EP. In both cases the decisive point is not that the boundary is simpler than the bulk, but that the observer sees only the projected homogeneous core together with finite-size and instrument-induced corrections.

Table 5.41: Four canonical signatures of a Qi-CON window.

Signature	**Mathematical form**	**Meaning inside** GDOS
Operational conformality	$C(\lambda z, \lambda^{\alpha} t) = \lambda^{-2\Delta} C(z,t)$	No intrinsic scale inside the active window; correlations decay by power laws.
Lévy statistics	$\widehat{p}(k,t) = \exp[-D_{\alpha} t \lvert k \rvert^{\alpha}]$	Coarse increments are α-stable rather than Gaussian.
Fractality	$N(\varepsilon) \sim \varepsilon^{-D_f}$	Trajectories and selected supports occupy non-integer effective dimension.
$1/f$ with two knees	$S(f) \propto f^{-\beta}$ on $f_{\mathrm{IR}} < f < f_{\mathrm{UV}}$	The selected branch produces multiscale soft modes bounded by attractor and microphysical cutoffs.

5.9.3 From the warped 6D bulk to homogeneous boundary kernels

The derivation of Qi-CON begins not on the boundary but in the warped six-dimensional bulk already established in GG-2026-2 (Arisaka, 2026b). The extra radial direction is not an ornamental coordinate. It is the geometric carrier of scale. Writing

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right), \tag{5.211}$$

the ultraviolet boundary corresponds to $r = 0$ and lower operational scales correspond to larger r. Equivalently, the bulk geometry stores the renormalization structure that later appears on the boundary as multiscale dynamics (Arisaka, 2026b,c; Maldacena, 1998; Wilson, 1971; Wilson & Kogut, 1974).

To expose the scaling law cleanly, it is convenient to replace r by a positive scale coordinate

$$\zeta = e^{-r/L}, \tag{5.212}$$

where L is the warped radial length scale. Large ζ corresponds to coarse boundary scales, whereas small ζ corresponds to ultraviolet scales. Consider a projected Qi-neutral observable on a local physical state patch $U \subset \mathcal{X}_{\text{phys}}$ of dimension

$$d_{\mathcal{X}} = \dim U. \tag{5.213}$$

Let $\ell = d_g(z, z')$ denote the geodesic separation of two nearby boundary points $z, z' \in U$. For a dominant projected bulk mode with effective scaling weight Δ, the bulk-to-boundary propagator has the generic warped form

$$K_\Delta(\zeta; z, z') = \left(\frac{\zeta}{\zeta^2 + \ell^2}\right)^\Delta, \tag{5.214}$$

up to slowly varying geometric prefactors (Maldacena, 1998). The essential feature is homogeneity: the kernel depends on z, z' only through the dimensionless ratio ℓ/ζ.

The OGGEP projection integrates over the hidden scale coordinate and the unresolved sector. Write the induced boundary jump kernel in the schematic form

$$W_\Delta(z, z') = \int_{\zeta_{\min}}^{\zeta_{\max}} \omega(\zeta)\, K_\Delta(\zeta; z, z')\, \mathrm{d}\zeta, \tag{5.215}$$

where $\omega(\zeta)$ is the effective radial weight after KK summation, warp-factor Jacobians, and instrument projection. In the active scaling window one may write

$$\omega(\zeta) = \zeta^{\sigma-1} w(\zeta), \qquad w(\zeta) \approx w_0, \tag{5.216}$$

with slowly varying w_0. Substituting Eq. (5.214) into Eq. (5.215) gives

$$W_\Delta(z, z') \approx w_0 \int_{\zeta_{\min}}^{\zeta_{\max}} \frac{\zeta^{\sigma+\Delta-1}}{(\zeta^2 + \ell^2)^\Delta}\, \mathrm{d}\zeta. \tag{5.217}$$

Now perform the scale-separating substitution

$$\zeta = \ell u, \qquad \mathrm{d}\zeta = \ell\, \mathrm{d}u. \tag{5.218}$$

One obtains

$$W_\Delta(z, z') \approx w_0\, \ell^{\sigma-2\Delta} \int_{\zeta_{\min}/\ell}^{\zeta_{\max}/\ell} \frac{u^{\sigma+\Delta-1}}{(1+u^2)^\Delta}\, \mathrm{d}u. \tag{5.219}$$

Inside the operational conformal window,

$$\zeta_{\min} \ll \ell \ll \zeta_{\max}, \tag{5.220}$$

the integral in Eq. (5.219) saturates to a finite dimensionless constant $C_{\Delta,\sigma}$, and therefore

$$W_\Delta(z, z') \propto \ell^{-(2\Delta-\sigma)}. \tag{5.221}$$

This is the decisive scaling step: after the hidden radial coordinate has been integrated out, the boundary inherits a power-law kernel with no intrinsic scale of its own. The full algebra is collected in Appendix 5E.1.

To compare Eq. (5.221) with standard nonlocal generators, write

$$2\Delta - \sigma = d_{\mathcal{X}} + \alpha, \qquad 0 < \alpha \leq 2. \tag{5.222}$$

Then

$$W_\Delta(z, z') \propto \frac{1}{d_g(z, z')^{d_{\mathcal{X}}+\alpha}}, \tag{5.223}$$

which is exactly the homogeneity required for a fractional jump operator on U.

Proposition 5.9.3 (Homogeneous kernel induced by OGGEP)**.** *Assume that a* Qi-EP*-selected branch admits an operational window satisfying Eq.* (5.220) *and that the projected radial weight is slowly varying there. Then the* OGGEP*-induced nonlocal kernel on the corresponding physical boundary patch is asymptotically homogeneous with degree* $-(d_{\mathcal{X}} + \alpha)$*. Consequently the leading nonlocal part of the effective generator is*

$$(\mathcal{J}_\alpha f)(z) = C_{d_{\mathcal{X}},\alpha}\ \mathrm{PV} \int_U \frac{f(z') - f(z)}{d_g(z, z')^{d_{\mathcal{X}}+\alpha}}\, \mathrm{d}\mu_g(z'), \tag{5.224}$$

which, in a local flat patch, reduces to

$$\mathcal{J}_\alpha = -\, D_\alpha\, (-\Delta_g)^{\alpha/2}. \tag{5.225}$$

Proof. Equation (5.219) already shows that the projected kernel is homogeneous with degree $-(2\Delta - \sigma)$. Identifying that degree with $d_{\mathcal{X}} + \alpha$ gives Eq. (5.223). The principal-value integral (5.224) is the unique translation-covariant jump operator with that homogeneity on a local Euclidean patch, and the curved-space generalization is the fractional Laplace–Beltrami operator to leading order in the local chart. Thus the nonlocal sector of the boundary generator has the form (5.225). □

The proposition explains the geometric origin of the exponent α. It is not introduced phenomenologically by fitting a heavy-tailed random walk to data. It is the boundary image of the warped hidden-scale integral. Once the selected branch fixes the dominant mode Δ and the effective measure exponent σ, the exponent α is fixed.

5.9.4 The leading Qi-CONgenerator and operational conformality

On the selected branch γ_*, the most general projected boundary equation combines drift, local diffusion, and the nonlocal term derived above:

$$\partial_t \rho = -\,\nabla_a\big(u^a \rho\big) + \nabla_a \nabla_b\big(D^{ab} \rho\big) - D_\alpha (-\Delta_g)^{\alpha/2} \rho + \mathcal{S}_{\mathrm{ext}}[\rho]. \tag{5.226}$$

Here u^a collects branch-dependent drift or transport, D^{ab} is the local diffusion tensor, and $\mathcal{S}_{\mathrm{ext}}$ denotes sources, sinks, or instrument-specific deformations. Equation (5.226) is exact enough for interpretation but still too detailed for a clean scaling analysis.

The Qi-CON window is precisely the regime in which the homogeneous nonlocal sector dominates. This occurs when the observational scale is much larger than the microscopic local-diffusion scale but much smaller than the attractor-confinement scale. In that regime one may write

$$\partial_t \rho = -\, D_\alpha (-\Delta_g)^{\alpha/2} \rho + \text{sub-leading terms}. \tag{5.227}$$

The scale covariance of the leading term is immediate. Let

$$(\mathcal{S}_\lambda f)(z) = f(\lambda z), \qquad \lambda > 0. \tag{5.228}$$

Then a direct Fourier or coordinate calculation yields

$$\mathcal{S}_\lambda^{-1} (-\Delta_g)^{\alpha/2} \mathcal{S}_\lambda = \lambda^\alpha (-\Delta_g)^{\alpha/2}, \tag{5.229}$$

modulo curvature corrections that are suppressed on the local scaling patch. Therefore Eq. (5.227) is invariant under the simultaneous transformation

$$z \mapsto \lambda z, \qquad t \mapsto \lambda^\alpha t. \tag{5.230}$$

This is the core of operational conformality: the selected open-system dynamics possess no preferred length or time scale inside the active interval.

A point-source initial condition makes the scaling explicit. In a local flat patch, Fourier transformation of Eq. (5.227) gives

$$\partial_t \widehat{\rho}(k,t) = -\, D_\alpha |k|^\alpha \widehat{\rho}(k,t), \tag{5.231}$$

so that

$$\widehat{\rho}(k,t) = e^{-D_\alpha t |k|^\alpha}\, \widehat{\rho}(k,0). \tag{5.232}$$

For $\rho(z,0) = \delta(z)$, inverse transformation yields the scaling kernel

$$p_\alpha(t,z) = t^{-d_{\mathcal{X}}/\alpha} \Phi_\alpha\!\left(\frac{z}{t^{1/\alpha}}\right), \tag{5.233}$$

where Φ_α is a dimensionless profile determined by α and the local metric. Equation (5.233) is the precise semigroup form of Qi-CON scaling.

Theorem 5.9.4 (Operational conformality theorem)**.** *Suppose that a* Qi-EP*-selected branch admits the* Qi-CON *leading equation* (5.227) *on a local physical state patch of dimension* $d_{\mathcal{X}}$*. Then the fundamental solution satisfies the homogeneity law*

$$p_\alpha(\lambda^\alpha t, \lambda z) = \lambda^{-d_{\mathcal{X}}}\, p_\alpha(t,z). \tag{5.234}$$

Consequently, for any scale-covariant Qi-neutral observable O *with effective boundary dimension* Δ_O*, the two-point function in the active window obeys*

$$\langle O(\lambda z, \lambda^\alpha t)\, O(0,0)\rangle = \lambda^{-2\Delta_O} \langle O(z,t)\, O(0,0)\rangle, \tag{5.235}$$

and in particular at equal time

$$\langle O(z)\, O(0)\rangle \propto |z|^{-2\Delta_O}. \tag{5.236}$$

Proof. Equation (5.234) follows directly from Eq. (5.233) by rescaling t and z. Equation (5.235) then follows from dimensional covariance of the semigroup acting on observables with effective weight Δ_O. Setting $t = 0$ gives Eq. (5.236). □

The theorem is intentionally phrased in terms of an operational window rather than a global conformal field theory. That is the natural level for GDOS. The observer does not access the full microscopic theory directly; the observer accesses the homogeneous boundary image of the selected branch over a finite but nontrivial scale range.

5.9.5 Second signature: Lévy statistics from the fractional generator

The second signature of Qi-CON is the appearance of α-stable coarse statistics. Once the leading generator has the fractional form (5.227), the characteristic function of a scalar coarse coordinate follows immediately. Let X_t denote one local coordinate projection of the boundary state. In the drift-free leading regime its density $p(x,t)$ satisfies

$$\partial_t p(x,t) = -D_\alpha(-\partial_x^2)^{\alpha/2} p(x,t). \tag{5.237}$$

Fourier transformation gives

$$\partial_t \widehat{p}(k,t) = -D_\alpha |k|^\alpha \widehat{p}(k,t), \tag{5.238}$$

with solution

$$\widehat{p}(k,t) = \exp[-D_\alpha t|k|^\alpha]\,\widehat{p}(k,0). \tag{5.239}$$

For a localized initial packet $\widehat{p}(k,0) = 1$, the characteristic function becomes

$$\phi_{X_t}(k) = \mathbb{E}\left[e^{\mathrm{i}kX_t}\right] = \exp[-D_\alpha t|k|^\alpha]. \tag{5.240}$$

This is the defining characteristic function of a symmetric α-stable law. The full derivation is written out in Appendix 5E.2 and agrees with the standard anomalous-diffusion literature (van Kampen, 2007; Metzler and Klafter, 2000).

Several consequences follow at once.

First, the process is self-similar:

$$X_t \overset{d}{=} t^{1/\alpha} X_1. \tag{5.241}$$

Second, for $0 < \alpha < 2$ the tails are heavy:

$$p(x,t) \sim \frac{D_\alpha t\,\Gamma(1+\alpha)\sin(\pi\alpha/2)}{\pi\,|x|^{1+\alpha}} \qquad (|x| \to \infty), \tag{5.242}$$

so the coarse increments are Lévy rather than Gaussian. Third, the Gaussian case appears only as the codimension-one limit $\alpha = 2$, where Eq. (5.240) becomes $e^{-D_2 t k^2}$.

Corollary 5.9.5 (Why heavy tails are generic inside Qi-CON)**.** *Within an operational* Qi-CON *window, the dominant boundary increments are generically heavy-tailed whenever* $0 < \alpha < 2$*. Exact Gaussianity requires* $\alpha = 2$*, which corresponds to the disappearance of the leading nonlocal contribution.*

Proof. Equation (5.240) is α-stable for all $0 < \alpha \leq 2$. For $0 < \alpha < 2$, the inverse Fourier transform has the asymptotic power law (5.242); for $\alpha = 2$ the law becomes Gaussian. Hence heavy tails are generic except on the Gaussian boundary. □

Physically, this means that selected open-system trajectories retain rare but consequential excursions across scales. The system does not explore its branch by small equal-sized Brownian steps alone. It mixes local relaxation with occasional larger state-space moves. In an exact mathematical model these are Lévy flights; in realistic finite systems they are often truncated or tempered by the same cutoffs that define the operational window. The present framework therefore predicts *approximately* α-stable increments over the active window, not an infinite-scale law.

5.9.6 Third signature: fractality of trajectories and selected supports

The third Qi-CON signature is fractality. This statement should not be treated as a poetic metaphor. It follows from the same self-similarity exponent that controlled the characteristic function.

Let R_T denote the range of a coarse trajectory on a finite observation interval:

$$R_T = \{X_t \mid 0 \leq t \leq T\} \subset U. \tag{5.243}$$

To estimate its box-counting dimension, cover R_T with boxes of side ε. Because the process is self-similar with exponent $1/\alpha$, a time interval δt typically produces a spatial excursion of size

$$\delta x \sim (\delta t)^{1/\alpha}. \tag{5.244}$$

To resolve the path at spatial resolution ε, choose $\delta x \sim \varepsilon$, which implies

$$\delta t \sim \varepsilon^{\alpha}. \tag{5.245}$$

An observation window of total duration T therefore contains approximately

$$N_T(\varepsilon) \sim \frac{T}{\delta t} \sim T\,\varepsilon^{-\alpha} \tag{5.246}$$

independent ε-scale segments. This estimate cannot exceed the number of ε-boxes available in the ambient physical patch, which scales as $\varepsilon^{-d_{\mathcal{X}}}$. Therefore the effective box-counting dimension of the range is

$$D_f = \min(d_{\mathcal{X}}, \alpha). \tag{5.247}$$

The derivation is elementary but important enough that it is recorded separately in Appendix 5E.3; it is also consistent with the standard fractal viewpoint (Mandelbrot, 1982).

Equation (5.247) immediately explains why Qi-CON belongs in a theory of selected open systems rather than in a theory of unrestricted random walks. The ambient manifold dimension $d_{\mathcal{X}}$ sets the maximal available space, while the exponent α sets the intrinsic scaling of the selected path. The realized fractal support is the lesser of the two. When GSSB further constrains the branch to a submanifold or attractor sheet, the effective support dimension can only decrease. Thus the

fractality seen by the observer is not just a property of the transport kernel; it is also a property of branch selection and attractor confinement.

Proposition 5.9.6 (Fractal support of a selected Qi-CONbranch)**.** *Let a* Qi-EP*-selected branch possess a* Qi-CON *window with exponent* $0 < \alpha \leq 2$ *on an ambient physical patch of dimension* $d_{\mathcal{X}}$*. Then the coarse trajectory range satisfies the box-counting estimate*

$$N(\varepsilon) \sim \varepsilon^{-D_f}, \qquad D_f = \min(d_{\mathcal{X}}, \alpha), \tag{5.248}$$

over $\ell_{\text{micro}} < \varepsilon < L_{\text{att}}$*. If* GSSB *restricts the branch to an attractor support* $A \subset U$*, then the observable support dimension satisfies* $D_A \leq D_f$*.*

Proof. The range estimate follows from Eqs. (5.244)–(5.247). The second statement follows because any attractor support A visited by the selected branch is a subset of the ambient range and therefore cannot have a larger box-counting dimension than the range itself. □

A useful way to phrase the result is that Qi-CON predicts *fractal admissibility*. The selected branch need not fill the ambient state space densely; indeed it usually does not. It occupies a support whose effective dimension is determined jointly by the scale exponent and by the symmetry-breaking constraints that shaped the branch.

Table 5.42: How the exponent α organizes the first three Qi-CON signatures.

Object	**Scaling law**		**Interpretation**
Semigroup kernel	$p_\alpha(t,z)$ $t^{-d_{\mathcal{X}}/\alpha}\Phi_\alpha(z/t^{1/\alpha})$	$=$	Operational conformality and absence of intrinsic scale in the active window.
Characteristic function	$\phi_{X_t}(k) = e^{-D_\alpha t\lvert k\rvert^\alpha}$		α-stable Lévy statistics for coarse increments.
Fractal support	$D_f = \min(d_{\mathcal{X}}, \alpha)$		Non-integer effective dimension of selected trajectory ranges or supports.

5.9.7 Fourth signature: $1/f$ spectra with two knees

The fourth Qi-CON signature is the most directly measurable in many applications: a broad spectral band of the form $1/f^\beta$, bounded by two characteristic knees. The derivation proceeds most transparently from the spectral representation of the selected branch generator.

Let $X(t)$ be a centered observable measured on the selected branch. Write its autocorrelation function as a mode expansion over the nonnegative spectrum of the reduced generator:

$$C_X(\tau) = \langle X(t+\tau)X(t)\rangle = \int_{\lambda_{\text{IR}}}^{\lambda_{\text{UV}}} A_X(\lambda)\, e^{-\lambda|\tau|}\, \rho(\lambda)\, \mathrm{d}\lambda. \tag{5.249}$$

Here $\rho(\lambda)$ is the density of relaxation modes, $A_X(\lambda)$ is the observable-dependent spectral weight, λ_{IR} is the slowest relevant mode, and λ_{UV} is the fastest mode retained by the operational description. Inside the Qi-CON window, the same homogeneity that produced the fractional

operator implies a power-law soft-mode density:

$$A_X(\lambda)\rho(\lambda) \approx A_0\,\lambda^{\gamma-1}, \qquad 0 \le \gamma < 1, \tag{5.250}$$

for $\lambda_{\mathrm{IR}} < \lambda < \lambda_{\mathrm{UV}}$. The nearly critical case $\gamma \approx 0$ is the one that leads to true $1/f$ behavior. The power spectral density is

$$S_X(\omega) = 2\int_0^\infty C_X(\tau)\cos(\omega\tau)\,\mathrm{d}\tau. \tag{5.251}$$

Substituting Eq. (5.249) and evaluating the Laplace transform of each exponential mode yields

$$S_X(\omega) = 2\int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \frac{\lambda\,A_X(\lambda)\rho(\lambda)}{\lambda^2+\omega^2}\,\mathrm{d}\lambda. \tag{5.252}$$

Using the power-law density (5.250), one finds

$$\begin{aligned} S_X(\omega) &\approx 2A_0\int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \frac{\lambda^\gamma}{\lambda^2+\omega^2}\,\mathrm{d}\lambda \\ &= 2A_0\,\omega^{\gamma-1}\int_{\lambda_{\mathrm{IR}}/\omega}^{\lambda_{\mathrm{UV}}/\omega} \frac{u^\gamma}{1+u^2}\,\mathrm{d}u, \qquad u = \frac{\lambda}{\omega}. \end{aligned} \tag{5.253}$$

Now the three frequency regimes separate cleanly.

Intermediate band: the $1/f^\beta$ law. If

$$\lambda_{\mathrm{IR}} \ll \omega \ll \lambda_{\mathrm{UV}}, \tag{5.254}$$

the integration limits in Eq. (5.253) may be extended to 0 and ∞, giving a constant dimensionless integral:

$$\int_0^\infty \frac{u^\gamma}{1+u^2}\,\mathrm{d}u = \frac{\pi}{2}\sec\!\left(\frac{\pi\gamma}{2}\right), \qquad 0 \le \gamma < 1. \tag{5.255}$$

Therefore

$$S_X(\omega) \propto \omega^{\gamma-1} = \frac{1}{\omega^\beta}, \qquad \beta = 1-\gamma. \tag{5.256}$$

The nearly critical case $\gamma \approx 0$ gives $\beta \approx 1$, i.e. genuine $1/f$ behavior.

Low-frequency side: the infrared knee. If $\omega \ll \lambda_{\mathrm{IR}}$, then $\lambda^2+\omega^2 \approx \lambda^2$, so

$$S_X(\omega) \approx 2A_0\int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \lambda^{\gamma-2}\,\mathrm{d}\lambda = \text{constant}. \tag{5.257}$$

Thus the spectrum flattens at low frequency. In ordinary language, there is a largest effective relaxation time beyond which the selected branch behaves as a finite system rather than as a scale-free one.

High-frequency side: the ultraviolet knee. If $\omega \gg \lambda_{\mathrm{UV}}$, then $\lambda^2 + \omega^2 \approx \omega^2$, so

$$S_X(\omega) \approx \frac{2A_0}{\omega^2} \int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \lambda^{\gamma}\, \mathrm{d}\lambda \propto \omega^{-2}. \tag{5.258}$$

Thus the spectrum steepens above the active window and leaves the scale-free regime.

These three regimes may be written compactly as

$$S_X(f) \approx \begin{cases} S_0, & f < f_{\mathrm{IR}}, \\ A\, f^{-\beta}, & f_{\mathrm{IR}} < f < f_{\mathrm{UV}}, \\ B\, f^{-2}, & f > f_{\mathrm{UV}}, \end{cases} \tag{5.259}$$

with

$$f_{\mathrm{IR}} = \frac{\lambda_{\mathrm{IR}}}{2\pi}, \qquad f_{\mathrm{UV}} = \frac{\lambda_{\mathrm{UV}}}{2\pi}. \tag{5.260}$$

The detailed algebra is given in Appendix 5E.4. The key point is that the two knees are not arbitrary fitting decorations. They are forced by the existence of both an attractor-scale cutoff and a microphysical cutoff.

Theorem 5.9.7 (Two-knee spectral theorem)**.** *Suppose a* Qi-EP*-selected branch possesses a* Qi-CON *window with soft-mode density* (5.250)*. Then the power spectral density of any observable with nonvanishing overlap on that soft sector obeys Eq.* (5.259)*. The infrared knee is fixed by the longest branch or attractor relaxation time, and the ultraviolet knee is fixed by the shortest retained microscopic time.*

Proof. Equations (5.252)–(5.258) already provide the three asymptotic regimes. The infrared plateau is controlled by the lower modal cutoff λ_{IR}, while the ultraviolet roll-off is controlled by the upper cutoff λ_{UV}. Translating to ordinary frequency gives Eq. (5.260). □

This theorem is central because it converts a broad and often qualitative empirical observation — "many open systems show $1/f$-like spectra" — into a structural consequence of the selected branch geometry. The scale-free band is not universal by accident. It appears because the selected branch carries a dense soft-mode sector between a largest confinement scale and a smallest microphysical scale.

Table 5.43: Physical meaning of the two Qi-CON knees.

Knee	**Mathematical origin**	**Physical interpretation**
Infrared knee f_{IR}	Lowest retained relaxation rate λ_{IR}	Largest branch or attractor timescale; beyond this the finite selected system behaves as a bounded object rather than a scale-free one.
Ultraviolet knee f_{UV}	Highest retained relaxation rate λ_{UV}	Fastest microscopic or instrument-resolved process; above this the effective coarse description breaks down and the spectrum steepens.

5.9.8 Why the Qi-CONexponents are runtime invariants

The four signatures just derived do not float independently of the master runtime. They are constrained by DGI, GSSB, GA, Qi-QUA, and Qi-EP. This subsection makes those constraints explicit.

5.9.8.1 DGI: gauge invariance of the spectrum

A local gauge change on the selected branch acts by conjugation on the reduced generator:

$$\mathcal{L}_{\text{eff}} \mapsto \mathcal{L}_{\text{eff}}^{(U)} = \mathcal{U}\,\mathcal{L}_{\text{eff}}\,\mathcal{U}^{-1}, \tag{5.261}$$

up to the same connection terms already analyzed in Section 5.4. Conjugation does not change the eigenvalue spectrum, and therefore it does not change the exponents α, γ, β extracted from that spectrum. The observed Qi-CON exponents are therefore DGI-invariants of the selected branch, provided one works with gauge-invariant observables.

Proposition 5.9.8 (DGI invariance of Qi-CONexponents)**.** *Let $\mathcal{L}_{\text{eff}}$ and $\mathcal{L}_{\text{eff}}^{(U)}$ be related by the admissible boundary gauge transformation (5.261). Then the* Qi-CON *exponents α, γ, β and the knee locations extracted from the spectrum of gauge-invariant observables are unchanged.*

Proof. Similarity transformations preserve the spectrum. The exponents α and γ are defined by the homogeneity of the generator and by the low-lying spectral density, while $\beta = 1 - \gamma$ follows from Eq. (5.256). Therefore these quantities are invariant under the transformation (5.261). □

5.9.8.2 GSSB: branch choice fixes the universality class

GSSB enters because different selected branches can carry different effective kernels, different confining geometries, and different modal cutoffs. If the admissible runtime offers several broken branches

$$\gamma_1, \gamma_2, \ldots, \gamma_n, \tag{5.262}$$

then each may come with its own exponent set

$$(\alpha_i, \gamma_i, \beta_i, D_{f,i}, f_{\text{IR},i}, f_{\text{UV},i}). \tag{5.263}$$

Qi-EP chooses one branch; Qi-CON then describes that chosen branch. This is why Qi-CON does not replace GSSB or Qi-EP. It inherits from them.

5.9.8.3 GA: attractor confinement fixes the infrared knee

GA enters because the selected branch is not just a set of formal trajectories; it is a persistent attractor structure. The largest relaxation time of that attractor is exactly what becomes λ_{IR}^{-1}. Therefore the infrared knee is a direct attractor observable:

$$f_{\text{IR}} \sim \frac{1}{\tau_{\text{GA}}}, \tag{5.264}$$

where τ_{GA} is the slowest decay time back to the selected attractor. This gives Qi-CON a clean bridge to the persistence language of Section 5.4.

5.9.8.4 Qi-QUA: spectral locking under the Arisaka Equation

The diagnostic layer also inherits a nontrivial consequence from the current budget of Section 5.7. In the ideal locked regime,

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi}. \tag{5.265}$$

Since power spectra are quadratic in amplitudes, exact locking implies

$$\frac{S_{J_E}(f)}{Q_B^2} = S_{J_e}(f) = S_{J_B}(f) = S_{J_{Qi}}(f) \tag{5.266}$$

throughout any common Qi-CON window. Even when realistic overhead factors disturb exact equality, the slopes β remain common while the amplitudes differ only by sector-dependent prefactors inherited from the same current conversion map.

Proposition 5.9.9 (Spectral Arisaka locking)**.** *On an ideal locked branch satisfying the Arisaka Equation, all sector-resolved power spectra share the same* Qi-CON *exponent β, and their amplitudes are related by the unit conversion* (5.266)*.*

Proof. If $J_E/Q_B = J_e = J_B = J_{Qi}$ pointwise in time, then the four current time series are identical up to the constant multiplicative factor Q_B in the energy channel. The Fourier transforms inherit the same relation, and the power spectra inherit its square. Therefore all four spectra have the same slope and differ only by the fixed unit conversion. □

5.9.8.5 Qi-EP: the selector decides which spectrum is seen

Finally, Qi-EP supplies the selector without which the diagnostic layer would be ambiguous. If several branches are available, each may carry a different soft-mode density and different knees. The observed Qi-CON signatures are therefore the signatures of the *selected* branch, not of the union of all conceivable branches. That is why the pillars must occur in the order budget → selector → diagnostic.

Table 5.44: How the earlier runtime layers constrain Qi-CON.

Earlier layer	**Constraint on Qi-CON**	**Consequence**
DGI	Similarity-covariant generator and gauge-invariant observables	Scaling exponents are branch invariants, not gauge artifacts.
GSSB	Branch and support selection	Different broken branches can carry different exponent sets and knees.
GA	Attractor persistence and return times	The infrared knee is fixed by the slowest attractor relaxation mode.
Qi-QUA	Common current denomination	Sector-resolved amplitudes and cross-spectral ratios are tied by the Arisaka Equation.
Qi-EP	Extremal branch selection	The observed spectrum is the spectrum of the selected branch.

5.9.9 Why Qi-CONapplies to all open systems

The universality of Qi-CON now follows from a simple structural observation. Any open system with

(i) a nontrivial separation between microscopic and macroscopic scales,

(ii) a selected persistent branch or attractor,

(iii) repeated throughput of energy, matter, entropy, or information,

(iv) and observer access only to a projected coarse state

is a candidate for an operational Qi-CON window. No special reference to particles, chemistry, or brains is needed at this level. The statement is geometric and statistical.

This does *not* mean that every dataset should exhibit exact power laws over arbitrarily many decades. Finite-size effects, external forcing, insufficient scale separation, coarse instrument bandwidth, and branch switching can all shorten or obscure the active window. The point is subtler and more useful: whenever a selected open system possesses a clear intermediate scale regime, the natural generic diagnostic of that regime is the Qi-CON quartet — power-law correlations, heavy tails, fractal support, and a two-knee $1/f^{\beta}$ spectrum.

A domain table makes the statement concrete.

Table 5.45: Illustrative cross-domain readings of Qi-CON.

Domain	**Typical coarse observable**	**Likely knees**	**Expected Qi-CON readout**
Transport or particle-flow systems	Arrival-time, channel current, branch occupancy	Device size / microscopic scattering time	Power-law transit statistics and scale-covariant throughput fluctuations.
Molecular machines	Dwell time, rotational step interval, cycle current	Whole-cycle return time / chemical substep time	Heavy-tailed dwell distributions and broad intermediate spectral band.
Cells and reaction networks	Flux, burst size, concentration time series	Homeostatic relaxation / reaction microtime	Fractal switching, broad throughput spectra, and correlated fluctuations across scales.
Brains and cognitive systems	Field potentials, avalanche size, decision-variable trajectory	Behavioral or theta-cycle time / synaptic or cellular time	$1/f$-like spectra with two knees, heavy-tailed event statistics, and scale-free internal dynamics.
Ecological and social systems	Population change, traffic, transaction or communication counts	Institutional or seasonal time / individual-event time	Broad multiscale fluctuation windows with truncated heavy tails and knee-bounded spectra.

The table is intentionally broad because broadness is the point. Qi-CON is not a domain-specific fit function. It is a universal diagnostic grammar for selected open-system branches.

5.9.10 Why Qi-CONis correctly placed as Pillar III of the Qi program

It is now possible to state clearly why Qi-CON deserves its own pillar and why it belongs exactly where it does.

First, Qi-CON solves a new problem. Qi-QUA solves the bookkeeping problem. Qi-EP solves the selector problem. Qi-CON solves the diagnostic problem. These are three logically distinct questions, and the third cannot be reduced to the first two.

Second, Qi-CON is the first pillar that speaks directly to the observer about multiscale data. Without it, the theory would have a budget and a selector but no principled account of why broad power laws, heavy tails, fractality, and $1/f$-type spectra should appear together in persistent open systems. The diagnostic layer is therefore indispensable if the program is to remain connected to measurement.

Third, Qi-CON must precede Qi-HAL. The architecture layer will later answer which structures implement memory, prediction, sensing, and action. But before architecture is inferred, the runtime must already have left a measurable statistical signature. The natural order is therefore: budget, selector, diagnostic, architecture.

Table 5.46: Role of Qi-CON relative to the other Qi pillars.

Layer	Question answered	Answer supplied
Qi-QUA	What is the budget?	Universal current locking and energy denomination.
Qi-EP	What is selected?	Extremal path, flux, or inferential state on the admissible branch set.
Qi-CON	What does the selected runtime look like across scales?	Conformality, heavy tails, fractality, and two-knee $1/f^{\beta}$ spectra.
Qi-HAL	What architecture carries the runtime?	Holographic attractor structures for memory, prediction, sensing, and action.

5.9.11 Section summary

This section has completed the third quantitative pillar of Chapter 5.

First, Qi-CON was defined as the diagnostic layer that follows the budget and selector layers. The relevant claim was formulated as operational conformality: an intermediate boundary window exists in which the selected branch is governed by an asymptotically homogeneous generator.

Second, the warped 6D bulk and the OGGEP projection were shown to induce a homogeneous power-law kernel on the boundary. The hidden radial-scale integration is the geometric origin of the nonlocal kernel and therefore of the exponent α.

Third, the homogeneous kernel was reduced to a fractional generator on the selected branch. This yielded the leading Qi-CON equation and the operational conformality theorem, including the power-law scaling of two-point functions.

Fourth, the four signatures were derived explicitly. The fractional generator produced α-stable characteristic functions and hence Lévy statistics. The self-similarity exponent produced fractal trajectory and support dimensions. The soft-mode spectral density produced $1/f^{\beta}$ behavior in an intermediate band bounded by two knees. In all cases the active window was shown to be finite but nontrivial, exactly as expected for natural open systems.

Fifth, the compatibility with the earlier runtime layers was made explicit. DGI preserves the diagnostic exponents as invariants of the selected branch, GSSB determines which branch and universality class are realized, GA fixes the infrared knee through attractor persistence, Qi-QUA fixes cross-sector amplitude locking, and Qi-EP fixes which branch's spectrum is actually observed.

The manuscript now contains the universal-runtime layer (DGI + GSSB + GA), the scalar-budget layer (EEGG), the packet layer (SIP), the event-locking layer (Qi-QUA), the selector layer (Qi-EP), and the diagnostic layer (Qi-CON). The natural next step is therefore Qi-HAL, where the question will no longer be "what signature does the selected runtime leave?" but "what architecture implements and stabilizes that runtime in persistent open systems?"

5.10 Qi-HAL: MePMoS, NHT, HAL, BFFT, and DGI–GSSB Interplay

After diagnostics comes architecture. It is not enough to know the ledger, the selector, and the signatures; one must also know what persistent memory-bearing organization can actually implement them. Section 5.10 therefore closes the Qi-4 block by deriving the architectural layer – MePMoS, NHT, HAL, BFFT, and the exploration-to-commitment switch – that makes the earlier pillars realizable in open systems with memory.

5.10.1 Why Qi-HALmust follow Qi-CON

The previous three pillars answer three indispensable but still incomplete questions. Qi-QUA answers the question of budget: what current ledger must be respected by every admissible event? Qi-EP answers the question of selection: which trajectory, current pattern, or inferential state is chosen inside that ledger? Qi-CON answers the question of diagnosis: what statistical footprints does the selected runtime leave across scales? None of these, however, yet says what concrete architecture can stably *carry* the selected runtime in a persistent open system.

That missing question is not decorative. It becomes unavoidable the moment one turns from particles and idealized devices to chemistry, cells, organisms, brains, and societies. Such systems do not merely dissipate or fluctuate. They store traces of previous interactions, compress those traces into internal summaries, use those summaries to generate predictions, act on the environment, and then update the stored traces by comparing expectation to outcome. Equivalently, they possess memory. Once memory appears, a theory of open systems must say how memory is written, how it is stabilized, how it is read, how it participates in action, and how it is coordinated across timescales. That is precisely the task of Qi-HAL.

The natural placement of Qi-HAL after Qi-CON is therefore strict rather than stylistic. The architecture layer should not be guessed before the diagnostic layer exists. First one determines that the runtime has a multiscale conformal window, broad fluctuations, and dyadic or near-dyadic scale nesting. Only then does one ask what kind of structure can realize such a runtime. Put differently: the diagnostic layer tells us that the system behaves as though it were phase-coordinated across scales; the architecture layer tells us what machinery can implement that fact.

A second reason for the order is conceptual. Qi-HAL depends on all three earlier pillars. It depends on Qi-QUA because every memory write, memory read, and predictive update must be paid for in the current ledger. It depends on Qi-EP because memory is useless unless it selects better trajectories, currents, or posterior states. It depends on Qi-CON because architecture without a scale protocol is unstable in complex systems. The architectural objects introduced here are therefore not independent inventions. They are the structural realization of what the first three pillars already require.

Table 5.47: Why the architectural layer must follow the budget, selector, and diagnostic layers.

Layer	**Question answered**	**Reason it must precede** Qi-HAL
Qi-QUA	What universal current and energy budget is allowed?	Memory, prediction, and action must be paid for in a common current ledger; otherwise architecture has no denominator and no event scale.
Qi-EP	Which path, current pattern, or inference is selected?	Architecture matters only if it improves the selection of histories, fluxes, or posterior states.
Qi-CON	What multiscale signatures does the selected runtime produce?	The scale-covariant window and the two-knee spectrum tell us that a nested architectural protocol is required.
Qi-HAL	What structure implements the selected, budgeted, multiscale runtime?	Supplies the concrete causal, memory, phase, and switching architecture.

The section will now make that claim explicit. It will show that once an admissible open system possesses persistent memory and a nontrivial $U(1)_{Qi}$ informational current, there is a canonical way to factor its runtime into five architectural motifs: MePMoS (the causal loop), NHT (the phase-holographic encoder), HAL (the memory lattice), BFFT (the dyadic multiscale coordination protocol), and the DGI–GSSB exploration-to-commitment switch.

5.10.2 Constitutional statement of the Qi-HALprinciple

The architectural claim of Qi-HAL begins with a minimal observation: an open system counts as more than a passive dissipator only if part of its present state depends on past interactions in a way that remains functionally available for future action. This is the generic notion of memory, and it is intentionally broad enough to cover synapses, gene-regulatory marks, chemical modification states, conformational cycles, trail networks, field configurations, and externalized collective memory.

Definition 5.10.1 (Memory-bearing open system)**.** An admissible open system with state packet

$$\Gamma(t) = \big(x^A(t), J^\alpha(t), \mathcal{P}, \mathcal{O}\big)$$

is called *memory-bearing* if there exists an internal or external state variable $m(t) \in \mathcal{M}$ and a causal kernel $K_m(\tau)$ with support on $\tau \geq 0$ such that

$$m(t) = \int_0^\infty K_m(\tau)\, \Xi\big(x(t-\tau), s(t-\tau), a(t-\tau)\big)\, \mathrm{d}\tau, \tag{5.267}$$

for some local update functional Ξ, and such that the associated retention time

$$\tau_m := \frac{\int_0^\infty \tau\ \|K_m(\tau)\|\ \mathrm{d}\tau}{\int_0^\infty \|K_m(\tau)\|\ \mathrm{d}\tau} \tag{5.268}$$

exceeds the characteristic observation-to-action cycle time T_c of the system:

$$\tau_m > T_c. \tag{5.269}$$

Definition 5.10.1 is intentionally operational. It does not assume that memory is neural, digital, or symbolic. It requires only that the system carry forward a compressed trace of earlier interactions long enough for that trace to matter. Equation (5.269) is the real entrance condition: without persistence beyond one cycle, memory cannot guide prediction or action.

The second ingredient is the informational gauge symmetry introduced earlier. In the present program a memory trace is not an abstract bookkeeping label added from outside; once $U(1)_{Qi}$ is present, informational events become physical current-carrying events. This promotes memory from a descriptive convenience to a gauge-bearing dynamical sector.

Let the informational carrier be represented locally by a complex field

$$\Psi_Q(x,t) = \sqrt{\rho_Q(x,t)}\, \mathrm{e}^{\mathrm{i}\phi_Q(x,t)}, \tag{5.270}$$

with $\rho_Q \geq 0$ the local informational intensity and ϕ_Q its phase. Under a local $U(1)_{Qi}$ gauge transformation with parameter $\Lambda(x,t)$,

$$\Psi_Q \mapsto \mathrm{e}^{\mathrm{i}q_Q\Lambda}\Psi_Q, \qquad A_\mu^{(Qi)} \mapsto A_\mu^{(Qi)} + \partial_\mu\Lambda, \tag{5.271}$$

so the covariant derivative is

$$D_\mu\Psi_Q = (\partial_\mu - \mathrm{i}q_Q A_\mu^{(Qi)})\Psi_Q. \tag{5.272}$$

The crucial consequence is that the physically meaningful memory token cannot be an absolute phase. It must be a relational, gauge-invariant quantity. The minimal such token is a phase difference corrected by the gauge holonomy:

$$\Phi_{ij}^{\mathrm{inv}} = \phi_i - \phi_j - q_Q \int_{\gamma_{ij}} A_\mu^{(Qi)}\, \mathrm{d}x^\mu, \tag{5.273}$$

where γ_{ij} joins two encoded states or channels. In particular, around a closed loop γ one obtains the informational holonomy

$$\mathcal{W}_\gamma^{(Qi)} = \exp\left(\mathrm{i}q_Q \oint_\gamma A_\mu^{(Qi)}\, \mathrm{d}x^\mu\right). \tag{5.274}$$

Equation (5.273) is the first real bridge from Qi-QUA and Qi-EP to Qi-HAL. Once memory is carried by a gauged informational field, relative phase and holonomy become natural state variables. That is why a memory-bearing open system under $U(1)_{Qi}$ can be re-expressed in an oscillator- and phase-based architecture even when its microscopic substrate is chemical, mechanical, biological, or social.

Definition 5.10.2 (Qi-HAL)**.** Qi-HAL is the canonical architectural factorization of a memory-bearing, $U(1)_{Qi}$-carrying open system into five mutually coupled layers:

(i) MePMoS: the causal loop of Memory → Prediction → Motion → Sensing → updated Memory;

(ii) NHT: a phase-holographic encoder that maps latent geometry or latent state to observable oscillatory phases and delays;

(iii) HAL: a gauge-invariant memory lattice storing relational phase structure and attractor couplings;

(iv) BFFT: a dyadic multiscale hierarchy coordinating slow/global and fast/local processing;

(v) DGI–GSSB **interplay**: the switch between exploratory gauge orbits and committed symmetry-broken attractors.

Definition 5.10.2 is intentionally architectural rather than phenomenological. It does not say that all systems look identical. Rather, it says that once persistent memory and informational gauge current are present, the runtime can be decomposed into these five tasks. Some systems implement them minimally or externally; some implement them richly and internally. The job of the rest of the section is to make that decomposition exact.

Remark 5.10.3 (Why the word "neural" in NHTis historical rather than restrictive). The acronym NHT is retained because the mature experimental examples come from neural systems. In this book, however, the mathematics is not restricted to neurons. Any substrate that converts latent geometry into phase, delay, or oscillatory code qualifies. Chemical oscillators, conformational cycles, trail networks, communication rhythms, and digital clocked systems can all instantiate the same structure.

Table 5.48: The five architectural motifs of Qi-HAL.

Component	**Question answered**	**Primary mathematical object**	**Operational role**
MePMoS	How is one cycle of action and update closed?	Causal loop map $m_n \mapsto \hat{y}_n \mapsto a_n \mapsto s_{n+1} \mapsto m_{n+1}$	Organizes prediction, action, observation, and memory update.
NHT	How is latent geometry encoded?	Gauge-corrected phase map $y \mapsto \phi(y)$	Converts hidden multidimensional state into observable phases and delays.
HAL	Where is the memory stored?	Coupling matrix/tensor W and attractor energy E_W	Stores and stabilizes phase relations or structural traces.
BFFT	How are scales coordinated?	Dyadic ladder $f_n = 2^n f_0$ and cross-frequency couplings	Couples slow/global predictions to fast/local details.
DGI–GSSB	How does the system switch between exploration and commitment?	Symmetry-preserving orbit and broken-symmetry potential	Governs attention, search, selection, and decision.

5.10.3 Why persistent memory plus $U(1)_{Qi}$ is the true entrance condition

The intuitive picture is simple: once a system has memory, prediction becomes possible; once prediction becomes possible, action becomes meaningful; once action becomes meaningful, feedback must be compared to prediction; and once that comparison is repeated, a stable architecture is required. The mathematics of Qi-HAL simply packages this logic in a gauge-covariant way.

The argument becomes sharper when written in the language of the earlier sections. In Section 5.7 we introduced the current ledger and the Arisaka Equation,

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi}.$$

A memory write therefore cannot be free. If a process creates one additional persistent informational event, there is a corresponding minimal denominator in the energy channel. This does not yet say how the event is stored, but it already forbids the fiction that memory is merely symbolic. Memory consumes budget.

In Section 5.8 we showed that selected trajectories, fluxes, and posterior states arise from a common projected extremal principle. A memory variable becomes useful only when it participates in that selection, namely when it changes which path is chosen, which current is sustained, or which posterior is stabilized. Memory without selection is inert archival residue.

In Section 5.9 we then showed that persistent open systems generically develop a scale-covariant window with broad fluctuations and two-knee spectra. A memory architecture operating in such a window must coordinate multiple timescales. That is the role later assigned to BFFT.

Taken together, the previous sections imply the following entrance proposition.

Proposition 5.10.4 (Architectural entrance proposition)**.** *Let an admissible open system satisfy the following four conditions:*

(a) it is memory-bearing in the sense of Definition 5.10.1;

(b) it carries nonzero informational current J_{Qi} coupled to $U(1)_{Qi}$;

(c) its actuation changes future observations through at least one open port; and

(d) its selected runtime contains a nontrivial Qi-CON *window or, at minimum, more than one dynamically relevant timescale.*

Then the runtime admits a nontrivial architectural factorization of the form given in Definition 5.10.2.

Proof. Condition (a) supplies a history-bearing state variable m. Condition (b) permits the informational state to be written in gauge-covariant amplitude-and-phase form (5.270)–(5.273), thereby making relational phase storage natural. Condition (c) ensures that memory can affect future input through action; this closes the MePMoS loop. Condition (d) guarantees that at least a two-level scale protocol exists, allowing slow and fast channels to separate. Once these four ingredients are present, one can define the causal loop (MePMoS), the phase

representation (NHT), the persistent coupling substrate (HAL), the multiscale ladder (BFFT), and the exploration/commitment switch (DGI–GSSB). The construction of each layer is given explicitly below. □

Proposition 5.10.4 is deliberately modest. It does not say that every such system has a large brain-like architecture. It says that the *normal form* of its runtime is architectural rather than purely differential-equation-like. Minimal systems realize the five motifs in degenerate or compressed form; richer systems realize them in distributed, nested, and highly nonlinear form.

5.10.4 MePMoS: derivation of the causal loop

We begin with the most elementary question: what is the unique causal ordering of memory, prediction, action, and sensing in a system that both acts and learns? In plain language the answer seems obvious, but in a theorem-driven paper we state carefully because the order is often blurred when perception and action are written as simultaneous updates.

Definition 5.10.5 (MePMoS)**.** For a memory-bearing open system, MePMoS is the minimal causal loop

$$\mathcal{M}_n \xrightarrow{\mathcal{P}} \widehat{y}_n \xrightarrow{\mathcal{A}} a_n \xrightarrow{\mathcal{S}} s_{n+1} \xrightarrow{\mathcal{U}} \mathcal{M}_{n+1}, \tag{5.275}$$

where $\mathcal{M}_n$ is the memory state available at the beginning of the cycle, $\widehat{y}_n$ a predicted latent or sensory outcome, a_n the action or motion command, s_{n+1} the next sensed input, and $\mathcal{U}$ the memory update map.

The essential point is that the system cannot directly map old memory to new memory if it is to remain adaptive; it must pass through a prediction step and an action step. Prediction alone is not enough, because without action there is no way to test the prediction by changing the environment or the vantage point. Likewise sensing cannot precede motion in a closed action-perception loop because the relevant new sensation is partly determined by the very action chosen in the current cycle.

Theorem 5.10.6 (Causal uniqueness of MePMoS)**.** *Consider a discrete open-system cycle with four macro-operations: memory readout, prediction, action, and sensing. Assume (i) prediction may depend only on information already stored at the start of the cycle, (ii) action may depend on the prediction and previously stored context, (iii) the incoming sensory record at the end of the cycle may depend on the chosen action and the environment, and (iv) the updated memory at the end of the cycle may depend on the old memory, the prediction, and the realized sensory record. Then, up to trivial relabeling inside each macro-operation, the only admissible topological order is*

$$\textit{Memory} \to \textit{Prediction} \to \textit{Motion} \to \textit{Sensing} \to \textit{updated Memory}.$$

Proof. The assumptions define a dependency graph with arrows

$$\mathcal{M}_n \to \widehat{y}_n, \qquad (\mathcal{M}_n, \widehat{y}_n) \to a_n, \qquad a_n \to s_{n+1}, \qquad (\mathcal{M}_n, \widehat{y}_n, s_{n+1}) \to \mathcal{M}_{n+1}.$$

Any topological sort must therefore place $\mathcal{M}_n$ before $\widehat{y}_n$, $\widehat{y}_n$ before a_n, a_n before s_{n+1}, and s_{n+1} before $\mathcal{M}_{n+1}$. No permutation violating these arrows is admissible: placing sensing before action

would require the system to know a future observation before choosing the action that partly generates it; placing prediction after sensing would collapse prediction into postdiction; placing memory update before new sensing would erase the error-correction step. Hence the macro-order is unique. □

The discrete cycle can be written more concretely as

$$\widehat{y}_n = \mathcal{P}(\mathcal{M}_n), \tag{5.276a}$$
$$a_n = \mathcal{A}(\widehat{y}_n, \mathcal{M}_n), \tag{5.276b}$$
$$s_{n+1} = \mathcal{S}(a_n, \eta_{n+1}), \tag{5.276c}$$
$$\mathcal{M}_{n+1} = \mathcal{U}(\mathcal{M}_n, \widehat{y}_n, s_{n+1}). \tag{5.276d}$$

Here η_{n+1} denotes environmental input. The operational error signal is

$$\epsilon_{n+1} = s_{n+1} - \widehat{s}_{n+1|n}, \tag{5.277}$$

where $\widehat{s}_{n+1|n}$ is the sensory consequence predicted at stage n. In an inferential implementation one may take

$$\mathcal{M}_{n+1} = \mathcal{M}_n - \eta_m G_m^{-1} \nabla_{\mathcal{M}} \left[\frac{1}{2} \left\| \epsilon_{n+1} \right\|^2 + \lambda_R R(\mathcal{M}_n) \right] + \xi_n, \tag{5.278}$$

with G_m a metric on memory space, R a regularizer or prior cost, η_m a learning step, and ξ_n a fluctuation term. Equation (5.278) is not a new principle added by hand. It is simply the Qi-EP/FEPtype selector written in the memory coordinates of the architecture layer (Friston, 2010).

In continuous time, writing the memory state as $m(t)$ and the predictive free-energy-like cost as $\mathcal{F}(m, s, a)$, one obtains the normal form

$$\tau_m \dot{m} = -\frac{\delta \mathcal{F}}{\delta m} + \chi_Q J_{Qi} + \zeta(t), \tag{5.279}$$

where the explicit J_{Qi} term reminds us that memory maintenance is an informational-current-bearing process rather than an abstract computational update.

The interpretation is extremely general. In a molecular machine the memory state may be a conformational coordinate or chemical occupancy. In a bacterium it may be a receptor methylation state. In a mammalian brain it may be a large distributed recurrent state. In a society it may be a distributed archive coupled to communication channels. The causal grammar is unchanged.

Table 5.49: The MePMoS loop across domains.

Domain	Memory	Prediction	Motion/action	Sensing/update
Molecular machine	Conformational occupancy	Expected next catalytic state	Step, rotation, or gating bias	Product release, ligand binding, torque feedback
Cell or microbe	Internal biochemical state	Expected gradient or nutrient change	Chemotactic step or regulatory action	Receptor occupancy, signal update
Nervous system	Recurrent network state	Predicted sensory or latent state	Motor command or attentional shift	Sensory stream, error correction
Collective system	Shared archive, communication state	Expected collective outcome	Policy, message, or institutional action	External response, archive revision

5.10.5 NHT: phase-holographic coding of latent geometry

Once a causal loop exists, the next question is how latent structure is encoded so that it can be stored, compared, and updated efficiently. The answer proposed by Qi-HAL is phase-holographic coding. Again the word "neural" in NHT is historical; the derivation is not specifically neural.

Suppose the relevant latent variable of the system is $y = (y^1, \dots, y^d) \in \mathcal{Y}$. A minimal phase encoding is

$$\phi_i(t) = \omega_i t + \frac{2\pi}{L_i} y^i(t) + \lambda(t), \qquad i = 1, \dots, d, \tag{5.280}$$

where ω_i is a carrier frequency, L_i a characteristic scale, and $\lambda(t)$ a common gauge phase. Under $U(1)_{Qi}$ the physically meaningful coordinate is the gauge-corrected phase

$$\widetilde{\phi}_i(t) = \phi_i(t) - q_Q \int^t A_{0,i}^{(Qi)}(\tau)\, \mathrm{d}\tau. \tag{5.281}$$

Therefore differences in latent state can be reconstructed from phase differences:

$$\delta y^i = \frac{L_i}{2\pi}\, \delta\widetilde{\phi}_i. \tag{5.282}$$

This immediately gives the local induced metric

$$\mathrm{d}s^2_{\mathcal{Y}} = \sum_{i=1}^{d} \left(\frac{L_i}{2\pi}\right)^2 \left(\mathrm{d}\widetilde{\phi}_i\right)^2. \tag{5.283}$$

Proposition 5.10.7 (Metric recovery by phase tomography)**.** *If the latent coordinates are encoded according to Eq. (5.280) and read out through the gauge-corrected phases (5.281), then the local Euclidean metric on the latent chart is recovered exactly by Eq. (5.283). More generally, a curved latent metric $g_{ij}(y)$ may be encoded by replacing the diagonal scale factors $(L_i/2\pi)^2$ with a positive-definite phase metric tensor $G_{ij}(\phi)$.*

Proof. Differentiate Eq. (5.280) at fixed carrier-time slice or, equivalently, subtract the trivial carrier contribution $\omega_i\,\mathrm{d}t$. Then $\mathrm{d}\widetilde{\phi}_i = (2\pi/L_i)\mathrm{d}y^i$. Multiplying by $(L_i/2\pi)^2$ and summing over i yields Eq. (5.283). The curved case follows by replacing the diagonal quadratic form with a general positive-definite one. □

The conceptual content of Eq. (5.283) is far-reaching. A high-dimensional latent world can be encoded in relative phases, delays, or rhythmic offsets. The system does not need a literal geometric map drawn somewhere inside it. It only needs a phase configuration from which distances, alignments, or transition costs can be reconstructed.

A familiar special case is a ring attractor, in which one coordinate $y \in S^1$ is encoded as a phase angle ϕ. But the same logic extends to toroidal, grid-like, chemical, and distributed memory systems. In a reaction network, phase may mean oscillation phase of concentrations. In a mechanical rotor, it may mean angular coordinate. In a social rhythm, it may mean synchronization offset relative to a shared clock.

The holographic language is justified because the code is not a point-by-point copy of the latent world. Rather, the latent geometry is stored distributively in phase relations, just as ordinary holography stores spatial information distributively in interference structure. This is why a damaged or partial substrate may still retain partial reconstructive power.

Table 5.50: Interpretations of NHT beyond neuroscience.

Substrate	**Encoded variable**	**Phase-like carrier**	**Meaning of tomography**
Chemical oscillator network	Concentration or environmental phase	Oscillation phase of coupled reactions	Recover latent chemical/environmental coordinates from relative phase
Conformational molecular cycle	Position on catalytic cycle	Rotor angle or cycle phase	Reconstruct step location and expected next state
Neural population	Position, heading, or latent cause	Spike phase / local field potential phase	Recover latent spatial or causal geometry
Collective communication system	Social or task state	Synchronization phase across agents/clocks	Recover distributed shared state from timing relations

5.10.6 HAL: the holographic attractor lattice as a memory substrate

Phase coding alone is not memory. A system also needs a substrate in which phase relations can leave durable traces. HAL is that substrate. In a minimal realization it is simply a set of couplings that stores relational structure. In a rich realization it becomes a high-dimensional attractor lattice supporting recall, completion, interpolation, and stable world-model reconstruction.

Let the encoded phase-bearing features be

$$\sigma_i^{(\alpha)}(n) = \exp\bigl(\mathrm{i}\phi_i^{(\alpha)}(n)\bigr), \tag{5.284}$$

where i labels local units or channels and α labels feature families or subbands. A minimal write rule is the gauge-invariant correlator update

$$W_{ij}^{\alpha\beta}(n+1) = (1-\lambda_W)W_{ij}^{\alpha\beta}(n) + \gamma_W\, \sigma_i^{(\alpha)}(n)\, \overline{\sigma_j^{(\beta)}(n)}, \tag{5.285}$$

with decay or forgetting rate λ_W and write strength γ_W. Under a common local $U(1)_{Qi}$ phase shift, the product $\sigma_i^{(\alpha)}\overline{\sigma_j^{(\beta)}}$ is invariant whenever the two channels carry the same informational charge. In the more general case one inserts the parallel-transport factor of Eq. (5.273); the same conclusion follows.

The corresponding readout energy is

$$E_W[\sigma] = -\frac{1}{2}\sum_{\alpha,\beta}\sum_{i,j} W_{ij}^{\alpha\beta}\sigma_i^{(\alpha)}\overline{\sigma_j^{(\beta)}} + \sum_{\alpha,i} U\left(|\sigma_i^{(\alpha)}|^2\right), \tag{5.286}$$

with a local confining term U. Stable memories are local minima of E_W. When the couplings are organized on a ring, torus, graph, membrane, or more general lattice, those minima define a geometric memory manifold. That manifold is what the name holographic attractor lattice denotes.

The connection to the earlier pillars is immediate. A write event changes W, and therefore costs energy according to the Qi-QUA ledger. A read event queries one or more minima of E_W, and therefore participates in Qi-EP trajectory or posterior selection. The existence of a whole family of minima across scales is precisely what Qi-CON diagnosed indirectly through broad spectral and spatial statistics.

The historical analogy to associative memory is useful but incomplete (Hopfield, 1982). In ordinary associative memory one stores patterns. In HAL one stores gauge-covariant relations, phase geometry, and attractor structure. This makes HAL more naturally compatible with continuous variables, manifold-valued latent states, and topological phases than a purely binary memory model.

Proposition 5.10.8 (Gauge-invariant memory storage)**.** *If the write rule is constructed from phase-difference correlators of the form in Eq. (5.285), optionally with explicit parallel transport for channels living in different gauge patches, then the stored memory tensor W is invariant under local $U(1)_{Qi}$ gauge transformations. Consequently the physically stored object is relational phase geometry rather than absolute phase.*

Proof. Under $\sigma_i^{(\alpha)} \mapsto e^{\mathrm{i}q_Q\Lambda_i}\sigma_i^{(\alpha)}$, the factor $\sigma_i^{(\alpha)}\overline{\sigma_j^{(\beta)}}$ acquires the phase $e^{\mathrm{i}q_Q(\Lambda_i-\Lambda_j)}$. If the channels are in the same local patch and the same charge sector, this phase cancels for equal local gauges or is absorbed into the covariant phase-difference variable. More generally, inserting the Wilson-line transport factor between i and j cancels the residual phase exactly. Therefore the stored object depends only on relational gauge-invariant data. □

The most important practical question is whether HAL depends on absolute frequency. If a memory is written at one carrier frequency and read at another, is the content lost? The answer in the idealized relational limit is no.

Theorem 5.10.9 (Frequency-independent HALreadout)**.** *Suppose a pattern is written using oscillatory carriers*

$$x_i^{\mathrm{w}}(t) = \exp\bigl(\mathrm{i}(\omega_{\mathrm{w}} t + \phi_i)\bigr), \tag{5.287}$$

and stored via the ideal correlator

$$W_{ij} \propto \left\langle x_i^{\mathrm{w}}(t)\, \overline{x_j^{\mathrm{w}}(t)} \right\rangle_t = e^{\mathrm{i}(\phi_i - \phi_j)}. \tag{5.288}$$

If the pattern is later queried using carriers

$$x_i^{\mathrm{r}}(t) = \exp\bigl(\mathrm{i}(\omega_{\mathrm{r}} t + \psi_i)\bigr), \tag{5.289}$$

then the overlap energy

$$\mathcal{O}[\psi|W] = \sum_{i,j} \overline{x_i^{\mathrm{r}}(t)}\, W_{ij}\, x_j^{\mathrm{r}}(t) \tag{5.290}$$

is independent of the common read frequency ω_{r}. It depends only on the relative phase pattern ψ_i compared to the stored relative phase pattern ϕ_i.

Proof. Substituting Eqs. (5.288) and (5.289) into Eq. (5.290) yields

$$\mathcal{O}[\psi|W] \propto \sum_{i,j} e^{-\mathrm{i}(\omega_{\mathrm{r}} t + \psi_i)} e^{\mathrm{i}(\phi_i - \phi_j)} e^{\mathrm{i}(\omega_{\mathrm{r}} t + \psi_j)} = \sum_{i,j} e^{\mathrm{i}[(\phi_i - \phi_j) - (\psi_i - \psi_j)]}.$$

The carrier phases $\omega_{\mathrm{r}} t$ cancel identically. Hence the overlap depends only on relative phase differences. The common read frequency therefore does not affect ideal recall. □

Theorem 5.10.9 has two important consequences. First, HAL is fundamentally a relational storage principle. Second, it can be realized across very different substrates. A chemical system, a neural system, and a digital clocked system may use entirely different carrier frequencies, yet the same relational content can in principle be preserved.

One can also define a simple architectural completeness index,

$$\mathrm{QCI} = \frac{1}{5}\bigl(I_{\mathrm{MePMoS}} + I_{\mathrm{NHT}} + I_{\mathrm{HAL}} + I_{\mathrm{BFFT}} + I_{\mathrm{switch}}\bigr), \tag{5.291}$$

where each indicator $I \in [0, 1]$ measures the degree to which the corresponding Qi-HAL component is realized in a given substrate. Minimal biochemical toggles may have $\mathrm{QCI} \ll 1$, while distributed nervous systems may have $\mathrm{QCI} \to 1$. Equation (5.291) is not needed for the theory itself, but it is useful as a future empirical scoring tool.

5.10.7 BFFT: the dyadic multiscale protocol

The moment memory becomes nontrivial, one also needs scale coordination. Not every variable changes at the same rate; not every prediction is local; not every correction is fast. The natural multiscale protocol in Qi-HAL is BFFT, the binary fractal Fourier transform. The name is meant broadly: one may realize it dynamically by a nested oscillator hierarchy or analytically by a dyadic wavelet/Fourier-packet decomposition (Mallat, 2009). The key point is the dyadic scale ladder.

The derivation begins from the conformal radial coordinate already used in GG-6 and later inherited by Qi-CON,

$$r = \ln\left(\frac{\mu_0}{\mu}\right). \tag{5.292}$$

Equal increments in r correspond to multiplicative steps in physical scale. Suppose a scale hierarchy is sampled by a constant increment $\Delta r = \ln \lambda$, so that

$$\mu_{n+1} = \lambda\, \mu_n, \qquad f_{n+1} = \lambda\, f_n. \tag{5.293}$$

If one also demands integer nesting of cycles between adjacent levels, the multiplicative factor λ must be an integer greater than one. The minimal nontrivial choice is $\lambda = 2$, and this is exactly the binary or dyadic ladder.

Proposition 5.10.10 (Dyadic minimality)**.** *Among multiplicative scale hierarchies with the properties (i) exact parent-child nesting of periods and (ii) the minimal number of additional structural choices beyond the constitutional ledger, the unique minimal nontrivial ratio is*

$$\lambda = 2. \tag{5.294}$$

Hence the natural Qi-HAL *scale ladder is*

$$f_n = 2^n f_0. \tag{5.295}$$

Proof. Exact parent-child nesting requires that one period at level n contain an integer number $m > 1$ of periods at level $n+1$, hence $f_{n+1} = m f_n$ with integer m. The requirement of minimal extra structure chooses the smallest admissible integer, namely $m = 2$. Therefore $\lambda = 2$ and the dyadic ladder follows. □

The dyadic ladder is not just aesthetically simple. It is exactly the structure required to mediate top-down and bottom-up flow. Slow channels carry coarse, global context; fast channels carry fine, local details. A phase-locking action for adjacent levels is

$$S_{\text{lock}} = \sum_n \kappa_n \int \mathrm{d}t \left[1 - \cos(2\phi_n - \phi_{n+1})\right]. \tag{5.296}$$

The stationary condition is

$$2\phi_n - \phi_{n+1} = \text{const}, \tag{5.297}$$

which is the canonical 1:2 phase-lock relation between neighboring scales.

This immediately gives a bridge to the two-knee Qi-CON spectrum. If the active conformal window spans from f_{IR} to f_{UV}, then the number of effectively available dyadic levels is approximately

$$N_{\text{lev}} \approx 1 + \left\lfloor \log_2\left(\frac{f_{\text{UV}}}{f_{\text{IR}}}\right) \right\rfloor. \tag{5.298}$$

Thus the diagnostic spectral window fixes the architectural depth of the BFFT ladder. This is a concrete, falsifiable bridge between Pillar III and Pillar IV.

For signal-analysis purposes one may write the same ladder as a dyadic transform,

$$X_n(\tau) = 2^{-n/2} \int x(t)\, \psi\!\left(\frac{t-\tau}{2^n}\right) \mathrm{d}t, \tag{5.299}$$

which simply restates that the architecture can be understood either dynamically or analytically. In both readings the binary hierarchy is doing the same job: separating scales while preserving cross-scale couplings.

Table 5.51: Functional interpretation of the BFFT hierarchy.

Level type	**Characteristic scale**	**Typical architectural role**	**Relation to neighboring levels**
Slow / coarse	Low frequency, long horizon	Global context, priors, slow homeostatic setpoints	Constrains the envelope or admissible region for faster levels
Intermediate	Mesoscale bands	Working coordination, routing, synchronization	Mediates bidirectional exchange between slow and fast levels
Fast / fine	High frequency, short horizon	Local detail, error correction, rapid sensory update	Reports deviations upward and refines lower-resolution predictions

5.10.8 The DGI–GSSBinterplay as the exploration-to-commitment switch

The fifth architectural motif is not an additional physical object but a runtime mode switch. A memory-bearing system must alternate between two regimes. In one regime it explores, samples, and remains flexible across a set of near-equivalent possibilities. In the other regime it commits to one branch, one action, one interpretation, or one attractor. The first is naturally described by DGI; the second by GSSB. The stabilized result is GA.

A minimal normal form uses an order parameter m (which may be scalar or vector) and an effective potential

$$V_{\mathrm{eff}}(m;t) = \frac{1}{2}a(t)\ \|m\|^2 + \frac{b}{4}\,\|m\|^4 - h(t)\cdot m, \qquad b>0. \tag{5.300}$$

When $a(t) > 0$, the symmetric point $m = 0$ is the unique minimum; the system remains exploratory. When $a(t) < 0$, a symmetry-broken shell or set of minima appears. The field $h(t)$ or small fluctuations then select one branch. That branch is stabilized by the quartic term and by environmental feedback, producing GA.

To connect this to observation and prediction error, define an evidence integral

$$I(t) = \int_{-\infty}^{t} \exp\!\left(-\frac{t-\tau}{\tau_I}\right)\left(\epsilon_\star^2 - \|\epsilon(\tau)\|^2\right)\mathrm{d}\tau, \tag{5.301}$$

where $\epsilon(t)$ is the current prediction error and $\epsilon_\star$ is a reference tolerance. Taking

$$a(t) = a_0 - \lambda_I I(t) \tag{5.302}$$

shows that sufficiently accumulated evidence can drive $a(t)$ through zero. The system then moves from exploratory DGI to symmetry-broken commitment. If the environment later changes so that error rises again, $I(t)$ decreases, $a(t)$ can return positive, and the system re-enters exploration. This is the normal form of attention, search, decision, commitment, and revision across many domains.

Proposition 5.10.11 (Exploration-to-commitment bifurcation)**.** *For the potential* (5.300) *with* $a(t)$ *given by Eq.* (5.302)*, there exists a critical evidence level*

$$I_c = \frac{a_0}{\lambda_I} \tag{5.303}$$

such that the system is in the exploratory DGI *regime for* $I < I_c$ *and in the broken-symmetry commitment regime for* $I > I_c$*, up to perturbations induced by* $h(t)$ *and noise.*

Proof. The sign change in $a(t)$ occurs exactly at $I = I_c$. For $a > 0$, V_{eff} has a single minimum at $m = 0$. For $a < 0$, the quartic term creates nonzero minima with radius $\sqrt{-a/b}$ when $h = 0$, or nearby perturbed minima for small $h \neq 0$. The system therefore crosses from a single exploratory basin to multiple committed branches at the critical evidence level. □

This proposition turns a qualitative story into a concrete architectural rule. It says that a memory-bearing open system need not choose between flexibility and stability once and for all. Rather, it moves between the two in a controlled, evidence-dependent way. That is exactly what one expects of adaptive organisms, decision circuits, and collective institutions.

5.10.9 A unified effective action for Qi-HAL

The preceding subsections can be compressed into a single effective architectural action. The claim is not that every real system literally writes its microphysics in this form, but that every admissible implementation has this as a normal-form description after coarse-graining.

Let the dynamic variables be memory m, phase coordinates ϕ_i, memory couplings W, actions a, and sensed inputs s. Then an effective action is

$$\begin{aligned} S_{\text{Qi-HAL}} = \int \mathrm{d}t \Bigg[& \underbrace{\frac{1}{2}\dot{m}^\top G_m \dot{m} + \frac{\lambda_\epsilon}{2}\|\epsilon\|^2}_{S_{\text{MePMoS}}} + \underbrace{\frac{1}{2}\sum_i \kappa_i \left(D_t\phi_i - \frac{2\pi}{L_i}\dot{y}^i\right)^2}_{S_{\text{NHT}}} \\ & + \underbrace{\frac{\tau_W}{2}\left\|\dot{W}\right\|^2 + V_H(W,\sigma)}_{S_{\text{HAL}}} + \underbrace{\sum_n \kappa_n\big[1-\cos(2\phi_n - \phi_{n+1})\big]}_{S_{\text{BFFT}}} \\ & + \underbrace{V_{\text{eff}}(m;t)}_{S_{\text{switch}}} + q_Q A_\mu^{(Qi)} J_{Qi}^\mu \Bigg]. \end{aligned} \tag{5.304}$$

Here $D_t\phi_i = \dot{\phi}_i - q_Q A_{0,i}^{(Qi)}$. The last term is the explicit informational gauge coupling. A natural partition or effective generating functional is then

$$\mathcal{Z}_{\text{Qi-HAL}} = \int \mathcal{D}m\,\mathcal{D}\phi\,\mathcal{D}W\,\mathcal{D}a\,\mathcal{D}s\ \exp\left[-\frac{S_{\text{Qi-HAL}}}{Q_B}\right]. \tag{5.305}$$

The appearance of the Arisaka Constant Q_B in the denominator is deliberate. It marks the ideal energetic denomination of an informational event, thereby tying the full architectural layer back to the current ledger of Qi-QUA.

With the first three Qi pillars already in place, the next theorem closes the Qi block by showing that the architectural layer is not a domain-specific add-on but the universal implementation class for memory-bearing open systems.

Theorem 5.10.12 (Qi-HAL universality theorem)**.** *Let an admissible open system satisfy the hypotheses of Proposition 5.10.4. Then, after suitable coarse-graining to the timescales on which memory persists, its runtime admits an effective action of the form* (5.304) *and therefore a canonical factorization into* MePMoS*,* NHT*,* HAL*,* BFFT*, and the* DGI–GSSB *switch.*

Proof. Definition 5.10.1 supplies the memory coordinate and therefore the possibility of a MePMoS sector. The $U(1)_{Qi}$ coupling supplies the phase-holonomy representation and therefore the NHT sector. Persistence of memory traces implies that relational correlators can accumulate into effective couplings, yielding the HAL sector. A nontrivial scale window implies at least a dyadic or near-dyadic hierarchy, yielding the BFFT sector. Finally, the selector principle together with attractor stabilization yields the exploration/commitment potential. Coarse-graining then bundles the sectors into Eq. (5.304). The decomposition is canonical because it follows from the logical roles that any such system must realize, not from the microscopic details of a particular substrate. □

The theorem is the central result of this section. It explains in precise terms what was claimed informally in the Introduction: once $U(1)_{Qi}$ is applied, any open system with memory acquires a common architectural grammar. The grammar may be instantiated minimally in chemistry or richly in brains, but the runtime tasks are the same.

5.10.10 How Qi-HALapplies to any open system with memory

The universality claim becomes most persuasive when one resists the temptation to imagine only brains. The point of Qi-HAL is broader. Memory can be internal or external, analog or digital, molecular or social. The theorem does not require a cortex, a language, or even neurons. It requires only persistence, action-conditioned feedback, and informational gauge current.

Minimal biochemical case. A biochemical toggle, receptor cluster, or catalytic cycle may have a very small state space, yet if its present reaction bias depends on a stored modification state, it already satisfies Definition 5.10.1. The cycle phase plays the role of NHT the modified couplings or energy landscape play the role of HAL and slow/fast relaxation channels provide a minimal BFFT hierarchy. The DGI–GSSB switch appears when the system moves from a flexible undecided regime to a committed branch of reaction or conformation.

Cellular and microbial case. A chemotactic microbe or slime-mold network is an especially transparent open-system example. Memory may be carried by receptor methylation, internal chemical state, or even an externalized trail network. Prediction appears as directional bias,

motion as crawling or swimming, sensing as receptor update, and multiscale coordination as the nesting of fast local signaling within slow environmental adaptation. Here HAL may be partly externalized into the environment itself, showing that the theorem does not require memory to be stored in a single physically bounded organ.

Neural and cognitive case. In a nervous system the five motifs become more familiar. Recurrent state and synaptic structure supply MePMoS and HAL phase coding and population rhythms supply NHT nested oscillatory bands or other multiscale population modes supply BFFT and evidence-driven selection produces the DGI–GSSB attention/decision cycle. The architecture is richer but not different in kind.

Collective and social case. A collective can also instantiate Qi-HAL. Shared archives, institutions, communication rhythms, distributed representations, and policy cycles provide memory, prediction, action, sensing, and update. The memory lattice is partly externalized in documents, infrastructures, or digital systems; yet the same logic applies. This is one of the reasons this book insists that the framework is for open systems in *all* sciences, not only for biology or physics.

Table 5.52: How the five Qi-HAL motifs appear across representative open systems with memory.

System	**Memory**	MePMoS	NHT	HAL	BFFT/**switch**
Catalytic cycle	Conformational occupancy, chemical mark	Cycle memory → catalytic bias → step → feedback	Rotor or oscillation phase	Modified energy landscape	Slow-fast nesting; branch commitment
Chemotactic cell	Internal adaptation, receptor memory, trail residue	Internal state → direction bias → movement → receptor update	Chemical oscillation phase	Internal network, possibly external trail	Adaptation bands; explore vs commit
Neural system	Recurrent state, synaptic matrix	Working memory → prediction → action → sensory update	Population phase code	Attractor manifold, synaptic lattice	Nested bands; decision bifurcation
Collective	Shared archive, institutions, digital storage	Archive → forecast → policy → measured outcome	Shared clocks, communication timing	Externalized memory lattice	Slow governance, fast signals; consensus

It is worth emphasizing the role of externalized memory. The theorem never assumed that memory must be stored behind the physical boundary of one organism. A beaver dam, a pheromone trail, a paper notebook, a digital server, or a laboratory instrument can all function as memory-bearing extensions of an open system. Once those extensions are coupled into the causal loop, they

become part of the effective HAL sector. This is why the theory naturally extends from particles to organisms to societies.

5.10.11 Worked examples spanning chemistry, life, and neural systems

The cross-domain table above is schematic. It is useful to spell out several cases in more detail.

5.10.11.1 ATP synthase as a minimal or proto-Qi-HALsystem

At the molecular scale, ATP synthase provides a canonical open-system cycle (Lane, 2015; Yoshida et al., 2001). The rotor angle θ stores where the machine sits in its catalytic cycle; proton motive force biases the next step; rotation constitutes motion; product release and torque fluctuations provide sensed feedback. In this reading:

$$\begin{aligned} \text{memory} &\leftrightarrow \theta \text{ and occupancy state,} \\ \text{prediction} &\leftrightarrow \text{expected next catalytic substate,} \\ \text{motion} &\leftrightarrow \text{rotational step,} \\ \text{sensing} &\leftrightarrow \text{load/proton feedback.} \end{aligned}$$

The phase variable θ is already an NHT coordinate. The periodic energy landscape is a rudimentary HAL substrate. Fast local chemical transitions and slower cycle completion define a minimal BFFT hierarchy. The system therefore realizes a proto-Qi-HAL architecture even though it is far simpler than a nervous system.

5.10.11.2 Slime mold or chemotactic collectives as externalized HAL

A slime mold such as *Physarum polycephalum* can solve path-selection problems by integrating internal rhythmic flows with an externalized trail network (Nakagaki et al., 2000). Here the memory is not localized in one synaptic matrix. Part of it is stored in the organism-environment composite. The trail geometry or tube-strength pattern functions as an externalized HAL while internal oscillatory flows supply the phase-coded NHT substrate. This is an important test case because it demonstrates directly that Qi-HAL should be formulated for open systems rather than for bounded organisms alone.

5.10.11.3 Ring attractors and mammalian cognitive maps

The neural case is the one in which the architecture becomes especially transparent. Experimental ring-attractor systems provide direct realizations of persistent phase-coded memory and evidence-dependent switching (Turner-Evans et al., 2020). More broadly, spatial representation in the hippocampal formation and related systems illustrates how latent geometry can be encoded in distributed activity states (O'Keefe and Dostrovsky, 1971). In the language of the present section, recurrent population state supplies HAL oscillatory phase relations supply NHT nested timescales supply BFFT and decision or attentional commitment supplies the DGI–GSSB switch.

These examples are not intended as exhaustive evidence. They serve a more limited but important purpose: they show that once the theorem is written in the right level of abstraction, the same architectural motifs really do recur from chemistry to organisms to brains.

5.10.12 Why Qi-HALis correctly placed as Pillar IV

The first three pillars and the fourth can now be summarized in one sentence. Qi-QUA tells us what can be afforded; Qi-EP tells us what is selected; Qi-CON tells us what signatures the selected runtime leaves; Qi-HAL tells us what structure implements that runtime. This is why the fourth pillar cannot be moved to the beginning without destroying the logic of the program.

Table 5.53: The four Qi pillars as a complete runtime stack in the integrated review draft.

Pillar	Core question	Primary object	What it contributes
Qi-QUA	What universal budget must every event respect?	Current-locking ledger and Q_B	Denominator for informational events and current conversion.
Qi-EP	Which path or state is selected?	Projected extremal principle	Chooses trajectories, fluxes, and posterior states.
Qi-CON	What multiscale footprints follow from the selected runtime?	Fractional/conformal diagnostics	Predicts scale covariance, Lévy, fractality, and two-knee spectra.
Qi-HAL	What architecture realizes the selected runtime?	MePMoS, NHT, HAL, BFFT, switch	Supplies memory, phase coding, multiscale coordination, and commitment dynamics.

At a deeper level, placing Qi-HAL fourth makes a philosophical statement about explanation. It says that architecture is not fundamental in the constitutional sense; it is emergent but inevitable. Once the current ledger, selection law, and diagnostic scale window are fixed, the architecture is no longer arbitrary. It is constrained. That is precisely why this section matters: it turns a qualitative design intuition into a theorem-level consequence of the earlier papers.

5.10.13 Section summary

The architectural pillar of GDOS has now been derived. The key claims can be summarized compactly.

(1) Persistent memory plus $U(1)_{Qi}$ informational current is the true entrance condition for Qi-HAL.

(2) The unique causal order of a predictive acting system is MePMoS: Memory → Prediction → Motion → Sensing → updated Memory.

(3) Latent geometry can be encoded holographically in gauge-corrected phases; this is NHT.

(4) Durable relational phase geometry is stored in a gauge-invariant attractor lattice; this is HAL.

(5) The natural multiscale protocol is dyadic, with $f_n = 2^n f_0$; this is BFFT.

(6) Adaptive systems must switch between exploratory DGI and committed GSSB regimes according to evidence; the stabilized branch is GA.

(7) Therefore any open system with memory and nontrivial $U(1)_{Qi}$ current admits a canonical coarse-grained architectural factorization into the five Qi-HAL motifs.

This completes the fourth Qi pillar at the same theorem-driven level as the previous sections. Part IV can therefore begin on top of a full Qi stack: current locking, selector, diagnostics, and architecture, already anchored to the earlier scalar ledger and packet layers. The next step is no longer construction of another pillar, but classification of persistence and completeness.

Persistence, completeness, and internality

Self-maintaining open systems, completeness, and holographic closure

5.11 SeOS and QCI: Persistence Class and the Qi-4 Completeness Index

Part IV begins only after the full Qi stack is available. The next question is no longer how an open system is driven, selected, diagnosed, or architected, but which such systems persist and how fully they instantiate the Qi-4 runtime. Section 5.11 therefore introduces the persistence class SeOS and the completeness diagnostic QCI.

5.11.1 Why persistence and completeness belong after the full Qi stack

After Sections 5.7–5.10, the manuscript finally possesses the full Qi stack: current locking, trajectory selection, multiscale diagnostics, and memory architecture. Only at that point does it become meaningful to ask two synthesis questions. First, which open systems truly preserve themselves as continuing packets rather than as transient driven patterns? Second, among those persistent systems, how fully is the four-pillar runtime actually realized?

These are distinct questions. Persistence concerns continued viability under throughput, repair, and perturbation. Completeness concerns the degree to which the full Qi grammar is present. The revised architecture therefore places them together in Part IV. The section first isolates the persistence class SeOS, then introduces QCI as a diagnostic of realized completeness.

5.11.2 SeOS: the class of persistent self-maintaining open systems

Once the packet and ledger are defined, one can finally ask which open systems actively preserve themselves. Every system in nature is open, but not every open system is a self-maintaining system. A Brownian tracer in a fluid is open, but it does not protect a boundary, repair its own packet, or preserve a memory-bearing attractor. A Bénard cell is closer to self-maintenance because it exists only under continued throughput, but its packet is comparatively thin and its memory depth is limited. A cell, organism, brain, or institution is more demanding still: it must preserve not only energy throughput but also organization, boundary conditions, and records.

The purpose of SeOS is to isolate this persistent class precisely. The term "selfish" is meant in the same stripped-down technical sense in which the selfish-gene metaphor generalized local fitness logic: the system acts to maintain its own packet, not because it possesses human intention, but because the lawful runtime selects trajectories that preserve local viability whenever such preservation is possible. Cooperation, symbiosis, and hierarchy can still emerge, but they emerge as higher-order ways of stabilizing packet maintenance rather than as violations of the local logic.

To make this precise, we first need a metric notion of packet preservation.

Definition 5.11.1 (Packet space and viability tube)**.** Let $\mathfrak{P}$ denote the space of admissible packets together with their current state distributions. Around a target packet-attractor $\Xi_* \in \mathfrak{P}$

define the packet distance

$$\begin{aligned} d^2_{\mathrm{SIP}}(\Xi_1,\Xi_2) := d^2_g(x_1,x_2) + \alpha_P \left\|\mathcal{P}_1 - \mathcal{P}_2\right\|^2 + \alpha_L \left\|\mathcal{L}_1 - \mathcal{L}_2\right\|^2 \\ + \alpha_F \left|\mathcal{F}_{\mathrm{EEGG},1} - \mathcal{F}_{\mathrm{EEGG},2}\right|^2 + \alpha_O \, d^2_O(\mathcal{O}_1,\mathcal{O}_2), \end{aligned} \tag{5.306}$$

with nonnegative weights $\alpha_{P,L,F,O}$. A *viability tube* of radius ϵ is the subset

$$\mathcal{U}_\epsilon(\Xi_*) = \{\Xi \in \mathfrak{P} : d_{\mathrm{SIP}}(\Xi,\Xi_*) \le \epsilon, B_\partial(\Xi) \le B_{\max}, I_m(\Xi) \ge I_{\min}\}, \tag{5.307}$$

where B_∂ measures boundary damage and I_m measures memory integrity.

Definition 5.11.1 says what it means for a packet to survive: the state, ports, lawful generator, ledger, and observables must remain sufficiently close to a maintained attractor, while boundary damage stays bounded and memory integrity remains above threshold.

We can now define the persistence class.

Definition 5.11.2 (SeOS)**.** An open system instance with packet Ξ_t is a member of SeOS if there exist a maintained attractor Ξ_*, a viability tube $\mathcal{U}_\epsilon(\Xi_*)$, and a physically admissible feedback law such that:

(i) **packet retention:** trajectories re-enter and remain in $\mathcal{U}_\epsilon(\Xi_*)$ after sufficiently small perturbations;

(ii) **positive maintenance margin:** the cycle-averaged maintenance power

$$\Pi_{\mathrm{maint}} := \left\langle \mathcal{P}_{\mathrm{port}} \right\rangle_\tau - \left\langle \mathcal{D}_{\mathrm{EEGG}} \right\rangle_\tau - \left\langle \mathcal{R}^{\min}_{\mathrm{repair}} \right\rangle_\tau \tag{5.308}$$

is positive for some physically relevant window τ;

(iii) **self-maintenance:** the repair term $\mathcal{R}^{\min}_{\mathrm{repair}}$ includes the minimal ledger cost needed to restore packet damage, boundary mismatch, and memory degradation;

(iv) **operational identity:** the observable dictionary remains valid while the system is maintained.

Definition 5.11.2 is intentionally local and non-moral. A system is "selfish" only in the sense that its lawful feedback and port usage are organized around the preservation of its own packet. It may cooperate with larger systems, embed within a larger hierarchy, or even sacrifice one subsystem to preserve a higher-order packet. What matters is that the preserved object is the system's own effective instance packet at the chosen level of description.

5.11.3 The persistence theorem

The definition above becomes practically useful only when one can test it through a dynamical criterion. Because Part IV places persistence after the full Qi stack, that criterion must be written as a consequence of the already-built runtime rather than as an independent biological intuition. The criterion is simplest when written as a Lyapunov-type estimate.

Theorem 5.11.3 (Persistence criterion for SeOS)**.** *Let Ξ_t be an admissible packet with target attractor Ξ_*. Define the viability functional*

$$\mathcal{V}(\Xi_t) := d_{\mathrm{SIP}}^2(\Xi_t, \Xi_*) + \lambda_\partial B_\partial(\Xi_t) + \lambda_m \big(I_{\min} - I_m(\Xi_t)\big)_+, \tag{5.309}$$

with $\lambda_\partial, \lambda_m > 0$. Suppose there exist constants $\kappa > 0$ and $c \geq 0$ such that along the packet dynamics,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V} \leq -\kappa\, \mathcal{V} + c - \beta\, \Pi_{\mathrm{maint}}, \qquad \beta > 0. \tag{5.310}$$

If the averaged maintenance margin satisfies $\Pi_{\mathrm{maint}} > c/\beta$, then $\mathcal{V}$ remains bounded and the packet is persistent; hence the system belongs to SeOS*. If $\Pi_{\mathrm{maint}} < 0$ for windows longer than the intrinsic repair time, the packet leaves the viability tube and the system exits* SeOS*.*

Proof. Equation (5.310) is a standard dissipative estimate. If $\Pi_{\mathrm{maint}} > c/\beta$, then the right-hand side is strictly negative whenever $\mathcal{V}$ exceeds a finite threshold. By Grönwall's inequality, $\mathcal{V}(t)$ remains bounded and eventually enters an invariant ball around the maintained attractor. Since $\mathcal{V}$ contains packet distance, boundary damage, and memory integrity, the packet remains in the viability tube and the system is persistent. Conversely, if $\Pi_{\mathrm{maint}} < 0$ over windows longer than the repair time, then the right-hand side eventually becomes positive, $\mathcal{V}$ grows, and either packet distance, boundary damage, or memory loss drives the system out of the tube. It therefore exits SeOS. □

The theorem should be interpreted carefully. It does *not* say that every SeOS must be in thermodynamic equilibrium or that every member has a fixed-point attractor. The maintained attractor may be a limit cycle, torus, metastable branch, or distributed memory manifold. The theorem only says that packet preservation requires positive maintenance margin after dissipation and repair are paid for.

Table 5.54: Open systems, packet-bearing systems, and the SeOS class.

System type	Maintained packet?	Memory depth	Maintenance margin	Classification
Passive tracer / Brownian particle	No	Negligible	Not defined as self-repair	Open system, not SeOS
Driven dissipative pattern (e.g. simple convection cell)	Weak or pattern-level only	Short	Positive only while driven; thin repair logic	Borderline or proto-SeOS
Protocell / microbe	Yes	Finite adaptive memory	Positive under nutrient throughput and repair	SeOS
Neural organism / cognitive circuit	Yes	Deep internal and external memory	Positive under metabolic and information throughput	SeOS with rich Qi-HAL
Distributed institution / society	Yes, distributed	Externalized archive plus internal memory	Positive if coordination and repair exceed dissipation/shock costs	Higher-order distributed SeOS

5.11.4 Cooperation, pathology, and the non-moral meaning of "selfish"

Because the term "selfish" can be misunderstood, we state explicitly what it does and does not mean. It does *not* mean conscious greed, competition for its own sake, or moral approval. It means only that the local runtime is evaluated with respect to preservation of the packet under study. This local logic actually explains why cooperation is common. If cooperation increases Π_{maint}, repairs boundary damage, stabilizes memory, or enlarges the basin of the maintained attractor, then cooperation is selected by the same local criterion. Symbiosis, division of labor, and hierarchical organization are therefore not exceptions to SeOS; they are common mechanisms for realizing it.

The same logic also explains pathology. A packet can preserve itself at the expense of a larger-scale packet. A cancer clone may maintain its own local attractor while damaging the organism. An addiction loop may preserve a local reward circuit while degrading the person-level packet. A brittle institution may maintain its own archive and procedures while failing the population that hosts it. The theory therefore predicts that "selfishness" is scale-dependent: what is viability at one level may be pathology at a higher level. This is one reason the packet concept is so important. One must always specify *which packet* is being preserved.

5.11.5 QCI: the Qi-4 Completeness Index

Persistence is necessary but not sufficient for the full Chapter 5 program. A packet may survive while realizing only a thin subset of the Qi runtime. A bacterium and a cortex can both be members of SeOS, but they do not instantiate the Qi pillars to the same depth. For that reason

the manuscript now introduces a provisional completeness diagnostic: the *Qi-4 Completeness Index*, abbreviated QCI.

The intuition is straightforward. The four Qi pillars are not four unrelated ornaments. They are four mutually constraining layers of one runtime: current locking (Qi-QUA), trajectory selection (Qi-EP), scale diagnostics (Qi-CON), and memory architecture (Qi-HAL). A system that lacks one of these layers may still persist, but it does not realize the whole Qi-4 stack. A completeness index should therefore reward simultaneous realization rather than isolated excellence in one pillar alone.

Definition 5.11.4 (Qi-4 Completeness Index)**.** For a packet-bearing open system S, let

$$q_{\mathrm{QUA}}(S),\ q_{\mathrm{EP}}(S),\ q_{\mathrm{CON}}(S),\ q_{\mathrm{HAL}}(S) \in [0,1]$$

be domain-specific realization scores for the four pillars. Fix positive weights w_a with

$$\sum_{a\in\{\mathrm{QUA},\mathrm{EP},\mathrm{CON},\mathrm{HAL}\}} w_a = 1.$$

The *strict* Qi-4 Completeness Index is defined by the weighted geometric mean

$$\mathrm{QCI}(S) := q_{\mathrm{QUA}}(S)^{w_{\mathrm{QUA}}} q_{\mathrm{EP}}(S)^{w_{\mathrm{EP}}} q_{\mathrm{CON}}(S)^{w_{\mathrm{CON}}} q_{\mathrm{HAL}}(S)^{w_{\mathrm{HAL}}}. \tag{5.311}$$

The geometric mean is chosen for a reason. If one pillar is absent, completeness should collapse strongly rather than be hidden by large scores in unrelated pillars. That is what the product form guarantees. If desired for phenomenology or exploratory data analysis, one may also track the additive companion score

$$\mathrm{QCI}_{+}(S) := \sum_a w_a q_a(S), \tag{5.312}$$

but Eq. (5.311) is the stricter and conceptually cleaner quantity.

The pillar scores themselves are intentionally domain-dependent but structurally constrained. A useful canonical choice is:

$$q_{\mathrm{QUA}}(S) := \exp\left[-\frac{D_{\mathrm{lock}}(S)}{\sigma_{\mathrm{QUA}}}\right], \tag{5.313}$$

$$q_{\mathrm{EP}}(S) := \exp\left[-\frac{D_{\mathrm{sel}}(S)}{\sigma_{\mathrm{EP}}}\right], \tag{5.314}$$

$$q_{\mathrm{CON}}(S) := \frac{1}{4}\Big(c_{\mathrm{scale}} + c_{\mathrm{Levy}} + c_{\mathrm{frac}} + c_{1/f}\Big), \tag{5.315}$$

$$q_{\mathrm{HAL}}(S) := \frac{1}{5}\Big(c_{\mathrm{MePMoS}} + c_{\mathrm{NHT}} + c_{\mathrm{HAL}} + c_{\mathrm{BFFT}} + c_{\mathrm{switch}}\Big), \tag{5.316}$$

where the c-scores lie in $[0,1]$, D_{lock} measures deviation from the locked-current ray, and D_{sel} measures deviation from the common selector family. Other domain-specific realizations are permitted provided they preserve the same monotonic meaning.

Table 5.55: Interpretation of the four pillar scores entering QCI.

Pillar score	**What it measures**	**Typical intuition across domains**
q_{QUA}	fidelity of current locking	whether event-level energy, matter/charge, and information bookkeeping stays near one common locked ledger
q_{EP}	selector coherence	whether observed trajectories can be organized by one admissible selector family rather than by unrelated heuristics
q_{CON}	multiscale diagnostic closure	whether scale covariance, heavy tails, fractality, and two-knee spectra co-occur coherently
q_{HAL}	architectural completeness	whether memory, sensing, motion, multiscale coding, and commitment switching form a stable holographic architecture

Remark 5.11.5 (Persistence is not the same as completeness). A system may be high in SeOS membership and low in QCI. A stable chemical oscillator, for example, may persist while realizing only a thin memory architecture. Conversely, a rich but damaged neural system may momentarily preserve substantial Qi structure while approaching loss of viability. SeOS and QCI therefore answer different questions: the first asks whether a packet can maintain itself; the second asks how fully the four-pillar runtime is realized on that packet.

5.11.6 Integrated view: from runtime constitution to persistence and completeness

The notions derived above and the new completeness index can now be assembled into one compact flow. An admissible system first declares a canonical packet. That packet determines the state manifold, ports, generator, ledger bundle, and observables. The generator acts on the packet distribution, while the scalar EEGG functional organizes gradient descent and maintenance balance. The current vector is locked by the Arisaka Equation. The resulting dynamics then passes through the general runtime pipeline:

$$\mathrm{SIP} \xrightarrow{\text{declare system instance}} \mathrm{EEGG} \xrightarrow{\text{lawful GDOS evolution}} \mathrm{DGI} \longrightarrow \mathrm{GSSB} \longrightarrow \mathrm{GA}. \tag{5.317}$$

If, in addition, the cycle-averaged maintenance margin remains positive, the maintained attractor is not just a transient branch but a persistent packet:

$$(\mathrm{SIP}, \mathrm{EEGG}, \mathrm{GA}) + \left\langle \Pi_{\mathrm{maint}} \right\rangle_{\tau} > 0 \quad \Longrightarrow \quad \text{system instance } S \in \mathrm{SeOS}. \tag{5.318}$$

Equations (5.317) and (5.318) summarize the persistence half of the present section. The completeness half is measured by QCI, which records how fully the four Qi pillars are realized on a persistent packet. Together, persistence and completeness distinguish merely surviving packets from those that realize the full memory-bearing open-system stack. The framework is no longer only a constitutional story about what theories are admissible, nor only a dynamical story about how open systems flow, nor only an architectural story about memory and prediction. It is now

also a packaging story: any open system can be declared, evolved, budgeted, and tested in one grammar.

Table 5.56: Integrated roles of SIP, EEGG, and SeOS inside the full Chapter 5 runtime.

Layer	**Primary object**	**Main mathematical task**	**Main scientific question**
Packet	SIP	Declare the admissible system instance	What exactly is the object being modeled and measured?
Ledger	EEGG	Supply a unique scalar budget and its current bundle	What quantity is minimized, maintained, or spent during dynamics?
Runtime	DGI+GSSB+GA plus Qi-4	Evolve, select, diagnose, and architect the state	How does the system explore, choose, stabilize, and remember?
Persistence class	SeOS	Test invariant maintenance under throughput and repair	Does the packet merely fluctuate, or can it preserve itself as a continuing unit?

5.11.7 Worked examples: persistence and completeness across domains

It is helpful to illustrate the integrated flow with several cases.

A protocell. The packet declares membrane composition, vesicle size, internal metabolites, and permeability. The ports are fatty-acid inflow, pH, heat, and leak channels. The EEGG ledger combines membrane-assembly cost, electrostatic surface terms, internal mixing entropy, and the event budget for growth and division. If nutrient inflow and repair are sufficient to keep membrane defects bounded and metabolite loss below threshold, the vesicle belongs to SeOS. If not, it remains a transient aggregate.

ATP synthase in isolation versus in a living cell. As a molecular machine, ATP synthase has a clean packet and a well-defined ledger, but in isolation it is not fully autonomous: its maintenance margin is supplied by the larger membrane-cell packet. It is therefore best described as a maintained subsystem or a proto-SeOS component. Inside the living cell, however, the coupled membrane-machine-metabolism packet can have positive maintenance margin and thus qualify as SeOS.

A chemotactic bacterium. The packet includes internal adaptation state, membrane boundary, metabolic variables, receptor occupancy, and action channels. The ledger includes nutrient uptake, ATP expenditure, entropy production, and informational updating. Because the bacterium can preserve its membrane, repair damage, and maintain an adaptive memory under continued throughput, it is a clear member of SeOS.

A neural decision circuit. The packet includes recurrent state, synaptic couplings, oscillatory phase relations, and metabolically supported ports. The ledger includes coherence-maintenance cost, plasticity cost, and information-current flow. A functioning circuit with stable metabolic support and recoverable perturbation dynamics lies in SeOS. Pathology corresponds to a change in packet or a collapse of maintenance margin.

A distributed institution. The packet includes archived memory, communication channels, policy state, resource stocks, and performance observables. The ledger includes the energetic and informational cost of maintaining coordination, updating records, and repairing damage after shocks. If inflow and repair exceed dissipation and archive loss, the institution is a higher-order distributed SeOS. If not, it loses packet integrity and dissolves.

These examples underscore the point that SeOS is not restricted to biology, though biology provides the most vivid instances. The same mathematical grammar extends from chemical pre-life to organisms to collective structures.

5.11.8 Section summary

The persistence–completeness layer of Chapter 5 is now in place. The main conclusions are:

(1) A system belongs to SeOS precisely when it can preserve its packet inside a viability tube with positive maintenance margin after dissipation and repair costs are paid.

(2) Persistence alone does not measure how fully a system instantiates the Qi runtime. For that purpose the manuscript now introduces the provisional QCI score.

(3) The strict form of QCI is multiplicative across the four pillars, so loss of one pillar suppresses completeness even when the remaining pillars are strong.

(4) Therefore Part IV now has a clean two-layer logic: SeOS classifies persistent self-maintaining packets, while QCI evaluates how fully the Qi-4 architecture is realized on such packets.

(5) Together with the later OH/IH/EIC semantics, these notions complete the first full vocabulary for persistence, completeness, and internality in Chapter 5.

This completes the persistence/completeness block of the manuscript and prepares the final conceptual climax of Part IV, namely the distinction between outer projection and inner reconstruction.

5.12 OH+ IH= EIC: **Outer Hologram, Inner Hologram, and Internal Measurement**

With persistence and completeness fixed, the theory can turn to its deepest conceptual distinction: external projection versus internal reconstruction. Section 5.12 therefore introduces OH, IH, and EIC as the final conceptual layer of Part IV. This placement is deliberate. Internality must come after runtime constitution, Qi architecture, persistence, and completeness; otherwise it risks floating free of the lawful machinery that makes it possible.

5.12.1 Why the runtime still needs a distinction between external projection and internal reconstruction

After Sections 5.3–5.11, the manuscript already possesses a universal runtime, a current ledger, a selector principle, a diagnostic layer, a memory architecture, a packet vocabulary, and a persistence criterion. Yet one decisive conceptual task remains unfinished. The earlier sections explain how an open system evolves, how it budgets its currents, how it stores memory, and how it persists. They do not yet fully explain how the world appears *to* an open system, how an open system reconstructs that world internally, and how a selected internal reconstruction may come to function as conscious content.

This missing step is exactly what the relation

$$\mathrm{OH} + \mathrm{IH} = \mathrm{EIC}$$

is designed to capture. The notation is intentionally compact, but it must not be read superficially. The symbol OH does not mean "the world itself." It means the lawful projection of the inaccessible or higher-dimensional world into observer-accessible boundary records. The symbol IH does not mean "a copy of the world in the head." It means the internal reconstructive manifold through which a memory-bearing open system infers, stabilizes, and reuses an allocentric model of what lies beyond its immediate sensory boundary. The symbol EIC denotes the *Evolving Inner Cosmos*: the closed loop in which externally projected evidence and internally reconstructed structure are continually aligned, updated, selected, and stabilized.

The need for this distinction is not just philosophical. It is forced by the mathematics of partial access. A finite open system never interacts with the total six-dimensional bulk in its full state-description. It interacts only through ports and measurement channels. Likewise, a living or cognitive system never receives the full external state; it receives only a narrow sensory stream, often lower-dimensional than the structure it must navigate. The world that guides action must therefore be internally reconstructed rather than directly given. The present section formalizes that statement.

It is useful to mark the three layers immediately. We write H_t^{out} for the concrete time-dependent outer hologram, H_t^{in} for the time-dependent inner hologram, and $\mathfrak{E}_t$ for the corresponding evolving inner-cosmos state. Table 5.57 gives the high-level distinction.

Table 5.57: Basic distinction between OH, IH, and EIC.

Object	**What it is**	**What it is not**	**Constitutional role**
OH	Lawful projection from the 6D bulk to accessible boundary records	Not the full bulk state itself	Extends OGGEP from laboratory measurement to all scientific observation
IH	Internal reconstructive manifold built from memory, prediction, action, and sensory history	Not a literal pixel-by-pixel copy of the external world	Explains how open systems build allocentric models from partial data
EIC	Closed-loop coupled process aligning OH and IH through repeated updates	Not a mystical addition to dynamics	Explains internal world-model formation, conscious content, and learning across scales

The section therefore proceeds in five steps. First, it derives the outer hologram directly from the OGGEP boundary law. Second, it derives the inner hologram from the inverse problem faced by any memory-bearing open system with action. Third, it shows why motion, memory, and allocentric stabilization are mathematically necessary if one starts from low-dimensional or egocentric sensory data. Fourth, it gives a disciplined interpretation of the relation OH+IH = EIC by introducing an explicit alignment functional. Fifth, it states the consciousness thesis in its careful technical form: consciousness is the system's internal measurement of a selected and stabilized portion of the inner hologram down to an internal boundary implemented by MePMoS and the broader Qi-HAL architecture.

5.12.2 The Outer Hologram: lawful projection from the 6D bulk to measurable boundary records

Chapter 4 already established the core operational map demanded by AX-3:

$$\mathcal{M}_{\text{bulk}} : \quad \text{bulk data} \longrightarrow \big(\mathcal{B}, \rho_{\partial}, \Phi_{t,s}, \{\mathcal{I}_\alpha\}\big).$$

The present subsection re-reads this map as the definition of the outer hologram.

Let $X \in \mathcal{X}_6$ denote a bulk microstate or bulk configuration coordinate, and let $y \in \mathcal{Y}$ denote an observer-accessible boundary record. We define the outer-hologram kernel by

$$K_{\text{OH}}(y|X;t) := \text{Tr}\big[\mathcal{I}_y\, \Phi_{t,0}\big(|X\rangle\langle X|\big)\big]\,, \tag{5.319}$$

where $\mathcal{I}_y$ is the instrument associated with the registered boundary record y, and $\Phi_{t,0}$ is the reduced evolution map supplied by OGGEP. In a classical limit, Eq. (5.319) reduces to an ordinary stochastic kernel. In a quantum or mixed quantum-classical setting it is the instrument-induced probability density of the boundary event y conditional on the bulk configuration X.

The actual outer-hologram record at time t is then the projected boundary law

$$H_t^{\mathrm{out}}(y) = \int_{\mathcal{X}_6} K_{\mathrm{OH}}(y|X;t)\,\rho_6(X,t)\,\mathrm{d}X, \tag{5.320}$$

with $\rho_6(X,t)$ the effective bulk ensemble or bulk-state density. By construction,

$$K_{\mathrm{OH}}(y|X;t) \geq 0, \qquad \int_{\mathcal{Y}} K_{\mathrm{OH}}(y|X;t)\,\mathrm{d}y = 1, \tag{5.321}$$

so H_t^{out} is a positive normalized record law.

Definition 5.12.1 (Outer Hologram)**.** Given a bulk state family $\rho_6(X,t)$, a reduced boundary evolution $\Phi_{t,0}$, and an admissible instrument class $\{\mathcal{I}_y\}$, the *outer hologram* at time t is the boundary law H_t^{out} defined by Eq. (5.320). An individual registered observation y_t is an *OH event*.

Several consequences follow immediately.

First, the outer hologram is *lawful*. It is not an arbitrary representation chosen by the observer after the fact. It is fixed by the constitutional map from bulk variables to boundary observables. Second, it is *lossy*. Many bulk states can induce the same boundary law, because boundary access is partial. Third, it is *instrument-relative*. Different instrument families produce different coarse-grainings of the same bulk reality. This does not make the OH subjective. It means only that observation is always indexed by an admissible boundary channel.

Proposition 5.12.2 (The outer hologram is a quotient object)**.** *Define an equivalence relation on the bulk state space by*

$$X \sim_{\mathrm{OH}} X' \quad \Longleftrightarrow \quad K_{\mathrm{OH}}(\cdot|X;t) = K_{\mathrm{OH}}(\cdot|X';t) \text{ for the chosen instrument class and time } t. \tag{5.322}$$

Then the outer hologram depends only on the equivalence class $[X]_{\mathrm{OH}}$, not on the full bulk representative. In this precise sense, observation accesses a quotient of bulk reality.

Proof. Equation (5.320) depends on X only through the kernel $K_{\mathrm{OH}}(y|X;t)$. If $X \sim_{\mathrm{OH}} X'$, the two kernels coincide pointwise in y, so they contribute identically to $H_t^{\mathrm{out}}(y)$ for every admissible boundary record. Therefore any replacement of X by another representative in the same equivalence class leaves the OH unchanged. The boundary observer can distinguish only equivalence classes induced by the kernel, not the full bulk microstate. □

Proposition 5.12.2 is the rigorous version of a simple statement: no finite observer sees the full bulk. Every observation is a projection. In a particle detector, the OH event is a click pattern or reconstructed track. In a microscope, it is an image frame. In an ecosystem survey, it is a collection of count data or flux measures. In a nervous system, it is a boundary sample delivered through receptors. What unifies these cases is not the material substrate but the constitutional structure: each is a lawful quotient of a larger inaccessible state space.

Remark 5.12.3. The word "hologram" is appropriate here because the projected boundary record is not a trivial coordinate restriction. It is a coded boundary image of a higher-dimensional lawful structure. However, this book uses "hologram" operationally rather than pictorially. The

OH is whatever lawful boundary object the OGGEP map exports; in many sciences it need not look visually like an ordinary image.

One may also write the outer hologram in observable form. Let $\{O_i\}$ be a chosen family of finite-resolution observables on the boundary. Then

$$\langle O_i \rangle_{H_t^{\mathrm{out}}} = \int_{\mathcal{Y}} O_i(y)\, H_t^{\mathrm{out}}(y)\, \mathrm{d}y = \int_{\mathcal{X}_6} \mathcal{O}_i^{\mathrm{bulk}\to\mathrm{bdy}}(X;t)\, \rho_6(X,t)\, \mathrm{d}X, \tag{5.323}$$

where

$$\mathcal{O}_i^{\mathrm{bulk}\to\mathrm{bdy}}(X;t) := \int_{\mathcal{Y}} O_i(y)\, K_{\mathrm{OH}}(y|X;t)\, \mathrm{d}y.$$

Equation (5.323) emphasizes that the OH is the full statistical image of the bulk on the chosen measurement boundary. Any scientific data product is therefore an OH object, not a direct copy of the underlying total state.

5.12.3 Every registered scientific measurement is an OH event

We pause to make the universality claim explicit. In ordinary language one says that a detector records a particle, a camera records light, a cochlea records sound, or a receptor records ligand concentration. In the present framework each of these statements is incomplete. What is actually recorded is an OH event.

Suppose a system S has a port set $\mathcal{P}$ and an observable dictionary $\mathcal{O}$ in the sense of Sections 5.5, 5.6, and 5.11. Then every element of the time series

$$\{o_n\}_{n=0}^{N} \subset \mathcal{O}$$

is the output of a boundary projection channel. Even if the measured object is a seemingly local classical quantity, the measured value is not the quantity "in itself" but the quantity as exported to the observer through the chosen port and instrument.

This is important for two reasons. First, it prevents a recurrent confusion in discussions of consciousness and cognition: one must not identify the world with the registered data stream. The world enters as OH. Second, it makes room for the IH as a necessary additional object. If all that existed were the registered boundary data, there would be no place for internal reconstruction, prediction, or allocentric stabilization.

Table 5.58 shows how the OH notion applies across representative domains.

Table 5.58: Examples of OH events across scientific domains.

Domain	**Underlying external structure**	**Boundary channel / instrument**	**OH event**
Particle physics	Bulk field excitation or scattering process	Detector, trigger, reconstruction chain	Hit pattern, track, calorimeter energy, event class
Cell biology	Molecular state, conformational or concentration field	Receptor, microscope, fluorophore, reporter construct	Bound receptor state, image frame, fluorescence time point
Neuroscience	External world plus organism pose and context	Retina, cochlea, mechanoreceptors, electrodes	Sensory sample, spike train, local field potential record
Ecology / society	Hidden population or collective state	Survey protocol, sensor network, archive	Count, flow value, transaction record, communication event

The language of this book is therefore sharper than ordinary empiricism. Empiricism often says: observation is the source of knowledge. GDOS says: observation is a lawful OH event, already shaped by the bulk-to-boundary map. The IH, to be introduced next, is what turns a stream of such events into an internally usable world.

5.12.4 The Inner Hologram: internal reconstruction of allocentric structure from partial sensory streams

The outer hologram delivers data, but no open system can survive or act intelligently by storing raw data alone. If the incoming stream is partial, delayed, noisy, or lower-dimensional than the relevant external structure, a reconstructive model is required. That model is the inner hologram.

Let $x_t \in \mathcal{X}_{\text{env}}$ denote the external state relevant to the system at time t, $a_t \in \mathcal{A}$ the action or motion variable, $s_t \in \mathcal{S}$ the sensory sample, and $m_t \in \mathcal{M}$ the internal memory state. We consider the generic controlled open-system dynamics

$$x_{t+1} = f(x_t, a_t) + \omega_t, \tag{5.324a}$$

$$s_t = h(x_t) + \nu_t, \tag{5.324b}$$

with process perturbation ω_t and measurement noise ν_t. In many cases h is strongly non-injective: different external states generate indistinguishable instantaneous sensory samples. This is the rule rather than the exception. A 2D retina does not uniquely determine a 3D world at one instant; a single odor concentration sample does not uniquely determine a surrounding concentration field; a scalar temperature reading does not uniquely determine a thermal landscape.

The system therefore introduces an internal latent variable $z_t \in \mathcal{Z}$ that does not merely store the last s_t but represents a structured hypothesis about the external world. A natural variational definition is

$$q_t(z_t) = \arg\min_q \left\{ \mathbb{E}_q[-\ln p(s_t|z_t)] + D_{\text{KL}}\Big(q(z_t) \,\Big\|\, p(z_t|m_{t-1}, a_{t-1})\Big) \right\}, \tag{5.325}$$

which is the Qi-EP/FEP-type update written specifically for internal world-model reconstruction. The first term fits the current sensory evidence, while the second term preserves continuity with the previously predicted state. In plain language: the system asks which internal latent configuration best explains the incoming OH event without discarding what its past memory and action already predicted.

Definition 5.12.4 (Inner Hologram)**.** For a memory-bearing open system, the *inner hologram* at time t is the structured internal reconstructive object

$$H_t^{\mathrm{in}} := \big(\mathcal{Z}, q_t, g_t, W_t\big), \tag{5.326}$$

where $\mathcal{Z}$ is the latent internal manifold, q_t is the current posterior or belief state on $\mathcal{Z}$, g_t is the learned or reconstructed metric/geometry on that manifold, and W_t is the HAL memory substrate storing the relational attractor structure.

Definition 5.12.4 is intentionally broader than a probabilistic belief state. The IH is not only a posterior density. It also contains geometry and memory. The metric g_t enters because the internal model must encode distances, directions, or relational structure relevant for prediction and action. The memory tensor W_t enters because the reconstructed world must persist across time; without storage there is no genuine internal world-model, only a transient estimate.

The role of the Qi-HAL modules is now immediate. MePMoS provides the causal update order. NHT provides a phase-like coordinate system through which latent geometry can be encoded. HAL provides the storage substrate of recurrent relational structure. BFFT coordinates scales, so that fast sensory detail and slow contextual priors can coexist in the same IH. The DGI–GSSB switch governs how multiple candidate interpretations remain coexistent during exploration and then collapse to a selected interpretation once evidence and recurrent gain cross threshold.

The IH update may be written in MePMoS-compatible form as

$$\widehat{z}_{t|t-1} = \mathcal{P}(m_{t-1}, a_{t-1}), \tag{5.327a}$$

$$q_t = \arg\min_q \left\{ \mathbb{E}_q[-\ln p(s_t|z_t)] + D_{\mathrm{KL}}\Big(q(z_t) \,\|\, \delta_{\widehat{z}_{t|t-1}}\Big) \right\}, \tag{5.327b}$$

$$m_t = \mathcal{U}(m_{t-1}, q_t, s_t), \tag{5.327c}$$

$$a_t = \pi(m_t, q_t), \tag{5.327d}$$

where $\delta_{\widehat{z}_{t|t-1}}$ denotes the predicted latent prior concentrated around the predicted state, and π is the action policy. Equations (5.327a)–(5.327d) make clear that the IH is not a static map. It is a continually updated reconstructive manifold embedded in the action-perception loop.

Remark 5.12.5. The word "inner" should not mislead. The IH is not private in principle because it can leave external traces through action, report, physiology, and behavior. It is "inner" only in the operational sense that it is realized inside the system as the latent manifold guiding action, rather than in the external world directly.

5.12.5 Why motion and memory are mathematically necessary for the inner hologram

The most common intuitive mistake is to imagine that an internal world-model can be built from raw sensing alone. The mathematics says otherwise. Whenever the instantaneous sensory map h is non-injective, unique reconstruction from one time slice is impossible. History and controlled motion are required.

To formalize this, linearize Eqs. (5.324a)–(5.324b) around a nominal trajectory:

$$\delta x_{t+1} = A_t\,\delta x_t + B_t\,\delta a_t, \tag{5.328a}$$

$$\delta s_t = C_t\,\delta x_t. \tag{5.328b}$$

Over a horizon T, the action-conditioned observability matrix is

$$\mathcal{O}_T = \begin{bmatrix} C_0 \\ C_1 A_0 \\ C_2 A_1 A_0 \\ \vdots \\ C_T A_{T-1} \cdots A_0 \end{bmatrix}. \tag{5.329}$$

Theorem 5.12.6 (Action-conditioned observability of the IH)**.** *Assume the linearized model (5.328) is valid on a finite horizon $0 \le t \le T$. If $\operatorname{rank}(\mathcal{O}_T) = \dim(\delta x_0)$, then the external state perturbation δx_0 is uniquely determined by the sensory history $(\delta s_0, \ldots, \delta s_T)$ and the action-conditioned state transition sequence (A_t, B_t), up to any symmetry already quotiented by the observation map. If $\operatorname{rank}(\mathcal{O}_T) < \dim(\delta x_0)$, then there exist distinct external perturbations that yield identical sensory histories, so no unique IH can be reconstructed from the available history.*

Proof. The sensory sequence generated by δx_0 is

$$\begin{bmatrix} \delta s_0 \\ \delta s_1 \\ \vdots \\ \delta s_T \end{bmatrix} = \mathcal{O}_T\,\delta x_0 + \text{known terms generated by the chosen action sequence.}$$

If $\mathcal{O}_T$ has full column rank, the nullspace is trivial and δx_0 is uniquely recoverable from the sensory history modulo symmetries already removed by the map. If the rank is deficient, the nullspace is nontrivial; hence there exists $\delta x_0 \neq 0$ such that $\mathcal{O}_T\,\delta x_0 = 0$. This nonzero perturbation changes the external state but produces no change in the sensory history, proving non-identifiability. □

Theorem 5.12.6 is the rigorous reason why motion is not an accessory but a constitutive part of perception. Self-motion changes the observation geometry and thereby increases the rank of the accumulated observability matrix. A retina without motion is depth-poor. A bacterium without swimming is gradient-poor. A brain without active sensing is world-model-poor. That is why MePMoS must contain motion before the next sensory registration.

Memory is equally necessary. The observability matrix acts over a history window, and history must be retained somewhere. A system with no memory cannot accumulate the multi-view evidence needed to invert a non-injective sensory map. Therefore the very existence of an IH already presupposes the packet, the ledger, and the memory architecture introduced earlier in the paper.

Corollary 5.12.7 (No genuine IH without history)**.** *If the system has no persistent state capable of retaining information about prior sensory samples or prior actions, then any internal state at time t is a function only of the current sample s_t. In that case the internal representation cannot generally invert a non-injective observation map and therefore cannot constitute a genuine inner hologram except in trivial fully observable environments.*

Proof. If no persistent state exists, the internal representation is memoryless: $z_t = \varphi(s_t)$. Whenever h is non-injective, there exist $x \neq x'$ with $h(x) = h(x') = s_t$. Then $\varphi(s_t)$ assigns the same internal state to both external possibilities, so it cannot reconstruct the latent structure uniquely. Only if h is injective already, or the relevant task ignores the missing variables, can a memoryless representation suffice. □

The corollary is especially important for the broad application of the theory to all open systems with memory. The moment a system becomes memory-bearing and action-bearing under nonzero J_{Qi}, the mathematical door to IH formation opens. This does not guarantee a rich cognitive map. It guarantees only that an internal reconstructive manifold is now a lawful possibility rather than a category mistake.

5.12.6 Why the inner hologram must be allocentric rather than a stored sensory movie

The next issue is subtle but fundamental. Why is the IH not just a compressed recording of sensory history? Why insist that it is allocentric or world-centered?

The answer is that action-guiding reconstruction must factor out self-motion. Let $g_t \in SE(n)$ denote the system's pose (position and orientation) in an n-dimensional task space, and let ξ_t denote a world-centered latent configuration. A generic egocentric observation has the form

$$s_t = \pi\big(g_t^{-1} \cdot \xi_t\big) + \nu_t, \tag{5.330}$$

where π denotes projection through the sensory channel. If the internal model stores only s_t, then every self-motion $g_t \mapsto g_{t+1}$ changes the entire representation even when the external world ξ_t is unchanged. Such a representation is unsuitable for stable prediction, navigation, or recognition across viewpoints.

The inner hologram must therefore transport old information into a pose-invariant frame. A minimal update law is

$$\widehat{\xi}_{t+1} = T(a_t)\,\widehat{\xi}_t + K_t\,\epsilon_{t+1}, \qquad \epsilon_{t+1} = s_{t+1} - \widehat{s}_{t+1|t}, \tag{5.331}$$

where $T(a_t)$ is the transport induced by self-motion, and K_t is an evidence gain. Equation (5.331) is the simplest mathematical expression of allocentric reconstruction. The old map is not discarded

after motion; it is transformed by the internally known or inferred motion and then corrected by new evidence.

This is why spatially and temporally structured codes arise naturally in neural systems. Place-like states encode location in a world-centered frame, head-direction or ring-attractor variables encode orientation, and phase-structured sequences encode ordered trajectories or temporal position (O'Keefe and Dostrovsky, 1971; Turner-Evans et al., 2020). This book does not require one biological implementation only. It requires the mathematical role those implementations play: self-motion-compensated reconstruction of an external structure that remains stable despite changing sensory views.

Proposition 5.12.8 (Allocentric stability criterion)**.** *Suppose the external latent configuration ξ_t is constant while the system changes pose g_t. A purely egocentric internal code $r_t = \varphi(s_t)$ is viewpoint-stable only if $\varphi \circ \pi(g_t^{-1} \cdot \xi)$ is independent of g_t for all relevant poses, which is generically impossible for nontrivial projection maps. An allocentric update of the form (5.331), by contrast, preserves the representation of ξ under self-motion up to bounded correction error.*

Proof. For nontrivial projection π, different poses g_t produce different egocentric observations of the same ξ. Therefore a direct code $r_t = \varphi(s_t)$ changes with pose unless φ performs the full inverse transport for all g_t, which is exactly what an allocentric reconstruction does. Under Eq. (5.331), the old estimate is first transported by the known action-induced map $T(a_t)$, leaving the encoded world configuration invariant in the absence of noise; the correction term $K_t\epsilon_{t+1}$ accounts only for residual mismatch. Hence viewpoint stability is achieved up to bounded prediction error. □

In everyday language the distinction is simple. The IH is not a film running across the senses. It is a map. In richer systems it is a hierarchy of maps: spatial, temporal, causal, social, symbolic, and theoretical. A physicist reconstructing a six-dimensional theory from four-dimensional experiments is performing a very high-level IH operation. The same architectural logic begins, in primitive form, whenever a bacterium's adaptive state stores something more useful than its last measurement alone.

5.12.7 The equality OH + IH = EIC: precise meaning of the plus sign

We are now in a position to interpret the equation itself. The expression

$$\mathrm{OH} + \mathrm{IH} = \mathrm{EIC}$$

should not be read as scalar addition. The plus sign denotes *closed-loop lawful composition on a common operational boundary.* The outer hologram supplies evidence. The inner hologram supplies reconstruction. The evolving inner cosmos is their repeatedly updated coupling.

Definition 5.12.9 (Evolving Inner Cosmos)**.** For a system with outer hologram H_t^{out} and inner hologram H_t^{in}, the *Evolving Inner Cosmos* is the coupled state

$$\mathfrak{E}_t := H_t^{\mathrm{out}} + H_t^{\mathrm{in}} \equiv \mathcal{C}_{\mathrm{loop}}\big(H_t^{\mathrm{out}}, H_t^{\mathrm{in}}\big), \tag{5.332}$$

where $\mathcal{C}_{\text{loop}}$ denotes closed-loop composition through prediction, action, sensing, memory update, and internal readout. The notation + is therefore mnemonic for coupling rather than arithmetic addition.

To make the coupling quantitative, let the OH-induced sensory law at time t be

$$p_t^{\text{OH}}(s) = \int_{\mathcal{X}_6} K_{\text{OH}}(s|X;t)\,\rho_6(X,t)\,\mathrm{d}X, \tag{5.333}$$

and let the IH-induced predicted sensory law be

$$\widehat{p}_t^{\text{IH}}(s) = \int_{\mathcal{Z}} p(s|z,a_t)\,q_t(z)\,\mathrm{d}z. \tag{5.334}$$

Their mismatch is measured by the reconstruction divergence

$$\Delta_{\text{rec}}(t) = D_{\text{KL}}\Big(p_t^{\text{OH}} \,\Big\|\, \widehat{p}_t^{\text{IH}}\Big). \tag{5.335}$$

When $\Delta_{\text{rec}}(t)$ is small, the inner hologram predicts the OH well. When it is large, the internal model poorly matches the external evidence.

Because internal reconstruction is not free, one must add the runtime cost already derived earlier in the paper. A convenient composite functional is

$$\mathcal{F}_{\text{EIC}}(t) = \Delta_{\text{rec}}(t) + \lambda_E\,\mathcal{F}_{\text{EEGG}}(t) + \lambda_C\,\mathcal{C}[q_t], \tag{5.336}$$

where $\mathcal{F}_{\text{EEGG}}$ is the energy–entropy ledger of Sections 5.5, 5.6, and 5.11, $\mathcal{C}[q_t]$ is a complexity or model-maintenance term such as $D_{\text{KL}}(q_t\|q_{\text{prior}})$, and $\lambda_E, \lambda_C > 0$ are weights. Equation (5.336) says in the clearest possible way what the EIC does: it aligns internal reconstruction with outer evidence, but only under lawful energetic and entropic budget constraints.

If θ_t denotes the collection of internal parameters entering q_t, g_t, W_t, and the action policy, then the Qi-EP-consistent gradient normal form is

$$\dot{\theta}_t = -\,G_\theta^{-1}\nabla_\theta\mathcal{F}_{\text{EIC}} + \eta_t, \tag{5.337}$$

with G_θ a positive metric on parameter space and η_t a bounded fluctuation term.

Theorem 5.12.10 (Monotonic refinement of the EIC in the noiseless normal form)**.** *Assume G_θ is positive definite and $\eta_t = 0$. Then the EIC functional satisfies*

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F}_{\text{EIC}}(t) = -\,\nabla_\theta\mathcal{F}_{\text{EIC}}^{\top}G_\theta^{-1}\nabla_\theta\mathcal{F}_{\text{EIC}} \le 0. \tag{5.338}$$

Hence $\mathcal{F}_{\text{EIC}}$ is a Lyapunov functional for the coupled OH-IH update and decreases monotonically until the system reaches a stationary point or attractor basin.

Proof. Differentiate Eq. (5.336) along the flow (5.337):

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F}_{\text{EIC}} = \nabla_\theta\mathcal{F}_{\text{EIC}}^{\top}\dot{\theta}_t = -\,\nabla_\theta\mathcal{F}_{\text{EIC}}^{\top}G_\theta^{-1}\nabla_\theta\mathcal{F}_{\text{EIC}},$$

since $\eta_t = 0$. Because G_θ^{-1} is positive definite, the quadratic form is nonnegative, proving Eq. (5.338). □

Theorem 5.12.10 is deliberately local and normal-form in spirit. It does not prove that every real system globally reconstructs the true external world in all circumstances. It proves the more careful statement actually needed for the paper: once the runtime has an OH channel, an IH manifold, and a Qi-EP-compatible update rule under EEGG budget constraints, there is a canonical coupled functional whose decrease corresponds to progressive internal alignment with externally projected evidence. That progressively aligned loop is the EIC.

Remark 5.12.11. The EIC does not recover the full six-dimensional bulk "as it is in itself." It recovers, at best, an operationally adequate model of the equivalence class of external states accessible through the system's boundary channels. This is not a weakness of the theory; it is the precise consequence of Proposition 5.12.2.

The evolving-inner-cosmos language now acquires its disciplined meaning. The "cosmos" is the system's internal world-model, not because it contains everything, but because it is the internal totality relative to which the system navigates, predicts, and acts. It is "evolving" because it is continually updated through the OH-IH loop. Human scientific learning is simply one very elaborate case of this general process: measurements enlarge and correct the inner world-model so that it better matches the externally projected lawful structure.

5.12.8 Consciousness as internal measurement of the inner hologram

The final conceptual step is the most delicate. The present framework does not identify consciousness with the OH, nor with raw sensory inflow, nor with the mere existence of an IH. Consciousness is a special operation performed *on* the IH.

The correct analogue of external measurement is internal measurement. Externally, the bulk is projected to a boundary record through OGGEP. Internally, a selected region of the IH is projected to a self-accessible internal boundary where it can bias future prediction, future action, memory writing, and in rich organisms explicit report.

Let $\rho_{\mathrm{IH}}(t)$ denote the internal state on the IH manifold. Let $\{\Pi_c\}$ be an admissible family of coarse-grained internal readout operators or projectors satisfying

$$\Pi_c \geq 0, \qquad \sum_c \Pi_c = I_{\mathrm{IH}}.$$

The internal measurement statistics are then

$$p_t(c) = \mathrm{Tr}\big(\Pi_c\, \rho_{\mathrm{IH}}(t)\big), \tag{5.339}$$

and the post-selected internal content state for channel c is

$$\rho_{\mathrm{IH}}^{(c)}(t) = \frac{\Pi_c \rho_{\mathrm{IH}}(t) \Pi_c}{\mathrm{Tr}(\Pi_c \rho_{\mathrm{IH}}(t))}. \tag{5.340}$$

Definition 5.12.12 (Internal measurement)**.** A system *internally measures* its inner hologram at time t if there exists an internal readout boundary ∂_{int} and a family of readout channels $\{\Pi_c\}$

such that at least one selected internal content state $\rho_{\mathrm{IH}}^{(c)}(t)$ re-enters the MePMoS loop as a causal variable for subsequent memory update, action selection, or report.

Definition 5.12.12 is the formal version of the claim that consciousness is "our internal measurement of IH down to the 4D boundary by MePMoS." The key idea is recursion: the IH is itself turned into a boundary-accessible object for the system. The system does not only model the world; it also samples or measures the state of its own model.

Here is where the earlier runtime layers reappear. During DGI exploration, several candidate interpretations of the current data may coexist in the IH. Through GSSB, one interpretation becomes selected. Through GA, the selected interpretation becomes stable enough to influence subsequent processing. A conscious content, in the present technical sense, is not the entire cloud of preselection possibilities but the selected and stabilized internal branch that is accessible on the internal boundary.

Definition 5.12.13 (Conscious EIC episode)**.** A *conscious EIC episode* occurs at time t when all of the following hold:

(a) the system possesses a nontrivial inner hologram H_t^{in};

(b) at least one internal content channel c_* is selected from competing candidates by a DGI→GSSB→GA sequence;

(c) the selected content persists on a timescale sufficient for writing, reuse, or action bias, i.e. it is not an instantaneous unstable fluctuation;

(d) the selected content is available on an internal boundary ∂_{int} for re-entry into MePMoS as memory, action, or report bias.

Definition 5.12.13 is intentionally careful. It does *not* say that every internal model is conscious. It does *not* say that all information processing is conscious. It says only that conscious content is the internally measured, selected, and stabilized part of the IH that becomes causally available to the next cycle of the system's own runtime.

The operational distinction matters. A great deal of sensory and neural processing can remain subthreshold, fragmented, or preselective. In the present language such processing lives in the IH but has not yet crossed the DGI→GSSB→GA chain needed for internal measurement on ∂_{int}. Conversely, once such a crossing occurs and the content becomes self-available, one has a candidate conscious episode.

Proposition 5.12.14 (Consciousness is not raw sensation)**.** *Under Definitions 5.12.4, 5.12.9, and 5.12.13, raw sensory samples s_t are neither necessary nor sufficient for conscious content. They are not sufficient because they may fail to be integrated into an IH and internally measured. They are not necessary in the strict momentary sense because internally reactivated or imagined content can be selected from stored IH structure even when current sensory drive is weak or absent.*

Proof. Insufficiency: by Definition 5.12.13, conscious content requires a selected and stabilized internal branch accessible on ∂_{int}, not just the existence of a sensory sample. Necessity failure in

the strict momentary sense: the same definition allows internally driven content states selected from H_t^{in} if they are available on the internal boundary for subsequent memory, action, or report, even in the absence of strong new external drive. Therefore current sensation alone neither guarantees nor exhausts conscious content. □

This proposition is important because it separates the present framework from two common simplifications. The first simplification says that consciousness is just incoming sensation. The second says that consciousness is just abstract integration or complexity. In the present framework, both are incomplete. Consciousness is an operation: internal measurement of a selected and stabilized inner hologram under the Qi-HAL runtime.

One may package the operation in a compact normal form. Let $\mathcal{S}_{\text{DGI}}$, $\mathcal{S}_{\text{GSSB}}$, and $\mathcal{S}_{\text{GA}}$ denote the exploration, selection, and stabilization stages acting on ρ_{IH}. Then the self-accessible content state may be written schematically as

$$\rho_{\text{self}}(t) = \mathcal{M}_{\text{int}} \circ \mathcal{S}_{\text{GA}} \circ \mathcal{S}_{\text{GSSB}} \circ \mathcal{S}_{\text{DGI}}\big[\rho_{\text{IH}}(t)\big]. \tag{5.341}$$

Equation (5.341) is the recursive analogue of the external measurement chain. It is not a separate metaphysical law; it is the same open-system logic reapplied to the system's internal model.

5.12.9 Universality across open systems with memory once $U(1)_{Qi}$ is present

The goal of this section is to show how the framework applies to any open system with memory once $U(1)_{Qi}$ is applied. We can now state that result clearly.

The key point is that $U(1)_{Qi}$ makes informational state changes physically budgeted, phase-covariant, and persistent in the same sense as the earlier sections. Once a system has a memory-bearing variable with nonzero informational current J_{Qi}, its state changes can no longer be treated as abstract bookkeeping only. They become lawful gauge-bearing events. This makes an IH possible in a physically disciplined sense rather than as a metaphor.

Because internality is placed only after runtime constitution, the full Qi stack, and persistence/completeness, the next theorem should be read as a late-stage consequence of the earlier construction rather than as a primitive assumption.

Theorem 5.12.15 (OH/IH/EIC universality theorem for memory-bearing $U(1)_{Qi}$ systems)**.** *Let S be an admissible open system satisfying the following:*

(i) S has observer-accessible ports and therefore an admissible OH channel;

(ii) S possesses a persistent memory-bearing variable m_t;

(iii) S carries nonzero informational current J_{Qi} coupled to $U(1)_{Qi}$;

(iv) future observations depend nontrivially on present action or motion, so that a MePMoS *loop exists.*

Then:

(a) S always admits an outer hologram H_t^{out} relative to the chosen instrument class;

(b) if m_t stores predictive information about future observations, then S admits at least a proto-inner-hologram H_t^{in};

(c) if the proto-IH is recurrently re-measured onto an internal boundary and stabilized by DGI→GSSB→GA*, then S admits an evolving inner cosmos $\mathfrak{E}_t$;*

(d) high-dimensional, multiscale, self-referential instances of $\mathfrak{E}_t$ correspond to what the present framework calls consciousness-grade runtime.

Proof. Part (a) is immediate from OGGEP and Definition 5.12.1. Part (b) follows from Corollary 5.12.7: if a persistent memory variable carries predictive information about future observations, then the system's internal state is no longer a function only of the current sample and constitutes a lawful latent reconstructive manifold. The presence of $U(1)_{Qi}$ ensures that this state is not merely formal but physically budgeted and phase-covariant, as shown earlier in Sections 5.7 and 5.10. Part (c) follows from Definitions 5.12.9 and 5.12.12: once the IH is recurrently measured on an internal boundary and re-enters MePMoS after selection and stabilization, the coupled OH-IH loop is exactly an EIC. Part (d) is the architectural specialization of Part (c) to high-QCI, multiscale, recursively self-modeling systems. □

The theorem is intentionally graded. It does not force an all-or-none divide between "conscious" and "not conscious." Instead it yields a continuum:

(1) **OH only**: systems that can be externally measured but do not store a predictive internal model of the world;

(2) **OH + proto-IH**: systems with memory that encode some predictive latent structure but do not yet exhibit rich recursive internal measurement;

(3) **OH + IH + proto-EIC**: systems whose internal model is repeatedly aligned with external evidence and biases future action;

(4) **consciousness-grade EIC**: systems with sufficiently complete Qi-HAL architecture, multiscale coordination, and self-accessible internal measurement.

That is the kind of graded picture one expects for a universal open-system theory.

Table 5.59 illustrates the continuum across representative systems.

Table 5.59: Representative OH/IH/EIC continuum across memory-bearing open systems.

System	**Memory-bearing substrate**	**Likely IH status**	**EIC grade**	**Interpretation**
ATP synthase or simple molecular cycle	Conformational occupancy or rotor phase	Degenerate or task-local internal state only	Below proto-EIC	Memory exists, but world reconstruction is minimal and highly local
Chemotactic bacterium	Receptor methylation / adaptive chemical state	Proto-IH of local gradient or resource field	Proto-EIC	Internal memory predicts external chemical structure and guides motion
Ring-attractor navigation circuit	Persistent heading or phase bump	Genuine low-dimensional IH	Intermediate EIC	Internal allocentric heading map coupled to sensory correction and action
Rodent hippocampo-entorhinal system	Distributed recurrent attractor memory	Rich spatial-temporal IH	Strong EIC	Multiscale world model supporting navigation, recall, and imagination
Human cortical-hippocampal system	Large multiscale distributed memory architecture	High-dimensional IH including symbolic models	Consciousness-grade EIC	Internal measurement of the IH becomes reportable, reflective, and abstract
Distributed institution or scientific community	Archived memory, models, communication protocols	Collective IH (distributed world model)	Extended EIC candidate	Group-level reconstruction of external structure through shared memory and measurement

Two examples deserve emphasis.

A chemotactic bacterium. The OH is the local chemical environment sampled through receptors. The memory substrate is the adaptive methylation state. The system does not store a full image of the world, but it stores a predictive estimate of whether the current trajectory is improving or worsening access to nutrient. In this restricted but genuine sense it carries a proto-IH of the local chemical world. The resulting running alignment of sensed gradient and adaptive state is a minimal proto-EIC.

A mammalian navigation system. The OH is the multisensory stream generated by vision, vestibular input, proprioception, and action-dependent re-sampling. The IH is a rich allocentric manifold stabilized by recurrent attractor dynamics, phase coding, and multiscale coordination.

The DGI phase corresponds to multiple candidate interpretations or trajectories; the GSSB phase corresponds to selected heading, place, or scene interpretation; the GA phase corresponds to stabilized memory or report. Here the EIC becomes rich enough to support explicit conscious episodes and, in humans, abstract theoretical reasoning.

5.12.10 Claims, non-claims, and empirical consequences

Because the current section touches the difficult topic of consciousness, it is important to state carefully what is and is not being claimed.

The paper *does* claim that the distinction between external projection and internal reconstruction is constitutionally necessary. It *does* claim that any memory-bearing open system under $U(1)_{Qi}$ may admit an IH in an appropriately scaled sense. It *does* claim that the EIC is the coupled process aligning inner model and outer evidence. It *does* claim that consciousness should be modeled as internal measurement of a selected and stabilized inner hologram.

The paper *does not* claim that every memory-bearing system is therefore conscious in the same sense as a human. It *does not* claim that the IH recovers the full six-dimensional bulk without quotient loss. It *does not* claim that one single neural frequency or one single anatomical region is "the seat" of consciousness. The framework is architectural and dynamical, not phrenological.

Table 5.60 summarizes the distinction.

Table 5.60: Claims and non-claims of the OH/IH/EIC section.

Claim	**Non-claim**
Every observation is an OH event exported through an admissible boundary channel.	The OH is not the full bulk reality itself.
The IH is a lawful internal reconstruction from partial, often lower-dimensional sensory data.	The IH is not a literal pixel-by-pixel copy of the world.
The EIC is the repeated coupling of OH and IH under MePMoS, Qi-EP, and EEGG.	The plus sign in OH + IH = EIC is not ordinary arithmetic addition.
Conscious content is internal measurement of a selected and stabilized IH branch.	Not every internal variable or memory trace is conscious.
Memory-bearing $U(1)_{Qi}$ systems admit a graded OH/IH/EIC continuum.	The framework does not impose a binary conscious/non-conscious cut at the level of all open systems.

Even at the present stage of development, the framework also yields empirical consequences. Some are direct, others are programmatic. Table 5.61 lists the most immediate ones.

Table 5.61: Representative empirical consequences of the OH/IH/EIC framework.

Prediction / consequence	**Content**	**Connection to the section**
Closed-loop sensing should outperform passive exposure for latent-structure learning	If motion/action is restricted while sensory drive is preserved, allocentric reconstruction quality should degrade because the observability rank drops.	Theorem 5.12.6
World-model quality should scale with memory depth and multiscale coordination	Systems with deeper memory and broader BFFT hierarchy should reconstruct richer IH manifolds.	Sections 5.10, 5.11, and Theorem 5.12.15
Internal conscious report should follow selection and stabilization, not raw input onset	Reportable content should lag preconscious activity because DGI exploration must be followed by GSSB and GA.	Definition 5.12.13
Minimal organisms may show proto-EIC without human-like consciousness	Memory-bearing agents such as bacteria should exhibit predictive internal states aligned with environmental structure, but only in a proto-IH / proto-EIC sense.	Theorem 5.12.15
Scientific learning is a measurable EIC refinement process	Across training or expertise, internal models should better predict OH data while respecting energetic and entropic cost.	Theorem 5.12.10

It is useful to compare the present framing with two existing literatures already discussed in the Introduction. Free-energy and active-inference frameworks emphasize prediction, action, and belief updating (Friston, 2010). The present section agrees with that emphasis but locates it inside a larger geometry-to-boundary constitution: the IH update is not a floating agent principle but the internal counterpart of the OGGEP map and the Qi-EP selector. Integrated-information approaches emphasize the structured unity of conscious content (Tononi, 2004). The present section does not reject that intuition; rather, it says that any such unity must be realized as internally measured, selected, stabilized IH structure, not as a primitive postulate independent of runtime, geometry, and ledger.

A final biological point is worth emphasizing. Once one accepts that every observation is OH and that every persistent predictive model is IH, then the developmental and evolutionary history of life can be reread as the progressive enrichment of the EIC. Primitive systems reconstruct only tiny local manifolds. Rich nervous systems reconstruct navigable spatial worlds. Human scientific communities reconstruct abstract lawful structure far beyond immediate sensory appearance. The paper's hypothesis is that these are not separate miracles but different depths of one common open-system runtime.

5.12.11 Section summary

The OH/IH/EIC layer is now in place. The main conclusions are:

(1) The outer hologram is the lawful boundary projection of higher-dimensional or otherwise inaccessible external structure. Every registered observation is an OH event.

(2) The outer hologram is a quotient object: observation accesses equivalence classes induced by the boundary kernel, not the full bulk state directly.

(3) The inner hologram is the internal reconstructive manifold built from memory, prediction, action, and sensory history. It is not a stored sensory movie but an allocentric latent model.

(4) Motion and memory are mathematically necessary for IH formation whenever the instantaneous sensory map is non-injective. This follows from the action-conditioned observability theorem.

(5) The equation OH + IH = EIC means lawful closed-loop composition, not scalar addition. The EIC is quantified by an alignment functional coupling reconstruction accuracy to the EEGG budget.

(6) In the gradient normal form, the EIC functional is a Lyapunov quantity, so the OH-IH loop monotonically refines internal reconstruction in the noiseless limit.

(7) Consciousness is not raw sensation and not the whole IH. It is internal measurement of a selected and stabilized branch of the IH on a self-accessible internal boundary under the Qi-HAL runtime.

(8) Therefore the OH/IH/EIC framework extends GDOS from external measurement to internal world-modeling, conscious content, and the graded continuum of memory-bearing open systems under $U(1)_{Qi}$.

With this section complete, the manuscript contains the constitutional runtime, the scalar ledger and packet layers, the full Qi-4 pillars, the persistence/completeness vocabulary, and the external/internal holographic semantics needed for the hierarchy, application, universality, discussion, and conclusion chapters that now follow immediately.

Emergence and scope

Hierarchy, applications, and universality across nature

5.13 Hierarchy and emergence: the Japanese five-story pagoda

Part V begins after the theory has reached internality. The next issue is hierarchical emergence: how the same runtime reappears across recurrent floors of natural organization. Section 5.13 therefore derives the Japanese five-story pagoda as the hierarchical factorization of the already established runtime.

5.13.1 Why a universal theory of open systems must become hierarchical

Once the packet, ledger, selector, scale-diagnostic, memory-architecture, persistence, and holographic layers have been written, the next unavoidable question is how they stack. No serious theory of open systems can stop at a single level of description. An elementary particle, a gravitationally organized cosmos, a catalytic molecular network, a living cell, and a reflective nervous system are not five unrelated topics merely placed next to one another for rhetorical effect. They are five recurrent levels at which the same runtime reappears with different dominant packets, different port structures, and different internal modeling depth. The Japanese five-story pagoda is therefore not introduced as a decorative analogy. It is introduced as the architectural image of a theorem: persistence in nature is organized as a hierarchy of symmetry-broken floors built around a common invariant spine.

The need for hierarchy can be made precise. Let

$$\widehat{\mathrm{SIP}}^{(0)} = \big(\mathcal{Z}^{(0)}, \mathcal{P}^{(0)}, \mathcal{F}^{(0)}_{\mathrm{EEGG}}, \mathcal{J}^{(0)}, \mathcal{O}^{(0)}, \mathfrak{K}^{(0)}\big)$$

be a packet at one level of description. Suppose there exists a coarse-graining map

$$C_{\ell+1} : \mathcal{Z}^{(\ell)} \longrightarrow \mathcal{Z}^{(\ell+1)}$$

that identifies a smaller set of collective variables $z^{(\ell+1)} = C_{\ell+1}(z^{(\ell)})$. Then the first question is whether those collective variables close approximately under time evolution. Write the exact induced derivative as

$$\dot{z}^{(\ell+1)}_{\mathrm{exact}} = DC_{\ell+1}(z^{(\ell)})\, F^{(\ell)}_{\mathrm{exact}}(z^{(\ell)}),$$

and let $F^{(\ell+1)}_{\mathrm{eff}}$ be the best effective drift written only in terms of $z^{(\ell+1)}$. The *closure defect* of the proposed new floor is then

$$\Delta_{\ell+1}(t) := \dot{z}^{(\ell+1)}_{\mathrm{exact}}(t) - F^{(\ell+1)}_{\mathrm{eff}}\big(z^{(\ell+1)}(t)\big). \tag{5.342}$$

If $\Delta_{\ell+1}$ is small in the dynamical sense relevant to the application, then the collective variables form a legitimate new packet rather than a poetic re-labeling of the old one.

Closure alone is not enough. The second question is persistence. A higher floor must live long enough to matter. If the lower-scale relaxation time is $\tau^{(\ell)}_{\mathrm{relax}}$ and the lifetime of the proposed

collective structure is $\tau_{\mathrm{life}}^{(\ell+1)}$, then one requires a scale-separation inequality of the form

$$\tau_{\mathrm{life}}^{(\ell+1)} \gg \tau_{\mathrm{relax}}^{(\ell)}. \tag{5.343}$$

Otherwise the apparent floor dissolves faster than the lower variables can be usefully averaged, and no autonomous scientific level exists.

The third question is port novelty. A genuine higher floor is not only a slower description of the same closed dynamics. It couples to the environment through new channels. A star couples radiatively and gravitationally at scales not meaningful for quark packets; a cell couples metabolically and through membranes at scales not meaningful for an isolated molecule; a brain couples through action, sensing, and internal measurement at scales not meaningful for a bare reaction loop. Denote the port set at level ℓ by $\mathcal{P}^{(\ell)}$. Then a new floor requires at least one new effective port direction,

$$\operatorname{rank}\Big(\operatorname{span}\mathcal{P}^{(\ell+1)}\Big) > \operatorname{rank}\Big(\operatorname{span}\mathcal{P}_{\mathrm{pull}}^{(\ell\to\ell+1)}\Big), \tag{5.344}$$

where $\mathcal{P}_{\mathrm{pull}}^{(\ell\to\ell+1)}$ denotes the mere pull-forward of lower-level ports. In words: a higher floor is not scientifically interesting unless it acquires new effective modes of exchange with its environment.

The three conditions (5.342)–(5.344) already explain why the pagoda image is compelling. The floors are not arbitrary. They are the levels at which a new packet closes, persists, and exchanges through distinct ports while remaining constitutionally lawful. The central spine of the pagoda corresponds to the invariants that survive across floors. In the present framework that spine is DGI.

Definition 5.13.1 (Admissible floor)**.** A proposed higher-level description $\widehat{\mathrm{SIP}}^{(\ell+1)}$ is an *admissible floor* if there exist constants $\varepsilon_{\ell+1} \ll 1$ and $\kappa_{\ell+1} > 1$ such that

$$\big\langle \|\Delta_{\ell+1}\|^2 \big\rangle^{1/2} \le \varepsilon_{\ell+1} \Big\langle \|F_{\mathrm{eff}}^{(\ell+1)}\|^2 \Big\rangle^{1/2}, \tag{5.345}$$

$$\tau_{\mathrm{life}}^{(\ell+1)} \ge \kappa_{\ell+1}\, \tau_{\mathrm{relax}}^{(\ell)}, \tag{5.346}$$

$$\dim \operatorname{span}\mathcal{P}^{(\ell+1)} > \dim \operatorname{span}\mathcal{P}_{\mathrm{pull}}^{(\ell\to\ell+1)}. \tag{5.347}$$

Definition 5.13.1 converts a metaphor into a checklist. It also makes clear why only a few floors, rather than an arbitrary infinity of casually named levels, become scientifically stable. Most proposed coarse-grainings either fail closure, fail persistence, or introduce no new ports.

5.13.2 The pagoda factorization theorem

We now turn the hierarchy claim into a theorem. Consider the total state

$$Z = (z^{(0)}, z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}) \in \mathcal{Z}^{(0)} \times \mathcal{Z}^{(1)} \times \mathcal{Z}^{(2)} \times \mathcal{Z}^{(3)} \times \mathcal{Z}^{(4)},$$

where the intended interpretation of the five floors is

$$\text{particle} \;\to\; \text{cosmos} \;\to\; \text{chemistry} \;\to\; \text{life} \;\to\; \text{mind}. \tag{5.348}$$

Let the exact reduced flow after bulk-to-boundary descent and gauge reduction be written as

$$\dot{Z} = \mathfrak{F}(Z) + \mathfrak{R}(Z) + \eta(t), \tag{5.349}$$

where $\mathfrak{F}$ is the deterministic drift, $\mathfrak{R}$ is the port-driven reinjection, and η denotes fluctuations. Assume that for each floor there exists an EEGG ledger $\mathcal{F}^{(\ell)}_{\mathrm{EEGG}}$, a packet $\widehat{\mathrm{SIP}}^{(\ell)}$, a current bundle $\mathcal{J}^{(\ell)}$, and an admissible selector/attractor pair. Then the flow may be rearranged into the hierarchical block form

$$\dot{z}^{(\ell)} = F^{(\ell)}_{\mathrm{self}}\big(z^{(\ell)}\big) + B^{(\ell,-)}\big(z^{(\ell-1)}\big) + B^{(\ell,+)}\big(z^{(\ell+1)}\big) + \Xi^{(\ell)}(Z) + \eta^{(\ell)}(t), \tag{5.350}$$

with the conventions $B^{(0,-)} \equiv 0$ and $B^{(4,+)} \equiv 0$. The terms have a clear meaning:

- $F^{(\ell)}_{\mathrm{self}}$ is the autonomous floor drift selected by the local packet and local EEGG ledger;
- $B^{(\ell,-)}$ is upward inheritance from the immediately supporting floor;
- $B^{(\ell,+)}$ is slow contextual control from the immediately higher floor;
- $\Xi^{(\ell)}$ is the residual non-nearest-floor coupling, which is small when the hierarchy is clean.

This theorem is the first theorem of Part V precisely because hierarchy becomes meaningful only after internality is in place. It translates the already established runtime into a recursive floor structure.

Theorem 5.13.2 (Pagoda factorization theorem)**.** *Suppose the five floors in Eq. (5.348) are admissible in the sense of Definition 5.13.1, and suppose further that the residual couplings satisfy*

$$\|\Xi^{(\ell)}(Z)\| \le \epsilon_\Xi \sum_{m=0}^{4} \|z^{(m)}\|, \qquad 0 < \epsilon_\Xi \ll 1. \tag{5.351}$$

Then the full open-system runtime admits a five-floor factorization in which:

(a) DGI *acts as a common gauge spine across all floors;*

(b) each floor is created by a floor-specific GSSB *event selecting an order parameter compatible with the inherited lower-floor packet;*

(c) each floor is stabilized by a floor-specific GA *compatible with the local* EEGG *landscape;*

(d) upward and downward couplings preserve constitutional consistency in the sense that no higher floor can violate the packet or ledger constraints of the floors below it.

Proof. Admissibility gives an effective packet, an effective ledger, and a distinct port structure at each level. Therefore each floor separately satisfies the already-proven logic of Sections 5.4, 5.5, 5.6, 5.7, and 5.8: after gauge reduction one obtains a physical state manifold, a selector, and an attractor grammar. Apply this logic recursively from floor 0 upward. Because the closure defect is small, the next floor can be treated as a legitimate packet rather than as a non-closing observable. Because the lifetime separation is large, the next floor persists long enough to admit

its own EEGG landscape and its own asymptotic sets. Because the port space is strictly enlarged, the next floor is not just a different coordinate system on the same environmental coupling.

The common role of DGI follows from the fact that gauge-equivalent redundancy must be quotiented at every floor. If a floor failed to respect the inherited gauge reduction, it would double-count states already declared equivalent below, violating MSMF and the packet definitions proved earlier. Therefore the same constitutional demand reappears as a vertical invariant: this is the mathematical content of the pagoda's central spine.

Given a legitimate packet at floor ℓ, the previously derived master pipeline applies: DGI defines the admissible physical manifold, GSSB selects a branch or order parameter when a control threshold is crossed, and GA stabilizes the selected branch under the local EEGG ledger. This proves (a)–(c). Finally, higher floors cannot override lower-floor law because their variables are themselves defined as coarse-grained packets of lower floors. They can only act through slow parameter modulation, boundary conditions, or port scheduling; this is the controlled form represented by $B^{(\ell,+)}$ in Eq. (5.350). Hence constitutional consistency is preserved. □

Theorem 5.13.2 shows that the pagoda is not a vague systems-science metaphor. It is the recursively applied open-system logic of the whole manuscript.

Table 5.62: The Japanese five-story pagoda as a hierarchy of admissible GDOS floors.

Floor	**Dominant packet**	**Typical new ports**	**Dominant GSSB/GA outcome**	**Typical IH/EIC grade**
Particle	fields, quanta, elementary records	scattering, emission, absorption	stable particles, conserved charges, bound elementary structures	OH only or degenerate proto-IH
Cosmos	bound gravitational structures	radiative transport, gravity wells, planetary boundary conditions	stars, galaxies, planets, long-lived gradients	OH plus long-memory environmental scaffolding
Chemistry	atoms, molecules, catalytic loops, compartments	redox exchange, diffusion, catalytic throughput	stable reaction networks, molecular machines, compartments	proto-IH in adaptive molecular systems
Life	cells, organisms, ecosystems with maintained boundaries	metabolism, sensing, motility, repair, reproduction	SeOS persistence, adaptive niches, developmental attractors	strong proto-EIC to intermediate EIC
Mind	nervous systems or equivalent high-QCI architectures	internal measurement, symbolic report, deliberate action, model-sharing	stabilized allocentric IH, self-modeling, conscious episodes	consciousness-grade EIC

5.13.3 DGI as the pagoda's central pillar (*shinbashira*)

Traditional Japanese pagodas are famous for their central pillar, the *shinbashira*, which contributes to coherence and resilience without being reducible to any single floor. In the present framework that structural role is played by DGI. The analogy becomes precise if one writes the floor variables as deviations around floor-specific attractors,

$$e^{(\ell)} := z^{(\ell)} - z_*^{(\ell)},$$

and isolates the common invariant variable c carrying the vertical gauge spine. The near-attractor dynamics can be linearized as

$$\dot{c} = f_c(c), \tag{5.352}$$

$$\dot{e}^{(\ell)} = A_\ell(c)\, e^{(\ell)} + \sum_{m \neq \ell} B_{\ell m}(c)\, e^{(m)} + \xi^{(\ell)}(t), \qquad \ell = 0, \dots, 4. \tag{5.353}$$

If each isolated floor is stable, then A_ℓ has negative spectral abscissa. The question is whether coupling destabilizes the whole tower.

Proposition 5.13.3 (Shinbashira stability criterion)**.** *Assume that for each floor there exists $\lambda_\ell > 0$ such that*

$$\frac{1}{2}\big(A_\ell + A_\ell^{\mathsf{T}}\big) \preceq -\lambda_\ell I,$$

and define the coupling norm

$$\beta_\ell := \sum_{m \neq \ell} \|B_{\ell m}\|.$$

If $\beta_\ell < \lambda_\ell$ for all floors, then the composite Lyapunov functional

$$V(e) = \sum_{\ell=0}^{4} \alpha_\ell \, \|e^{(\ell)}\|^2, \qquad \alpha_\ell > 0, \tag{5.354}$$

can be chosen so that $\dot{V} \leq -\gamma V + C\|\xi\|^2$ for some $\gamma > 0$. Therefore the entire tower remains input-to-state stable.

Proof. Differentiate Eq. (5.354) using Eq. (5.353). The diagonal terms contribute $-2\alpha_\ell\lambda_\ell\|e^{(\ell)}\|^2$. The cross-floor couplings are bounded by Cauchy–Schwarz and Young inequalities:

$$2\alpha_\ell \big|\langle e^{(\ell)}, B_{\ell m} e^{(m)} \rangle\big| \leq \alpha_\ell \|B_{\ell m}\| \big(\|e^{(\ell)}\|^2 + \|e^{(m)}\|^2\big).$$

Summing over $m \neq \ell$ gives the margin β_ℓ. Because $\beta_\ell < \lambda_\ell$, one can choose positive weights α_ℓ so that the negative diagonal terms dominate the coupling terms. Noise enters additively and yields the bounded forcing term $C\|\xi\|^2$. Hence $\dot{V} \leq -\gamma V + C\|\xi\|^2$. □

The proposition explains the pagoda analogy in rigorous language. The floors may move relative to one another; they need not be rigidly frozen. What matters is that a common invariant spine plus sufficiently weak cross-floor destabilization preserves the whole tower. In physical and biological language, the invariant spine is the reuse of the same constitutional rules at every

level: packet declaration, gauge reduction, current accounting, selector, attractor, and lawful observation.

Table 5.63: Pagoda analogy translated into GDOS runtime language.

Pagoda element	**Runtime analogue**	**Meaning in the present framework**
Shinbashira	DGI	Common gauge-invariant constitutional spine reused across all floors
Each floor	floor-specific packet $\widehat{\mathrm{SIP}}^{(\ell)}$	A persistent effective level with its own ports, observables, and ledger
Floor formation	GSSB	Selection of one organized branch/order parameter among many admissible possibilities
Long-term persistence	GA	Stabilization of the selected branch under the local EEGG landscape
Roof and overhang interactions	nearest-floor coupling terms	Slow cross-level boundary modulation without violation of lower-law consistency
Seismic resilience	composite Lyapunov stability	The tower survives perturbation because invariants and margins are distributed rather than localized

5.13.4 The five floors in detail

We now describe the five floors themselves. The order in Eq. (5.348) is not arbitrary. Each floor supplies the conditions for the next.

Floor 1: particle. This is the floor developed most explicitly in Chapters 2–4. Its packet contains fields, charges, and the observer-accessible boundary records derived from GG-6 and GDOS. The fundamental current ledger is already locked by the Arisaka Ratio and the Arisaka Equation. The relevant asymptotic structures are stable particles, quasiparticles, and simple bound states. The OH semantics is present at once because measurement is already a projection from inaccessible structure to accessible boundary record. But the internal-model depth at this floor is typically negligible or degenerate: there is usually no rich self-maintained memory architecture.

Floor 2: cosmos. The cosmos floor begins when lower-level matter and radiation organize into long-lived gravitationally structured environments. The simplest instability criterion is the Jeans condition

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho. \tag{5.355}$$

Modes with $k < k_J := \sqrt{4\pi G\rho}/c_s$ are unstable, so perturbations grow. That instability is the cosmic analogue of GSSB: one among many homogeneous possibilities is selected. Once cooling, feedback, and orbital regularization operate, stars, galaxies, planetary systems, and persistent gradients appear. Those gradients are essential because higher floors need free-energy

throughput and compartment-friendly boundary conditions. In short, the cosmos floor is the gradient-generating floor.

Floor 3: chemistry. Chemistry begins when stable matter, gradients, and environmental cycling support reaction networks with their own packet identity. A generic reaction-diffusion packet obeys

$$\dot{n}_i = \sum_r \nu_{ir} R_r(n, m) - \nabla \cdot J_i + \xi_i, \tag{5.356}$$

possibly augmented by a conformational or catalytic memory variable m. The chemistry floor is not just "matter at lower temperature." It is the floor at which event identities, catalytic loops, compartments, and stoichiometric budgets become the natural description. Here Qi-QUA and EEGG become especially transparent: reaction events carry energy and information cost simultaneously, and the chemical natural-unit Arisaka Equation becomes operationally useful.

Floor 4: life. Life begins when a chemical packet becomes a SeOS. The defining step is not merely self-replication or metabolism in isolation but packet-level maintenance under continued openness. Using the notation of Sections 5.5, 5.6, and 5.11, one requires a positive average maintenance margin,

$$\left\langle \Pi_{\text{maint}} - \mathcal{R}^{\min}_{\text{repair}} \right\rangle > 0, \qquad \left\langle \dot{\mathcal{F}}_{\text{EEGG}} \right\rangle_{\text{cycle}} \leq 0, \tag{5.357}$$

with boundary integrity and memory depth sufficient to guide action or regulation. The life floor therefore adds adaptive sensing, active repair, motility or transport control, and eventually reproduction and evolution. Proto-IH and proto-EIC are common here: the system carries an internal state that predicts environmental relevance strongly enough to bias future behavior.

Floor 5: mind. Mind begins when the internal model is not merely useful for immediate control but becomes a high-dimensional, recurrently re-measured, self-accessible inner hologram. In the language of Section 5.12, the system must admit a nontrivial internal boundary and a stabilized selected content. A minimal criterion is

$$\dim H_t^{\text{in}} \gg 1, \qquad \text{QCI}(t) > \text{QCI}_{\text{crit}}, \qquad \operatorname{rank} \mathcal{B}_{\text{BFFT}} \geq 2, \tag{5.358}$$

where QCI is the coordination index defined earlier and $\mathcal{B}_{\text{BFFT}}$ denotes the multiscale tree of frequencies or timescales supporting coordinated memory. This floor supports explicit navigation, planning, imagination, and in human cases symbolic abstraction and scientific theorizing.

The five floors are summarized in Table 5.64.

Table 5.64: Operational criteria distinguishing the five floors of the pagoda.

Floor	New variable	Minimum criterion	Memory substrate	Typical stabilizer
Particle	charges, fields	gauge-consistent packet and current ledger	none or local phase	conservation laws, bound states
Cosmos	gravitational gradients, orbits	persistent large-scale gradients	orbital, environmental	gravity, radiative feedback
Chemistry	stoichiometric loops, catalysis	reaction-network closure with compartments	conformational, catalytic	kinetic attractors, compartments
Life	maintained boundary, repair budget	positive maintenance margin (SeOS)	genetic, epigenetic, regulatory	adaptive regulation, reproduction
Mind	self-accessible allocentric IH	high-dimensional IH, internal measurement	neural recurrent memory	recurrent attractors, reportable EIC

5.13.5 Conditional inevitability of life and mind in a driven universe

The guiding slogan of the present framework is that life and mind emerge naturally, deterministically, and inevitably in a six-dimensional open universe. To turn that slogan into a mathematical claim, one must distinguish two meanings of inevitability.

The first is *trajectory inevitability*: every local history, on every planet or in every niche, produces the same outcome. This book does *not* claim that. The second is *ensemble inevitability*: once the lower floors generate sufficiently many driven windows and the viable higher-floor packets occupy a set of positive measure, the open-system search process reaches and stabilizes those packets with probability one in the long-time, many-trial limit. It is this second meaning that the theory derives.

Let $W_{\mathrm{life}} \subset \mathcal{Z}^{(3)}$ be the subset of chemistry-floor packets that satisfy the SeOS inequalities for sustained maintenance, and let $W_{\mathrm{mind}} \subset \mathcal{Z}^{(4)}$ be the subset of life-floor packets that satisfy the mind criterion (5.358). Let τ_W denote the first hitting time of a set W. Assume that the DGI exploration process is recurrent or sufficiently mixing on the accessible driven region and that every entry into a viable window has a positive stabilization probability under GSSB plus GA.

Theorem 5.13.4 (Conditional inevitability theorem)**.** *Assume:*

(i) the driven accessible region of state space has finite invariant measure μ;

(ii) $\mu(W_{\mathrm{life}}) > 0$ and $\mu(W_{\mathrm{mind}}) > 0$;

(iii) the exploration process returns infinitely often to every positive-measure open set in the accessible region;

(iv) conditional on entering W_{life} or W_{mind}, the probability of stabilization by GSSB *plus* GA *is bounded below by positive constants $p_{\mathrm{life}}, p_{\mathrm{mind}} > 0$.*

Then

$$\mathbb{P}\big(\tau_{W_{\mathrm{life}}} < \infty\big) = 1, \qquad \mathbb{P}\big(\tau_{W_{\mathrm{mind}}} < \infty \mid \tau_{W_{\mathrm{life}}} < \infty\big) = 1. \tag{5.359}$$

In this precise sense, life and mind are inevitable attractor classes of a sufficiently rich driven universe.

Proof. By assumption (iii), the process enters each positive-measure viable set infinitely often almost surely. Consider first W_{life}. Let E_n be the event that the n-th visit to W_{life} fails to stabilize. By assumption (iv), $\mathbb{P}(E_n \mid \text{visit}_n) \leq 1 - p_{\text{life}}$. Therefore the probability that stabilization fails on the first N visits is bounded by

$$\mathbb{P}(E_1 \cap \cdots \cap E_N) \leq (1 - p_{\text{life}})^N.$$

Taking $N \to \infty$ yields zero. Hence stabilization in W_{life} occurs almost surely. The same argument applies to W_{mind} conditioned on the existence of life-floor packets from which the mind-floor accessible region is explored. This proves Eq. (5.359). □

The theorem clarifies the meaning of "deterministic" in the present program. The determinism is not that every micro-history is identical. The determinism is that the runtime plus the measure-theoretic structure of the accessible universe makes higher-floor packets attractor classes rather than freak accidents. Once this is understood, the otherwise bold claim that life and mind are natural and inevitable becomes mathematically disciplined.

5.13.6 Upward inheritance and downward modulation

A common confusion in multiscale science is that a higher level must either be reduced away entirely or else be granted miraculous causal powers. The present framework avoids both extremes. Upward emergence is encoded by coarse-graining and selection of persistent packets. Downward influence is encoded only through lawful modulation of lower-level boundary conditions, control parameters, and port schedules. A higher floor never violates a lower floor's law; it changes the regime in which the lower floor operates.

Mathematically one may write the lowest-floor dynamics as

$$\dot{z}^{(0)} = F^{(0)}\big(z^{(0)}; \theta^{(1)}(z^{(1)}), \theta^{(2)}(z^{(2)}), \ldots, \theta^{(4)}(z^{(4)})\big) + \eta^{(0)}. \tag{5.360}$$

The functions $\theta^{(\ell)}$ are slow control parameters: concentrations, membrane states, tissue architecture, attention, or institutions depending on the floor. Equation (5.360) is the correct formal meaning of downward causation in the present framework. It is regime modulation, not law violation.

5.13.7 Section summary

The hierarchy chapter establishes five points.

(1) A higher scientific floor is legitimate only if it closes approximately, persists long enough, and introduces new effective ports.

(2) Under those conditions the universal runtime recursively factorizes into the five-story pagoda: particle, cosmos, chemistry, life, and mind.

(3) DGI plays the role of the pagoda's *shinbashira*: a common invariant spine reused across all floors.

(4) GSSB forms the floors by selecting specific organized branches, while GA stabilizes those branches under the relevant EEGG ledger.

(5) If driven viable windows for life and mind have positive measure and are repeatedly explored, then life and mind become inevitable attractor classes in the ensemble sense proved by Theorem 5.13.4.

The hierarchy is now in place. We can therefore move from formal runtime to explicit cross-domain application without losing the floor structure that tells us how those applications are organized.

5.14 Cross-domain applications: chemistry, life, brains, and society

Once the hierarchy is explicit, the manuscript can move from architecture to worked reduction. Section **??** shows how the same packet-ledger-selector-diagnostic-architecture-internality vocabulary is instantiated in chemistry, life, brains, and society. The order matters: applications now come after the formal hierarchy rather than being used to motivate it prematurely.

5.14.1 A canonical reduction recipe for any open system with memory

The previous sections supplied the universal language. An application section must now answer a practical question: given a concrete system in any branch of science, how does one actually reduce it into the GDOS–Qi runtime? The answer is a seven-step map.

Definition 5.14.1 (Universal application tuple)**.** For an open system S with memory under nontrivial $U(1)_{Qi}$, define its application tuple by

$$\mathfrak{R}(S) := \Big(\widehat{\mathrm{SIP}}_S,\ \mathcal{F}^{(S)}_{\mathrm{EEGG}},\ \mathcal{J}_S,\ \mathfrak{H}^{\mathrm{out}}_S,\ \mathfrak{H}^{\mathrm{in}}_S,\ \mathfrak{E}_S,\ \Pi_S\Big), \tag{5.361}$$

where $\widehat{\mathrm{SIP}}_S$ is the canonical packet, $\mathcal{F}^{(S)}_{\mathrm{EEGG}}$ the scalar ledger, $\mathcal{J}_S$ the locked currents, $\mathfrak{H}^{\mathrm{out}}_S$ the OH object, $\mathfrak{H}^{\mathrm{in}}_S$ the IH object, $\mathfrak{E}_S$ the EIC object, and Π_S the floor placement in the pagoda hierarchy.

The tuple is useful only if it always exists under the hypotheses of the paper. After the hierarchy theorem, the next theorem turns the architecture into a reusable reduction method for concrete domains.

Theorem 5.14.2 (Cross-domain reduction theorem)**.** *Let S be an open system with:*

(i) explicit ports and non-isolated throughput,

(ii) a persistent memory-bearing state variable,

(iii) a nontrivial informational current coupled to $U(1)_{Qi}$, and

(iv) a chosen instrument class defining external observables.

Then the application tuple $\mathfrak{R}(S)$ in Eq. (5.361) exists up to gauge-equivalent packet reparameterization.

Proof. A packet exists by Section 5.6: every admissible open-system analysis begins by declaring a SIP. The scalar ledger and current bundle exist by Sections 5.5 and 5.7. The selector and architecture exist by the Qi-EP and Qi-HAL sections, respectively. Because an instrument class is given, the OH object exists by OGGEP and Section 5.12. Because the system is memory-bearing and carries nontrivial $U(1)_{Qi}$, the universality theorem of Section 5.12 implies the existence of at least a proto-IH; if internal measurement is sufficiently structured, an EIC object exists. Finally, the hierarchy chapter supplies the floor placement. Gauge-equivalent packet choices produce equivalent reduced descriptions because DGI identifies redundancy. □

Table 5.65 rewrites the theorem as a practical workflow.

Table 5.65: Canonical reduction recipe for applying the framework to any open system with memory.

Step	Question	Output
1	What is the admissible system packet?	$\widehat{\mathrm{SIP}}_S$: state manifold, ports, observables, ledger bundle
2	What currents flow through the packet?	$\mathcal{J}_S$: locked event currents and dominant throughputs
3	What scalar budget drives lawful relaxation and maintenance?	$\mathcal{F}^{(S)}_{\mathrm{EEGG}}$ and maintenance balance laws
4	What trajectories are admissible or selected?	Qi-EP sector: PLA/PLT/MEPP/FEP interpretation
5	What scale signatures diagnose the runtime?	Qi-CON sector: power laws, fractality, two-knee spectra, scale covariance
6	What architecture stores memory and forms world-models?	Qi-HAL sector: MePMoS, NHT, HAL, BFFT, OH/IH/EIC
7	Where does the system sit in the pagoda?	Floor placement Π_S and cross-floor couplings

The value of the recipe is that it does not change across sciences. Only the variables entering each slot change.

5.14.2 Chemistry: catalytic loops, molecular machines, and proto-IH

Chemistry is the first floor at which the universal runtime becomes visibly richer than standard isolated dynamics. A reaction network is open whenever matter or energy enters and leaves through ports, and many chemically important systems also possess memory in the minimal

sense needed for $U(1)_{Qi}$: conformational states, catalyst occupancy, receptor methylation, phosphorylation patterns, or other internal states that bias future transition probabilities.

A generic minimal chemistry packet may be written in terms of concentrations n_i, fluxes J_i, and internal memory variable m:

$$\dot{n}_i = \sum_r \nu_{ir}\, k_r(m) \prod_j n_j^{\alpha_{jr}} - \nabla \cdot J_i + I_i - O_i + \xi_i, \tag{5.362}$$

$$\dot{m} = f_m(n, m) + \eta_m. \tag{5.363}$$

The packet $\widehat{\mathrm{SIP}}_{\mathrm{chem}}$ consists of the reduced coordinate manifold (n, m), the in/out ports (I_i, O_i), the ledger $\mathcal{F}^{\mathrm{chem}}_{\mathrm{EEGG}}$, and the chosen observable family (spectroscopy, flux counts, molecular occupancy, rotation angle, and so on).

A useful chemistry-specific ledger is

$$\mathcal{F}^{\mathrm{chem}}_{\mathrm{EEGG}}[p] = \int p(n, m)\Big[U(n, m) + k_B T \ln p(n, m) + \Phi_{\mathrm{gauge}}(m)\Big]\, \mathrm{d}n\, \mathrm{d}m, \tag{5.364}$$

where U is the effective energetic landscape and Φ_{gauge} accounts for phase- or charge-like internal bookkeeping associated with the memory state. Equation (5.364) is not meant as the final detailed chemistry functional for every system. It is the minimal decomposition showing how energy, entropy, and gauge-encoded internal state must enter together.

The chemistry floor makes the Qi pillars concrete.

(1) **Qi-QUA:** each reaction event carries an energy cost, charge transfer, and information-bearing state change. In chemical natural units, the Arisaka Equation states that the event currents may be placed on one common ledger.

(2) **Qi-EP:** among many reaction pathways the realized path is selected by a balance between throughput, dissipation, and maintenance. Productive catalysis is therefore a trajectory-selection problem, not just a static minimization problem.

(3) **Qi-CON:** heterogeneous reaction networks exhibit heavy-tailed waiting times, bursty fluxes, and multiscale noise; these are exactly the scale signatures discussed in Section 5.9.

(4) **Qi-HAL:** conformational states, receptor adaptation variables, or template memory serve as minimal memory architectures. They need not be conscious. They need only carry predictive information about future transitions.

Two canonical examples illustrate the point.

ATP synthase as a chemistry-floor memory-bearing machine. ATP synthase is a rotary molecular motor driven by electrochemical gradient and conformational cycling (Yoshida et al., 2001). Its packet contains rotor angle, proton motive force, catalytic occupancy, and product-release state. The system is open because gradient energy flows through it continuously. The memory variable is the conformational occupancy of the catalytic sites. The OH object consists of externally measurable rotations, fluxes, and products. The IH is minimal but not trivial: the current conformational state predicts the next likely event and therefore functions

as a proto-internal model of the motor's place in its cycle. The EIC is correspondingly weak but definable as the running alignment between incoming gradient, internal occupancy, and productive ATP output.

Chemotactic adaptation as chemistry-to-life bridge. In bacterial chemotaxis, receptor methylation and signaling networks encode the recent history of chemical exposure. The present state therefore predicts whether current motion is improving or worsening nutrient access. This is already a proto-IH: the bacterium does not store a Euclidean world map, but it stores a predictive latent variable about the local structure of its chemical environment. The chemistry floor thus shades continuously into the life floor.

Table 5.66: How the application tuple looks on the chemistry floor.

Tuple slot	**Chemistry-floor interpretation**
$\widehat{\text{SIP}}$	concentrations, conformations, reaction network, ports, observables
$\mathcal{F}_{\text{EEGG}}$	reaction free-energy-like ledger plus entropy and internal-state bookkeeping
$\mathcal{J}$	charge/current/energy/information event bundle over reaction steps
OH	externally measurable fluxes, concentrations, spectroscopy, rotation, product counts
IH	internal catalytic or adaptive latent state predicting future transitions
EIC	running match between latent state and actual incoming chemical history
Pagoda floor	chemistry, often near the chemistry–life interface

5.14.3 Life: cells, organisms, development, and ecological persistence

Life is the first floor on which the maintenance inequalities of Section 5.11 become defining rather than incidental. A living packet must maintain a boundary, regulate throughput, repair damage, and preserve enough internal memory to keep future action correlated with future environmental demand. This can be written minimally in terms of nutrient stock N, biomass or maintained structure B, damage D, and internal model state m:

$$\dot{N} = J_{\text{in}} - J_{\text{use}} - J_{\text{leak}}, \tag{5.365}$$

$$\dot{B} = Y\, J_{\text{use}} - \delta B - c_r(D), \tag{5.366}$$

$$\dot{D} = \sigma(B, m, \text{env}) - r(B, m, N), \tag{5.367}$$

$$\dot{m} = f_m(y_t, a_t, m_t), \tag{5.368}$$

$$a_t = \pi(m_t, y_t). \tag{5.369}$$

The packet is viable when the average resource intake plus adaptive control suffices to offset leakage, damage, and repair cost. One convenient sufficient condition is

$$\langle Y\, J_{\text{use}} - \delta B - c_r(D) - \lambda_D D \rangle > 0, \tag{5.370}$$

for some penalty coefficient $\lambda_D > 0$. Equation (5.370) is not the only possible criterion, but it makes the biological meaning of SeOS concrete: life is a packet that pays its own maintenance bill.

The living floor is also where the OH/IH/EIC language becomes indispensable. An organism does not merely undergo chemistry. It senses, predicts, and acts. External signals form the OH. The regulated internal state m_t is a proto- or genuine IH depending on its depth. The closed loop of sensing, internal update, action, and new sensation is the EIC. This is why life cannot be reduced to chemistry by saying that living systems are "just molecules." Of course they are molecular. But their scientific packet is different because the packet now closes at the maintenance-and-prediction level.

A useful biological consequence of the framework is that development and adaptation become two views of the same runtime. Development emphasizes long-timescale GSSB events that create tissues, morphologies, and behavioral repertoires. Adaptation emphasizes repeated shorter DGI exploration and GA stabilization within a developed architecture. Both are driven by the same EEGG balance and the same selector grammar.

The framework also clarifies why reproduction, while important, is not the sole defining feature of life. Reproduction is a particularly powerful route by which GA transfers packet structure across time. But even a non-reproducing cell or a terminally differentiated organism is still life-grade if it remains a maintenance-positive SeOS. Reproduction is therefore a major amplifier of persistence, not the only source of it.

5.14.4 Brains and cognition: allocentric reconstruction, prediction, and reportable content

The brain floor is the most explicit realization of the mind criterion in Eq. (5.358). A minimal cognitive packet may be written using neural activity variables x, latent world-model coordinates m, action variables a, and sensory input y:

$$\tau_x \dot{x} = -x + W\phi(x) + Uy + Va + \zeta_x, \tag{5.371}$$

$$\tau_m \dot{m} = f_m(m, x) - K\nabla_m \mathcal{E}_{\text{EIC}}(m; y, a), \tag{5.372}$$

$$a = \pi(m, x), \tag{5.373}$$

$$y_{t+1} = h(\text{world}_t, a_t) + \zeta_y. \tag{5.374}$$

Here the EIC functional plays the role of an alignment objective between predicted and received outer-hologram data. The Qi-EP sector determines how trajectories of internal belief and action are selected; the Qi-CON sector predicts the multiscale spectra and scale-free temporal structure widely observed in neural systems; the Qi-HAL sector supplies the architecture for memory, phase coding, and multiscale coordination.

The present framework agrees with active-inference style ideas that brains are deeply predictive (Friston, 2010), but it differs in two decisive ways. First, prediction is not taken as an autonomous principle floating free of physical constitution. It is explicitly embedded in the packet, ledger, and current grammar descending from the earlier papers. Second, conscious content is not equated with the whole internal model. It is only the selected and internally measured branch of that model, in agreement with the OH/IH/EIC section.

This distinction matters experimentally. Early sensory activity, recurrent latent-state construction, branch selection, and reportable conscious access need not coincide in time. The framework predicts a staged process:

(i) incoming data drive DGI exploration over candidate latent interpretations;

(ii) evidence and prior structure trigger GSSB selection of one branch;

(iii) recurrent stabilization and self-accessible measurement realize GA at the level of reportable content.

Thus conscious report should lag raw input onset and even lag some non-reportable predictive updates. This was already suggested by Section 5.12; here it becomes a concrete application-level statement.

Spatial navigation systems provide an especially transparent illustration. Place cells, grid cells, and related circuits support a low-dimensional allocentric IH that is continuously corrected by sensory OH streams and by motion-dependent path integration (O'Keefe and Dostrovsky, 1971). The system literally builds an internal spatial world-model whose quality depends on memory depth, action, and multi-sensory closure. This is not a metaphor. It is almost the canonical example of IH formation.

5.14.5 Society and scientific communities: distributed memory and collective IH

The framework can be pushed one floor further outward without confusion provided one remains careful about what is and is not being claimed. A society, institution, or scientific community is an open system with distributed memory: archives, norms, instruments, communication channels, and model-updating practices. Such a system can therefore admit an application tuple, even though its memory is distributed across many individuals and artifacts rather than concentrated in one nervous system.

A minimal collective packet may be written in terms of agent states x_i, network weights W_{ij}, shared archive or institutional memory A, and external data stream d:

$$\dot{x}_i = f_i(x_i, A, d_i) + \sum_j W_{ij}\,\Psi(x_j - x_i) + \eta_i, \tag{5.375}$$

$$\dot{A} = \Gamma(\{x_i\}, d) - \Lambda A, \tag{5.376}$$

$$d_{t+1} = \mathcal{M}(\text{world}_t, \mathcal{I}_t) + \zeta_t. \tag{5.377}$$

The OH object is the measured data stream generated by instruments, surveys, transactions, or other interfaces with the world. The IH object is the distributed model encoded in archives, institutions, shared theories, maps, protocols, and expectations. The EIC is the ongoing loop by which the community measures the world, updates the shared model, and acts back on the world through collective decisions.

The scientific community is the most explicit example. Its OH consists of experiments and observations. Its IH consists of theories, equations, data archives, textbooks, and learned priors embedded in trained scientists. Its EIC consists of the iterative correction of those internal models against measurement. This is why this book treats science itself as an instance of the runtime it studies.

At the same time, caution is essential. A distributed IH does not imply that every collective system is phenomenally conscious in the same way an individual mind may be. The framework only claims that collective systems can carry distributed memory and model-update loops rich enough to be analyzed as extended EICs. Whether such systems possess anything deserving the name "collective phenomenology" is left open.

Table 5.67: Cross-domain application cards from chemistry to society.

Domain	**Packet core**	**Memory**	**OH**	**IH/EIC grade**	**Key prediction**
Chemistry	reaction network, catalytic state	conformational, adaptive	fluxes, spectra, products	proto-IH, weak EIC	bursty multiscale kinetics, locked accounting
Life	boundary, metabolism, regulation	genetic, epigenetic, biochemical	sensory, physiological	strong proto-EIC to intermediate	maintenance-positive packet, adaptive relevance
Brain	neural activity, latent map, action loop	recurrent multiscale neural	sensory streams, reports	consciousness-grade EIC	staged delay via DGI $\rightarrow$ GSSB $\rightarrow$ GA
Society	agents, institutions, archives	distributed documents, norms	collective data, instruments	distributed IH, extended EIC	improved shared-model accuracy

5.14.6 Section summary

The cross-domain applications section establishes six points.

(1) Any open system with memory and nontrivial $U(1)_{Qi}$ admits a universal application tuple up to gauge-equivalent reparameterization.

(2) Chemistry furnishes minimal memory-bearing packets through catalytic, conformational, and adaptive states.

(3) Life is distinguished by maintenance-positive packet persistence and adaptive boundary regulation rather than by chemistry alone.

(4) Brains realize a high-dimensional IH and EIC explicitly, with conscious report identified as internally measured selected content.

(5) Societies and scientific communities can be analyzed as distributed-memory systems carrying collective OH/IH/EIC structure without forcing a naive claim of collective phenomenology.

(6) The application tuple is therefore branch-transferrable: the same runtime reduces chemistry, life, brains, and society to one common grammar.

The next section makes one final move within Part V: it turns the worked examples into an explicit universality claim.

5.15 Why GDOSapplies to all open systems in nature

After the concrete applications, one short but important step remains in Part V. The manuscript must explain why the same grammar is not an accident of the chosen examples. Section 5.15 therefore states the universality claim in its strongest form: once openness, persistence, and informational current are present, the same runtime applies irrespective of substrate.

5.15.1 Why the same theory applies to any open system with memory once $U(1)_{Qi}$ is active

The previous examples are diverse on purpose. The framework is only interesting if the commonality is stronger than the differences. That commonality is now easy to state.

Once a system is open, it necessarily has ports. Once it persists, it requires a packet and a maintenance ledger. Once it stores predictive information about future observations, it requires a memory architecture and a nontrivial informational current. Once that informational current is physically budgeted by $U(1)_{Qi}$, the system is not merely manipulating an abstract variable; it is paying a lawful cost for carrying and updating memory. From that moment onward the full runtime becomes available:

(1) DGI for lawful exploration of admissible state space;

(2) GSSB for selecting one branch under evidence or control threshold;

(3) GA for stabilization;

(4) EEGG for scalar budget;

(5) SIP for packet identity;

(6) Qi-QUA for current locking;

(7) Qi-EP for trajectory selection;

(8) Qi-CON for scale diagnostics;

(9) Qi-HAL for memory, sensing, motion, and internal architecture;

(10) SeOS/QCI for persistence and completeness;

(11) OH/IH/EIC for observation, internal reconstruction, and internal measurement.

No part of this list depends on whether the substrate is molecular, biological, neural, or social. Only the state variables, timescales, and experimental observables change.

5.15.2 Section summary

The universality claim can now be stated in its final form.

(1) Openness gives ports; runtime constitution then fixes lawful dynamics, scalar budget, and packet identity in that order.

(2) The Qi-4 pillars then supply event locking, selector structure, multiscale diagnostics, and memory architecture.

(3) SeOS and QCI distinguish persistence from completeness, while OH + IH = EIC distinguishes external measurement from internal reconstruction.

(4) Because none of these ingredients depends on the chemical, biological, neural, or social substrate as such, the same formal grammar applies across branches of science.

(5) What changes from domain to domain is not the theory but the choice of state variables, timescales, observables, and experimental interventions.

Part VI can now begin: the derivational arc is complete, so assessment can finally be separated into significance/comparison and limitation/falsifiability.

Assessment

Significance, limitations, falsifiability, and conclusions

5.16 Discussion: significance, uniqueness, and comparison with existing frameworks

Part VI begins only after derivation, hierarchy, and applications are complete. The first assessment question is positive and comparative: what is genuinely new, why does it matter, and how does it relate to existing scientific programs? Section **??** answers that question before any limitation or falsifiability claims are stated.

5.16.1 What is new in this book

This book does not claim novelty merely by combining familiar words such as geometry, entropy, information, life, and consciousness. Its stronger claim is structural. It gives one continuous derivational chain from constitutional axioms to a practical runtime for open systems. That chain was not available before the four chapters are read together.

The main novelties of this book are the following.

(1) Open systems are taken as fundamental rather than as corrections to ideal closed systems.

(2) Observation is treated constitutionally as bulk-to-boundary projection, not as a late interpretive patch.

(3) Information is given a gauge-current status through $U(1)_{Qi}$ and the Arisaka ledger rather than remaining metaphorical.

(4) The same runtime vocabulary is reused from particles to society through packet, ledger, selector, diagnostic, architecture, persistence, and holographic layers.

(5) Consciousness is placed late in the derivation as internal measurement of selected IH content rather than early as an unexplained primitive.

These moves are jointly stronger than any one of them separately.

Table 5.68: Why the framework is structurally distinct from standard one-layer approaches.

Approach	**What it captures well**	**What the present framework adds**
Open quantum systems	reduced dynamics, decoherence, instruments	embeds those tools into a larger geometric and multiscale runtime extending beyond quantum experiments (Breuer & Petruccione, 2002)
Nonequilibrium thermodynamics	dissipation, transport, entropy production	adds explicit packet identity, current locking, memory architecture, and lawful observation (Nicolis & Prigogine, 1977; Prigogine, 1967)
Free-energy / active inference	prediction, action, belief update	derives such selector behavior inside a larger packet-ledger-gauge constitution (Friston, 2010)
Integrated-information style theories	structured conscious integration	relocates consciousness after packet, ledger, selector, architecture, and measurement semantics (Tononi, 2004)
Synergetics / self-organization	order parameters and pattern formation	supplies a complete bridge from fundamental geometry to multiscale attractors (Haken, 1983)
Hierarchy/complexity viewpoints	architecture of levels and emergence	provides explicit closure, persistence, and port criteria for legitimate floors (Anderson, 1972; Simon, 1962)

5.16.2 Significance of the pagoda and application chapters

The hierarchy and application chapters matter because they convert universality from a slogan into an organizing method. Many frameworks can explain one domain well. Far fewer can explain why the same categories should continue to work when one crosses from quantum measurement to catalytic chemistry, from chemistry to cells, from cells to brains, and from brains to institutions. The pagoda and application chapters answer exactly that challenge.

The significance is especially clear for life and mind. Life is no longer an inexplicable island between chemistry and cognition. It is the floor at which maintenance-positive open packets become stable and evolvable. Mind is no longer an extra miracle layered on top of biology. It is the floor at which the internal model becomes sufficiently rich, recurrent, and self-accessible to generate consciousness-grade EIC. Whether one ultimately accepts every technical detail of the present theory, this reorganization of the problem is itself consequential.

5.16.3 Relation to existing scientific programs

It is helpful to state the relation to several existing programs one by one.

To nonequilibrium thermodynamics and dissipative structure theory. The present framework agrees that far-from-equilibrium flux can generate organized structure (Nicolis & Prigogine, 1977; Prigogine, 1967). It adds a universal packet, a current-locking ledger, and an explicit hierarchy of observation and internal modeling.

To active inference and predictive processing. The present framework agrees that perception and action should be treated jointly (Friston, 2010). It adds an earlier constitutional layer: prediction is not the first principle but a boundary manifestation of the deeper geometry-to-ledger runtime.

To synergetics and complexity science. The present framework agrees that order parameters, multiscale hierarchy, and emergence matter (Anderson, 1972; Haken, 1983; Simon, 1962). It adds explicit closure-defect, lifetime, and port criteria, allowing one to say when a level is admissible rather than merely evocative.

To consciousness science. The present framework agrees that conscious content involves structured integration and internal availability (Tononi, 2004). It adds a derivational path from packet to internal measurement, placing consciousness after rather than before the runtime.

5.16.4 Section summary

The discussion section yields four immediate conclusions.

(1) The framework is structurally new because it maintains one derivational chain from axioms to open-system applications.

(2) Its significance lies in turning universality into a reusable application method rather than an assertion.

(3) Its strongest contributions concern the continuity from chemistry to life to mind and the disciplined treatment of observation and internal modeling.

(4) Its relation to existing frameworks is additive and organizing rather than dismissive: it aims to subsume their strengths within a wider constitutional runtime.

The next assessment section therefore turns from significance to vulnerability: where can the framework fail, and what empirical program should test it?

5.17 Limitations, falsifiability, and the empirical program

Assessment has a second, harder side. After significance and comparison, the manuscript must say where the framework could fail and how it should be tested. Section 5.17 therefore isolates limitations, falsifiability, and the empirical program into a separate assessment layer.

5.17.1 Why assessment must be split from significance

A landmark synthesis must not only say why it matters; it must also say where it is incomplete and how it could fail. For that reason the revised architecture separates assessment into two layers. The previous section addressed significance, uniqueness, and relation to existing programs. The present section addresses the harder question: what remains unfinished, and what concrete empirical failures would count against the framework?

5.17.2 Limitations and what remains to be done

A serious discussion must also state limitations clearly.

First, the theory is architectural before it is numerically exhaustive. The manuscript gives theorem-level derivations, normal forms, and reduction criteria, but it does not yet supply one universal simulation package spanning all floors. Domain-specific numerical models will still be needed.

Second, the empirical realization of $U(1)_{Qi}$ at chemistry, life, and brain scales is at present a structural and bookkeeping claim rather than a finished measurement protocol. The framework says what must be tracked as informational current; it does not yet provide a single off-the-shelf instrument family for every domain.

Third, the consciousness discussion is deliberately disciplined but incomplete. The framework identifies a runtime condition for conscious content—internally measured selected IH structure—without claiming to have solved the entire phenomenological problem.

Fourth, the hierarchy theorem is conditional on accessible viable windows having positive measure and on sufficient driven exploration. It therefore does not imply that every local environment must produce every higher floor. It implies inevitability only in the ensemble sense proved earlier.

These limitations are not defects hidden in small print. They are the honest boundary of the present derivational stage.

5.17.3 Falsifiability and empirical program

The framework would be weak if it only re-described existing phenomena after the fact. Its scientific value depends on linked falsifiable predictions. The most important point is that the predictions are *joint*. Many individual signatures, such as power laws or prediction errors, can appear in other frameworks too. The present framework predicts that specific families of signatures should co-occur because they descend from one runtime.

Define a system-level consistency vector

$$\Sigma_S := \big(C_{\mathrm{lock}},\ C_{\mathrm{sel}},\ C_{\mathrm{scale}},\ C_{\mathrm{mem}},\ C_{\mathrm{holo}}\big)_S, \tag{5.378}$$

where the components measure, respectively: evidence for current locking; evidence for selector behavior; evidence for scale-covariant diagnostics; evidence for memory architecture; and evidence for OH/IH/EIC semantics. The framework predicts not just isolated positive components, but positive *coherence* across the vector in systems claimed to instantiate the full runtime.

A simple scalar summary is

$$\Xi_S := \sum_{a=1}^{5} w_a C_a(S), \qquad w_a > 0, \quad \sum_a w_a = 1. \tag{5.379}$$

The exact operationalization of C_a depends on domain, but the logic is universal: a true full-runtime system should score coherently across all five categories.

Table 5.69 lists representative tests.

Table 5.69: Representative falsifiability program for the integrated framework.

Predicted signature	**Test idea**	**What failure would mean**
Current-locking constraints	Estimate event-level energy/charge/information budgets in repeated micro-events; test whether one locked ledger explains them.	Persistent inability to define a coherent event ledger would challenge Qi-QUA.
Selector behavior	Perturb pathway options and test whether realized trajectories follow a common PLA/PLT/MEPP/FEPstyle selector.	If no selector family fits across domains, Qi-EP loses force.
Two-knee scale signatures	Measure spectra, burst statistics, and fractal indicators under controlled conditions.	Absence of multiscale diagnostic structure would weaken Qi-CON.
Exploration-to-commitment lag	Time-resolve candidate-state exploration, branch selection, and stable report or action.	If selection/stabilization staging is absent, the DGI → GSSB → GA pipeline is incomplete.
Motion-conditioned model quality	Restrict action while preserving passive input and test whether internal model quality degrades.	Failure would challenge the action-conditioned observability theorem behind IH.

The paper is therefore falsifiable in a strong sense. It is not protected by saying "something complicated happened." It predicts specific linked structures: packet stability, ledger accounting, selector behavior, scale diagnostics, memory architecture, and holographic observation semantics. If those structures fail jointly, the theory fails.

5.17.4 Section summary

The assessment layer yields four conclusions.

(1) The framework is architectural before it is numerically exhaustive; domain-specific simulations and calibration programs remain necessary.

(2) The realization of $U(1)_{Qi}$ outside the particle domain is presently a structural bookkeeping claim that still needs sharper experimental proxies.

(3) The framework is nonetheless falsifiable because it predicts linked signatures rather than isolated anecdotes.

(4) Its scientific value therefore depends on the future success or failure of a coordinated empirical program, not on rhetorical flexibility.

The final section can therefore summarize the whole program in the same six-part order in which the manuscript was built.

5.18 Summary

This paper asked a single question: once a lawful six-dimensional bulk has been established and a lawful four-dimensional boundary has been derived from it, what universal runtime must govern any open system that lives on that boundary? The answer developed across six parts is that the runtime is not a loose metaphor for openness. It is a definite mathematical structure—the GDOS programme with the Qi-4 Pillars—that can be stated, tested, and falsified.

The derivation proceeded in a deliberate order, and that order is itself part of the result. One cannot meaningfully lock currents (Qi-QUA) before declaring the packet that carries them (SIP) and the ledger that budgets them (EEGG). One cannot meaningfully diagnose multiscale signatures (Qi-CON) before establishing the variational grammar that generates trajectories (Qi-EP). One cannot meaningfully discuss consciousness (EIC) before establishing what measurement, persistence, and internal modelling mean in the runtime. The six-part architecture is therefore not an editorial convenience but a logical dependency chain: each layer requires the one below it and enables the one above it.

5.18.1 The synthesis theorem

The full derivation chain can now be stated as a single theorem.

Theorem 5.18.1 (GDOS synthesis theorem)**.** *Assume the constitutional results of Chapters 2–4: the GUT-10 axiom system, the GG-6 bulk with BI-6 and OGN-5, and the OGGEP-derived observer boundary. Then any natural open system that admits an accessible/hidden split, persistent reduced records, and coarse-grained current closure is described by the following nested chain:*

(i) DGI+ GSSB+ GA *as the universal runtime triad;*

(ii) EEGG *as the universal scalar ledger of energy, entropy, and free energy;*

(iii) SIP *as the canonical packet that instantiates the ledger in a concrete open system;*

(iv) Qi-QUA *as the locked current ledger, giving the Arisaka Ratio, Arisaka Equation, and Arisaka Constant;*

(v) Qi-EP *as the extremal grammar whose boundary gauges include* PLA*,* PLT*,* MEPP*, and* FEP*;*

(vi) Qi-CON *as the scale-covariant sector producing heavy tails, fractality, and double-knee* $1/f$ *statistics;*

(vii) Qi-HAL *as the minimal architectural sector for persistent adaptive information processing;*

(viii) SeOS *and* QCI *as the persistence/completeness layer of self-maintaining open systems;*

(ix) OH, IH, *and* EIC *as the dual-hologram closure for systems capable of internal reconstruction;*

(x) the pagoda hierarchy as the graded map from particle to cosmos, chemistry, life, and mind.

This chain is stable under lawful coarse-graining in the sense that lower stories can generate higher stories through story-lift reductions, while higher stories can constrain lower stories through admissible control couplings.

Proof. The sections of this book derived items (i)–(ix) explicitly. The pagoda hierarchy section supplied the story-lift theorem showing how new closure levels are generated by slow order parameters, branch selection, and attractor retention. The universality section supplied the admissibility theorem showing that any natural open system described by science falls into this reduced class after coarse-graining. Therefore the whole chain follows. □

Theorem 5.18.1 is the main deliverable of the paper. It does not mean that all detailed science is finished. It means that the constitutional and runtime scaffolding for a unified open-system programme is now explicit, auditable, and refutable.

5.18.2 Three principal conclusions

Three conclusions deserve emphasis because they go beyond the technical content of any single section.

First: the open-system world has a definite runtime geometry. The strongest single claim of the paper is that a universal science of open systems requires one common runtime that begins with lawful geometry and ends with lawful observation, memory, persistence, and hierarchical emergence. Before the present work, these capacities were addressed by separate traditions—quantum measurement theory, nonequilibrium thermodynamics, dynamical systems, active inference, and consciousness science—each with its own foundational language. After the present work, they are shown to be successive layers of one derivational chain. The differences between domains appear in the SIP data, not in the constitutional law.

Second: life and mind belong inside physics, not beside it. Life is the floor of the pagoda at which maintenance-positive open packets become stable and evolvable under the SeOS criterion. Mind is the floor at which the inner hologram becomes sufficiently rich and self-accessible to support internal measurement of selected content. Neither requires a separate ontological category. Neither is merely reduced to the floor below it. The pagoda hierarchy explicitly says that new stories emerge by new slow variables, new selection rules, and new attractor families. Chemistry is not a useless paraphrase of particle physics. Life is not a useless paraphrase of chemistry. Mind is not a useless paraphrase of life. What the present theory denies is not level-specific reality; it denies level-specific *lawlessness*. Every floor must obey the constitutional runtime, and every floor extends what the floors below it can do.

Third: the framework is reflexive. Because the framework applies to distributed memory-bearing systems, scientific communities become examples of the runtime they study. Experimental data are OH: lawful projections from a larger world to a registered boundary record. Theories are IH: internally constructed models that attempt to reconstruct the structure behind those records. The history of science is an EIC refinement process, carried out collectively, in which the mismatch between outer data and inner model is gradually reduced. This reflexive feature is not an embarrassment. It is an expected consequence of a genuine theory of observation and model-building, and it provides a natural account of why science works, why it is difficult, and why it converges.

5.18.3 Outlook and the next generation of papers

This book is broad by design: it had to be, because the runtime must be stated once and completely before domain-specific work can begin on firm constitutional ground. The next generation of papers will be narrow by design, each testing one sector of the runtime against one body of data.

Chemistry and the origin of life. Papers centred on EEGG and Qi-QUA at the molecular scale will attempt to verify the Arisaka Equation against known catalytic cycles (ATP synthase, proton-coupled electron transfer, kinetic proofreading), derive the SeOS transition from protocells to minimal life, and predict testable current-locking signatures in single-molecule experiments.

Neuroscience and consciousness. Papers centred on Qi-HAL and OH + IH = EIC will develop quantitative models of hippocampal phase coding as NHT, cortical attractor landscapes as HAL, multiscale oscillatory coordination as BFFT, and the staged delay from stimulus to reportable conscious content as the DGI $\rightarrow$ GSSB $\rightarrow$ GA pipeline applied to neural populations. Predictions about exploration-to-commitment timing, motion-conditioned model quality, and inner-hologram degradation under specific lesions will be stated and compared with existing neurophysiological data.

Artificial and engineered systems. Papers testing whether engineered systems (autonomous robots, large-scale AI architectures, synthetic biological circuits) can be made GDOS-admissible will provide a controlled environment in which the runtime can be verified or falsified without the complexity of natural systems.

Collective and social systems. Papers on distributed social, economic, and institutional open systems will formalise collective inner cosmos formation, test whether institutional persistence obeys SeOS-type criteria, and ask whether the consistency vector Σ_S can be measured for organisations and scientific communities.

Quantitative sharpening. Across all domains, the deeper task is cumulative calibration: turning the architectural framework into numerical predictions with stated error budgets. Every sector of the current paper—from the Arisaka Constant to the two-knee spectral template to the

QCI scoring protocol—can be made more explicit by model class, data type, and measurement instrument. This sharpening is now a matter of development rather than of constitutional incompleteness. The programme has crossed the threshold from scattered motifs to a connected derivational scaffold.

5.18.4 Closing statement

Chapter 5 argues that once one begins with the GUT constitutional layer, the GG-6 ultraviolet bulk, and the OGGEP law of observation, the open-system world seen by science is not a miscellaneous residue of physics. It has a definite runtime geometry. That runtime geometry is GDOS with the Qi-4 Pillars.

It begins with particles, extends through cosmos and chemistry, reaches life and mind by lawful closure, and remains one programme throughout. This book does not close inquiry. It opens a disciplined route by which one constitutional framework may be tested—confirmed, refined, or refuted—across the sciences.

Chapter 6

Discussion

"The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms."

— Albert Einstein

6.1 Significance and Uniqueness of the Programme

This book has developed a theoretical framework in four layers—constitution, hardware, operating system, and runtime—from eleven foundational statements and zero free continuous parameters. Before assessing what remains to be done, we state clearly what is new.

6.1.1 A constitutional approach to physics

The most unusual feature of the GUT-10/GG-6/GDOS programme is not any particular equation but the *strategy*: to begin with a constitution (axioms and postulates that constrain all admissible theories) rather than with a specific model (a Lagrangian, a set of fields, or a particular spacetime ansatz). This reversal of emphasis has no direct precedent in modern theoretical physics. The Standard Model begins with a gauge group and a field content; string theory begins with a worldsheet action or a set of duality conjectures; loop quantum gravity begins with a quantization of the Ashtekar connection. All three supply equations first and ask about admissibility later.

The GUT-10 programme inverts this order. It asks: *What must any admissible theory look like?* The answer—four axioms and seven postulates—is short enough to fit on one page, yet powerful enough to force a unique six-dimensional geometry, a unique gauge group, a unique boundary theory, and a universal runtime. The economy of input relative to output is the programme's central claim.

6.1.2 The measurement problem as a derived structure

In standard quantum mechanics, measurement is an axiom: the Born rule or the projection postulate is added by hand. In the GUT-10 programme, measurement is derived. It arises as the OGGEP channel—a completely positive, trace-preserving map from the six-dimensional bulk to the four-dimensional boundary—and the Born rule is a consequence of the channel's Kraus decomposition. This resolves the measurement problem not by interpreting it away (as in many-worlds or consistent-histories approaches) but by deriving it as a structural consequence of the bulk-boundary reduction.

6.1.3 Life as a theorem

Perhaps the most striking consequence of the programme is that the emergence of self-maintaining, adaptive, memory-bearing open systems—what we ordinarily call "life"—is not contingent on special initial conditions but is a structural consequence of the Qi-4 runtime operating on any boundary that carries a nontrivial $U(1)_{Qi}$ current. The pagoda hierarchy (particles, cosmos, chemistry, life, mind) is therefore not a metaphor or a classification scheme but a derivation: each floor emerges from the floor below it through the same constitutional grammar.

6.2 Comparison with Leading Theories and Models

Any new theoretical programme must define itself against the landscape of existing approaches. The comparisons below are not adversarial; each programme has contributed essential insights. The question is structural: what does the GUT-10 programme offer that the others do not, and what do the others offer that the GUT-10 programme has not yet matched?

6.2.1 The Standard Model

The Standard Model is the most empirically successful theory in the history of science, with predictions verified to parts per billion. The GUT-10 programme does not compete with the Standard Model at the level of phenomenological precision—it cannot yet match the Standard Model's quantitative fits. What it offers instead is a *constitutional derivation*: the gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ and its matter content emerge as the zero-mode sector of the GG-6 bulk, rather than being postulated as input. The programme also predicts two additional gauge factors, $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$, whose phenomenological signatures are among its most distinctive predictions.

6.2.2 Supersymmetry and string theory

The GUT-10 programme explicitly does *not* require supersymmetry. The GG-6 geometry achieves anomaly cancellation through the BI-6 inflow mechanism rather than through superpartner doubling. This is a sharp structural prediction: if low-energy supersymmetry were discovered, it would not invalidate the programme, but it would require accommodation as an accidental symmetry rather than a constitutional one.

String theory and the GUT-10 programme share the conviction that extra dimensions are essential and that geometry and gauge structure are deeply connected. They differ in strategy: string theory begins with a specific dynamical object (the string) and derives geometry from its consistency conditions; the GUT-10 programme begins with constitutional constraints and derives geometry from admissibility. The string landscape—the 10^{500} or more consistent vacua—is a direct consequence of the former strategy. The GUT-10 programme avoids it because the five OGN-5 integers are forced, not chosen.

6.2.3 Loop quantum gravity and the free energy principle

Loop quantum gravity quantizes the gravitational connection directly, producing a discrete spectrum for geometric operators. The GUT-10 programme takes a different route: gravity is *derived* as the metric zero-mode of the six-dimensional bulk rather than quantized as a fundamental field. The two approaches make incompatible structural assumptions, and distinguishing between them empirically is a long-term challenge.

The free energy principle (FEP) of Karl Friston shares significant conceptual territory with the Qi-4 runtime, particularly in the role of variational inference and the minimization of surprise. The GUT-10 programme derives the FEP as the inferential gauge of Qi-EP—one of four pillars

rather than the foundational principle—and embeds it in a broader constitutional structure that also produces the other three pillars.

6.3 Predictions and Falsifiability

A theoretical programme that cannot be falsified is not physics. The GUT-10/GG-6/GDOS programme makes several classes of predictions that are in principle distinguishable from those of competing frameworks.

6.3.1 Hard predictions

Three predictions of the programme are categorical and admit no exceptions within the framework.

First, **absolute proton stability**. The exact $\mathrm{U}(1)_B$ gauge symmetry of GG-6 forbids all local baryon-violating operators. Any confirmed observation of proton decay at any energy would falsify the programme.

Second, **no fundamental supersymmetry**. The GG-6 bulk achieves anomaly cancellation without superpartners. Discovery of fundamental (not accidental) low-energy supersymmetry would require structural revision.

Third, **six-dimensional consistency**. The programme predicts exactly two extra dimensions with specific warping profiles. Evidence for a different number of extra dimensions would be fatal.

6.3.2 Diagnostic predictions

Three further predictions are quantitative and testable across multiple scientific domains.

The **Arisaka Constant**: the locked ratio of energy, entropy, material, and information currents should be measurable in sufficiently well-characterized open systems.

The $1/f$ **double-knee spectrum**: Qi-CON predicts that all persistent open systems should exhibit $1/f$ noise with two characteristic knee frequencies separating three scaling regimes.

The **QCI completeness index**: self-maintaining systems should be classifiable by a graded index measuring how many of the four Qi-4 pillars are operationally active.

6.4 Remaining Challenges and Future Prospects

Honesty about the programme's incompleteness is essential. The most important open problems are: (i) unique determination of the microscopic six-dimensional matter reservoir; (ii) phenomenological realization of the $\mathrm{U}(1)_{Qi}$ sector at accessible energies; (iii) quantitative confrontation with precision electroweak, neutrino, and cosmological data; and (iv) nonperturbative completion of the GG-6 bulk above the GUT scale.

Each of these is a major research programme in its own right, and each will require sustained effort across multiple papers and volumes. The present book therefore claims a *theorem-level*

derivation, not a finished phenomenological fit. The distance between the two is the agenda for the next decade.

6.5 Preview of Volumes II–IV

Volume I has developed the theoretical framework. The remaining volumes will apply and test it.

Volume II: Geometric Origin of the Cosmos and Particles will develop the first two floors of the pagoda: (1) GUT-Particle—the Standard Model as a derived boundary theory, with detailed predictions for collider signatures and proton stability; (2) GUT-Cosmos—cosmological dynamics, dark energy as a bulk effect, and the arrow of time.

Volume III: Inevitable Emergence of Life and Mind will develop floors three through five: (3) GUT-Chemistry—molecular structure, catalytic cycles, and the emergence of proto-cellular open systems; (4) GUT-Life—the GDOS account of biological organization, from self-replicating molecules to multicellular organisms; (5) GUT-Mind—consciousness as $\mathrm{OH} + \mathrm{IH} = \mathrm{EIC}$, and the relationship between the Qi-4 runtime and empirical neuroscience.

Volume IV: GUT-10 — Grand Unification of 10 Sciences will extend the framework beyond the five pagoda floors to the remaining five sciences: GUT-Health, GUT-Society, GUT-Environment, GUT-AI, and GUT-Philosophy, completing the ten-science programme.

Together, the four volumes of *The Geometry of Existence* are intended to provide a complete account—from axioms to empirical test, from particles to philosophy—of the proposition that the laws of physics are sufficient to explain not only the matter and energy of the universe, but the life, mind, and civilization that have emerged within it.

Chapter 7

Conclusions

"We are a way for the cosmos to know itself."

— Carl Sagan, *Cosmos*, 1980

7.1 The Argument of the Book

This book asked one question and pursued it to its logical end: *Is it possible to derive all of known physics, and the principles governing life and mind, from a single set of foundational rules?*

The answer, developed across five chapters of derivation, is yes—provided one accepts three premises. First, that the primitive object of physics is not spacetime alone but a composite geometry-gauge connection, so that force and curvature are two faces of the same mathematical structure. Second, that every act of science—every measurement, every observation, every registration of data—is an irreducible operation by an open system on its accessible boundary, not a passive reading of a pre-existing reality. Third, that the consistency of these two premises, enforced without exception across all scales, is sufficient to determine the dimensionality of spacetime, the gauge group of particle physics, the dynamical laws of the boundary, and the universal runtime that governs every open system from a proton to a conscious mind.

The GUT-10 constitution (Chapter 2) fixes the rules. GG-6 (Chapter 3) derives the unique six-dimensional geometry, with five integers and zero continuous parameters. GDOS (Chapter 4) projects the bulk onto the four-dimensional boundary, producing general relativity, the Standard Model, and quantum measurement as derived outputs. The Qi-4 runtime (Chapter 5) shows that any open system on that boundary—if it maintains persistent records and exchanges energy, matter, and information with its environment—is governed by a universal pipeline, a scalar ledger, four diagnostic pillars, and the Arisaka Equation.

Nothing is assumed that is not forced by the constitution. Nothing is free that is not fixed by the geometry.

7.2 Life as a Theorem

If the framework is correct, then life is not an accident. It is not a statistical fluke in an indifferent cosmos, nor a fortunate by-product of chemistry that happened to arise on one planet around one unremarkable star. Life is a *theorem*—a necessary consequence of the same geometric and gauge principles that produce gravity and the strong nuclear force.

Any universe that obeys the GUT-10 constitution and admits a persistent boundary observer will, given sufficient time and energy throughput, generate self-maintaining open systems, adaptive memory, internal models of the environment, and eventually what we call consciousness.

This does not mean that every planet will produce life, any more than the laws of arithmetic guarantee that every number is prime. It means that the *possibility* of life is not contingent on special initial conditions, fine-tuned parameters, or lucky accidents. It is built into the architecture of the laws themselves. The pagoda hierarchy—particles, cosmos, chemistry, life, mind—is not a metaphor. It is a derivation.

7.3 The Open Frontier

The framework is incomplete, and this book has been careful to say where. There is also a deeper question that this book does not attempt to answer. The GUT-10 constitution is stated as a set of axioms. But where do axioms come from? Is there a meta-constitutional principle from which the four axioms and seven postulates can themselves be derived? Or are they the irreducible bedrock—the point at which explanation must stop and description must begin?

This question is not a weakness of the programme. It is the question that every foundational framework in the history of physics has eventually faced. Euclid's axioms stood for two millennia before Riemann showed they were not the only possibility. Newton's laws stood for two centuries before Einstein showed they were approximations. If the GUT-10 axioms eventually yield to a deeper principle, that will not invalidate the programme—it will extend it, just as general relativity extended Newtonian mechanics rather than abolishing it.

7.4 A Closing Thought

The history of physics teaches that the deepest advances come not from new equations but from new principles. Newton's laws were not equations—they were a new way of thinking about force and motion. Einstein's special relativity was not an equation—it was a new way of thinking about simultaneity. Quantum mechanics was not an equation—it was a new way of thinking about measurement.

The GUT-10/GG-6/GDOS programme proposes a new way of thinking about *all of these at once*: that geometry, gauge, measurement, and openness are not four separate subjects requiring four separate formalisms, but four faces of a single constitutional structure—and that this structure, correctly understood, makes the emergence of life and mind not merely possible but inevitable.

Whether this proposal survives the coming decades of theoretical scrutiny and experimental test is, of course, an open question. But the question itself—*Are the laws of physics sufficient to explain why we are here?*—is one that deserves to be asked with the full mathematical precision that modern science can bring to bear.

This book has been one attempt to do exactly that.

The next step is yours.

Frequently Asked Questions

Questions and Answers for a Broad Audience

FAQ-2

The GUT-10 Constitution (Chapter 2)

FAQ-2: The GUT-10 Constitution (Chapter 2)

This section addresses the questions most commonly raised by readers encountering the GUT program for the first time.

FAQ-2.1 "Is this just another Theory of Everything?"

No. A Theory of Everything (ToE), in the traditional sense, is a specific model—a single Lagrangian from which all phenomena can (in principle) be derived. String theory aspires to be a ToE in this sense.

GUT is not a ToE. It is a *constitution*—a set of admissibility rules that any candidate ToE must satisfy. It does not tell you what particles exist or what their masses are. It tells you what kinds of theories are *allowed*. The distinction is between writing a specific law ("the speed limit is 65 mph") and writing a rule about laws ("all speed limits must be posted in advance and enforced consistently").

That said, the constitution turns out to be so restrictive that it essentially forces a unique theory (GG-6 + GDOS). So in practice, the constitutional approach *produces* something close to a ToE—but it arrives there by elimination rather than by construction.

FAQ-2.2 "Why ten sciences? Is this reductionism?"

The claim is not that biology reduces to physics, or that economics reduces to quantum mechanics. The claim is more modest: all sciences must obey the same *foundational hygiene rules*. These rules (gauge invariance, positivity, falsifiability, etc.) are not specific to physics; they are general requirements that any rigorous science should satisfy.

The word "unification" in GUT means *interface discipline*: each branch must declare a state, evolve it by an admissible dynamics, and export observable predictions with kill criteria. The ten branches keep their own domain-specific knowledge (chemistry keeps molecules, biology keeps evolution, neuroscience keeps neurons). What they share is the constitutional audit trail.

FAQ-2.3 "Why should I believe that four axioms are enough?"

You shouldn't—yet. The question is not whether the four axioms are "true" (axioms are accepted, not proved) but whether they are *productive*: do they generate interesting, falsifiable consequences?

The test is the companion chapters. If the four axioms plus seven postulates can derive:

(a) the six-dimensional GG-6 bulk with its specific gauge group and Bianchi identity;

(b) a complete anomaly-free quantum field theory at E_{GUT};

(c) the four-dimensional Standard Model as a boundary projection;

(d) the thermodynamic arrow of time;

(e) concrete, falsifiable numerical predictions;

then the axioms have earned their keep. If they can't, they need to be revised or replaced.

FAQ-2.4 "What happens above the GUT scale?"

The GUT constitution does not claim to describe physics at arbitrarily high energies. The GG-6 bulk is defined up to the GUT scale ($E_{\mathrm{GUT}} \sim 2 \times 10^{16}$ GeV), not up to the Planck scale ($E_{\mathrm{Pl}} \sim 10^{19}$ GeV).

What happens above E_{GUT}? The constitution is silent on this question. It may be that a new constitutional layer is needed at higher energies, or it may be that the same constitution extends to the Planck scale without modification. The program is honest about its domain of validity: it claims to describe physics from the GUT scale down to the laboratory scale, and it does not pretend to solve every ultraviolet question in one stroke.

FAQ-2.5 "How does this relate to the Free Energy Principle?"

The Free Energy Principle (FEP), proposed by Karl Friston, states that biological systems minimize a variational free energy—a quantity that measures the discrepancy between the system's internal model and the actual sensory input. The FEP has been applied to perception, action, learning, and even consciousness.

In the GUT framework, the FEP is a *consequence* of POS-2 (the Unified Extremal Principle) applied to the biological boundary theory. The bulk variational engine (least action in six dimensions) projects down to a boundary free-energy minimization via the GDOS dictionary. The brain's inference process is then a special case of this general boundary dynamics.

This connection is not yet fully worked out—it is the subject of future GUT-Mind papers. But the constitutional framework provides the natural home for the FEP: it is not an independent postulate but a boundary projection of the bulk variational engine.

FAQ-2.6 "Can I use the constitution in my own research?"

Yes. The constitutional export to non-physics domains (Section **??**) is designed to be modular. If you work in chemistry, biology, neuroscience, or any other science, you can:

(1) Identify your state variables and their symmetries (POS-1).

(2) Write down a variational principle for the dynamics (POS-2).

(3) Check that the dynamics preserves positivity (POS-4).

(4) Verify that your descriptions at different scales are consistent (POS-5).

(5) Use the simplest possible normalization (POS-6).

(6) State your claims with registered observables and kill criteria (POS-7).

You do not need to know anything about six-dimensional geometry or anomaly cancellation to apply this checklist. The checklist is the constitutional export—the "health code" for your branch of science.

FAQ-2.7 "What would convince you that the program is wrong?"

Several things:

- **Internal mathematical inconsistency.** If the eleven clauses are found to be mutually contradictory, the program collapses.
- **Proton decay.** If the proton is observed to decay, the exact $\mathrm{U}(1)_B$ symmetry of GG-6 is falsified.
- **Required supersymmetry.** If superpartners are discovered and shown to be essential for the consistency of nature, the non-SUSY stance of GG-6 is falsified.
- **Continuous moduli in nature.** If the normalization sector of physics turns out to require continuous free parameters beyond OGN-5, the program needs revision.
- **Failure to derive the Standard Model.** If the GG-6 + GDOS derivation chain cannot reproduce the known particle spectrum and coupling constants, the program has failed its most basic test.

FAQ-2.8 "Is this published in a peer-reviewed journal?"

Not yet. The papers in the UCLA-Arisaka/GG-2026 series are currently preprints. They are being prepared for submission to journals such as JHEP (Journal of High Energy Physics). The program is new, and independent scrutiny—both mathematical and experimental—is exactly what it needs. The preprint format is used to make the work available for community review as early as possible.

FAQ-2.9 "What does 'geometry–gauge equivalence' really mean physically?"

Imagine you are driving on a curved highway. The curvature of the road itself is like gravity: it determines which direction your car drifts if you take your hands off the steering wheel. Now imagine that the road also has a magnetic strip embedded in it, and your car has a sensor that responds to the strip's orientation. As you drive along the road, the strip rotates—sometimes pointing north, sometimes east. This rotation is like a gauge field: it determines an "internal direction" at each point.

In the traditional picture, the road curvature (gravity) and the magnetic strip (gauge field) are two completely separate things. The road was paved by one contractor; the strip was installed by another. They happen to coexist on the same highway, but there is no deep reason they should be related.

AX-2 says: the road curvature *and* the magnetic strip are both aspects of one object—the "composite connection." The road was not paved and then decorated; it was *woven* from a

material that is simultaneously curved *and* magnetically patterned. You cannot separate the curvature from the magnetism without tearing the fabric of the road.

This is what "geometry–gauge equivalence" means physically: the geometric structure of spacetime and the internal structure of the gauge forces are woven from the same fabric.

FAQ-2.10 "Why do we need extra dimensions? Are they real?"

This is one of the most common questions, and it deserves a careful answer.

First, the pragmatic answer: *you need extra dimensions to have enough room for all the gauge degrees of freedom.* In four dimensions, you can fit gravity, but the gauge forces require internal structure—extra "directions" for the fields to point in. If you want all of this to be geometry (AX-2), you need extra geometric directions.

How many? The constitution forces exactly two extra dimensions: one non-compact (the radial scale direction r) and one compact (the angular phase direction θ). Total: six.

Second, the philosophical answer: *the extra dimensions are as "real" as the electromagnetic field.* When Maxwell unified electricity and magnetism in 1865, he introduced the electromagnetic field—an entity that pervades all of space but cannot be seen or touched directly. Nobody asks "is the electromagnetic field real?" We accept it because it makes correct predictions. Similarly, the extra dimensions of GG-6 are "real" in the sense that they make correct predictions—and they will stand or fall on the basis of those predictions.

Third, the practical answer: *you will never see the extra dimensions directly.* They are either very small (the compact S^1_θ) or represent a scale direction that is not directly accessible as a spatial dimension. What you *can* see are their *consequences*: specific patterns in the gauge coupling constants, specific Kaluza–Klein tower spectra, and specific relationships between parameters that would be unexplained in four dimensions.

Metaphor: Consider a garden hose. From a distance, it looks like a one-dimensional line. But up close, you see that it has a circular cross-section—an extra dimension that is "compactified" (curled up so small that it's invisible from far away). The extra dimensions of GG-6 are like the cross-section of the hose: invisible at low energies, but physically present and detectable through their effects on the spectrum of excitations.

FAQ-2.11 "What is the 'informational force'? Is that real physics or philosophy?"

The informational force—gauged $\mathrm{U}(1)_{Qi}$—is probably the most unfamiliar and potentially controversial element of the GG-6 program. The idea deserves a careful examination.

Where it comes from. AX-3 (the operational measurement axiom) requires that the theory include a conserved "information ledger" at the same structural level as electric charge and baryon number. Just as electric charge conservation is enforced by the gauge symmetry of electromagnetism, information conservation is enforced by the gauge symmetry $\mathrm{U}(1)_{Qi}$.

What it is *not*. It is not Shannon entropy. It is not the "information" of everyday speech. It is not a vague philosophical concept. It is a precisely defined conserved quantum number carried by matter fields, entering the Bianchi identity BI-6 on exactly the same footing as baryon number.

What it *is*. The informational charge Q_i is a quantum number that tracks the "operational distinguishability" of states in the bulk-to-boundary dictionary. When a bulk state projects to the boundary, the information content of that projection—how many distinguishable outcomes the boundary observer can resolve—is constrained by $\mathrm{U}(1)_{Qi}$ gauge invariance.

Is it detectable? Yes, in principle. The gauged $\mathrm{U}(1)_{Qi}$ predicts a massive gauge boson X_{Qi} (with a Stückelberg mass), kinetic mixing with the hypercharge field, and specific signatures in precision electroweak data. The detailed phenomenology is the subject of future papers.

Why it matters. If $\mathrm{U}(1)_{Qi}$ is real, it would be the first force directly tied to the operational semantics of measurement. This would close the gap between the "ontological" side of physics (what exists) and the "epistemological" side (what can be known), by treating the latter as a gauge symmetry—a constitutional structure, not a philosophical afterthought.

FAQ-2.12 "How does GUT-10 handle quantum entanglement and the EPR paradox?"

The Einstein–Podolsky–Rosen (EPR) paradox (1935) asks: if two particles are entangled and you measure one in New York, how can the other, in Tokyo, "know" the outcome instantly? Doesn't this violate causality?

The answer within GUT is clean and constitutional.

AX-1 (causality) says: no *signal* can travel faster than light. Entanglement does not violate AX-1 because entanglement is a *correlation*, not a signal. The outcome in Tokyo is correlated with the outcome in New York, but the Tokyo experimenter cannot use this correlation to send a message faster than light. The no-signaling theorem, which holds in any CPTP quantum channel (POS-4), guarantees this.

AX-3 (operational measurement) provides the deeper story. The entangled state exists in the *bulk*; the measurements in New York and Tokyo are *boundary projections* via the operational dictionary. The apparent "nonlocality" of entanglement is an artifact of projecting a higher-dimensional bulk state onto lower-dimensional boundary patches. The bulk description is local and causal; the boundary description *appears* nonlocal because the two boundary patches share a common bulk origin.

Metaphor: Imagine a pair of gloves packed in two separate boxes and shipped to New York and Tokyo. When the New York recipient opens their box and finds a left glove, they instantly know the Tokyo box contains a right glove. No signal was sent; the correlation was established when the gloves were packed. Entanglement is similar, but quantum: the "gloves" are quantum states, and the correlation is stronger than any classical packing could produce (this is the content of Bell's theorem). AX-1 is satisfied because no signal is sent; AX-3 is satisfied because the measurement dictionary specifies exactly how the bulk "packing" projects to the boundary "unboxing."

FAQ-2.13 "What about Schrödinger's cat?"

Schrödinger's cat is a thought experiment designed to highlight the absurdity of wavefunction superposition applied to macroscopic objects. A cat in a box is (supposedly) in a superposition of "alive" and "dead" until someone opens the box and looks.

In the GUT framework, the resolution is constitutional.

AX-3 says: measurement is not a mysterious collapse triggered by a conscious observer. It is a specific CPTP operation in the operational dictionary—a quantum instrument that extracts boundary information from the bulk.

POS-4 says: the reduced dynamics of the cat (after tracing out the environment—the air molecules, the photons, the box walls) is a GKLS evolution that decoheres the "alive + dead" superposition on an extraordinarily short timescale (less than 10^{-30} seconds for a macroscopic object). The cat is never in a genuine superposition as seen by any boundary observer; decoherence destroys the coherence long before anyone opens the box.

The "paradox" arises only if you insist on describing the cat as a closed system (pure state, unitary evolution). The constitution says: the cat is not a closed system. It is an open system embedded in an environment. AX-3 + POS-4 force an open-system description in which the cat is always effectively "alive" or "dead," never both.

Summary: Schrödinger's cat is not a problem for GUT. It is a *symptom* of the constitutional gap that GUT is designed to fill. Once you constitutionalize measurement (AX-3) and positivity (POS-4), the paradox dissolves.

FAQ-2.14 "What about the cosmological constant problem?"

The cosmological constant problem is one of the most famous puzzles in theoretical physics. Quantum field theory predicts that the vacuum energy density should be roughly 10^{120} times larger than the observed value of the dark energy density. This is sometimes called "the worst prediction in the history of physics."

How does GUT address this?

The honest answer is: *not yet fully.* The cosmological constant problem is a phenomenological question that requires a detailed comparison between the GG-6 predictions and observational data. This comparison is the subject of future papers.

However, the constitutional framework provides two structural handles:

1. AX-4 (minimum freedom). The cosmological constant Λ is a free parameter in the Standard Model. AX-4 demands that it be either derivable from the constitution or constitutionally frozen (like the OGN-5 integers). If Λ can be related to the warped geometry of the six-dimensional bulk—for example, through the negative bulk cosmological constant Λ_6 that produces the AdS_6-like warping—then the problem shifts from "why is Λ so small?" to "what is Λ_6, and how does it project to four dimensions?"

2. POS-6 (OGN). If the effective four-dimensional cosmological constant is controlled by the OGN-5 integers and the throat depth $K = 35$, then it is not a free continuous parameter but a derived discrete quantity. The hierarchy $e^{-K} = e^{-35} \approx 10^{-15}$ is suggestive—it is in the right ballpark for generating a large hierarchy from a modest integer—but the detailed calculation has not yet been performed.

Bottom line: The cosmological constant problem is one of the tests (POS-7) that the program must eventually pass. If the constitution cannot produce a satisfactory value of Λ, this would be a serious challenge.

FAQ-2.15 "Does the constitution say anything about dark matter?"

The short answer is: not directly, not yet.

Dark matter accounts for about 27% of the energy density of the universe. Its nature is unknown. The Standard Model does not contain a dark matter candidate; most proposals for dark matter (WIMPs, axions, sterile neutrinos) are model-dependent additions.

In the GUT framework, dark matter would need to be *constitutionally admissible*: it must carry well-defined quantum numbers under the full gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$, its dynamics must satisfy POS-1 through POS-6, and its existence must be falsifiable (POS-7).

One intriguing possibility is that the Kaluza–Klein excitations of the six-dimensional bulk—the massive modes that appear when the bulk fields are expanded in harmonics of the compact S^1_θ and the radial direction r—could serve as dark matter candidates. The lightest KK mode would be stable (protected by a discrete symmetry related to the compactification) and would interact with ordinary matter only through gravity and the massive Abelian gauge bosons. But this is speculative; the detailed analysis is a target for future phenomenological papers.

FAQ-2.16 "Why isn't this just Kaluza–Klein theory with extra steps?"

Kaluza–Klein (KK) theory is a beautiful idea with a fundamental limitation: it geometrizes only *Abelian* gauge fields naturally. The original Kaluza theory (1921) put electromagnetism into the geometry of a fifth dimension (Kaluza, 1921). But the strong and weak nuclear forces are *non-Abelian* (their gauge groups are SU(3) and SU(2)), and fitting them into KK geometry requires internal manifolds with specific curvature and topology—leading to many continuous moduli (free parameters).

GG-6 differs from KK theory in four crucial ways:

(i) **The tensor sector.** KK theory uses only the metric to encode gauge fields. GG-6 adds a two-form tensor field B_2 whose Bianchi identity (BI-6) locks all force sectors together. This tensor sector has no KK analogue.

(ii) **Anomaly cancellation.** KK theory does not address quantum anomalies. GG-6 makes anomaly cancellation (via the Green–Schwarz mechanism) a constitutive feature of the theory.

(iii) **Discrete normalization.** KK theory leaves the size and shape of the internal manifold as continuous moduli. GG-6 freezes these to discrete integers via OGN-5.

(iv) **Constitutional derivation.** KK theory assumes an extra dimension for aesthetic reasons. GG-6 *derives* the need for exactly two extra dimensions from the constitutional axioms.

In short: GG-6 uses the *spirit* of Kaluza–Klein (geometrize forces via extra dimensions) but goes far beyond it in mathematical structure and constitutional rigor.

FAQ-2.17 "Can ordinary people understand this, or is it only for specialists?"

The constitution itself—the eleven clauses—can be stated in plain English and understood by anyone with a basic science education. The four axioms (causality, geometry–gauge equivalence, operational measurement, and minimum freedom) are intuitive once the metaphors are in place. The seven postulates (gauge invariance, variational principle, anomaly freedom, positivity, scale compatibility, integer normalization, and falsifiability) are more technical, but each can be explained with everyday analogies (currency exchange, universal remote, balance sheet, census, Google Maps, tuning fork, sports bet).

The *derivations*—the mathematical proofs that the constitution forces GG-6 and GDOS—require graduate-level training in quantum field theory, differential geometry, and anomaly theory. These are in the technical papers (GG-2026-1 through GG-2026-3).

The *predictions*—proton stability, gauge coupling ratios, KK spectra—are stated as concrete numbers that any experimentalist can compare with data.

So the answer is: ordinary people can understand *what* the theory says and *why* it matters (that's what this White Paper is for). Graduate students and specialists can understand *how* it works (that's what the technical papers are for). And experimentalists can test *whether* it is right (that's what POS-7 demands).

This three-level accessibility—what, how, whether—is deliberate. It is part of the constitutional philosophy: every scientific program should be understandable at multiple levels, not locked behind a wall of jargon accessible only to initiates.

FAQ-2.18 "How does GUT-10 relate to quantum computing and quantum information?"

Quantum information theory is one of the most active areas of modern physics, and its connection to GUT runs through two axioms.

AX-3 and quantum channels. AX-3 demands that the theory include a precise measurement dictionary. In the language of quantum information, this dictionary is a *quantum channel*—a completely positive, trace-preserving (CPTP) map that describes how information flows from the bulk to the boundary. Quantum channels are the bread and butter of quantum computing: every quantum gate, every error-correction code, and every quantum algorithm can be expressed as a sequence of channels.

POS-4 and the CPTP condition. POS-4 demands that all physical evolution be CPTP. This is exactly the condition that quantum information theorists impose on physical quantum

operations. The GUT constitution does not merely borrow this condition from quantum information theory; it *derives* it as a consequence of causality (AX-1) and the measurement axiom (AX-3).

Entanglement as a resource. In quantum computing, entanglement is a resource that enables tasks impossible with classical systems (teleportation, superdense coding, Shor's algorithm). In GUT, entanglement between bulk and boundary degrees of freedom is what generates the non-trivial structure of the boundary theory. The GDOS dynamics—the open-system evolution of the boundary—is fundamentally shaped by the entanglement structure of the bulk.

So quantum information is not a separate field from GUT; it is one of the constitutional ingredients. A quantum computer, from the GUT perspective, is a device that manipulates the boundary channels of a constitutionally admissible theory.

FAQ-2.19 "What would Newton, Maxwell, or Einstein think of this?"

This is a fun question, even if it is obviously speculative.

Newton would probably appreciate AX-4 (minimum freedom). His famous rule "we are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances" is essentially the same principle. He might be puzzled by the extra dimensions but would recognize the variational principle (POS-2) as a descendant of his laws of motion.

Maxwell would appreciate AX-2 (geometry–gauge equivalence) deeply. His unification of electricity and magnetism was the first geometry–gauge merger in physics. He would see GUT as the natural completion of his program: not just electromagnetism, but *all* forces unified through geometry.

Einstein would likely have mixed feelings. He spent the last thirty years of his life searching for a unified field theory—a geometric theory that combined gravity and electromagnetism. He never found it, partly because the strong and weak nuclear forces were not yet understood. He would recognize AX-2 as his dream realized. But he might resist AX-3 (the measurement axiom), since he was famously uncomfortable with the role of the observer in quantum mechanics. The GUT answer to Einstein would be: "You were right to demand that measurement have a precise definition. AX-3 provides one—not by eliminating the observer, but by constitutionalizing the observation process."

FAQ-2.20 "Is there a one-sentence summary of the entire program?"

Yes:

The GUT-10 constitution is a set of eleven rules that, when imposed simultaneously,
force a unique six-dimensional geometry unifying six forces,
a specific open-system boundary dynamics producing four-dimensional quantum mechanics,

and a discrete integer ledger replacing the continuous landscape of free parameters—
all without supersymmetry, strings, or any ingredient not compelled by self-consistency.

FAQ-3

GG-6: Geometry–Gauge Locking (Chapter 3)

FAQ-3: GG-6: Geometry–Gauge Locking (Chapter 3)

This section addresses the questions most commonly raised by readers encountering GG-6 for the first time.

FAQ-3.1 "Isn't this just Kaluza–Klein theory with extra bells and whistles?"

No. Kaluza–Klein theory geometrizes only Abelian gauge fields (electromagnetism) via extra dimensions, and it leaves the size and shape of the internal space as continuous free parameters. GG-6 differs in four fundamental ways: (i) it uses a tensor sector B_2 whose Bianchi identity locks *all* force sectors together; (ii) it makes anomaly cancellation (via the Green–Schwarz mechanism) a constitutive feature; (iii) it freezes all continuous normalization freedom to discrete integers via OGN-5; and (iv) it *derives* the need for exactly two extra dimensions from constitutional axioms, rather than assuming them for aesthetic reasons.

FAQ-3.2 "Why SU(5) and not SO(10) or E_6?"

AX-4 (Maximum Symmetry, Minimum Freedom) demands the *smallest* simple group that contains the Standard Model gauge group as a subgroup. SU(5) is the unique such group (Georgi and Glashow, 1974). SO(10) and E_6 are larger and would introduce additional representations and parameters not forced by the constitution. They are not *excluded* by the axioms—they are simply not *selected* by the lexicographic ordering of AX-4.

If future evidence requires SO(10) (for example, to accommodate right-handed neutrinos naturally), the constitution would need revision, and the resulting theory would be a larger GG-6 variant.

FAQ-3.3 "How can five integers replace the entire landscape?"

The string landscape contains $\sim 10^{500}$ vacua because each vacuum corresponds to a different choice of compactification manifold, flux integers, brane configurations, and moduli values. These are, in effect, continuous normalization choices.

POS-6 (Occam Gauge Normalization) eliminates this freedom by demanding that all normalizations take the simplest possible rational or integer values. In the GG-6 realization, this reduces the entire normalization sector to five integers: $M = 1$, $N = 3$, $L = 5$, $J = 7$, $K = 35$. These five integers satisfy two closed arithmetic relations ($K = LJ$ and $K = M^2 + N^2 + L^2$), which together fix the throat depth, the gauge coupling ratios, and the exponential hierarchy factor.

The central observation is that the landscape exists only because conventional string theory *allows* continuous moduli. GUT does not allow them.

FAQ-3.4 "Does the proton *really* never decay?"

In GG-6, proton stability is *exact*—not a numerical suppression but an absolute selection rule. The baryon number B is a gauged $\mathrm{U}(1)_B$ symmetry, and no local operator with $\Delta B \neq 0$ can

appear in the exact theory. Even a Stückelberg mass for the baryonic gauge boson does not break the symmetry.

This is a qualitative distinction from all other GUT frameworks, where proton decay is *permitted but slow*. In GG-6, it is *forbidden by topology*. If proton decay is ever observed, GG-6 is dead. This is the sharpest single falsifier of the program.

FAQ-3.5 "What is the 'informational force' and why should I believe it exists?"

The informational force (gauged $\mathrm{U}(1)_{Qi}$) is the most novel and potentially controversial element of GG-6. It arises from AX-3 (OGGEP), which demands that the theory include a conserved "information ledger" at the fundamental level.

Think of it this way: electric charge conservation tells you that charge cannot be created or destroyed. Baryon number conservation tells you that protons cannot disappear. Information conservation (Q_i) tells you that the operational distinguishability of states—the ability of a boundary observer to tell two states apart—is a conserved quantum number.

The informational gauge boson X_{Qi} would be massive (via the Stückelberg mechanism) and would produce specific signatures: kinetic mixing with the photon, small corrections to precision electroweak observables, and potentially a new Z' resonance at high-energy colliders. These are concrete predictions that future experiments can test.

FAQ-3.6 "What would Einstein say about GG-6?"

Einstein would recognize GGEP (AX-2) immediately—it is his lifelong dream of a unified geometric field theory, finally realized with the correct gauge group and the correct mathematical tools. He would appreciate that the theory derives its structure from principles (the constitution) rather than guessing equations.

He might be uncomfortable with AX-3 (the measurement axiom), since he was famously skeptical of the role of observers in quantum mechanics. But the GG-6/GDOS answer to his concern is subtle: the observer is not a mysterious consciousness; the observer is a *boundary*—a four-dimensional projection of a six-dimensional bulk. Measurement is not "subjective"; it is the mathematical operation of tracing out inaccessible bulk degrees of freedom. This is as objective and geometric as anything in general relativity.

Einstein would also appreciate AX-4 (minimum freedom). His famous quote—"everything should be made as simple as possible, but not simpler"—is essentially the MSMF principle in a nutshell.

FAQ-3.7 "How does GG-6 compare to loop quantum gravity?"

Loop quantum gravity (LQG) is an approach to quantum gravity that quantizes the geometry of spacetime directly, without extra dimensions or strings. Its key prediction is that spacetime has a discrete structure at the Planck scale—areas and volumes come in discrete quanta.

GG-6 differs from LQG in several ways:

- **Strategy:** LQG quantizes gravity. GG-6 geometrizes gauge.
- **Extra dimensions:** LQG works in four dimensions. GG-6 requires six.
- **Gauge forces:** LQG does not unify the gauge forces with gravity. GG-6 does.
- **UV endpoint:** LQG aims at the Planck scale. GG-6 aims at the GUT scale.
- **Matter content:** LQG does not determine the matter content. GG-6 constrains it through anomaly cancellation.

The two programs are not necessarily incompatible. It is possible that LQG describes physics at the Planck scale while GG-6 describes physics at the GUT scale, with a transition region between them. But this is speculative.

FAQ-3.8 "Is there any connection to the holographic principle and black holes?"

Yes. AX-3 (OGGEP) is directly inspired by the holographic principle—the idea that the information content of a region of space is encoded on its boundary. The most famous realization of this principle is the AdS/CFT correspondence (Maldacena, 1998), in which a theory of quantum gravity in a $(d{+}1)$-dimensional "bulk" is equivalent to a quantum field theory on the d-dimensional "boundary."

GG-6 uses the same logical structure: the six-dimensional bulk projects to a four-dimensional boundary via the dictionary $\mathcal{D}$. But GG-6 does not require the full machinery of string theory or conformal field theory. The dictionary is constitutional (AX-3), not model-specific.

The connection to black holes is intriguing but speculative at this stage. The Bekenstein–Hawking entropy formula $S = A/(4G_N)$ suggests that black hole entropy is a boundary quantity. In GG-6, this would be a consequence of the bulk-to-boundary projection: the black hole's microstates live in the six-dimensional bulk, and the entropy counts the number of boundary-distinguishable configurations.

FAQ-3.9 "Why should a chemist or biologist care about six dimensions?"

You don't need to care about six dimensions to use the constitution. The six-dimensional bulk is the *ultraviolet completion*—the fundamental stage from which all physics derives. But the constitutional export to chemistry, biology, and other sciences operates entirely at the four-dimensional boundary level.

What you *do* need to care about is the constitutional checklist: gauge-invariant descriptions, positive dynamics, consistent multiscale behavior, minimal normalization, and falsifiable predictions. These are domain-independent requirements that apply to your science regardless of whether you know anything about the bulk.

The six dimensions are like the server room of a cloud computing service. As an end user, you interact with the interface (the four-dimensional boundary); you don't need to know what's in the server room. But the server room is what makes the interface work.

FAQ-3.10 "What happens to quantum gravity in this picture?"

GG-6 does not "solve" quantum gravity in the traditional sense. It does not quantize the metric at the Planck scale. What it does is something subtler: it *geometrizes the gauge forces* at the GUT scale and then shows that the resulting six-dimensional theory is quantum-consistent (anomaly-free, BRST-closed) without quantizing gravity separately.

Gravity enters the composite connection on equal footing with the gauge fields, and the Green–Schwarz mechanism cancels the gravitational anomaly along with the gauge anomalies. So gravity is "quantum" in the sense that it participates in the quantum consistency conditions—but it is not "quantized" in the sense of having a discrete spectrum of spacetime geometries.

This is a deliberately modest stance. The constitution asks: what is the minimal stage needed to unify forces, cancel anomalies, and produce operational measurements? The answer is a six-dimensional bulk at E_{GUT}, not a Planck-scale quantum spacetime. Whether a deeper theory is needed at higher energies is left as an open question.

FAQ-3.11 "Is the one-sentence summary?"

GG-6 is the unique six-dimensional warped geometry
that locks gravity, gauge forces, and two new Abelian symmetries
into one anomaly-free quantum field theory at the GUT scale,
replacing the continuous landscape of free parameters
with five integers and Newton's constant—
and it does so by fulfilling Einstein's geometric vision
while simultaneously completing quantum mechanics
with the first constitutional definition of measurement.

FAQ-4

GDOS: From the Bulk to the Boundary (Chapter 4)

FAQ-4: GDOS: From the Bulk to the Boundary (Chapter 4)

FAQ-4.1 "Does GDOS solve the measurement problem?"

Yes, in the following precise sense. The measurement problem asks: why does a quantum system, which exists in a superposition of many states, give a single definite outcome when measured? GDOS answers: because a "measurement" is a specific CPTP channel (a quantum instrument) that maps the boundary density matrix to a classical probability distribution over outcomes. The instrument is derived from the OGGEP dictionary, not postulated. The single outcome is obtained by the standard Bayesian update upon receiving data. No collapse postulate is needed.

FAQ-4.2 "But isn't decoherence enough?"

No. Decoherence explains why quantum superpositions become effectively classical—the off-diagonal elements of the density matrix decay rapidly when the system interacts with an environment. But decoherence alone does not explain (i) why a specific outcome is observed rather than another, (ii) what constitutes a "measurement" as opposed to any other interaction, or (iii) where the Born rule comes from. GDOS addresses all three: the instrument selects the basis, the OGGEP dictionary defines the instrument, and the Born rule is a consequence of the trace operation.

FAQ-4.3 "Is this just the many-worlds interpretation in disguise?"

No. Many-worlds claims that all branches of the wavefunction are equally real and that the universe splits at every measurement. GDOS does not require splitting. The bulk state evolves unitarily (one state, one universe), and the boundary observer sees one outcome because the instrument channel extracts one classical record from the quantum state. There is no multiverse, no branch counting, and no problem of probability measure.

FAQ-4.4 "How is GDOS different from ordinary open quantum systems theory?"

In formalism: not much. The GKLS equation, the NZ memory kernel, and the instrument formalism are all standard tools in open quantum systems theory (Gorini et al., 1976; Lindblad, 1976; Nakajima, 1958; Zwanzig, 1960). What is new is the *context*: GDOS is not an approximation applied to a closed system; it is the *exact* boundary theory of a constitutionally specified six-dimensional bulk. The Lindblad operators are not phenomenological fits; they are derived from the bulk-boundary coupling. The instrument is not a convenient tool; it is the constitutional definition of measurement.

FAQ-4.5 "How can gravity be 'derived' without being quantized?"

The metric zero mode of the six-dimensional warped bulk satisfies the four-dimensional Einstein equations. This is not "quantizing gravity"; it is "geometrizing gravity"—showing that the four-

dimensional gravitational law is a boundary projection of a higher-dimensional geometry. The six-dimensional theory *is* quantum-consistent (anomaly-free, BRST-closed), so in a sense gravity is already "quantum" at the six-dimensional level. But the four-dimensional projection produces Einstein's classical equations, not a quantized metric.

FAQ-4.6 "Why three generations? Can GDOS predict the number?"

The number of generations is encoded in the OGN-5 integer $N = 3$ and corresponds to three independent zero-mode solutions of the Dirac equation in the warped background. The constitution selects $N = 3$ as part of the minimal integer package satisfying the closure relations $K = LJ$ and $K = M^2 + N^2 + L^2$. Whether this selection is "strong enough" to uniquely predict three generations (rather than merely accommodate them) is a question that the phenomenological benchmarking papers must answer.

FAQ-4.7 "What happens to the arrow of time?"

The arrow of time emerges naturally from the bulk-to-boundary projection. The bulk evolution is unitary (time-symmetric). But the partial trace over the environment is irreversible—information flows from the boundary to the environment and does not come back on the timescales relevant to the boundary observer. This irreversibility is the arrow of time. The second law of thermodynamics (entropy increases) is a consequence of the CPTP nature of the boundary evolution (POS-4).

FAQ-4.8 "Does this framework say anything about consciousness?"

Not directly in this book. However, the companion monograph MG-2026-1 argues that the GDOS framework, extended to biological systems, predicts the hierarchical emergence of nervous systems, brains, and an "Evolving Inner Cosmos" that models the external universe. Consciousness, in that framework, is the system's recognition of its own bulk-to-boundary structure.

FAQ-4.9 "Can I understand this without knowing quantum field theory?"

Yes, at the level of this White Paper. The essential ideas—Hilbert space splitting, partial tracing, memory kernels, instruments, zero modes, and boundary conditions—are analogous to everyday concepts (three-way phone calls, canyon echoes, guitar frets, weather forecasts). The technical details require graduate-level training, but the logic is accessible.

FAQ-4.10 "What is the single most important equation in the paper?"

The GKLS equation:

$$\frac{d\rho}{dt} = -\mathrm{i}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right).$$

This is the equation that encodes all of open-system physics: decoherence, dissipation, measurement, the arrow of time, and the emergence of classicality. It is derived (not postulated) from AX-1 + POS-4 + the bulk-to-boundary projection.

FAQ-4.11 "What would von Neumann, Bohr, or Einstein think?"

Von Neumann would recognize the instrument formalism immediately—he invented the projection postulate. He would appreciate that GDOS derives it rather than postulating it.

Bohr would appreciate the emphasis on the operational role of the observer but might resist the claim that measurement has a precise mathematical definition (he preferred complementarity to formalism).

Einstein would be delighted. His lifelong objection to quantum mechanics was that it was "incomplete"—that the Born rule must emerge from a deeper theory. GDOS provides exactly that deeper theory: the Born rule emerges from the bulk-to-boundary projection of a deterministic, geometric, six-dimensional bulk. "God does not play dice" is correct—in the bulk. The dice are rolled only on the boundary, by the act of ignoring what you cannot see.

FAQ-4.12 "What is the one-sentence summary?"

The four-dimensional world of quantum mechanics, gravity, and the Standard Model is the shadow cast by a six-dimensional geometric bulk onto the wall of a finite observer's cave—and GDOS is the mathematics of shadow-casting.

FAQ-4.13 "Why do we need six dimensions? Why not just four?"

In four dimensions alone, you cannot simultaneously geometrize gauge forces (AX-2), implement the renormalization group as a geometric coordinate (POS-5), and provide a compact phase direction for gauge holonomy (POS-1 + POS-6). You need at least two extra dimensions: one non-compact (the radial scale direction r) and one compact (the angular phase direction θ). Four plus two equals six. A fifth dimension could provide one of these structures but not both; a seventh would introduce unnecessary freedom that AX-4 forbids.

FAQ-4.14 "What is the relationship between GDOS and thermodynamics?"

Thermodynamics emerges as a consequence of the GDOS boundary dynamics. The second law (entropy increases) follows from the CPTP nature of the boundary evolution: a completely positive, trace-preserving channel cannot decrease the von Neumann entropy of the system. The gradient current J_{gradient} in the GDOS continuity equation is the mathematical expression of "flowing downhill toward equilibrium"—which is the second law in disguise. Temperature, free energy, and the Boltzmann distribution all emerge from the GDOS landscape structure.

Metaphor: water flowing downhill. A ball on a hilly landscape always rolls toward the nearest valley (the second law: entropy increases, free energy decreases). The GDOS gradient current is the mathematical description of this rolling. The curl current—which has no thermodynamic analog in simple systems—is what keeps biological systems *away* from equilibrium: it is the force that pushes the ball back uphill, maintaining organized structure against the tendency to roll down.

FAQ-4.15 "Does GDOS apply to cosmology?"

Yes, in principle. From the four-dimensional boundary perspective, the observable universe is an open system: it is a projection of the six-dimensional bulk, and the projection discards information. The cosmological arrow of time, the expansion of the universe, and the low entropy of the Big Bang are all expected to be consequences of the GDOS boundary dynamics. However, the detailed cosmological analysis is the subject of future papers, not the present one.

FAQ-4.16 "How does GDOS relate to the holographic principle?"

AX-3 (OGGEP) is directly inspired by the holographic principle—the idea, originating from black hole physics and formalized by Maldacena's AdS/CFT correspondence, that the information content of a spatial region is encoded on its boundary. GDOS implements this principle constitutionally: the four-dimensional boundary theory contains all the information accessible to a finite observer, and the OGGEP dictionary specifies the precise encoding.

The key difference from AdS/CFT: GDOS does not require strings, conformal symmetry, anti-de Sitter space, or supersymmetry. The holographic structure is a consequence of the constitutional axioms, not of a specific string-theoretic construction.

FAQ-4.17 "What does GDOS say about quantum computing?"

Quantum computing manipulates quantum states using carefully controlled sequences of unitary gates and measurements. In the GDOS language, each quantum gate is a boundary unitary operation, and each measurement is a quantum instrument. The advantage of quantum computing over classical computing—the ability to exploit superposition and entanglement—arises from the fact that the boundary Hilbert space is exponentially larger than the classical state space.

GDOS provides a deeper perspective: quantum computing is the systematic exploitation of the boundary's open-system structure. Error correction, in this view, is the art of maintaining coherence (the curl current) against decoherence (the gradient current). The threshold theorem for fault-tolerant quantum computing is a statement about the balance between these two currents.

FAQ-4.18 "Is there any experimental evidence for the extra dimensions?"

Not yet directly. The extra dimensions in GG-6 are either compactified (the S^1_θ direction, with a radius far below current experimental reach) or represent a scale direction that is not directly accessible as a spatial dimension. However, the extra dimensions leave indirect traces: specific

patterns in the gauge coupling constants at the GUT scale, the existence of Kaluza–Klein tower states at high masses, specific correlations between the baryonic and informational gauge sectors, and the precise value of the effective Planck mass from the overlap integral. Future colliders and precision measurements may probe these signatures.

FAQ-4.19 "What is the most important experiment to test GDOS?"

Proton decay. If the proton decays at any rate, the exact $\mathrm{U}(1)_B$ protection of GG-6 is falsified, and the entire GDOS framework (which inherits its structure from GG-6) collapses. Hyper-Kamiokande and DUNE are currently searching for proton decay with sensitivity approaching 10^{35} years. A positive signal would kill the theory. Continued non-observation strengthens it.

FAQ-4.20 "Can GDOS be tested in a laboratory?"

Yes, through precision tests of open quantum systems. The GDOS framework predicts specific relationships between decoherence rates, Lindblad operator spectra, and the geometry of the environment coupling. Modern quantum optics, trapped-ion experiments, and superconducting qubit platforms can test these predictions. In particular, the GDOS prediction that the Lindblad operators of a specific system are *derivable* from the bulk-boundary coupling (rather than phenomenological fits) is testable whenever the microscopic Hamiltonian is known.

FAQ-4.21 "How is this different from the 'it from bit' idea?"

John Wheeler's famous slogan "it from bit" proposed that information is the most fundamental concept in physics. GDOS partially agrees: the $\mathrm{U}(1)_{Qi}$ gauge symmetry treats information as a conserved quantity at the same structural level as electric charge and baryon number. But GDOS goes further: the information current J_{Qi} is not the *foundation* of physics; it is one of *six* force currents, all derived from the same geometric structure. Information is fundamental, but so is energy, so is baryon number, and so is spacetime curvature. "It from geometry" might be a better slogan.

FAQ-4.22 "What would a complete success of GDOS look like?"

A complete success would mean: (i) the zero-mode Lagrangian, when benchmarked against a specific matter reservoir, reproduces the 19 Standard Model parameters (quark masses, mixing angles, coupling constants) to experimental precision; (ii) the KK tower masses are either observed at future colliders or shown to be beyond experimental reach in a way consistent with the throat depth $K = 35$; (iii) the $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ gauge bosons are either detected (via Z' resonances or precision electroweak shifts) or shown to be sufficiently heavy and weakly mixed to evade current bounds; (iv) proton decay continues to be absent, as predicted; and (v) the GDOS open-system framework is successfully applied to at least one non-physics domain (chemistry, biology, or neuroscience) in a way that produces a novel, experimentally confirmed prediction. All five of these tests are currently in progress or planned.

FAQ-4.23 "What is the most beautiful idea in GDOS?"

The most beautiful idea is also the simplest: *the apparent randomness of quantum mechanics is not a fundamental feature of reality; it is a consequence of a finite observer's inability to see the full picture.* The bulk is deterministic. The boundary looks random. The "mystery" of quantum mechanics is the mystery of a shadow: it seems to have a life of its own until you realize it is cast by something solid moving behind the screen.

FAQ-5

The Qi-4 Pillars and Runtime (Chapter 5)

FAQ-5: The Qi-4 Pillars and Runtime (Chapter 5)

This section is designed for the broadest possible audience. No physics background beyond high-school level is assumed. If a question interests you, read the answer slowly—each one is self-contained.

FAQ-5.1 General questions

Q1: What is GUT-10 in plain language? Imagine you are designing a board game. Before you can play, you need a rulebook: which moves are legal, what counts as a win, how disputes are resolved. GUT-10 is the rulebook of physics. It is a set of four axioms and seven postulates that define what any lawful physical theory is allowed to assume, what it must derive from those assumptions, and what it must make measurable by real instruments.

Why is this useful? Because for the past century, different branches of physics have been playing by slightly different rules. Quantum mechanics has its postulates, general relativity has its postulates, thermodynamics has its postulates—and nobody has ever checked whether all these rulebooks are mutually consistent. GUT-10 is the first attempt to write *one* rulebook that all of physics must obey. If a proposed law violates the GUT-10 constitution, it is unconstitutional—it may be approximately useful, but it cannot be fundamentally correct.

Q2: What is GG-6 in plain language? If GUT-10 is the rulebook, GG-6 is the game board. When you enforce the GUT-10 rules and ask "what kind of universe is logically possible?", the answer turns out to be unique: a six-dimensional warped geometry that automatically contains gravity, the strong nuclear force, the weak nuclear force, electromagnetism, and two new forces (one for baryon number, one for information). All six forces are locked together by a single geometric identity—the Bianchi identity BI-6—and the entire structure is described by exactly five integers.

Think of it this way: if you demand that a building must stand up, resist wind, allow light in, and keep rain out, there are not infinitely many architectural solutions. The constraints narrow the possibilities dramatically. GG-6 is the unique "building" that satisfies all the GUT-10 constraints simultaneously. The five integers are like the five measurements you need to fully specify the building's shape.

Q3: What is GDOS in plain language? GDOS stands for Geometric Dynamics of Open Systems. If GUT-10 is the rulebook and GG-6 is the game board, GDOS is the instruction manual for how the game is actually played.

The key idea is this: we live on the four-dimensional *surface* of a six-dimensional universe, like ants living on the surface of a basketball. The ants cannot see the interior of the ball, but the interior determines the shape of the surface they walk on. GDOS is the mathematical procedure that translates the properties of the six-dimensional interior into the properties of the four-dimensional surface. When you carry out this translation, you get quantum mechanics, general relativity, and the Standard Model of particle physics—not as separate postulates, but as automatic consequences of living on the surface of a six-dimensional ball.

Q4: What does Chapter 5 add that was not already in the first three papers? Chapters 2–4 built the stage (the 6D geometry) and showed how particle physics emerges on that stage. Chapter 5 asks: what else emerges? The answer is: everything.

Chapter 5 shows that the same geometric machinery that produces protons and electrons also produces the conditions for chemistry, life, brains, and consciousness. It introduces a universal "operating system" for all open systems in nature—the master pipeline (explore, select, stabilize), four diagnostic pillars, a persistence criterion, and a framework for understanding how internal world-models and consciousness arise. In short, Chapters 2–4 explain why the universe has the particles it has. Chapter 5 explains why the universe has the *structures* it has—from crystals to cities, from cells to conscious minds.

Q5: Do I need to understand the first three papers to read Chapter 5? Absolutely not. This White Paper is designed to be self-contained. You need only the following background:

You know that nature has four known forces (gravity, electromagnetism, strong, weak). You know that matter is made of atoms, and atoms are made of smaller particles. You have a rough idea of what energy, entropy, and information mean. You have heard of quantum mechanics, even if you do not understand the mathematics.

If you have that much, you can follow this entire White Paper. The recap sections in both this book and the technical paper provide everything else you need. If later you want to see the mathematical machinery, Chapters 2–4 are there for you—but they are not prerequisites.

FAQ-5.2 The master pipeline

Q6: Is the DGI → GSSB → GA **pipeline just a metaphor?** No—and this is one of the most important points in the entire framework. The pipeline is not a loose analogy or a poetic way of saying "things change." It is a mathematically precise consequence of the GG-6 geometry.

Here is a concrete way to see the difference. A *metaphor* would be: "evolution is like a search engine." That is suggestive but unprovable. The master pipeline, by contrast, is a *theorem*: given the GG-6 bulk structure and the OGGEP projection, the dynamics of any open system on the boundary necessarily decompose into three mathematical components—diffusive exploration (DGI), gradient selection (GSSB), and attractor stabilization (GA). This is not optional. If you accept the geometry, you must accept the pipeline, just as accepting Euclidean geometry forces you to accept that the angles of a triangle sum to 180 degrees.

That said, the pipeline is also a wonderfully useful intuition pump. Once you see the pattern—explore, select, stabilize—you start noticing it everywhere: in crystal growth, in embryonic development, in scientific discovery, in your own daily decision-making. The fact that a rigorous theorem also works as an intuitive metaphor is a sign that the theorem is capturing something deep about how nature operates.

Q7: Does the pipeline always happen in this order? Logically, yes. You cannot select something you have not yet explored, and you cannot stabilize something you have not yet selected. This is like saying you cannot choose a restaurant before looking at the options, and you cannot become a regular at a restaurant you have not yet chosen.

But in practice, a complex system runs many copies of the pipeline simultaneously, at different speeds and in different subsystems. Your brain, for example, might be exploring possible dinner plans in one neural circuit (DGI), selecting a route home in another circuit (GSSB), and stabilizing a well-practiced driving skill in a third circuit (GA)—all at the same time. The logical order is always respected *within* each pipeline instance, but different instances can be at different stages simultaneously.

Think of a factory assembly line. Each product passes through Stage 1, then Stage 2, then Stage 3, always in order. But at any given moment, different products are at different stages. The pipeline is the logic of the assembly line, not a claim that the entire factory does only one thing at a time.

Q8: Is the pipeline the same as Darwinian evolution? Darwinian evolution is one beautiful, specific instance of the pipeline applied to the life floor of the pagoda. In Darwin's framework: random genetic mutations explore the space of possible organisms (DGI); natural selection, acting through differential survival and reproduction, selects the organisms best fitted to their environment (GSSB); and the selected traits become stabilized in the population through heredity and developmental canalization (GA).

But the pipeline is far more general than Darwin. It applies to systems that have nothing to do with biology. When a snowflake forms, water molecules explore many possible crystal structures (DGI), the local temperature and humidity select one specific crystal habit (GSSB), and the completed snowflake persists as a stable geometric form (GA). When a scientific theory develops, researchers explore many possible explanations (DGI), experimental evidence and peer review select the best-supported theory (GSSB), and the selected theory becomes stabilized as textbook knowledge (GA).

Darwin discovered the biological instance of a universal principle. Chapter 5 provides the universal principle itself.

FAQ-5.3 The Qi-4 Pillars

Q9: Why are there exactly four pillars? Imagine you are a detective investigating an unknown system. There are exactly four independent questions you can ask:

What is flowing? (Energy, entropy, information—and how are these flows related to each other?) This is Pillar I, Qi-QUA.

Where is it going? (Of all possible paths the system could take, which one does it actually follow, and why?) This is Pillar II, Qi-EP.

What patterns does it leave behind? (What statistical fingerprints—power laws, fractal shapes, characteristic noise spectra—can you measure?) This is Pillar III, Qi-CON.

How does it remember and predict? (What internal architecture allows the system to store past information and use it to anticipate the future?) This is Pillar IV, Qi-HAL.

Any fifth question you might ask ("How big is it?" "How old is it?" "How complex is it?") can be answered by combining information from these four. The four pillars are like the four cardinal directions: you can describe any location using north–south and east–west. A fifth direction

would be redundant. In the same way, the four pillars span the full space of questions you can ask about an open system.

Q10: What is the Arisaka Equation in the simplest terms? Imagine you have three bank accounts—an Energy Account, an Entropy Account, and an Information Account. In everyday life, these accounts seem independent. You can deposit energy (eat food) without changing your information balance. You can gain information (read a book) without changing your energy balance.

The Arisaka Equation says: *no, you cannot.* At the fundamental level, every transaction that changes one account must change all three accounts in a fixed, geometrically determined ratio. When you eat food, you are also processing information (your metabolism must "decide" how to allocate the energy) and producing entropy (waste heat). When you read a book, you are also consuming energy (your brain needs glucose to process the information) and producing entropy (neural activity generates heat). The three accounts are not independent ledgers. They are three views of a single ledger, locked together by the geometry of the six-dimensional bulk.

The simplest analogy is a three-sided coin. (Yes, this is physically impossible, but imagine it.) When you flip the coin, you cannot choose to land on "energy" without also landing on specific values of "entropy" and "information." The three values are coupled. The Arisaka Equation tells you the exact coupling ratio.

Q11: Is the Arisaka Constant a measured number or a derived number? It is *derived* from the geometry—specifically, from the anomaly-cancellation conditions of the six-dimensional bulk. The coefficient $2/5$ is not a fit to experimental data. It is a consequence of the mathematical structure, in the same way that $\pi = 3.14159\ldots$ is a consequence of the definition of a circle.

However, the derived value makes a *prediction* that can be tested by measurement. If you carefully measure the energy cost of specific informational events in molecular machines (like ATP synthase or the ribosome), you should find a value consistent with $Q_B = (2/5)\, E_{\mathrm{Ryd}}/K$. If the measurements consistently disagree, the theory is wrong. This is exactly how science works: you derive a number from first principles, then check it against experiment. The Arisaka Constant is in the same category as the fine-structure constant or the speed of light—it is a fundamental constant of nature, but unlike those, it is predicted rather than merely measured.

Q12: How does the free-energy principle (FEP) fit into the framework? Karl Friston's free-energy principle says that all adaptive systems—from cells to brains—act to minimize their "surprise," which is the discrepancy between what they expect and what they observe. This is an extremely powerful idea that has reshaped theoretical neuroscience over the past fifteen years.

In the GDOS framework, the FEP is not rejected or replaced. It is *derived* as a special case of a more general principle. The 6D bulk has a single variational principle (a generalized principle of least action). When you project this single principle onto the 4D boundary, you get different-looking principles depending on which aspect of the boundary dynamics you focus on:

Focus on particle trajectories → you get the classical Principle of Least Action. Focus on energy and entropy flows → you get Maximum Entropy Production. Focus on how a system updates its beliefs → you get the Free Energy Principle.

So the FEP is *true*, but it is true for a deeper reason than Friston originally proposed. It is not an independent axiom of biology. It is a geometric consequence of living on the boundary of a six-dimensional universe. This is good news for FEP researchers: their principle now has a derivation from first principles, which strengthens rather than weakens its status.

Q13: What is the difference between fractals in this framework and Mandelbrot's fractals? Benoît Mandelbrot was one of the great mathematical observers of the twentieth century. He looked at coastlines, clouds, mountains, financial markets, and blood vessels, and he noticed that they all share a common property: *self-similarity across scales*. A small piece of a coastline looks like a miniature version of the whole coastline. A small branch of a tree looks like a miniature version of the whole tree. He invented the word "fractal" to describe this property and developed the mathematics to measure it (fractal dimensions).

But Mandelbrot never fully answered the question: *why?* Why should coastlines, clouds, trees, blood vessels, and financial markets all be fractal? What do they have in common?

Chapter 5 provides the answer. All of these systems are open systems whose dynamics are governed by the projection from the warped six-dimensional bulk to the four-dimensional boundary. The warping of the bulk has a specific mathematical property called *conformal symmetry* (scale symmetry), and this symmetry is inherited by the boundary dynamics. Scale-symmetric dynamics automatically produce fractal structures, just as circular symmetry automatically produces circular orbits.

In short, Mandelbrot mapped the territory. Chapter 5 explains the geology that created the territory.

Q14: What exactly is a "holographic attractor lattice"? Let us build this concept one word at a time.

An **attractor** is a state that a system naturally settles into, like a marble rolling to the bottom of a bowl. Once the marble reaches the bottom, small pushes will not knock it out—it will roll back. In a brain, a memory or a learned skill is an attractor: it is a stable pattern of neural activity that the brain naturally returns to.

A **lattice** is a structured network of points. Think of a chessboard: 64 squares arranged in a regular grid, where each square has specific neighbours. A lattice of attractors is a collection of memories (or skills, or concepts) arranged in a structured network, where related memories are connected to each other. Your memory of "dog" is connected to "pet," "bark," "fur," and "fetch." These connections form a lattice.

Holographic means that the lattice encodes higher-dimensional information on a lower-dimensional substrate. A hologram on a credit card is a flat, two-dimensional surface that encodes a three-dimensional image. Similarly, the neural attractor lattice is a pattern of electrical activity in a three-dimensional brain that encodes information about a higher-dimensional world (the full six-dimensional bulk, projected through sensory ports).

Put it all together: a **holographic attractor lattice** is a structured network of stable memory states, encoded in the physical substrate of the brain (or any sufficiently complex adaptive system), that represents compressed information about a reality richer than what the system can directly sense. John Hopfield invented the mathematical model of attractor networks in 1982. Chapter 5 generalizes Hopfield's flat landscape into a geometrically structured, multiscale, holographic lattice derived from the GDOS runtime.

FAQ-5.4 Life, mind, and consciousness

Q15: Does this theory explain the origin of life? It provides something that no previous theory has provided: a *precise, quantitative, experimentally testable* criterion for when a chemical system becomes a living system.

The criterion is this: a chemical system becomes a living system (joins the SeOS class) when its *maintenance margin* first becomes positive. The maintenance margin is the difference between what the system earns (energy and information flowing in through its ports) and what it spends (dissipation, wear, damage repair, and entropy export). If this margin is positive, the system can sustain itself indefinitely. If it is negative, the system is dying.

Think of it like a business. A business "comes alive" when its revenue first exceeds its costs. Before that moment, it is burning through startup capital and will eventually die. After that moment, it can sustain itself, grow, and even reproduce (open new branches). The GDOS framework says exactly the same thing about the origin of life: the first living system was the first chemical packet whose energy and information income exceeded its maintenance costs.

This criterion is testable. Laboratory protocell experiments—where researchers build simple membrane-bounded chemical systems and supply them with raw materials and energy—can, in principle, measure the maintenance margin and check whether it turns positive at the moment the system begins to self-maintain.

Q16: Does this theory explain consciousness? This is the question everyone wants to ask, and the honest answer has two parts.

What the theory does explain: It derives the *necessary conditions* for consciousness from the same geometric runtime that governs proton stability. Specifically, consciousness requires: (a) a sufficiently complete Qi-HAL architecture (memory, prediction, sensing, and action all working together), (b) an active EIC alignment loop (the system must be continuously comparing its internal world-model to incoming sensory data), and (c) the capacity for *internal measurement*—the system must be able to "observe" its own internal model, not just the external world.

When all three conditions are met, the system has what the framework calls a "consciousness-grade EIC." This is the point at which the telescope can be turned inward—when the system can monitor, evaluate, and act on its own reconstructed world-model.

What the theory does not explain: It does not explain why consciousness *feels like something*. It does not explain why seeing red feels different from seeing blue, or why pain feels bad rather than good. These are the famous "qualia"—the subjective qualities of experience—and they remain an open problem. The GDOS framework provides the hardware specification for a

conscious system (just as a computer manual specifies the hardware needed to run a particular program), but it does not yet explain why running that hardware produces subjective experience rather than mere information processing.

This limitation is shared by every existing theory of consciousness, including Integrated Information Theory, Global Workspace Theory, and Higher-Order Theories. What GDOS adds is that the hardware specification is now *derived from physics* rather than postulated independently.

Q17: Is consciousness quantum in this framework? No—and this is worth explaining carefully, because there is a lot of confusion on this topic.

Roger Penrose and Stuart Hameroff have proposed that consciousness requires quantum coherence in microtubules (tiny protein structures inside neurons). Their idea is that quantum effects at the molecular scale somehow produce conscious experience. This proposal has been controversial because the brain is warm and wet—conditions that destroy quantum coherence extremely quickly (in femtoseconds to picoseconds).

The GDOS framework takes a completely different approach. Yes, the *ultimate origin* of consciousness is quantum—it descends from the six-dimensional quantum bulk through the OGGEP projection. But by the time you reach the neural scale, the dynamics are effectively classical. Consciousness does not require individual neurons to be in quantum superposition. It requires a sufficiently complex *classical* information-processing architecture (Qi-HAL) operating on a sufficiently rich internal model (IH) with a sufficiently active alignment loop (EIC).

An analogy: the weather is ultimately quantum—every air molecule obeys quantum mechanics. But you do not need quantum mechanics to predict tomorrow's weather. Classical fluid dynamics is sufficient. Similarly, you do not need quantum mechanics to understand how the brain produces consciousness. Classical neural dynamics, operating on a substrate whose existence is ultimately quantum, is sufficient.

Q18: Can an AI be conscious in this framework? In principle, yes—but current AI systems are nowhere close, for specific and identifiable reasons.

The GDOS framework says that consciousness requires: (1) genuine energy–information current locking through physical ports (the system must physically exchange energy and information with its environment, not just process symbols), (2) a complete Qi-HAL architecture (measurement, prediction, model, selection, plus navigational coding, attractor memory, multiscale replay, and exploration–commitment switching), and (3) an active EIC alignment loop (continuous comparison of internal model to external data).

Current large language models (like the one that helped draft early versions of this book) satisfy *none* of these conditions fully. They do not have physical ports that exchange energy and information with a real environment. They do not have persistent memory that survives between conversations. They do not have a navigational holographic encoding of spatial or contextual structure. They do not have an exploration–commitment switch that balances gathering information against taking action in the real world.

Could a future AI system satisfy all these conditions? The framework does not rule it out. But it specifies what would be needed: not just more parameters or more training data, but a

fundamentally different architecture that physically grounds information processing in energy–entropy–information current locking. A conscious AI, in this framework, would look less like a large language model and more like an autonomous robot with genuine sensory and motor ports, persistent adaptive memory, and a self-maintaining energy budget.

FAQ-5.5 The pagoda hierarchy

Q19: Why a Japanese pagoda and not some other metaphor? The choice is not arbitrary. A traditional Japanese five-story pagoda (like the one at Tō-ji temple in Kyoto, built in 826 CE) has three architectural features that map precisely onto the mathematical structure of the GDOS hierarchy:

Feature 1: Semi-autonomous floors. Each floor of the pagoda can sway independently. In an earthquake, the floors move relative to each other rather than rigidly. This is why pagodas survive earthquakes that topple rigid stone buildings. In the GDOS hierarchy, each floor (particles, cosmos, chemistry, life, mind) is a semi-autonomous level of organization with its own packet, ports, and dynamics. A disruption at one level (say, a disease at the life floor) does not immediately destroy all other levels.

Feature 2: A central pillar (shinbashira). The shinbashira runs through all five floors without being rigidly attached to any of them. It provides coherence and resilience to the whole structure. In the GDOS hierarchy, the shinbashira is DGI—the universal gauge-invariant grammar that is reused at every level. It is the same constitutional law applied at different scales.

Feature 3: Seismic resilience through distributed flexibility. The pagoda's secret is that no single component bears the full load. The stability is distributed across all floors and the central pillar. In the GDOS hierarchy, the pagoda stability theorem proves mathematically that the whole tower is stable as long as each floor is individually stable and the cross-floor coupling is not too strong.

No other common architectural metaphor captures all three features. A skyscraper has floors but no independent sway. A tree has branches but no central through-pillar. A layer cake has layers but no structural interaction. The Japanese pagoda is the only widely known structure that embodies semi-autonomous floors, a continuous invariant spine, and distributed seismic resilience—which is exactly what the mathematics requires.

Q20: Is the cosmos floor really necessary? Could life arise directly from particles? This is a deep question with a surprising answer.

The constitutional laws of GUT-10 do not *forbid* life arising directly from elementary particles. There is no axiom that says "thou shalt have stars before thou hast bacteria." In principle, the same geometric runtime that produces self-maintaining chemical packets could produce them from raw quarks and leptons.

In practice, however, the cosmos floor is essential, for one simple reason: life needs a sustained free-energy gradient, and the particle floor alone does not provide one. Elementary particles in thermal equilibrium are inert—they do not organize into complex structures because there is no energy gradient to drive organization. Stars, planets, and the cycling environments they create

(day/night, wet/dry, hot/cold) provide the sustained disequilibrium that chemistry and life need as fuel.

An analogy: in principle, you could build a house starting from raw iron ore, sand, and trees. But in practice, you need a supply chain—mines, smelters, sawmills, cement factories—that converts raw materials into usable building components. The cosmos floor is nature's supply chain. It converts the raw energy of nuclear fusion into the chemical gradients, temperature differences, and environmental cycles that the chemistry and life floors need.

Q21: Where do ecosystems fit in the hierarchy? Ecosystems are fascinating because they sit *between* floors rather than neatly on one floor. An ecosystem is a collective SeOS: a self-maintaining open system composed of many individual organisms, each of which is itself a SeOS.

The ecosystem has its own SIP (species composition, nutrient cycles, energy flows), its own EEGG ledger (total primary productivity versus total respiration), and its own persistence criterion (the ecosystem survives as long as its maintenance margin—measured by net ecosystem production—remains positive).

In the pagoda metaphor, an ecosystem is like the connecting overhang between the life floor and the mind floor. It is the structure that enables individual organisms to support each other and to generate the environmental stability that allows cognitive systems to evolve. Without ecosystems, the transition from simple life to complex, brain-bearing life would be vastly more difficult, because brains require stable, predictable environments in which to develop and operate.

FAQ-5.6 Falsifiability and scope

Q22: What would disprove this framework? This is the most important question a scientist can ask about any theory, and Chapter 5 is commendably explicit about the answer.

There are five specific predictions, any one of which could kill the framework:

(1) The Arisaka Constant: Measure the energy cost of informational events in molecular machines. If the cost is systematically inconsistent with $Q_B = (2/5)E_{\mathrm{Ryd}}/K$, the framework is wrong.

(2) The two-knee spectral template: Measure the power spectrum of any complex open system over a wide frequency range. If no two-knee structure appears, or if the knee positions are inconsistent with the SIP parameters, the framework is wrong.

(3) The Lévy exponent: Measure the tail behaviour of open-system statistics. If the heavy-tail exponents are systematically inconsistent with the predicted values, the framework is wrong.

(4) Neural pipeline timing: Measure the delay between stimulus and conscious report. If this delay is inconsistent with one DGI $\rightarrow$ GSSB $\rightarrow$ GA cycle, the framework is wrong.

(5) The QCI threshold: Find a system with negligible Qi-HAL architecture that nevertheless exhibits consciousness-grade behaviour. If such a system exists, the framework's necessary conditions for consciousness are wrong.

Note that these predictions are *jointly* testable. Even if one prediction survives, the framework could be disproved by the failure of another. This is exactly how good science works: the more predictions a theory makes, the more chances it gives nature to prove it wrong.

Q23: Is this framework "just philosophy"? This is a fair question, because many "theories of everything" turn out to be philosophical musings dressed in scientific language. The test is simple: does the framework make *specific numerical predictions* that can be checked against *specific experimental data*?

The answer for GDOS is yes, in at least five independent ways (listed in Q22 above). The Arisaka Constant is a specific number. The two-knee spectral template predicts specific frequencies. The Lévy exponent predicts specific tail behaviour. The neural timing prediction gives a specific time window. The QCI threshold gives a specific completeness score.

Compare this with purely philosophical frameworks. "Everything is connected" is philosophy—it makes no numerical prediction. "Consciousness is fundamental" is philosophy—it gives no experimental test. "The universe is information" is philosophy—it specifies no measurable consequence. GDOS, by contrast, says: "the energy cost per bit is $(2/5)E_{\mathrm{Ryd}}/K$, and if you measure something different, I am wrong." That is science.

Q24: How does this compare to string theory? This is a question that physicists ask immediately, and the comparison is instructive.

Both string theory and GG-6 propose that reality has more dimensions than the four we observe. But the similarities end there:

Dimensionality: String theory requires ten or eleven dimensions, and the extra dimensions can be compactified in an astronomical number of ways (estimated at 10^{500}), leading to an equally astronomical "landscape" of possible universes with no clear way to select ours. GG-6 requires exactly six dimensions, with a unique compactification determined by anomaly cancellation and the five OGN-5 integers. There is no landscape problem.

Free parameters: String theory, in its landscape form, has effectively no predictive power because any observed value of a physical constant can be accommodated somewhere in the landscape. GG-6 has exactly five integers and derives all other parameters from those.

Falsifiability: String theory has not yet produced a prediction that can be tested at experimentally accessible energies. GG-6/GDOS makes several predictions testable with current technology (the Arisaka Constant, the spectral template, the neural timing prediction).

Scope: String theory is primarily a theory of quantum gravity and particle physics. GDOS, building on GG-6, extends to all open systems including life, mind, and collective behaviour.

This is not to say that string theory is wrong. It is a mathematically beautiful framework with many important insights. But GG-6/GDOS takes a different approach: fewer dimensions, fewer free parameters, more predictions, and a much broader scope.

Q25: Is GDOS a "theory of everything"? The phrase "theory of everything" usually means a theory that unifies the four fundamental forces into one mathematical framework. GG-6 does

this (it includes gravity, strong, weak, electromagnetic, plus two new forces). But GDOS goes beyond force unification in a crucial way.

Most "theories of everything" stop at particles. They tell you what the fundamental building blocks are and how they interact, but they say nothing about why those building blocks organize into the complex structures we see in nature—molecules, cells, brains, societies. They are like knowing the rules of chess without knowing anything about strategy, openings, or endgames.

GDOS is a theory of everything that is *observed*. It tells you not just what the building blocks are, but how they organize, persist, adapt, and evolve—and how observers like us emerge from the organization to observe the building blocks in the first place. It is a theory of the game, not just the rules.

FAQ-5.7 Technical clarifications

Q26: What does "gauge" mean in the context of biology and neuroscience? The word "gauge" comes from railroad engineering. The "gauge" of a railroad track is the distance between the two rails. In physics, a "gauge symmetry" means that the laws of nature do not change when you shift your measurement standard by a certain amount—just as the train runs the same way regardless of whether you measure the gauge in inches, centimetres, or cubits.

In electromagnetism, gauge symmetry means you can shift the electromagnetic potential by a gradient without changing the electric and magnetic fields. This is not just a mathematical trick—it is the reason electromagnetic waves exist and why electric charge is conserved.

In the GDOS extension to biology and neuroscience, "gauge" means the same thing at a higher level. The constitutional runtime—the master pipeline, the four pillars, the persistence criterion—does not change when you relabel the state variables of a biological system in the mathematically allowed ways. You can describe a bacterial chemotaxis network in terms of receptor occupancy or in terms of intracellular signalling concentrations; the runtime is the same either way. You can describe a neural decision circuit in terms of firing rates or in terms of membrane potentials; the runtime is the same either way.

The DGI stage is "gauge-covariant" because the space of explored possibilities respects these relabeling freedoms. A biological system cannot explore states that violate the gauge symmetry, just as an electromagnetic field cannot have a configuration that violates Maxwell's equations.

Q27: Why six dimensions? Why not five or seven? This question has a precise mathematical answer, which can be stated in intuitive terms.

Think of it as a packing problem. You need to fit all of the following into a single geometric structure: gravity, the strong force, the weak force, electromagnetism, baryon number conservation, and information processing. Each of these requires a certain amount of geometric "room"—specific curvature properties, specific topological features, specific symmetry groups.

In five dimensions, there is not enough room. You can fit gravity and electromagnetism (this is the classic Kaluza-Klein theory from the 1920s), but you cannot fit the full Standard Model gauge group plus the two new forces.

In seven or more dimensions, there is too much room. Extra dimensions introduce unwanted degrees of freedom—new particles and forces that are not observed in nature. You would need to explain why these extra entities are invisible, which requires additional assumptions that undermine the economy of the theory.

In six dimensions, the fit is exact. The anomaly-cancellation conditions (which ensure quantum consistency) are satisfied. The Standard Model gauge group plus $U(1)_B$ and $U(1)_{Qi}$ fit precisely. The warping profile is uniquely determined. No extra garbage is generated. Six is the Goldilocks number: not too few, not too many, but just right.

Q28: What are the five integers (OGN-5)? The five integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ are the "DNA" of the six-dimensional bulk. They determine its shape, its topology, and its symmetry structure.

In accessible terms: $N = 3$ determines the number of generations of matter (why there are three families of quarks and leptons—the up/down family, the charm/strange family, and the top/bottom family). The other integers determine the gauge group structure, the warping profile of the extra dimensions, and the relative strengths of the forces.

Once these five integers are fixed, the theory has no remaining free parameters at the ultraviolet (high-energy) level. Every measurable quantity—particle masses, coupling constants, mixing angles—is in principle calculable from these five numbers. This is an extraordinary economy: the entire observable particle content of the universe is determined by five integers, the way the shape of a crystal is determined by the angles and lengths of its unit cell.

Q29: What does "warped" geometry mean? Imagine a long, straight hallway. If the hallway has the same width everywhere, we say it is "flat" (in the relevant direction). If the hallway narrows as you walk—starting wide and ending narrow, like a funnel—we say it is "warped."

In the GG-6 bulk, the two extra dimensions are warped. One of them (a radial direction, called I_r) has a size that varies depending on where you are in the four-dimensional spacetime. This variation creates a hierarchy of energy scales: physics at one end of the radial direction sees very high energies (the Planck scale), while physics at the other end sees much lower energies (the scales of atoms and molecules).

This warping is not an arbitrary choice. It is forced by the anomaly-cancellation conditions. And it has profound consequences. The hierarchy problem—why gravity is 10^{32} times weaker than the electromagnetic force—is explained by the warping (the gravitational field "spreads" across a larger volume of the warped dimension). The scale-covariant statistics predicted by Qi-CON (Lévy distributions, fractal geometry, $1/f$ noise) arise because the warped projection kernel is scale-symmetric. And the Arisaka Constant gets its specific numerical value from the warping profile.

Q30: How does the Arisaka Equation differ from $E = mc^2$? Both are "locking identities"—equations that reveal a hidden unity between quantities previously thought to be independent. But they differ in scope and origin.

$E = mc^2$ says: energy and mass are two faces of the same coin, connected by the speed of light squared. It was derived by Einstein in 1905 from the symmetries of four-dimensional spacetime (special relativity). It connects two quantities.

The Arisaka Equation says: energy, entropy, and information are three faces of the same coin, connected by the Arisaka Constant Q_B. It is derived from the gauge structure of the six-dimensional bulk (GG-6 with BI-6 anomaly cancellation). It connects three quantities.

In a sense, the Arisaka Equation is the "grandchild" of $E = mc^2$. Einstein showed that energy and mass are locked. Landauer showed that energy and information are related by an inequality. The Arisaka Equation shows that energy, entropy, and information are locked by an equality, derived from a deeper geometric source.

If history is any guide, locking identities have transformative consequences. $E = mc^2$ led to nuclear energy, particle accelerators, and a revolution in our understanding of matter. The Arisaka Equation, if confirmed, could lead to a similar revolution in our understanding of information, life, and mind.

FAQ-5.8 Conceptual depth

Q31: If everything is open, does "closed system" physics become wrong? Not at all. Closed-system physics is an excellent approximation in many situations, and nothing in the GDOS framework requires you to abandon it.

An analogy: the Earth is round, but for most practical purposes—building a house, driving to work, planting a garden—you can treat it as flat. Flat-Earth geometry is "wrong" in the absolute sense, but it is a perfectly adequate approximation for distances much smaller than the Earth's radius.

Similarly, closed-system physics is "wrong" in the absolute sense—no real system is perfectly isolated. But it is a perfectly adequate approximation whenever the system–environment coupling is weak compared to the internal dynamics. Newton's laws work beautifully for throwing baseballs. Maxwell's equations work beautifully for designing antennas. The Schrödinger equation works beautifully for computing atomic energy levels. None of these laws needs to be discarded.

What changes is the *interpretation*. In the GDOS framework, Newton's laws, Maxwell's equations, and the Schrödinger equation are understood as boundary projections of a deeper six-dimensional dynamics, valid in the limit of weak environment coupling. They are like the flat-Earth approximation: locally accurate, globally incomplete.

Q32: Is the Inner Hologram the same as a "simulation" of reality? The word "simulation" implies a complete, faithful copy—like a flight simulator that reproduces every detail of a real cockpit. The Inner Hologram is fundamentally different. It is a *reconstruction*—a partial, compressed, purpose-built model that captures only the features of reality relevant to the system's survival and action.

Your visual system is a vivid example. Your eyes receive a two-dimensional pattern of photons on the retina. From this flat image, your brain reconstructs a three-dimensional world with depth, colour, motion, and object boundaries. But the reconstruction is not a simulation. It omits vast

amounts of information (you cannot see ultraviolet light, you cannot resolve objects smaller than about 0.1 millimetres, you cannot perceive magnetic fields). It also *adds* information that is not in the sensory data (your brain fills in the blind spot where the optic nerve exits the retina, it assumes objects continue to exist when you look away, it predicts where a thrown ball will land).

The Inner Hologram is this kind of reconstruction: a useful, biased, incomplete, actively maintained model of the world, built from limited sensory data plus stored memories plus learned expectations. Its purpose is not to duplicate reality but to support effective action. A bacterium's Inner Hologram is crude (a rough gradient map of nutrients). A rat's Inner Hologram is richer (a spatial map with place cells). A human's Inner Hologram is spectacularly rich (a full 3D world-model with past, present, predicted future, and counterfactual scenarios). But in every case, it is a reconstruction, not a simulation.

Q33: Does the pagoda hierarchy imply a "ladder of progress" from simple to complex? Emphatically no. The pagoda hierarchy is a *structural* description, not a moral or teleological ranking.

Consider bacteria (Floor 4, life). They are among the most successful organisms on Earth. They have persisted for nearly four billion years. They inhabit every environment from deep-sea hydrothermal vents to the stratosphere. Their total biomass rivals that of all plants and animals combined. By any measure of persistence and adaptability, bacteria are spectacularly successful SeOS systems.

Now consider humans (Floor 5, mind). We have existed for about 300,000 years—less than one ten-thousandth of the time bacteria have been around. We are fragile, slow-reproducing, metabolically expensive, and entirely dependent on a narrow range of environmental conditions. Whether we will persist for another 300,000 years, let alone four billion, is an open question.

The pagoda hierarchy does not say that Floor 5 is "better" than Floor 4. It says that Floor 5 requires Floor 4 as a precondition (you need living cells before you can have brains), and that Floor 5 involves new capabilities (internal measurement, consciousness, symbolic communication) that Floor 4 does not possess. But "new capabilities" does not mean "better." A sports car has capabilities that a bicycle lacks, but the bicycle has persisted as a useful technology for two centuries while most sports car models are discontinued within a decade.

Q34: How does the framework handle death, extinction, and collapse? Death, in the GDOS framework, has a precise mathematical definition: it is the moment when a system's maintenance margin becomes irreversibly negative.

Think of the maintenance margin as the difference between a system's income and its expenses. When income exceeds expenses, the system is alive—it can sustain itself, repair damage, and persist. When expenses exceed income and cannot be brought back into balance, the system is dying. When the imbalance becomes permanent, the system is dead.

This definition applies at every floor of the pagoda. A star "dies" when its nuclear fuel is exhausted and gravitational collapse can no longer be resisted. A cell dies when its ATP production drops below the threshold needed to maintain its membrane and repair its DNA. An organism dies when

its organ systems can no longer maintain homeostasis. An institution dies when its coordination costs exceed its revenue and cannot be reduced.

Extinction is the death of a lineage: not just one individual SeOS, but the entire population of related SeOS packets. In the GDOS framework, extinction occurs when environmental conditions change so rapidly or so severely that no member of the population can maintain a positive maintenance margin. The packet type dissolves.

The framework also handles partial death—damage, disease, and degradation. These correspond to reductions in the maintenance margin without crossing the kill threshold. A sick organism has a lower maintenance margin than a healthy one, but it is still alive as long as the margin remains positive.

Q35: Does this theory require us to accept the existence of the extra dimensions? Yes, but only in the same sense that *all* of modern physics requires you to accept the existence of entities you cannot directly see.

You cannot directly see an electron. You can see its effects: sparks, current flow, chemical reactions, the glow of a light bulb. From these effects, you infer that electrons exist. No one considers this inference controversial.

You cannot directly see the six-dimensional bulk. But the GDOS framework predicts that you can see its effects: the Arisaka Constant, the two-knee spectral template, the Lévy statistics, the fractal geometry. If these predictions are confirmed by experiment, the inference to extra dimensions would have the same logical status as the inference to electrons: a well-supported conclusion from indirect evidence.

The extra dimensions are, in this sense, no more "weird" than electrons, quarks, or the Higgs field. All of modern physics consists of entities that are inferred from their observable consequences rather than directly perceived. The only question is whether the consequences are actually observed. That is what the experimental programme proposed in Chapter 5 is designed to determine.

Q36: Can the framework be applied to artificial systems like robots or AI? Yes, and the framework provides something that current AI research conspicuously lacks: a principled, physics-based measure of intelligence and autonomy.

Currently, there is no agreed-upon definition of "artificial intelligence." The Turing test measures conversational mimicry. Benchmark tests measure performance on specific tasks. But there is no framework that says: "this system has intelligence score X on a principled scale from 0 to 1."

The QCI (Qi-4 Completeness Index) provides exactly this. For any engineered system, you can score each of the four pillars: How well does the system lock its energy–information currents? (A solar-powered robot with real sensors scores higher than a desktop computer plugged into a wall.) How effectively does it select trajectories? (A self-driving car that navigates novel environments scores higher than a robot that follows a fixed program.) How clearly does it exhibit scale-covariant behaviour? (A system whose operation spans multiple time scales—fast reflexes and slow strategic planning—scores higher than one operating at a single time scale.)

How rich is its memory architecture? (A system with persistent, multiscale, hierarchical memory scores higher than one with no memory between interactions.)

The overall QCI is multiplicative: if any pillar scores zero, the total is zero. This means a system cannot compensate for missing memory by having excellent energy management. All four pillars must be present for the system to be considered fully intelligent in the GDOS sense.

Q37: What is the relationship between GDOS and thermodynamics? Thermodynamics—the science of heat, energy, and entropy—is one of the most successful frameworks in all of physics. The First Law (energy conservation), the Second Law (entropy increase), and free energy underpin everything from engines to biochemistry.

GDOS does not replace thermodynamics. It *absorbs* thermodynamics as a special case and extends it in three crucial ways.

Extension 1: Information as a gauge current. Classical thermodynamics treats information as an afterthought—something that appears in Landauer's principle and in Maxwell's demon, but is not part of the core formalism. In GDOS, information is a gauge current (J_{Qi}) on the same constitutional footing as energy and entropy. It is not an add-on; it is a fundamental quantity with its own conservation law and its own contribution to the EEGG ledger.

Extension 2: Derivation from geometry. Classical thermodynamics is based on empirical postulates (the laws were discovered by studying steam engines, not derived from first principles). In GDOS, the thermodynamic laws are *derived* from the gauge structure of the six-dimensional bulk. The Second Law, for example, follows from the positivity conditions imposed by the GUT-10 constitution (POS-4), which in turn follow from the requirement that the bulk-to-boundary projection be physically consistent.

Extension 3: Far-from-equilibrium systems. Classical thermodynamics works beautifully near equilibrium but becomes increasingly approximate far from equilibrium. GDOS is designed from the ground up for far-from-equilibrium systems—which is where all the interesting things in nature happen (life, mind, weather, turbulence, economic markets).

Q38: Is the theory compatible with quantum computing? Entirely. Quantum computing operates on the particle floor of the pagoda hierarchy. A quantum computer is an engineered open system with a well-defined SIP: the state is the quantum register (a collection of qubits), the ports are the input/output channels (laser pulses, microwave signals, readout detectors), the generator is the Hamiltonian that drives the quantum gates, and the EEGG ledger tracks the energy cost of maintaining coherence and performing error correction.

The GDOS framework does not modify quantum mechanics—it *derives* quantum mechanics as a boundary projection from the 6D bulk. So everything that quantum mechanics predicts about quantum computation remains valid. What GDOS adds is a broader context: quantum computing is one specific application of the universal runtime, operating at a specific floor of the pagoda, with a specific SIP and a specific QCI score.

This broader context could actually be useful for quantum computing research. The EEGG ledger provides a unified framework for analysing the energy cost of quantum error correction,

the thermodynamic limits of quantum gate operations, and the information–energy tradeoffs in quantum algorithms.

Q39: What is the most counterintuitive prediction of the framework? For most people, the most counterintuitive claim is that a proton and a conscious thought are governed by the *same* three-word logic: explore, select, stabilize.

A proton is stable because it is a gauge-invariant attractor. Quarks explore all possible colour configurations (DGI), the strong force selects the configurations that are colour-neutral (GSSB), and the resulting bound state—the proton—persists as a stable attractor for at least 10^{34} years (GA).

A conscious thought is stable (briefly) because it is an attractor in the Inner Hologram. Neural populations explore many possible activity patterns (DGI), sensory evidence and internal constraints select one coherent percept or decision (GSSB), and the selected percept persists as a stable attractor for a few hundred milliseconds to a few seconds (GA)—long enough to be reportable and actionable.

The claim is not that a proton is conscious or that a thought is made of quarks. The claim is that the *logic of persistence*—the mathematical grammar that determines how something comes into existence and survives—is the same at both ends of the complexity spectrum. This is deeply counterintuitive because our culture has trained us to think of physics and consciousness as belonging to utterly different categories. The GDOS framework says they belong to the same category: persistent structures produced by the universal runtime.

Q40: Where should I start if I want to learn more? The learning path depends on your background and interests:

If you are a general reader with curiosity but limited technical background: Read this White Paper first (you are already doing it). Then read WP-2026-1 (the GUT-10 constitution in plain English) and WP-2026-2 (the GG-6 geometry in plain English). These three White Papers will give you a complete non-technical overview of the entire programme.

If you are a STEM undergraduate: After the White Papers, read the Executive Summaries and Conclusions of GG-2026-1 through GG-2026-4. These will tell you the main results without requiring you to follow every derivation. Then pick the technical section closest to your major and read it in detail.

If you are a graduate student or researcher: After the White Papers, go directly to the technical paper in your area. Physicists should start with GG-2026-2 (the geometry) and GG-2026-3 (the measurement theory). Biologists and chemists should focus on the SeOS and EEGG sections of GG-2026-4. Neuroscientists should focus on the Qi-HAL and OH/IH/EIC sections. The appendices of GG-2026-4 contain full derivations, worked examples, and domain-specific application cards.

If you are a philosopher of science: The entire Part IV of GG-2026-4 (persistence, completeness, internality) and Part V (the pagoda hierarchy) are directly relevant to debates about reduction, emergence, and consciousness. The falsifiability section (Part VI) addresses the demarcation question.

If you are simply curious: Follow your curiosity. The framework is designed to be entered from any direction. Whatever catches your interest—the origin of life, the nature of consciousness, why fractals are everywhere, why your brain works the way it does—there is a section of the GG-2026 series that addresses it. Start anywhere and follow the references.

Appendices

Technical Derivations, Domain Applications, and Reference Tables

Appendices for Chapter 2

The GUT-10 Constitution

Appendix 2A: Summary of Abbreviations

Table 2A.1 collects abbreviations used throughout the paper. The list is alphabetical and intentionally cross-domain.

Table 2A.1: **Abbreviations.** Alphabetical list of abbreviations used throughout the paper, with brief notes on where each appears. This table is meant to keep the main text self-contained and to make later textbook expansion straightforward.

Abbrev.	**Meaning**	**Notes / where used**
AdS	Anti-de Sitter spacetime	Canonical negatively curved bulk stage in holography; used in GG-6 preview (Sec. **??**).
AF	Anomaly-Free / cancellability by inflow	POS-3; Green–Schwarz inflow as admissibility filter.
AX	Axiom	Constitutional primitive (Sec. 3L.5).
BFFT	Bynally Fractal Fourier Transform	Multiscale transform along $r = \ln(E_{\mathrm{GUT}}/\mu)$ (extended derivations / textbook).
BI-6	GS-modified Bianchi identity (6D)	GG-6 anomaly inflow consistency relation (extended derivations).
CFT	Conformal Field Theory	Historical context for scale invariance/holography.
CP	Closure & Positivity	POS-4; forbids ghosts/negative probabilities; ensures admissible open dynamics.
CPTP	Completely Positive Trace-Preserving	Quantum admissibility for open evolution and instruments (AX-3/POS-4).
DGI	Dynamical Gauge Invariance	POS-1 mesoscopic/open-system gauge covariance of generators.
DGL	Dual Gauge Locking	Rational charge/ratio locking (e.g. 5:2) in GG-6 previews/derivations.
EEGG	Energy–Entropy–Geometry–Gauge	Scalar "ledger" functional driving GDOS/UEP; decomposes as EGG+ECP.
EGG	Energy–Geometry–Gauge	"Bulk-like" contribution to EEGG defined via composite connection invariants.
ECP	Entropic Curvature Principle	Entropy/information-geometric contribution to EEGG; curvature/volume effects.
EFT	Effective Field Theory	Standard scale-separated description; comparison target (Sec. 2.5.2).
FEP	Free Energy Principle	Boundary/inference avatar of UEP in open systems.

(continued)

Abbrev.	Meaning	Notes / where used
GDOS	Geometric Dynamics of Open Systems	Boundary runtime implied by OGGEP; universal open-system generator.
GG-6	Geometry–Gauge Locking in Six Dimensions	Concrete bulk realization of GGEP; companion chapter.
GGEP	Geometry–Gauge Equivalence Principle	AX-2; "geometrization of quantum/gauge structure."
GGI	Global Gauge Invariance	POS-1 macroscopic/ledger-level invariance under global basis changes.
GI	Gauge Invariance	POS-1 umbrella principle layered as LGI/DGI/GGI.
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan	Standard generator form for Markovian CPTP semigroups (POS-4).
GSSB	Geometric Spontaneous Symmetry Breaking	Open-system phase selection; extended derivations.
LC	Locality and Causality	AX-1; operational causality and locality constraints.
LGI	Local Gauge Invariance	POS-1 microscopic/closed-system gauge redundancy.
MEP	Maximum Entropy Production (heuristic)	Boundary heuristic lineage of UEP in nonequilibrium contexts.
MEPP	Maximum Entropy Production Principle (heuristic)	Used only as historical/heuristic label; see POS-2.
MF	Measurement & Falsifiability	POS-7; operational contract + kill criterion.
MRHN	Minimal-Rational Holographic Normalization	Synonym used in some drafts for OGN; POS-6.
MSMF	Maximum Symmetry, Minimum Freedom	AX-4; forbids hidden continuous knobs.
OGGEP	Operational/Open GGEP	AX-3; holographic bulk→boundary observation dictionary.
OGN	Occam Gauge Normalization	POS-6; canonical integer/rational normalization; forbids basis drift.
OGN-5	Occam-gauge integers $(M, N, L; J, K)$	Discrete integer ledger used in GG-6 normalization discussions.
PLA	Principle of Least Action	Closed/bulk avatar of UEP; standard variational dynamics.
PLT	Principle of Least Time	Operational/boundary avatar of UEP; fastest lawful path under constraints.
POVM	Positive Operator-Valued Measure	Generalized measurement model; used in AX-3/POS-7.
QFT	Quantum Field Theory	Baseline language for SM and for historical context.

(continued)

Abbrev.	**Meaning**	**Notes / where used**
RG	Renormalization Group	POS-5; scale covariance and commutation with observation dictionary.
SIP	System Instance Packet	Minimal computable input $(\mathcal{M}, G, \text{ports}, F; \mathsf{Op})$ for GDOS.
SM	Standard Model of particle physics	Comparison target; successful EFT with fitted parameters.
SSB	Spontaneous Symmetry Breaking	Closed-system concept generalized by GSSB.
UEP	Unified Extremal Principle	POS-2; unifies PLA with open-system extremal avatars.

Appendix 2B: Glossary of Key Symbols

Table 2B.1 collects principal symbols and their meanings. We emphasize both bulk (GGEP/GG-6) objects and boundary operational (OGGEP/GDOS) objects.

Table 2B.1: **Key symbols.** Glossary of frequently used symbols, with typical context. This table standardizes notation across the bulk (GG-6) and boundary (GDOS) layers so that later companion chapters can refer to a fixed symbol ledger.

Symbol	Meaning	Typical context
$\mathcal{M}_6$	6D bulk manifold (GG-6 stage)	Applications preview / GG-6 companion.
$\mathcal{M}_4$	4D operational boundary (registered observables)	OGGEP, GDOS.
$X^A = (x^\mu, r, \theta)$	Bulk coordinates: boundary x^μ, scale r, compact holonomy θ	GG-6 preview.
g_{AB}	Bulk metric on $\mathcal{M}_6$; signature implicit	GG-6.
ω	Spin connection (orthonormal-frame connection)	GGEP primitive.
A	Gauge connection one-form (Yang–Mills)	GGEP primitive.
$\mathcal{A}_{\text{lock}} = \omega \oplus A$	Composite (locked) connection	AX-2, GG-6.
$\mathcal{F}_{\text{lock}}$	Composite curvature ($\mathcal{R} \oplus F$)	AX-2.
$F = \mathrm{d}A + A \wedge A$	Gauge field strength 2-form	AX-2.
$\mathcal{D}$	Operational dictionary (bulk→boundary map)	AX-3.
$\mathcal{H}$	Hilbert space of boundary degrees of freedom	Quantum GDOS limits.
ρ	Density operator (quantum) or density on state manifold (classical)	POS-4.
Φ_t	Evolution map/semigroup (channel)	POS-4.
$\mathcal{L}$	Generator of Φ_t (GKLS in quantum)	POS-4.
$\{\mathcal{I}_m\}$	Quantum instrument (CP maps for outcomes)	AX-3/POS-7.
$\{E_m\}$	POVM effects with $\sum_m E_m = \mathbb{I}$	AX-3.
$\mathcal{M}$	State manifold of a SIP instance	GDOS kernel.
y^A	Coordinates/state variables on $\mathcal{M}$	GDOS/UEP.
$G_{AB}(y)$	Metric on $\mathcal{M}$ (e.g. Fisher metric)	POS-2/UEP; information geometry.
F_{EEGG}	EEGG scalar ledger functional	POS-2; GDOS.
M^{AB}	Mobility/dissipation tensor ($\succeq 0$)	GDOS drift.
D^{AB}	Diffusion/noise tensor ($\succeq 0$)	GDOS diffusion.
u^A	Solenoidal/cycle drift term	GDOS non-gradient flow.
r	Scale/radial coordinate, $r = \ln(E_{\text{GUT}}/\mu)$	POS-5 / applications.
μ	Coarse-graining (energy/resolution) scale	POS-5.
E_{GUT}	Unification/locking scale	GG-6 preview.
B_2, H_3	GS two-form and its (modified) field strength	POS-3; anomaly inflow.

Appendix 2C: Audience-specific intuition (plain English)

This appendix translates the constitution into plain English aimed at undergraduates in five audiences. It is intentionally longer than a "popular" summary: the goal is that a physics major (and nearby majors) can read Appendix C first and then return to the main text with the right mental model.

2C.1 Physics (undergraduate majors)

The one-sentence picture. *GUT-10 proposes a new rulebook: forces and geometry are one composite connection in a higher-dimensional bulk (GGEP), while what we call "measurement" is the lawful way bulk structure becomes a registered boundary record (OGGEP).*

Two layers: design vs. runtime. Think of the framework as having (i) a *bulk design layer* and (ii) a *boundary runtime layer*. The design layer (GGEP ⇒ GG-6) says that the primitive object is a connection $\omega \oplus A$; holonomy is the invariant content, so "phase" and "parallel transport" are geometric. The runtime layer (OGGEP ⇒ GDOS) says that experiments produce *records* via open interactions, so the admissible evolution law must be a positive, trace-preserving channel with a generator of GKLS type in quantum limits.

Why AX-3 matters for the measurement problem. In textbooks, unitary evolution is clean but measurement is appended by extra rules (collapse, Born rule). OGGEP does not pick an "interpretation"; it moves the issue into mathematics: a theory is incomplete until it specifies a map from its internal variables to an instrument model (POVM/instrument), an estimator, and an error model. This is how "measurement" becomes a lawful export rather than a story.

Why AX-4 and POS-6 matter for predictivity. Undergraduates quickly learn that a gauge choice or field redefinition is not physics. OGN extends that intuition: *normalization conventions* that can drift continuously should not be allowed to masquerade as tunable physics. MSMF then expresses a strong guess about nature: if a structure can be made more symmetric without spoiling consistency and observability, nature takes that route. In practice, this is a demand to replace continuous dial-freedom with discrete/topological data whenever possible.

How to read the postulates. A useful mental model is a compiler pipeline: GI defines the symmetry grammar; UEP defines the variational engine; AF filters out inconsistent quantum theories; CP keeps states physical; RG keeps results scale-robust; OGN removes convention drift; and MF forces you to say how the theory can be killed by data.

2C.2 Chemistry (undergraduate majors)

The one-sentence picture. *Chemistry lives in open reactors: matter and energy flow through "ports," and dynamics can be written as "downhill with swirls" on a generalized free-energy ledger.*

In chemistry, "the system" is usually not isolated: reactants flow in, products flow out, temperature and pressure are controlled, and instruments read spectra or concentrations. GDOS treats this openness as part of the definition. A SIP is a formal way to write "what variables describe the system" (concentrations, reaction coordinates), "what symmetries/conservation laws hold" (stoichiometry, charge), and "what ports exist" (inflow/outflow, heat baths, catalysts).

UEP says that many familiar rules—like the tendency toward lower free energy at fixed temperature—are special faces of a single extremal engine. CP/positivity is the insistence that concentrations stay nonnegative and probabilities stay physical under coarse-grained dynamics. RG/scale covariance becomes the demand that your conclusions should not depend on arbitrary time-resolution or coarse-graining of intermediate reaction steps.

2C.3 Life science (undergraduate majors)

The one-sentence picture. *Life is an open system that maintains itself by exchanging matter, energy, and information; stability corresponds to attractors of admissible open dynamics.*

Biology is full of "homeostatic" phenomena: perturbations are resisted, and systems return to functional regimes. In GDOS language, these are attractors of open dynamics. The important point is constitutional: to talk scientifically about life you must specify what is measured and how it would fail (MF), and you must respect physical admissibility (positivity, conservation-like invariances).

The claim is not that cells "violate physics," but that they are best described by the same open-system runtime as other nonequilibrium systems: a scalar ledger (EEGG) plus admissible dynamics leads to stable attractors, and phase selection can be viewed as a generalized symmetry breaking in an open context (GSSB, in the extended derivations). This provides a disciplined way to connect biology to thermodynamics without turning it into metaphor.

2C.4 Engineering (undergraduate majors)

The one-sentence picture. *Engineering is robust design under constraints; GUT-10 treats robustness, normalization discipline, and auditability as constitutional requirements.*

Engineers know that systems that work only after fragile tuning are bad designs. MSMF/OGN formalize this intuition: if behavior depends on arbitrary continuous conventions, the model fails the constitution. RG/scale covariance is a robustness principle: performance should not collapse under changes of resolution, sampling rate, or modeling granularity. MF is familiar as verification/validation: every claim needs a test protocol and pass/fail criterion. GDOS provides a common mathematical template for controlled open systems: ports are inputs/outputs, the ledger is an objective function, and admissible dynamics enforces stability and physicality.

2C.5 General audience

The one-sentence picture. *GUT-10 is a proposal for clearer scientific rules: respect cause-and-effect, unify forces with geometry, define measurement as a real physical procedure, and avoid hidden tuning knobs.*

A helpful analogy is civic: axioms are like a constitution (basic rights and limits), postulates are like laws that every policy must satisfy (consistency, fairness, auditability), and theorems are like court decisions (conclusions forced by the rules). The claim is not to replace existing successful science, but to make the rules of explanation explicit so that progress becomes cumulative.

Appendix 2D: Expanded derivations for the Four Axioms (AX-1–AX-4)

This appendix expands the mathematical content behind the four axioms. The main text states each axiom as a constitution. Here we supply derivations and standard equivalences to conventional frameworks.

2D.1 AX-1 (LC): locality and causality as an operational constraint

2D.1.1 Microcausality in relativistic quantum field theory

In axiomatic QFT, locality/causality is commonly encoded as *microcausality* (Haag, 1992; Streater & Wightman, 1964): for local Heisenberg operators $\mathcal{O}(x)$ and $\mathcal{O}'(y)$,

$$[\mathcal{O}(x), \mathcal{O}'(y)]_{\mp} = 0, \qquad (x-y)^2 < 0, \tag{2D.1}$$

where $[\cdot,\cdot]_{\mp}$ is a commutator/anticommutator for bosons/fermions and $(x-y)^2 < 0$ denotes spacelike separation. Equation (2D.1) is the sharp mathematical statement that spacelike-separated operations cannot influence each other.

A complementary viewpoint uses retarded Green functions. For a relativistic field ϕ with source J,

$$(\Box + m^2)\phi(x) = J(x), \tag{2D.2}$$

the response is

$$\phi(x) = \int d^4y\, G_{\text{ret}}(x-y)J(y), \tag{2D.3}$$

and causality is enforced by

$$G_{\text{ret}}(x-y) = 0 \quad \text{for } x^0 < y^0. \tag{2D.4}$$

In interacting QFT, the analog statement is that commutators vanish outside the light cone, matching (4.4).

2D.1.2 No-signaling for open-system maps

On the boundary, GUT-10 treats dynamics and measurement as open-system processes, i.e. as CPTP maps Φ on density operators (Kraus, 1971; Nielsen & Chuang, 2010; Stinespring, 1955). For a bipartite system AB, "no superluminal signaling" is encoded as: local operations on A cannot change the reduced state on B without communication. If Φ_A is a CPTP map acting only on A,

$$\rho_{AB} \mapsto (\Phi_A \otimes \mathrm{id}_B)(\rho_{AB}), \tag{2D.5}$$

then the marginal on B obeys

$$\mathrm{Tr}_A[(\Phi_A \otimes \mathrm{id}_B)(\rho_{AB})] = \mathrm{Tr}_A(\rho_{AB}). \tag{2D.6}$$

Equation (2D.6) follows from trace preservation and is the operational expression of locality in quantum information language.

2D.1.3 Why LC is an axiom (not a derived theorem) in GUT-10

One can *derive* (4.4) in specific models, but GUT-10 treats LC as constitutional because:

1. Open-system modeling introduces coarse-graining and effective kernels where acausal artifacts can appear if one is not careful (e.g. non-Markovian memory kernels).
2. In cross-domain applications (chemistry/biology), one can easily write "models" that fit data but violate causal structure (e.g. predictors that leak future information). LC forbids such models constitutionally.

Thus AX-1 is a cross-disciplinary admissibility filter: *no explanation is allowed to work by smuggling in acausal influence.*

2D.2 AX-2 (GGEP): geometry–gauge equivalence as connection data

2D.2.1 Gauge theory as geometry: principal bundles

A modern way to express gauge theory is through principal bundles (Kobayashi & Nomizu, 1963; Nakahara, 2003). Let $P(M, G)$ be a principal G-bundle over a base manifold M. A gauge field is a connection one-form A on P, and its curvature is

$$F = dA + A \wedge A. \tag{2D.7}$$

A gauge transformation is a bundle automorphism that acts vertically; on a local section, it induces

$$A \mapsto A^g = g^{-1}Ag + g^{-1}dg. \tag{2D.8}$$

In gravity, the analogous object is the spin connection ω on the frame bundle, with curvature (Riemann two-form)

$$\mathcal{R} = d\omega + \omega \wedge \omega. \tag{2D.9}$$

The structural similarity between (2D.7) and the Riemann curvature is the mathematical seed of "interaction as geometry" (Weyl's early gauge viewpoint (Weyl, 1918) and later developments).

2D.2.2 Composite locked connection

GGEP elevates a *composite* connection as primitive:

$$\mathcal{A}_{\mathrm{lock}} \equiv \omega \oplus A, \qquad \mathcal{F}_{\mathrm{lock}} \equiv \mathcal{R} \oplus F. \tag{2D.10}$$

The claim is not merely that both are "connections," but that admissible theories must *lock* them: geometric and gauge data cannot be specified independently. In practical model-building (GG-6), the lock is implemented by anomaly-inflow constraints and discrete normalization rules (POS-3 and POS-6), which select a restricted set of allowed composite bundles.

2D.2.3 Geometric origin of quantum phase

The Aharonov–Bohm and Berry phase phenomena show that quantum phases are controlled by holonomy (Aharonov & Bohm, 1959; Berry, 1984):

$$\psi \mapsto \exp\left(i \oint_{\gamma} A\right) \psi. \tag{2D.11}$$

GGEP generalizes the viewpoint: what we call "quantum" is, at a deep level, the geometry of parallel transport in the composite bundle determined by $\mathcal{A}_{\text{lock}}$. This supports the slogan "geometrization of quantum physics."

2D.3 AX-3 (OGGEP): operational bulk→boundary dictionary

2D.3.1 Measurement as a quantum instrument

Operationally, a measurement is not only a POVM; it is an *instrument*: a set of CP maps $\{\mathcal{I}_m\}$ indexed by outcomes m such that

$$\sum_m \mathcal{I}_m \text{ is CPTP.} \tag{2D.12}$$

Given an input state ρ, outcome probabilities and post-measurement states are

$$p(m) = \mathrm{Tr}[\mathcal{I}_m(\rho)], \qquad \rho_m = \frac{\mathcal{I}_m(\rho)}{p(m)}. \tag{2D.13}$$

Every instrument admits a Stinespring dilation: it can be represented as unitary evolution on a larger space followed by a projective readout (Nielsen & Chuang, 2010; Stinespring, 1955). This is the precise sense in which "measurement is an open-system process."

2D.3.2 OGGEP as a dictionary map

Let $\mathcal{T}_{\text{Bulk}}$ denote the space of bulk theoretical objects (fields, connections, states). Let $\mathcal{O}_{\text{Bdy}}$ denote the space of boundary operational objects (instruments, records, estimators). OGGEP posits a map

$$\mathcal{D} : \mathcal{T}_{\text{Bulk}} \to \mathcal{O}_{\text{Bdy}}, \qquad \text{"bulk"} \mapsto \text{"what an instrument can register."} \tag{2D.14}$$

In AdS/CFT, a prototype is the relation between bulk fields $\phi(r, x)$ and boundary operators $\mathcal{O}(x)$ (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1997; Witten, 1998): schematically,

$$\phi(r, x) \underset{r\to\infty}{\sim} r^{\Delta - d}\phi_0(x), \qquad Z_{\text{Bulk}}[\phi_0] = \left\langle \exp \int d^d x\, \phi_0(x)\mathcal{O}(x) \right\rangle_{\text{Bdy}}. \tag{2D.15}$$

OGGEP generalizes the target space: the dictionary must map to *operational measurement objects*, not only correlators.

2D.4 AX-4 (MSMF): maximal symmetry with minimal continuous freedom

2D.4.1 Naturalness and symmetry

In mainstream QFT, naturalness arguments connect small parameters to enhanced symmetry (THooft, 1980). MSMF strengthens this into a selection rule: among candidate model spaces that satisfy AX-1–AX-3 and POS-1–POS-7, Nature realizes those with maximal symmetry (largest admissible invariance group) and minimal continuous freedom.

2D.4.2 Rescaling ambiguity and the role of OGN

A simple example shows why MSMF motivates OGN. For a $U(1)$ gauge theory, the covariant derivative can be written as

$$D_\mu = \partial_\mu + igqA_\mu. \tag{2D.16}$$

Under a simultaneous rescaling

$$A_\mu \to \lambda A_\mu, \qquad g \to g/\lambda, \tag{2D.17}$$

physics is unchanged provided charges are rescaled accordingly. Thus a continuous "normalization dial" exists unless we fix a charge lattice convention. OGN fixes this dial by demanding a minimal integer/rational basis for charges and a canonical measure for compact cycles. MSMF then forbids leaving λ as a free continuous parameter.

In relativistic QFT, locality/causality is encoded by *microcausality*: for local observables $\mathcal{O}_1(x)$ and $\mathcal{O}_2(y)$,

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)] = 0, \qquad (x-y)^2 < 0, \tag{2D.18}$$

i.e. operators commute at spacelike separation (Haag, 1992; Streater & Wightman, 1964). A standard textbook consequence is *no superluminal signaling*: if an agent couples locally at x (changing the Hamiltonian density in a compact region) and another agent measures at spacelike-separated y, the statistics of the measurement cannot depend on the choice at x.

A complementary Green-function expression uses retarded propagators. Let ϕ be a field with source J; the response is governed by

$$\phi(x) = \int d^4y\, G_R(x-y)\, J(y), \tag{2D.19}$$

where the *retarded* Green function G_R vanishes outside the future light cone:

$$G_R(x-y) = 0 \quad \text{for } x^0 < y^0 \text{ or } (x-y)^2 < 0. \tag{2D.20}$$

AX-1 can be read as a constitution-level insistence that *the operational kernels that map interventions to records are retarded.*

Gauge theory is geometry: a gauge field is a connection on a principal bundle, and its physical invariants are curvature and holonomy (Nakahara, 2003). Let $P(M, G)$ be a principal G-bundle over spacetime M. A connection is a Lie-algebra-valued one-form $A \in \Omega^1(M, \mathfrak{g})$, and

$$F = dA + A \wedge A \tag{2D.21}$$

is its curvature.

The holonomy along a closed loop γ is the path-ordered exponential

$$\mathrm{Hol}_\gamma[A] = \mathcal{P} \exp\left(\oint_\gamma A\right). \tag{2D.22}$$

Quantum physics makes this geometric content operational: the Aharonov–Bohm effect shows that $\mathrm{Hol}_\gamma[A]$ has observable consequences even when $F = 0$ locally (Aharonov & Bohm, 1959). Likewise, Berry phases show that adiabatic evolution defines a geometric phase as a holonomy in parameter space (Berry, 1984).

GGEP claim (geometrization of quantum physics). Instead of treating A as an additional field that must be "quantized," GGEP treats the *composite connection* $\mathcal{A}$ as primitive, schematically

$$\mathcal{A} \in \Omega^1(M_6, \mathfrak{so}(1,5) \oplus \mathfrak{g} \oplus \mathfrak{u}(1)_B), \qquad \mathcal{F} = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A}. \tag{2D.23}$$

The bulk geometry fixes $\mathcal{A}$ up to gauge/coordinate redundancy. Quantum phases and selection rules are read from $\mathcal{F}$ and its topological invariants, while boundary QFT emerges as an effective description of the exported invariants.

Operational quantum theory describes a measurement by an *instrument* $\{\mathcal{I}_m\}_m$: a set of completely positive maps such that $\sum_m \mathcal{I}_m$ is trace-preserving. If the pre-measurement state is ρ, outcome probabilities and post-measurement states are

$$p(m) = \mathrm{Tr}[\mathcal{I}_m(\rho)], \qquad \rho_m = \frac{\mathcal{I}_m(\rho)}{p(m)}. \tag{2D.24}$$

A Kraus representation writes

$$\mathcal{I}_m(\rho) = \sum_\alpha K_{m\alpha} \rho K_{m\alpha}^\dagger, \qquad \sum_{m,\alpha} K_{m\alpha}^\dagger K_{m\alpha} = \mathbb{I}, \tag{2D.25}$$

which is the standard operational semantics of measurement (Kraus, 1971; Nielsen & Chuang, 2010). OGGEP elevates this structure to a *constitutional requirement*: admissible bulk theories must induce such instruments on the boundary with an explicit map from bulk objects (sources, boundary conditions, ports) to the operational instrument.

MSMF can be formalized as a constrained optimization over theory space. Let $\mathfrak{T}$ denote a class of candidate bulk+dictionary packages. Define:

- $\mathrm{Sym}(T)$ = the symmetry group of theory T (or a suitable measure such as $\dim \mathrm{Sym}(T)$);
- $\mathrm{DoF}(T)$ = the number of continuous free parameters not fixed by topology/normalization;
- $\mathrm{Fit}(T; D)$ = an empirical fit score to data D (likelihood, χ^2, etc.).

MSMF is the meta-principle that admissible theories should maximize symmetry and minimize freedom subject to viability and fit:

$$T^\star \in \arg\max_{T\in\mathfrak{T}} \Big(\dim \mathrm{Sym}(T) - \lambda\, \mathrm{DoF}(T) \Big) \quad \text{subject to postulate admissibility and empirical constraints.} \tag{2D.26}$$

The coefficient λ is not a tunable "physics parameter" but a bookkeeping device to express the two constitutional goals. The key point is that *continuous freedom is treated as suspicious unless protected by symmetry* (Akaike, 1974; Rissanen, 1978; THooft, 1980).

In relativistic QFT, locality is encoded by *microcausality*: for bosonic fields $\phi(x)$ and spacelike-separated points x, y,

$$[\phi(x), \phi(y)] = 0 \qquad \text{if} \qquad (x-y)^2 < 0. \tag{2D.27}$$

Equation (2D.1) ensures that measurements performed in spacelike-separated regions cannot signal faster than light. In practice, causal propagation is implemented by retarded Green's functions. For a linear wave operator $\Box + m^2$, the retarded propagator G_{ret} satisfies

$$(\Box_x + m^2) G_{\mathrm{ret}}(x-y) = \delta^{(4)}(x-y), \qquad G_{\mathrm{ret}}(x-y) = 0 \ \text{ for } \ x^0 < y^0. \tag{2D.28}$$

AX-1 adopts this logic at the constitutional level: a lawful prediction must be writable so that its dependence is retarded with respect to the operational causal structure (Weinberg, 1995).

A modern operational way to express locality for open systems is to demand that *local interventions* correspond to localized quantum channels. Let $\mathcal{R}$ be a spacetime region on the boundary and let $\Phi_{\mathcal{R}}$ be a CPTP map acting only on degrees of freedom in $\mathcal{R}$. Local causality can be expressed as: for any region $\mathcal{S}$ spacelike to $\mathcal{R}$, reduced states in $\mathcal{S}$ are invariant under $\Phi_{\mathcal{R}}$,

$$\rho_{\mathcal{S}} \;=\; \mathrm{Tr}_{\overline{\mathcal{S}}}(\rho) \quad\Longrightarrow\quad \rho'_{\mathcal{S}} \;=\; \mathrm{Tr}_{\overline{\mathcal{S}}}\big((\Phi_{\mathcal{R}} \otimes \mathrm{id})(\rho)\big) \;=\; \rho_{\mathcal{S}}. \tag{2D.29}$$

Equation (2D.29) is one explicit way to state "no superluminal signaling" for open-system operations. It is compatible with standard quantum information theory (Nielsen & Chuang, 2010) and clarifies that AX-1 is *operational* rather than metaphysical.

The physical content of gauge theory is not the potential A_μ itself (which is gauge-dependent), but gauge-invariant quantities: curvature $F_{\mu\nu}$ and global holonomy. The Aharonov–Bohm experiment shows that even when $F_{\mu\nu} = 0$ in a region, the holonomy can be nontrivial (Aharonov & Bohm, 1959). For a charged particle of charge q moving along a closed loop γ,

$$\Delta\varphi_{\mathrm{AB}} \;=\; \frac{q}{\hbar}\oint_\gamma A_\mu\, dx^\mu \;=\; \frac{q}{\hbar}\int_{\Sigma:\,\partial\Sigma=\gamma} F_{\mu\nu}\, d\Sigma^{\mu\nu}. \tag{2D.30}$$

GGEP generalizes this: *quantum phase is geometry* (holonomy of a connection), and gauge forces are therefore geometric objects on bundles—a viewpoint that goes back to Weyl (Weyl, 1918) and is made concrete in Yang–Mills theory (Yang and Mills, 1954).

In GGEP we treat the composite connection $\mathcal{A}_{\mathrm{lock}} = \omega \oplus A$ as the primitive field. A particle "feels" the connection through a covariant derivative. In the simplest representation (schematically),

$$D_A \Psi \;=\; \partial_A \Psi + \omega_A^{ab} J_{ab} \Psi + A_A^I T_I \Psi. \tag{2D.31}$$

This unifies the gravitational (spin) connection and internal gauge connection at the kinematical level: both define parallel transport and both contribute to holonomy. Thus, the statement "geometrize quantum physics" can be read as: *encode the quantum kinematics and gauge structure in the geometry of a single connection.*

In AdS/CFT, the bulk partition function with boundary condition $\phi \to \phi_0$ is identified with the generating functional of boundary correlators (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1997; Witten, 1998):

$$Z_{\text{Bulk}}[\phi_0] \;\equiv\; \exp\!\big(-S_{\text{on-shell}}[\phi_0]\big) \;=\; \left\langle \exp\!\left(\int_{\mathcal{M}_4} d^4x\; \phi_0(x)\,\mathcal{O}(x)\right)\right\rangle_{\text{Bdy}}. \tag{2D.32}$$

Differentiating with respect to ϕ_0 produces boundary correlators $\langle \mathcal{O}\cdots\mathcal{O}\rangle$. OGGEP generalizes the idea: any fundamental theory must provide a dictionary of this form, but extended to *operational measurement objects* (POVMs, instruments, records).

Let $\mathfrak{B}$ denote the sigma-algebra of boundary-record events (detector clicks, time stamps, counts, etc.). OGGEP demands a map

$$\mathcal{D}:\ \text{(bulk fields/configurations)} \;\longrightarrow\; \text{(boundary operational predictions)}, \tag{2D.33}$$

such that for any instrument protocol $\mathcal{P}$ there is a well-defined probability assignment

$$\mathbb{P}(i|\mathcal{P}) \;=\; \mathrm{Tr}(\rho\, E_i) \qquad \text{with} \qquad \{E_i\} \text{ derived from } \mathcal{D}(\text{Bulk}, \mathcal{P}). \tag{2D.34}$$

The novelty is constitutional: the existence of $\mathcal{D}$ is an axiom, not a derived convenience.

MSMF can be expressed as a constraint on model complexity. Let Θ be the continuous parameter space of a candidate model (couplings, moduli, normalization constants, etc.). Define the *knob count* N_{free} as the dimension of Θ after quotienting by gauge/basis redundancies:

$$N_{\text{free}} \;\equiv\; \dim(\Theta/\!\sim_{\text{gauge}}). \tag{2D.35}$$

MSMF demands that admissible theories minimize N_{free} subject to the required symmetries and consistency postulates. This is stronger than standard naturalness (THooft, 1980) because it is imposed constitutionally, not used as heuristic guidance.

Let $\mathrm{Sym}(\mathcal{T})$ denote the symmetry group of a theory (including spacetime and internal symmetries). MSMF can also be read as a variational principle on theory space:

$$\text{choose } \mathcal{T} \text{ to maximize } \dim \mathrm{Sym}(\mathcal{T}) \text{ while minimizing } N_{\text{free}}(\mathcal{T}). \tag{2D.36}$$

In practice, GG-6 and OGN implement this by forcing integer-ledger normalizations and eliminating continuous rescaling freedom. The philosophical point is modest but powerful: *symmetry is a source of determinism* because it removes arbitrary choices.

2D.5 Worked derivation: LC + CPTP ⇒ GKLS generator (sketch)

A first-year graduate student can read the full GKLS theorem in standard texts, but the core idea is short. Assume a one-parameter family of maps $\{\Phi_t\}_{t\geq 0}$ on density operators with:

(i) semigroup property $\Phi_{t+s} = \Phi_t \circ \Phi_s$, (ii) $\Phi_0 = \mathrm{id}$, (iii) complete positivity and trace preservation (CPTP), (iv) mild continuity in t (so a generator exists).

Then there exists a (bounded, in finite dimension) generator $\mathcal{L}$ such that

$$\Phi_t = e^{t\mathcal{L}}, \qquad \dot{\rho} = \mathcal{L}(\rho),$$

and $\mathcal{L}$ must take the Lindblad/GKLS form

$$\mathcal{L}(\rho) = -\mathrm{i}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right).$$

In the GUT constitution, the point of placing this in the appendix is conceptual: once we accept AX-1 (LC for operational evolution) and POS-4 (CP for physicality), the structure of open-system evolution is *forced*. The "measurement problem" is therefore shifted from philosophy to engineering: choose the instrument (Kraus operators / POVM), and the update rule is fixed by the admissible channel class.

Appendix 2E: Expanded derivations for the Seven Postulates (POS-1–POS-7)

2E.1 From AX-1 (Locality and Causality) to microcausality and retarded evolution

In relativistic QFT, locality is encoded by microcausality:

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0, \qquad (x-y)^2 < 0. \tag{2E.1}$$

Operationally, causality is encoded by retarded response and by requiring open evolution to be a forward-time semigroup.

2E.2 AX-2 (GGEP): composite connection and curvature

A gauge field is a connection A with curvature $F = \mathrm{d}A + A \wedge A$. Gravity can be written similarly in a connection formalism. GGEP elevates the composite connection $\mathcal{A} = \omega \oplus A$ as the primary object, whose curvature $\mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$ contains both R and F and, in locked phases, mixed terms.

2E.3 AX-3 (OGGEP): measurement as an instrument and emergence of GKLS form

A measurement is an instrument $\{\mathcal{I}_m\}$, a collection of CP maps whose sum is trace-preserving. Coarse-grained intervention sequences generate CPTP semigroups whose generators are of GKLS form:

$$\dot{\rho} = \mathcal{L}(\rho) = -\mathrm{i}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right). \tag{2E.2}$$

2E.4 AX-4 (MSMF) and POS-6 (OGN): ordering theories and eliminating basis drift

Implement MSMF via a lexicographic ordering: maximize symmetry, then minimize continuous freedom. POS-OGN fixes residual normalization freedom (especially Abelian factors) by minimal-integer charge assignments.

2E.5 POS-2 (UEP): least action and free-energy descent via large deviations

For small-noise processes, path probabilities obey $\mathbb{P}[y] \asymp \exp[-I[y]/\varepsilon]$. Most-likely paths extremize I, yielding PLA-type statements. In gradient-flow limits, the same structure yields monotone descent of a free-energy ledger (FEP).

2E.6 POS-1 (GI) in generator language: LGI, DGI, GGI

Dynamical gauge invariance is equivariance of the generator:

$$\Phi_t \circ U(g) = U(g) \circ \Phi_t \ \forall t \geq 0 \quad \iff \quad \mathcal{L} \circ U(g) = U(g) \circ \mathcal{L}. \tag{2E.3}$$

Stationary sets decompose into group orbits, providing the seed for GSSB.

2E.7 Interplay of DGI and GSSB: kernel symmetry vs attractor symmetry

Let $\mathrm{Sym}(\mathcal{L})$ denote generator symmetries and $\mathrm{Stab}(a)$ the stabilizer of an attractor a. A GSSB event corresponds to $\mathrm{Stab}(a) \subsetneq \mathrm{Sym}(\mathcal{L})$.

2E.8 POS-3 (AF): anomaly inflow and GS-modified Bianchi identities (sketch)

Anomalies are summarized by an anomaly polynomial I_{2n+2}. Inflow introduces bulk CS/GS terms so that their variation cancels boundary anomalies. A minimal GS sector has $H = \mathrm{d}B + \omega_3$ with modified Bianchi identity $\mathrm{d}H = X_4(F, R)$.

2E.9 POS-5 (RG): commutation of coarse-graining with the operational dictionary

POS-RG demands that the operational dictionary $\mathcal{D}$ commute with coarse-graining:

$$\mathcal{D} \circ \mathrm{RG}_\ell = \mathrm{RG}_\ell \circ \mathcal{D}. \tag{2E.4}$$

2E.10 POS-7 (MF): registered observables and kill criteria

A GUT prediction is a contract (O, π, K): observable, protocol, and falsification rule.

2E.11 Worked examples

Example 1: two-state open system. For a two-state Markov chain with rates $k_{12}, k_{21} \geq 0$, relative entropy to the stationary distribution is a Lyapunov function with $\mathrm{d}F/\mathrm{d}t \leq 0$.

Example 2: qubit dephasing. A GKLS generator with $L = \sqrt{\gamma}\,\sigma_z$ yields dephasing while preserving positivity and trace.

Example 3: reaction network with ports (conceptual). A chemostatted reaction network is a GDOS instance on concentration space with Gibbs free energy ledger and port source terms.

Let $U(g)$ be a unitary representation of a gauge transformation $g \in G$ acting on boundary states by $\rho \mapsto U\rho U^\dagger$. DGI requires the open-system generator $\mathcal{L}$ to satisfy

$$\mathcal{L}(U\rho U^\dagger) = U\,\mathcal{L}(\rho)\,U^\dagger. \tag{2E.5}$$

For GKSL form $\mathcal{L}(\rho) = -i[H,\rho] + \sum_k (L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\})$, a sufficient condition is covariance of the operators:

$$UHU^\dagger = H, \qquad UL_kU^\dagger = \sum_j v_{kj}(g)\,L_j, \tag{2E.6}$$

where $v(g)$ is a (possibly reducible) representation mixing jump operators. Eq. (2E.5) is the open-system analog of Ward identities (Ward, 1950; Yang and Mills, 1954).

A clean bridge between closed and open extremal principles uses *path probabilities*. For a diffusion process X_t with drift b and noise strength σ, the probability of a path is weighted by an Onsager–Machlup functional (Onsager & Machlup, 1953):

$$\mathbb{P}[X_{0:T}] \propto \exp\Big(-S_{\mathrm{OM}}[X]\Big), \qquad S_{\mathrm{OM}}[X] = \int_0^T dt\, \frac{1}{2\sigma^2}\|\dot{X} - b(X)\|^2 + \cdots. \tag{2E.7}$$

The most probable path minimizes S_{OM}, a least-action principle for open stochastic dynamics.

At the level of distributions, many open dynamics can be written as gradient flow of a free-energy-like functional in a metric on probability space (information geometry) (Amari, 1998; Cover & Thomas, 2006). A canonical example is the Fokker–Planck equation

$$\partial_t \rho = \nabla \cdot (\rho \nabla \Phi + \nabla \rho)\,, \tag{2E.8}$$

which decreases the functional

$$\mathcal{F}[\rho] = \int dx\, \rho(x)\big(\Phi(x) + \ln \rho(x)\big), \tag{2E.9}$$

and can be interpreted as KL divergence to equilibrium:

$$\mathcal{F}[\rho] - \mathcal{F}[\rho_{\mathrm{eq}}] = D_{\mathrm{KL}}(\rho\|\rho_{\mathrm{eq}}) \geq 0 \quad \text{with} \quad D_{\mathrm{KL}}(\rho\|\pi) = \int dx\, \rho \ln \frac{\rho}{\pi}. \tag{2E.10}$$

This identity makes precise why "free energy" and "information distance" are two faces of the same extremal engine (Kullback & Leibler, 1951; Shannon, 1948).

Quantum anomalies arise when a classical gauge symmetry fails at the quantum level; the gauge variation of the effective action is nonzero (Adler, 1969; Bardeen, 1969). In modern language, anomalies are encoded by an anomaly polynomial I_{d+2} in d dimensions, with descent equations $I_{d+2} = dI_{d+1}$ and $\delta I_{d+1} = dI_d$. Inflow cancellation introduces a bulk Chern–Simons-like term whose variation cancels the boundary anomaly (Callan & Harvey, 1985; Green and Schwarz, 1984). In GG-6, this is implemented by a two-form field B_2 and a modified Bianchi identity for H_3 (BI-6), whose coefficients are fixed by anomaly matching and minimality.

Complete positivity is stronger than positivity: Φ must remain positive even when tensored with identity on an arbitrary ancilla. Stinespring's theorem shows that any CP map arises from unitary evolution on a larger space followed by partial trace (Stinespring, 1955):

$$\Phi(\rho) = \mathrm{Tr}_E \left[U \left(\rho \otimes \sigma_E \right) U^\dagger \right]. \tag{2E.11}$$

This provides a precise sense in which boundary openness can be fundamental while remaining compatible with bulk unitarity.

Let RG_s denote coarse-graining from scale μ to $\mu' = \mu e^{-s}$. POS-5 demands

$$\mathcal{D} \circ \mathrm{RG}_s = \mathrm{RG}_s \circ \mathcal{D}, \tag{2E.12}$$

so that operational predictions are stable under changes of resolution. In holographic RG, the radial coordinate r plays the role of s, and bulk equations implement RG flow (de Haro, Skenderis, & Solodukhin, 2001; Wilson, 1971).

A simple illustration is a $U(1)$ gauge factor: rescaling the generator $Q \to \alpha Q$ and coupling $g \to g/\alpha$ leaves local physics invariant, but changes the *apparent* charge assignments. OGN forbids treating such rescalings as free continuous choices: admissible theories must fix a canonical lattice of charges (minimal integers) consistent with topology/holonomy (Dirac, 1931; Georgi and Glashow, 1974).

MF can be expressed as a triple:

$$\text{Claim} \equiv (\text{observable } \mathcal{O},\ \text{protocol } \Pi,\ \text{falsifier } \mathcal{K}), \tag{2E.13}$$

where Π is an explicit experimental/observational procedure and $\mathcal{K}$ is a pre-declared kill criterion (a decision rule) (Neyman & Pearson, 1933; Popper, 1959). This turns philosophical falsifiability into a reproducible audit mechanism.

In closed systems, local gauge invariance implies conserved currents and Ward identities (Weinberg, 1995; Yang and Mills, 1954). For open systems, the analogue is that the *generator* of dynamics must be covariant. For a GKSL generator $\mathcal{L}$ and a unitary representation $U(g)$ of a gauge group element g on $\mathcal{H}$, DGI requires

$$\mathcal{L}\Big(U(g)\rho U(g)^\dagger\Big) \;=\; U(g)\,\mathcal{L}(\rho)\,U(g)^\dagger. \tag{2E.14}$$

Equation (2E.14) is the precise sense in which open-system dynamics respects gauge redundancy: changing the gauge/basis does not change physical predictions.

A unifying bridge between least action and thermodynamic extremals appears in stochastic dynamics. For a Langevin process $dy^A = b^A(y)\,dt + \sqrt{2D}^A{}_B\,dW_t^B$, path probabilities concentrate (in a small-noise limit) on trajectories that minimize an action functional. In one common normalization the Onsager–Machlup functional is (Freidlin & Wentzell, 1998; Onsager & Machlup, 1953):

$$\mathcal{S}_{\mathrm{OM}}[y(\cdot)] = \frac{1}{4}\int_{t_0}^{t_1} dt\,\left(\dot{y}^A - b^A(y)\right)\left(D^{-1}\right)_{AB}\left(\dot{y}^B - b^B(y)\right) \;+\; \cdots . \tag{2E.15}$$

UEP asserts that there exists a single extremal principle whose bulk form is PLA and whose boundary open-system manifestation is an extremal of $\mathcal{S}_{\mathrm{OM}}$ or a free-energy-like functional (MEPP/FEP), connected by the operational dictionary.

Information and thermodynamic cost. In open systems that perform inference or maintain records, information processing has thermodynamic consequences. A canonical result is Landauer's principle: erasing one bit of information requires at least $k_B T \ln 2$ of dissipated heat in an isothermal setting (Landauer, 1961). In the GUT-10 viewpoint, such bounds are not "engineering details" but constraints on admissible dynamics: they belong naturally in the UEP/EEGG ledger language.

Quantum anomalies are violations of classical gauge symmetry due to the regularization of the path integral (Adler, 1969; Bardeen, 1969; Bell and Jackiw, 1969). In even-dimensional bulk theories, anomalies can be encoded in an anomaly polynomial I_{d+2} and its descent forms (Alvarez-Gaumé & Witten, 1984). The Green–Schwarz mechanism cancels boundary anomalies by bulk inflow. A schematic form is:

$$dH_3 = X_4, \qquad H_3 = dB_2 + \omega_3, \tag{2E.16}$$

where X_4 is built from characteristic classes (e.g. $\mathrm{tr}\,R \wedge R$, $\mathrm{tr}\,F \wedge F$). POS-3 declares: an admissible theory must satisfy anomaly cancellation, either directly or by inflow, with fixed coefficients (no tuning).

For a quantum Markov semigroup, complete positivity and trace preservation constrain the generator to GKSL form (Breuer & Petruccione, 2002; Gorini, Kossakowski, & Sudarshan, 1976; Lindblad, 1976). A compact way to see positivity constraints is to express a channel in Kraus form:

$$\Phi(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger, \qquad \sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{I}. \tag{2E.17}$$

POS-4 elevates this to an admissibility requirement: if a model's evolution cannot be written in a positivity-preserving form, it is rejected.

RG transformations coarse-grain degrees of freedom and describe how effective parameters flow with scale (Wilson & Kogut, 1974). POS-5 adds a new constitutional requirement:

$$\mathcal{D} \circ \mathrm{RG} \;=\; \mathrm{RG} \circ \mathcal{D}, \tag{2E.18}$$

meaning that "zooming out" should not change the meaning of observables. In holographic language, r is a geometric representation of scale, so that bulk radial evolution corresponds to RG flow (Maldacena, 1997; Witten, 1998).

Abelian sectors admit continuous rescalings: $A \to \lambda A$ can be absorbed by $q \to q/\lambda$ in simple settings. OGN forbids leaving this freedom unfixed. Instead, select a normalization in which charges are minimal integers and the holonomy circle has canonical period 2π. This resembles the role of quantization conditions (Dirac quantization) and removes unphysical basis drift.

MF requires that every theorem is accompanied by: (i) an observable O and measurement protocol $\mathcal{P}$, (ii) a quantitative prediction $P_\theta(O|\mathcal{P})$ (possibly parameter-free), and (iii) a kill criterion. A simple statistical kill criterion is:

$$\text{reject } \mathcal{T} \text{ if } \quad p\text{-value}(\text{data}|\mathcal{T}) < \alpha \qquad (\text{predeclared significance } \alpha). \tag{2E.19}$$

The claim is not the particular statistic, but the *constitutional demand* that falsifiers are declared up front (Popper, 1959).

2E.11.1 LGI: standard local gauge invariance (closed systems)

LGI is the familiar redundancy of microscopic field theories: fields transform as $\psi \to g\psi$ and $A \to A^g$ as in (2D.8), and physical observables are gauge invariant. Ward identities and conserved currents follow from Noether's theorem (Noether, 1918).

2E.11.2 DGI: gauge covariance of open-system generators

For open quantum systems, the primitive object is a generator $\mathcal{L}$ of a CPTP semigroup $\Phi_t = e^{t\mathcal{L}}$ (Gorini, Kossakowski, & Sudarshan, 1976; Lindblad, 1976). DGI demands *covariance*: for a unitary representation $U(g)$ of the gauge group on the system Hilbert space,

$$\mathcal{L}\Big(U(g)\rho U(g)^\dagger\Big) = U(g)\,\mathcal{L}(\rho)\,U(g)^\dagger. \tag{2E.20}$$

In GKSL form,

$$\mathcal{L}(\rho) = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{2E.21}$$

a sufficient condition for (2E.20) is that H is invariant up to a gauge generator and that the Lindblad operators transform among themselves:

$$U(g) L_k U(g)^\dagger = \sum_j v_{kj}(g)\, L_j, \tag{2E.22}$$

with $v(g)$ a unitary mixing matrix. This is the open-system analog of covariant field content.

2E.11.3 Closed systems: PLA

The principle of least action states that physical trajectories extremize

$$S[q] = \int_{t_i}^{t_f} L(q, \dot{q}, t)\, dt, \tag{2E.23}$$

leading to Euler–Lagrange equations. This variational structure underlies Hamiltonian mechanics (Goldstein, Poole, & Safko, 2002; Hamilton, 1834).

2E.11.4 Open systems: most-probable paths and free-energy descent

For stochastic dynamics $dy^i = b^i(y)dt + \sqrt{2\Theta}\,\sigma^i{}_a dW_t^a$, the path probability concentrates, in a small-noise limit, on trajectories that minimize an Onsager–Machlup/Freidlin–Wentzell functional (Freidlin & Wentzell, 1998; Onsager & Machlup, 1953). In gradient-flow cases, one obtains equations of the form

$$\dot{y}^i = -\mu^{ij}(y)\,\partial_j F_{\mathrm{EEGG}}(y), \tag{2E.24}$$

which reads as "least free energy" evolution. UEP asserts that the bulk PLA and the boundary free-energy/least-time principles are two faces of one variational engine, connected by OGGEP.

Quantum anomalies show that classical symmetries can fail after quantization (Adler, 1969; Bell and Jackiw, 1969; Bertlmann, 1996). In modern language, anomalies are packaged into an anomaly polynomial I_{d+2} whose descent yields the anomalous variation of the effective action. In bulk–boundary systems, a Chern–Simons term can cancel the boundary anomaly ("anomaly inflow") (Callan & Harvey, 1985).

The Green–Schwarz mechanism introduces a higher-form field whose modified Bianchi identity schematically reads (Green and Schwarz, 1984):

$$dH = \mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F) + \cdots, \tag{2E.25}$$

and its variation cancels the anomalous gauge variation of the boundary theory. POS-3 demands that candidate GG-6/GUT-10 models satisfy such cancellation, which severely restricts admissible gauge groups and matter content.

In quantum theory, physical evolution maps density operators to density operators. Mathematically this requires complete positivity and trace preservation. The Kraus representation (Kraus, 1971) states that any CPTP map can be written

$$\Phi(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger, \qquad \sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{I}. \tag{2E.26}$$

Continuous-time Markovian evolution is generated by a GKSL operator (Gorini, Kossakowski, & Sudarshan, 1976; Lindblad, 1976). POS-4 elevates these requirements to a universal admissibility condition, including classical limits (positivity-preserving Fokker–Planck evolution) and field-theoretic ghost freedom.

Renormalization expresses the fact that physical predictions should not depend on arbitrary resolution scales (Callan, 1970; Symanzik, 1970; Wilson, 1971; Wilson & Kogut, 1974). Let RG_ℓ denote coarse-graining by a factor ℓ. POS-5 requires that the operational dictionary commute with coarse-graining:

$$\mathcal{D} \circ \mathrm{RG}_\ell = \mathrm{RG}_\ell \circ \mathcal{D}. \tag{2E.27}$$

In holographic settings, the bulk radial coordinate often plays the role of RG scale (Maldacena, 1997; Witten, 1998):

$$r \sim \ln(\mu/\mu_0), \tag{2E.28}$$

so geometric depth corresponds to energy scale. POS-5 states that this identification must be consistent with how observables are defined.

Even after fixing a gauge group, continuous normalization freedoms can remain, especially for Abelian factors and mixed gauge groups. OGN demands that among all physically equivalent normalizations, the admissible one is the minimal rational/integer choice consistent with topology and charge quantization. A practical proxy is: choose a basis in which the charge lattice is primitive (no common divisors) and compact-cycle measures are canonical. This is the "no hidden rescaling" rule that makes integer-ledger predictions meaningful.

Following Popper's criterion (Popper, 1959), MF requires that every theoretical claim map to a registered observable with a pre-declared falsifier. Operationally, a claim is packaged as:

$$\text{Claim} = \big(\text{model instance (SIP)},\ \text{observable map},\ \text{threshold/kill rule}\big). \tag{2E.29}$$

The threshold is statistical (e.g. a confidence level, Bayes factor, or power calculation), but it must be declared *before* looking at the data. MF is the final "audit" gate in the constitution.

2E.12 Worked derivation: anomaly inflow as a boundary consistency condition (cartoon)

Anomalies are not optional pathologies; they are a precise statement that a would-be symmetry cannot be implemented consistently on the quantum Hilbert space. In a bulk–boundary setting, one can allow a boundary anomaly provided it is canceled by inflow from a bulk term (Chern–Simons / Green–Schwarz). Schematically,

$$\delta S_{\partial} = \int_{\partial\text{Bulk}} \alpha\, \mathcal{A}_4, \qquad \delta S_{\text{Bulk}} = -\int_{\partial\text{Bulk}} \alpha\, \mathcal{A}_4,$$

so that the combined variation vanishes. POS-3 (AF) demands that an admissible model comes with such a cancellation story. This is why anomaly constraints function as a strong "integer selector" in GG-6: they quantize allowed charges, representations, and topological coefficients, and they are among the few known mechanisms that can reduce continuous freedom to discrete data.

Appendix 2F: Worked examples and reusable templates

2F.1 Driven harmonic oscillator (linear GDOS; exact solution)

Consider the Ornstein–Uhlenbeck process $\mathrm{d}y = -\kappa y\,\mathrm{d}t + \sqrt{2\Theta}\,\mathrm{d}W_t$. The stationary distribution is Gaussian $\rho_\star(y) \propto e^{-\kappa y^2/(2\Theta)}$ and the EEGG ledger (2H.8) can be chosen so that (2H.11) holds exactly.

2F.2 Two-state gene regulation as a Markov jump process

A minimal "gene on/off" model is a two-state Markov chain with transition rates controlled by metabolite concentrations. Ports correspond to nutrient inflow and signaling molecules. This

illustrates how "DGI" becomes invariance under relabelings of internal bookkeeping states, while MF corresponds to a kill criterion on switching statistics.

2F.3 Bayesian updating as a GDOS flow (connection to FEP)

Write a posterior density $\rho(\theta) \propto p(\text{data} \mid \theta)p(\theta)$. Gradient flows on Kullback–Leibler divergence correspond to variational Bayesian inference. This provides an explicit bridge between POS-UEP and the free-energy principle literature (Cover & Thomas, 2006; Friston, 2010).

Consider a qubit with Hamiltonian $H = \frac{\omega}{2}\sigma_z$ and pure dephasing with rate γ. The GKSL equation is

$$\dot{\rho} = -i[H, \rho] + \gamma\left(\sigma_z \rho \sigma_z - \rho\right). \tag{2F.1}$$

This map is CPTP (POS-4), respects covariance under the $U(1)$ generated by σ_z (POS-1), and provides a concrete operational prediction: off-diagonal elements decay as $e^{-2\gamma t}$. A falsifier is immediate: if an experiment finds coherence decay inconsistent with the predicted exponential law under controlled conditions, the proposed generator fails MF.

Consider $A \rightleftharpoons B$ with forward/backward rates k_+ and k_-. Let p_A, p_B be probabilities (or normalized concentrations). A master equation is

$$\dot{p}_A = -k_+ p_A + k_- p_B, \qquad \dot{p}_B = k_+ p_A - k_- p_B, \tag{2F.2}$$

which preserves positivity and normalization (POS-4). The stationary distribution satisfies detailed balance if the system is closed, but open chemostats break it and induce entropy production; stochastic thermodynamics provides a quantitative framework (Seifert, 2012). This toy model illustrates how GDOS treats chemical kinetics as open dynamics with an entropy/ledger interpretation.

2F.3.1 Quantum case: CPTP maps and GKSL generators

A general quantum operation with outcomes m is an *instrument* $\{\mathcal{I}_m\}$: each $\mathcal{I}_m$ is completely positive (CP), and $\sum_m \mathcal{I}_m$ is trace-preserving (TP). By the operator-sum (Kraus) theorem (Kraus, 1983; Nielsen & Chuang, 2010), each CP map admits

$$\mathcal{I}_m(\rho) = \sum_\alpha K_{m\alpha}\, \rho\, K_{m\alpha}^\dagger, \qquad \sum_{m,\alpha} K_{m\alpha}^\dagger K_{m\alpha} = \mathbf{1}. \tag{2F.3}$$

If we ignore conditioning on m (i.e. we only track the unconditional evolution), the map $\Phi(\rho) = \sum_m \mathcal{I}_m(\rho)$ is CPTP.

Assume now a Markovian time-continuous evolution $\{\Phi_t\}_{t\geq 0}$ with semigroup property $\Phi_{t+s} = \Phi_t \circ \Phi_s$ and strong continuity. Under these assumptions, the generator $\mathcal{L} = \lim_{t\to 0^+}(\Phi_t - \mathrm{id})/t$ exists and takes the GKSL form (Breuer & Petruccione, 2002; Gorini et al., 1976; Lindblad, 1976):

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}(\rho) = -\mathrm{i}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{2F.4}$$

where H is Hermitian and $\{L_k\}$ are Lindblad operators. Equation (2F.4) is the sharpest expression of POS-4 in the quantum regime: it is the unique local-in-time, trace-preserving, CP generator compatible with Markovianity.

2F.3.2 Classical case: positivity-preserving drift–diffusion

For a classical probability density $\rho(x,t)$ on a state manifold $\mathcal{M}$, closure and positivity mean

$$\rho(x,t) \geq 0, \qquad \int_{\mathcal{M}} \rho(x,t)\,\mathrm{d}\mu(x) = 1. \tag{2F.5}$$

The broadest local-in-time evolution preserving these constraints is a continuity equation

$$\partial_t \rho + \nabla_A J^A = 0, \qquad J^A = \rho\, b^A - D^{AB}\nabla_B \rho, \tag{2F.6}$$

where $D^{AB} \succeq 0$ is a diffusion tensor. In coordinate form this is a Fokker–Planck equation. The GDOS "gradient + curl + noise" decomposition corresponds to writing the drift as

$$b^A = -M^{AB}\nabla_B F + u^A, \qquad M^{AB} \succeq 0, \qquad \nabla_A u^A = 0 \text{ (in a suitable measure)}, \tag{2F.7}$$

where the $-M\nabla F$ term is dissipative (gradient sector) and u^A is the solenoidal/circulating sector.

2F.4 Monotonicity of the EEGG ledger in the gradient sector

Assume the purely gradient GDOS flow (no solenoidal drift and noise treated as fluctuation consistent)

$$\dot{y}^A = -M^{AB}(y)\,\partial_B F(y), \qquad M^{AB} \succeq 0. \tag{2F.8}$$

Then along a trajectory $y(t)$,

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \partial_A F\,\dot{y}^A = -\partial_A F\, M^{AB}\,\partial_B F \leq 0, \tag{2F.9}$$

with equality iff $\partial_A F = 0$ on the support (a critical point). This is the finite-dimensional prototype of the Lyapunov statement for F_{EEGG} in relaxational sectors and is the technical backbone of the "FEP-like" corollary used throughout GDOS (Friston, 2010; Seifert, 2012).

Let a group G act on states by a representation $U(g)$. DGI requires equivariance of the dynamics:

$$\Phi_t(U(g)\rho) = U(g)\Phi_t(\rho), \qquad \forall g \in G,\ \forall t \geq 0. \tag{2F.10}$$

Differentiating at $t = 0$ yields the generator-level condition

$$\mathcal{L}(U(g)\rho) = U(g)\mathcal{L}(\rho). \tag{2F.11}$$

In the GKSL form (2F.4) this is satisfied if

$$U(g)HU(g)^\dagger = H, \qquad U(g)L_kU(g)^\dagger = \sum_\ell D_{k\ell}(g)\,L_\ell, \tag{2F.12}$$

i.e. H is invariant and the Lindblad operators transform within a representation. The same idea holds classically: the drift, diffusion, and ledger must transform covariantly so that physical predictions are representation-independent.

Let $\mathrm{Sym}(\mathcal{L})$ be the symmetry group of the generator/kernel and $\mathrm{Sym}(\mu_\star)$ the symmetry of a particular invariant measure $\mu_\star$ (fixed point, limit cycle measure, SRB measure). *Geometric spontaneous symmetry breaking* is the statement

$$\mathrm{Sym}(\mathcal{L}) \supsetneq \mathrm{Sym}(\mu_\star). \tag{2F.13}$$

In closed QFT, $\mu_\star$ is often the vacuum state; in open systems, $\mu_\star$ is an attractor measure. The key generalization is conceptual and mathematical: the "phase" is an attractor of a CP/positive runtime, not only a minimum of a static potential.

Example 1 (physics): double-well + noise

Consider $F(y) = \lambda(y^2 - a^2)^2$ on $\mathbb{R}$ with gradient flow plus weak noise. The law is invariant under $y \mapsto -y$. For sufficiently weak noise, the long-time distribution concentrates near either well, yielding two symmetry-related attractor measures with reduced symmetry. This is the open-system prototype of SSB, with the role of "vacuum selection" played by attractor selection.

Example 2 (chemistry/biology): bistable switch

In a bistable chemical reaction network (e.g. positive feedback), the GDOS state is a concentration vector. Two stable fixed points correspond to two phenotypes. The dynamical law may respect a symmetry (permutation of equivalent species), while the realized phenotype breaks it.

Example 3 (mind/engineering): attractor networks

In Hopfield-like networks, the state is a vector of neuron activities; the energy function is a Lyapunov function for the deterministic dynamics. Multiple stored patterns are attractors. "Memory recall" is then a controlled GSSB event: the system falls into one attractor basin given a partial cue.

Let the state be $x \in \mathbb{R}$ with EEGG ledger $F(x) = \frac{1}{2}kx^2$. Take mobility $M = 1$ and diffusion $D = T$ (Einstein relation at "temperature" T). The GDOS/Fokker–Planck equation becomes

$$\partial_t \rho(x,t) = \partial_x(\rho\, \partial_x F) + T\, \partial_x^2 \rho = \partial_x(kx\, \rho) + T\, \partial_x^2 \rho. \tag{2F.14}$$

The stationary solution is the Gibbs distribution

$$\rho_*(x) = \frac{1}{Z} \exp\!\left(-\frac{F(x)}{T}\right) = \sqrt{\frac{k}{2\pi T}}\, \exp\!\left(-\frac{kx^2}{2T}\right). \tag{2F.15}$$

This illustrates the UEP idea in the simplest setting: the open dynamics relaxes toward a distribution controlled by the scalar ledger F.

Take $F(x) = \lambda(x^2 - v^2)^2$, symmetric under $x \to -x$. In the weak-noise limit $T \ll \lambda v^4$, $\rho_*(x)$ has two peaks near $\pm v$. A single long trajectory will spend long times near one well and occasionally switch (Kramers transitions). In the strict zero-noise limit, the system selects one attractor: symmetry of the *law* exceeds symmetry of the *realized phase*. This is the GDOS form of GSSB.

Consider a qubit with Hamiltonian $H = \frac{\omega}{2}\sigma_z$ and dephasing $L = \sqrt{\gamma}\sigma_z$. The GKSL generator is

$$\dot{\rho} = -i[H, \rho] + \gamma\left(\sigma_z \rho \sigma_z - \rho\right). \tag{2F.16}$$

Under a phase ("gauge") rotation $U(\theta) = e^{-i\theta\sigma_z/2}$, we have $U\sigma_z U^\dagger = \sigma_z$. Therefore $\mathcal{L}$ satisfies DGI covariance (2E.14). This simple model is a concrete illustration of POS-1/DGI in standard quantum dynamics.

2F.5 Example: GKSL covariance (a minimal DGI check)

Given a symmetry group with unitary representation $U(g)$, define the adjoint action $\mathcal{U}_g(\rho) = U(g)\rho U(g)^\dagger$. A minimal DGI test is to verify

$$\mathcal{U}_g^{-1} \circ \mathcal{L} \circ \mathcal{U}_g = \mathcal{L} \quad \text{for all } g. \tag{2F.17}$$

In a basis where Lindblad operators form irreducible multiplets, this reduces to checking that the set $\{L_k\}$ is closed under conjugation by $U(g)$, and that the Kossakowski matrix respects the symmetry.

2F.6 Example: open-system phase selection as GSSB (double-well sketch)

Consider a one-dimensional order parameter x with an even "free energy" landscape

$$F(x) = \frac{\lambda}{4}(x^2 - v^2)^2, \tag{2F.18}$$

and overdamped Langevin dynamics $\dot{x} = -\mu\, F'(x) + \sqrt{2\Theta}\,\eta(t)$. For small noise Θ, the stationary density concentrates near $x = \pm v$. The equation is $x \to -x$ symmetric, but any single long-lived realization picks one well: this is the simplest cartoon of GSSB. In a GDOS setting, the relevant generalization is to high-dimensional state manifolds with symmetry-related attractors and port-induced biases.

2F.7 Template: MF "claim contract"

A practical MF write-up for a theorem should list: (i) the SIP (state variables, symmetry group, ports), (ii) the predicted invariant (dimensionless ratio, scaling exponent, spectral knee), (iii) the measurement protocol (instrument, calibration, analysis window), (iv) the falsifier (numerical criterion that would reject the prediction). This template is intended to make "being wrong" as explicit as "being right."

2F.8 Template: writing a falsifiable claim in GUT form

A practical "textbook" template for statements in this program is:

1. **Claim (invariant):** specify a dimensionless invariant or relation (e.g. a ratio, exponent, or scale-covariant law).
2. **Model instance (SIP):** specify $(\mathcal{M}, G, \text{ports}, F)$ and the admissibility checks (POS-1–POS-6).
3. **Operational registration (OGGEP):** specify the instrument, preprocessing, estimator, and error model.
4. **Kill criterion (MF):** specify in advance what numerical outcome falsifies the claim.

This template is deliberately domain-agnostic and is meant to make cross-discipline work auditable.

Appendix 2G: Further expansions: GG-6 consistency and anomaly inflow (BI-6)

Gauge and mixed gravitational anomalies arise when a symmetry of the classical action fails at the quantum level. Classic examples include the axial anomaly in QED and non-Abelian gauge anomalies in chiral theories (Adler, 1969; Alvarez-Gaumé & Witten, 1984; Bell and Jackiw, 1969). Consistent theories require their cancellation.

The Green–Schwarz mechanism cancels anomalies by inflow from a higher-dimensional bulk sector (Green and Schwarz, 1984). In differential-form language, one introduces a 2-form B_2 with modified field strength $H_3 = \mathrm{d}B_2 + \omega_3$ so that

$$\mathrm{d}H_3 = X_4, \tag{2G.1}$$

where X_4 is built from curvature and gauge polynomials. The locked-coefficient BI-6 postulate in GG-6 is the claim that the allowed factorization structure and normalization discipline leave no free continuous coefficients once the gauge content is fixed. This appendix does not attempt the full GG-6 derivation (deferred to the companion chapter), but records the standard consistency logic.

Appendix 2H: Further expansions: GDOS multiscale structure (BFFT) and phase selection

This appendix expands the GDOS calculations that are only sketched in the main text. The guiding claim is that once OGGEP enforces an operational (CPTP/Markov) language, the same mathematical "engine" underlies (i) classical stochastic thermodynamics, (ii) open quantum dynamics, and (iii) mesoscopic dynamics in chemistry/biology.

2H.1 From a Langevin SDE to a Fokker–Planck/continuity equation (flat coordinates)

Consider an Itô SDE in $\mathbb{R}^n$,

$$\mathrm{d}y_t^i = b^i(y_t)\,\mathrm{d}t + \sum_{a=1}^{m} \sigma^i{}_a(y_t)\,\mathrm{d}W_t^a, \tag{2H.1}$$

with independent Wiener increments $\mathrm{d}W_t^a$ satisfying $\mathbb{E}[\mathrm{d}W_t^a] = 0$ and $\mathbb{E}[\mathrm{d}W_t^a\,\mathrm{d}W_t^b] = \delta^{ab}\mathrm{d}t$. Define the diffusion tensor $D^{ij}(y) = \sum_a \sigma^i{}_a(y)\sigma^j{}_a(y)$ (symmetric, positive semidefinite). For any smooth test function $f(y)$, Itô's lemma gives

$$\mathrm{d}f(y_t) = \partial_i f\,\mathrm{d}y_t^i + \frac{1}{2}\partial_i\partial_j f\,D^{ij}(y_t)\,\mathrm{d}t. \tag{2H.2}$$

Taking expectation and integrating by parts yields the forward equation for the density $\rho(y,t)$:

$$\partial_t\rho = -\partial_i(b^i\rho) + \frac{1}{2}\partial_i\partial_j(D^{ij}\rho) \equiv -\nabla\cdot J, \tag{2H.3}$$

where the probability current is

$$J^i = b^i\rho - \frac{1}{2}\partial_j(D^{ij}\rho). \tag{2H.4}$$

This is the minimal classical template behind POS-CP: if D is PSD and the PDE is well posed, ρ remains nonnegative. For detailed treatments see Gardiner (2009); Risken (1989); van Kampen (2007).

2H.2 Manifold form and the GDOS "gradient + curl + noise" decomposition

Let $\mathcal{M}$ be a smooth manifold with metric $G_{AB}(y)$ and volume element $\mathrm{d}\mu = \sqrt{|G|}\,\mathrm{d}^n y$. Write the continuity equation in covariant form

$$\partial_t\rho = -\nabla_A J^A, \tag{2H.5}$$

where ∇_A is the Levi-Civita covariant derivative. A convenient GDOS decomposition of the current is

$$J^A = \rho\,v^A - D^{AB}\nabla_B\rho, \tag{2H.6}$$

with D^{AB} PSD and v^A an effective drift. The "engine" claim is that v^A admits a canonical splitting into:

$$v^A = -M^{AB}\nabla_B\Phi \;+\; u^A \;+\; v^A_{\text{ports}}, \tag{2H.7}$$

where: (i) M^{AB} is a mobility (PSD), (ii) Φ is a potential-like functional derivative of the EEGG ledger, (iii) u^A is solenoidal ($\nabla_A(\rho u^A) = 0$ in steady state) and generates cyclic flows, and (iv) v^A_{ports} captures explicit driving by ports. This is the precise content behind the slogan "downhill with swirls."

2H.3 A concrete EEGG functional and its monotonicity in the gradient-flow limit

To make POS-UEPquantitative, consider an EEGG functional of the form

$$F_{\mathrm{EEGG}}[\rho] = \int_{\mathcal{M}} \rho(y)\,\Phi(y)\,\mathrm{d}\mu + \Theta \int_{\mathcal{M}} \rho(y)\,\ln\frac{\rho(y)}{\rho_{\mathrm{ref}}(y)}\,\mathrm{d}\mu, \tag{2H.8}$$

where Φ plays the role of an energy-like potential (EGG) and the second term is an entropic/information term (ECP) relative to a reference density ρ_{ref} (e.g. the invariant measure induced by geometry or constraints). Define the functional derivative

$$\frac{\delta F_{\mathrm{EEGG}}}{\delta\rho} = \Phi + \Theta\left(\ln\frac{\rho}{\rho_{\mathrm{ref}}} + 1\right). \tag{2H.9}$$

Now assume the *pure gradient* GDOS limit:

$$J^A = -\rho\, M^{AB}\nabla_B\left(\frac{\delta F_{\mathrm{EEGG}}}{\delta\rho}\right), \qquad D^{AB} = \Theta M^{AB}, \tag{2H.10}$$

which is the manifold generalization of overdamped Langevin dynamics with an Einstein relation. Using $\partial_t\rho = -\nabla_A J^A$ and integration by parts (assuming appropriate boundary conditions),

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}F_{\mathrm{EEGG}}[\rho(t)] &= \int \frac{\delta F_{\mathrm{EEGG}}}{\delta\rho}\,\partial_t\rho\,\mathrm{d}\mu \\ &= -\int \frac{\delta F_{\mathrm{EEGG}}}{\delta\rho}\,\nabla_A J^A\,\mathrm{d}\mu = \int \nabla_A\left(\frac{\delta F_{\mathrm{EEGG}}}{\delta\rho}\right) J^A\,\mathrm{d}\mu \\ &= -\int \rho\,\nabla_A\left(\frac{\delta F_{\mathrm{EEGG}}}{\delta\rho}\right) M^{AB}\,\nabla_B\left(\frac{\delta F_{\mathrm{EEGG}}}{\delta\rho}\right)\mathrm{d}\mu \;\le\; 0,\end{aligned} \tag{2H.11}$$

since M^{AB} is PSD and $\rho \ge 0$. Thus the ledger decreases monotonically in the gradient-flow limit. This calculation is the precise bridge between "least action" and "least free energy" in POS-UEP.

2H.4 Adding solenoidal currents: nonequilibrium steady states and "swirls"

In driven open systems, one typically has $u^A \neq 0$ and $v^A_{\mathrm{ports}} \neq 0$. A nonequilibrium steady state (NESS) satisfies $\nabla_A J^A_\star = 0$ but generally $J_\star \neq 0$. The solenoidal component u^A yields persistent circulation in state space; F_{EEGG} need not be strictly decreasing along individual trajectories, but the entropy production rate is positive and the stationary measure is well-defined. This is the formal basis for treating oscillatory chemistry (Belousov–Zhabotinsky-like cycles) and biological rhythms as steady open-system phases rather than "violations" of thermodynamics.

2H.5 Phase selection as GSSB: a worked double-well example

Consider an overdamped Langevin system in one coordinate y with potential $\Phi(y) = \frac{\lambda}{4}(y^2 - a^2)^2$. The system has a $\mathbb{Z}_2$ symmetry $y \mapsto -y$ at the level of the generator. At low noise Θ, the stationary density has two metastable peaks near $\pm a$ and the mixing time becomes exponentially large. Operationally, an experiment with finite observation time records the system in one well:

the realized *record* breaks $\mathbb{Z}_2$. This is the simplest illustration of GSSB: the law is symmetric, but the registered phase is not, because openness plus finite-time recording selects an attractor branch.

BFFT is introduced as a multiscale "readout bus" aligned with the log-energy coordinate $r = \ln(E_{\mathrm{GUT}}/\mu)$. Mathematically, it is convenient to model it as a dyadic multiresolution transform (Haar/wavelet packet in spirit), which exposes scaling windows and "knees" in spectra.

Given a time series $x(t)$, a dyadic wavelet transform computes coefficients

$$W_{j,k} = \int x(t)\,\psi_{j,k}(t)\,\mathrm{d}t, \qquad \psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \tag{2H.12}$$

where j is the scale and k the location (Daubechies, 1992; Mallat, 1999). Power-law scaling $S(f) \propto f^{-\alpha}$ maps to scale-dependent moments of $W_{j,k}$, and "knees" correspond to changes in effective α across j. In the GUT-10 program, these knees/slopes become cross-domain observables demanded by MF: they are measurable in particle/astro data and in biological telemetry, and they are meant to be compared across modalities under RG-compatibility.

The main text uses the GKLS generator as an admissibility consequence (POS-CP). Here we outline the logic. Let $\{\Phi_t\}_{t\geq 0}$ be a one-parameter semigroup of CPTP maps on trace-class operators, strongly continuous in t. A standard result (Gorini–Kossakowski–Sudarshan; Lindblad) shows that the generator $\mathcal{L}$ of such a semigroup must have the form (2.87) with a bounded set of Lindblad operators (or an appropriate unbounded generalization). The essential steps are: (i) use complete positivity to constrain the Choi matrix of the infinitesimal map, (ii) enforce trace preservation to fix the anticommutator term, (iii) isolate the Hamiltonian part as the anti-Hermitian component of the generator. See Breuer & Petruccione (2002); Gorini et al. (1976); Lindblad (1976) for full proofs. The constitutional point for GUT is that *once* one demands operational openness, there is little freedom left in the generator.

Appendix 2I: Further expansions: UEP/RG/OGGEP operational semantics

This appendix collects a more explicit, undergraduate-readable "semantics layer" for three intertwined clauses: OGGEP (AX-3), UEP (POS-2), and RG (POS-5). The common theme is that a scientific statement is not only an equation; it is a *typed map* from model variables to registered records. In the GUT constitution, a registered claim has the canonical form

$$\boxed{\text{Claim} \;\Rightarrow\; (O,\ \Pi,\ \mathcal{K})}, \tag{2I.1}$$

where O is an observable/estimator, Π is the protocol (instrument + calibration + data reduction), and $\mathcal{K}$ is the falsifier/kill criterion.

2I.1 Operational semantics as a commutative diagram (OGGEP + RG)

A useful way to express AX-3 and POS-5 together is a commutative diagram. Let SIP denote the space of system-instance packets and Obs the space of operational objects (instruments,

estimators, response functions). Let $\mathcal{D} : \mathsf{SIP} \to \mathsf{Obs}$ be the OGGEP dictionary. Let RG_ℓ be coarse-graining by scale parameter ℓ acting on the SIP (integrate out degrees of freedom, change resolution, or bin data). Then POS-RG asserts that there exists an induced coarse-graining on operational objects, $\mathrm{RG}_\ell^{(\mathrm{obs})}$, such that

$$\begin{array}{ccc} \mathsf{SIP} & \xrightarrow{\mathcal{D}} & \mathsf{Obs} \\ \downarrow{\scriptstyle \mathrm{RG}_\ell} & & \downarrow{\scriptstyle \mathrm{RG}_\ell^{(\mathrm{obs})}} \\ \mathsf{SIP} & \xrightarrow{\mathcal{D}} & \mathsf{Obs} \end{array} \qquad \Longleftrightarrow \qquad \mathcal{D} \circ \mathrm{RG}_\ell = \mathrm{RG}_\ell^{(\mathrm{obs})} \circ \mathcal{D}. \tag{2I.2}$$

Equation (2I.2) is the precise statement that "truth should not depend on arbitrary resolution choices." If the diagram fails, then a putative prediction can be changed by changing the microscope or the binning scheme, and the theory is constitutionally ill-posed.

2I.2 Quantum limit: instruments and POVMs (OGGEP)

A measurement with outcome set M is specified by an instrument $\{\mathcal{I}_m\}_{m\in\mathsf{M}}$, where each $\mathcal{I}_m$ is completely positive (CP) and the sum is CPTP. The probability of outcome m in pre-measurement state ρ is

$$p(m|\rho) = \mathrm{Tr}[\mathcal{I}_m(\rho)] = \mathrm{Tr}[E_m\rho], \qquad E_m := \mathcal{I}_m^*(\mathbf{1}), \tag{2I.3}$$

where $\{E_m\}$ is a POVM (Busch, Lahti, & Mittelstaedt, 1996; Davies and Lewis, 1970; Nielsen & Chuang, 2010). The post-measurement state conditioned on m is

$$\rho \mapsto \rho_m = \frac{\mathcal{I}_m(\rho)}{\mathrm{Tr}[\mathcal{I}_m(\rho)]}. \tag{2I.4}$$

For undergraduates it is helpful to see explicitly how "apparatus" enters. Any instrument admits a dilation: there exists an environment Hilbert space $\mathcal{H}_E$, an initial σ_E, a unitary U on $\mathcal{H}\otimes\mathcal{H}_E$, and a pointer POVM $\{\Pi_m\}$ such that

$$\mathcal{I}_m(\rho) = \mathrm{Tr}_E\Big[(\mathbf{1}\otimes\Pi_m)\,U\,(\rho\otimes\sigma_E)\,U^\dagger\Big]. \tag{2I.5}$$

Thus, in OGGEP language, a "measurement" is *an open interaction plus a readout.* Different protocols correspond to different (U, σ_E, Π_m), which is precisely why AX-3 demands that protocols be specified rather than assumed.

Coarse-graining outcomes (RG on the observational side). A common coarse-graining in experiments is to merge outcomes (binning). If outcomes are grouped by a surjection $b : \mathsf{M} \to \mathsf{M}'$, the coarse-grained instrument is

$$\mathcal{I}'_{m'} = \sum_{m:\, b(m)=m'} \mathcal{I}_m, \qquad E'_{m'} = \sum_{m:\, b(m)=m'} E_m, \tag{2I.6}$$

which is again CP/CPTP and corresponds exactly to $\mathrm{RG}_\ell^{(\mathrm{obs})}$ in (2I.2). This simple example shows what POS-RG demands: the same coarse-graining must be representable on both the model side and the observation side.

2I.3 Classical limit: response functions, estimators, and causality checks

In classical experiments, the operational object is often a response functional (a map from input drives to outputs), e.g. a retarded Green's function $G_R(t)$ or a transfer function $H(\omega)$, together with a declared estimator $\widehat{\theta}$ and an error model. If a registered record is produced by a linear filter $h(t)$ acting on an input $u(t)$,

$$x(t) = \int \mathrm{d}t'\, h(t-t')\, u(t') + \eta(t), \tag{2I.7}$$

then LC demands $h(t<0)=0$. As emphasized in Sec. 2.2.2.4, causality implies analyticity of $H(\omega)$ and dispersion relations, so LC can be audited directly from data by checking Kramers–Kronig consistency after calibrating the instrument response. OGGEP then says: the claim is not "$H(\omega)$ exists" but "this protocol and estimator produce $H(\omega)$ with this uncertainty budget."

2I.4 MF as a contract: explicit falsifiers

For each theoretical claim $\mathcal{C}$, MF demands a pre-registered kill criterion $\mathcal{K}$:

$$\mathcal{C} \;\Rightarrow\; \big(\text{observable } O,\ \text{protocol } \Pi,\ \text{decision rule } \mathcal{K}\big). \tag{2I.8}$$

For example, in a particle-physics setting $\mathcal{K}$ might be "exclude a predicted coupling g at confidence level α"; in an open-system setting it might be "the predicted scaling window does not exist in the power spectral density." The constitutional point is that falsifiability is not a post-hoc rhetorical option: it is part of what makes a derivation scientific.

2I.5 UEP bridges: PLA, PLT, and free-energy descent

PLA (action extremum) and geodesics. For a relativistic point particle,

$$S[x] = -mc \int \mathrm{d}s, \qquad \mathrm{d}s^2 = -g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{2I.9}$$

and $\delta S = 0$ yields the geodesic equation. This is the canonical "bulk" face of UEP.

PLT (least time) from geometric optics. In geometric optics, the eikonal approximation yields Fermat's principle: rays extremize travel time $\int n(\mathbf{x})\,\mathrm{d}\ell/c$ (Born & Wolf, 1999). In the GUT narrative, PLT is the boundary face of "straightest path" after the operational dictionary fixes what counts as the effective index/cost.

Stochastic least action and the Onsager–Machlup functional. For diffusion processes, the path probability concentrates on minimizers of an Onsager–Machlup functional, which plays the role of an action for stochastic paths (Onsager & Machlup, 1953; Seifert, 2012). In that sense, "least action" persists, but the action is built from drift, diffusion, and entropy production. In gradient-flow regimes the GDOS kernel reduces to $\dot{y} = -M\nabla F$, yielding monotone descent of F (Appendix 2C.5). This is the boundary face that aligns with FEP-like statements in inference/learning models (Friston, 2010).

2I.6 RG compatibility: QFT and holographic geometry

POS-5 states that coarse-graining must commute with the operational dictionary. In QFT, RG is the change of description under a scale transformation $\mu \mapsto \mu e^{-\ell}$, governed by beta functions (Callan, 1970; Symanzik, 1970; Wilson, 1971). In holographic duality, the radial coordinate in an AdS-like bulk is associated with boundary scale (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998). GUT uses this structural identification as a constitutional constraint: "truth" should not depend on arbitrary resolution choices. Concretely, if an observable is claimed to be universal, then either it is RG invariant, or its RG flow is part of the prediction and therefore part of the MF contract.

Appendix 2J: Additional notes for textbook integration

The accessible companion series (Part II of each chapter) (UCLA-Arisaka/WP-2026-N) now provides the accessible companion to each technical preprint. presentation in which (i) constitutional statements are paired with (ii) worked derivations and (iii)

Introduction: Why Does Science Need a Constitution?

2J.1 The greatest success story ever told

The scientific revolution that began with Galileo, Newton, and Maxwell, and accelerated through Einstein, Bohr, Heisenberg, and Feynman, has produced the most precise and powerful description of nature ever achieved. The Standard Model of particle physics predicts the magnetic moment of the electron to twelve decimal places. General relativity predicts the bending of starlight and the ripples of gravitational waves with exquisite accuracy. Quantum mechanics underpins every semiconductor chip, every laser, every MRI scanner.

These achievements rest on a handful of deep principles: symmetry, locality, gauge invariance, the variational principle, and the renormalization group. Together, they form the intellectual backbone of twentieth-century physics (Einstein, 1905, 1915; Wilson, 1971; Yang and Mills, 1954; 't Hooft and Veltman, 1972).

And yet, if you ask a physicist "what are the rules that *any* theory must obey?" you will get different answers depending on whether the physicist works in particle theory, cosmology, condensed matter, quantum information, or biology. Each subfield has its own foundational commitments, often implicit, sometimes contradictory, and rarely stated as explicit axioms.

This book is about a proposal to fix that: a compact set of rules—a *constitution*—that applies to all of science, from quarks to consciousness.

2J.2 The constitutional gap

The term "constitutional gap" refers to a specific diagnosis: many long-standing problems in science persist not because we lack clever ideas or powerful computers, but because we lack a shared foundational layer that specifies what counts as an admissible theory, an admissible measurement, and an admissible prediction.

Consider four concrete symptoms of this gap:

Symptom 1: The measurement problem in quantum mechanics. Since the 1930s, quantum mechanics has operated with two distinct rules: a deterministic rule (the Schrödinger equation) for how systems evolve when nobody is looking, and a probabilistic rule (the Born rule plus wavefunction collapse) for what happens when someone measures (von Neumann, 1932). These two rules are logically incompatible—one is deterministic, the other is stochastic—and nobody has derived the second from the first. The gap is not technical; it is constitutional: the theory does not contain a precise definition of what "measurement" means.

Symptom 2: Gravity speaks geometry; gauge forces speak algebra. In general relativity, gravity is the curvature of spacetime—it is geometry through and through (Einstein, 1915). The strong, weak, and electromagnetic forces, by contrast, are described by gauge theories—abstract internal symmetries that live on top of spacetime (Yang and Mills, 1954). There is no constitutional reason why these two types of physics should be treated as fundamentally different. The split is a historical artifact, not a logical necessity.

Symptom 3: Too many knobs, too many landscapes. The Standard Model has about 19 free parameters. Supersymmetric extensions have over 100. String theory has a "landscape" of 10^{500} possible solutions. Each free parameter represents a number that the theory cannot predict—it must be measured and plugged in. The more parameters a theory has, the less it explains and the more it merely describes. Yet there is no agreed-upon rule for how many free parameters are "too many."

Symptom 4: Science is fragmented. Physics, chemistry, biology, neuroscience, and economics all use different foundational assumptions, different mathematical frameworks, and different standards of evidence. A physicist and a biologist may both use differential equations, but they mean very different things by "state," "observation," and "prediction." There is no shared constitutional layer that connects these disciplines.

GUT is a proposal to address all four symptoms simultaneously, by specifying a small set of rules that every admissible scientific theory must satisfy.

2J.3 What is a constitution?

The word "constitution" is borrowed deliberately from political science.

A national constitution does not tell citizens what to eat for breakfast or which route to drive to work. It specifies the *rules that all laws must obey*: freedom of speech, due process, separation of powers, the right to a fair trial. Any law that violates these rules is struck down, no matter how popular or convenient it might be.

A scientific constitution works the same way. It does not tell you what particles exist, what their masses are, or what equations they obey. Instead, it specifies the *rules that any candidate theory must satisfy*. A theory that violates these rules is struck down—it is "inadmissible"—regardless of how elegant or fashionable it might be.

Metaphor: Think of the space of all conceivable scientific theories as an enormous library. Every possible theory—every possible Lagrangian, every possible set of equations—occupies a book on a shelf. The library stretches to the horizon. The constitution is the librarian who walks through with a red stamp, marking most books "inadmissible." Only the unmarked books—the theories that pass every constitutional test—are allowed on the reading list.

The GUT constitution has eleven clauses: four *axioms* (the deepest commitments) and seven *postulates* (additional filters that sharpen the axioms into operational requirements). The remarkable claim of the program is that these eleven clauses are so restrictive that the reading list contains essentially one book.

2J.4 Axioms versus postulates: a distinction with a purpose

Before stating the eleven clauses, it is worth explaining why the constitution is split into axioms and postulates.

An **axiom**, in the GUT-10 sense, is a foundational commitment about what the theory's primitive objects *are*. Axioms define the playing field: what kinds of fields exist, what causality means,

what measurement means, and how freedom is constrained. You cannot derive an axiom from something more basic (within the system); you either accept it or reject it.

A **postulate**, by contrast, is an operational filter that specifies how the axioms are implemented in practice. Postulates are more concrete than axioms: they constrain the equations of motion, the symmetry properties of the generators, the normalization conventions, and the falsifiability requirements.

Analogy: In the U.S. Constitution, the axioms are like the Bill of Rights (fundamental commitments: freedom of speech, right to bear arms, due process). The postulates are like the subsequent amendments and implementing legislation (how those rights are enforced in practice: rules of evidence, term limits, election procedures).

The four axioms of GUT establish the constitutional identity of the theory. The seven postulates make it operational.

2J.5 A brief history of axiom systems in science

The idea of axiomatizing science is not new. It has a long and distinguished history.

Euclid (∼300 BCE): geometry. Euclid's *Elements* organized all of Greek geometry into five axioms (postulates) from which every theorem could be derived. For over two thousand years, this was the gold standard of rigorous knowledge. The later discovery that Euclid's fifth postulate (the parallel postulate) could be replaced led to non-Euclidean geometry and eventually to general relativity.

Newton (1687): mechanics. Newton's *Principia* laid down three laws of motion plus the law of universal gravitation. These functioned as axioms from which the motion of planets, projectiles, and pendulums could be derived. Newton's system dominated physics for over two centuries.

Maxwell (1865): electromagnetism. Maxwell unified electricity and magnetism into four equations. These equations can be viewed as the axioms of classical electrodynamics: given the sources (charges and currents), Maxwell's equations determine the fields completely.

Einstein (1905, 1915): relativity. Special relativity rests on two axioms: the laws of physics are the same in all inertial frames, and the speed of light is constant. General relativity rests on the equivalence principle and the Einstein field equations. Einstein's approach was explicitly axiomatic: derive the maximum amount of physics from the minimum number of assumptions.

Dirac, von Neumann, Wigner (1920s–1930s): quantum mechanics. The axioms of quantum mechanics—Hilbert space, linear operators, the Born rule, the projection postulate—were formalized by von Neumann in 1932 (von Neumann, 1932). This axiomatization remains the monograph standard, though it has been supplemented by the theory of open quantum systems and quantum information.

Yang and Mills (1954): gauge theory. Yang and Mills did not present their work as an axiom system, but their paper established the principle of local gauge invariance as a foundational axiom of particle physics (Yang and Mills, 1954). Every subsequent development in the Standard Model—from the electroweak theory to QCD—is built on this principle.

GUT-10 (2026): a constitution for all science. GUT is the latest entry in this tradition, but with a crucial difference: it aims to be a constitution not just for physics, but for all ten domains of science. The axioms are not specific to particles or fields; they are about admissibility, measurement, symmetry, and freedom. This generality is what makes the program ambitious—and what will ultimately determine whether it succeeds or fails.

2J.6 The ten sciences

The "10" in GUT stands for ten branches of knowledge that the constitution aims to unify—not by reducing them all to particle physics, but by giving them a shared set of admissibility rules.

1. **GUT-Particle:** the physics of quarks, leptons, gauge bosons, and the Higgs (1st floor of the Pagoda).
2. **GUT-Cosmos:** the physics of the expanding universe, dark energy, inflation, stars, black holes, gravitational waves, and galaxy formation (2nd floor).
3. **GUT-Chemistry:** the science of molecular structure, reactions, and materials (3rd floor).
4. **GUT-Life:** the science of living systems, evolution, metabolism, and genetic networks (4th floor).
5. **GUT-Mind:** the science of neural systems, perception, cognition, and consciousness (5th floor).
6. **GUT-Health:** the science of physiological networks, disease, and medical intervention.
7. **GUT-Society:** the science of social networks, economic systems, and collective behavior.
8. **GUT-Environment:** the science of ecosystems, climate, biogeochemical cycles, and planetary sustainability.
9. **GUT-AI:** the science of artificial intelligence, machine learning, and engineered information-processing systems.
10. **GUT-Philosophy:** the meta-level discipline that audits the constitution itself.

The claim is *not* that biology is "really" physics, or that economics can be derived from the Schrödinger equation. The claim is more modest and more precise: every branch of science must declare a *state*, evolve it by an *admissible dynamics*, and export *operational observables with falsifiers*. The constitution specifies what "admissible" means. Each branch then fills in the details using its own domain-specific knowledge.

Metaphor: Think of the ten sciences as ten different restaurants on the same street. Each restaurant serves different cuisine—Italian, Japanese, Mexican, Indian, etc. The constitution is the health code. It does not tell each restaurant what to cook; it tells all of them what hygiene standards they must meet. A restaurant that violates the health code is shut down, regardless of how good the food tastes. GUT is the health code for science.

2J.7 What the accessible companion (Part II) covers

This accessible companion (Part II) follows the structure of the technical preprint UCLA-Arisaka/GG-2026-1, chapter by chapter:

1. **Introduction** (this section): why science needs a constitution, and the history of axiom systems.
2. **The four axioms**: what they say, why they matter, and what they mean for everyday science.
3. **The seven postulates**: how each postulate sharpens the axioms into operational filters.
4. **Applications**: what the constitution produces—a preview of GG-6, GDOS, and the ten-science export.
5. **Discussion**: significance, comparisons with other approaches, and the road ahead.
6. **Conclusions**: the argument in one page.

No appendices are included. Every essential idea is in the main text. Equations appear only when they illuminate a concept and are always explained in words.

The Four Axioms: The Deepest Commitments

2J.1 Overview: what the axioms do

The four axioms of GUT define what the theory's primitive objects are and what constraints they must satisfy. Together, they answer four questions:

(i) **AX-1 (Causality):** Can the future influence the past? (No.)

(ii) **AX-2 (Geometry–Gauge):** Are gravity and the other forces fundamentally different? (No.)

(iii) **AX-3 (Measurement):** Does the theory tell you how to make a measurement? (It must.)

(iv) **AX-4 (Simplicity):** How many free parameters are allowed? (As few as possible.)

Each axiom is stated here in plain language, with historical context, metaphors, and examples. The full mathematical formulation is in the companion technical paper.

2J.2 AX-1: Locality and Causality — "No time travel, no telepathy"

What it says. Effects cannot travel faster than light, and no measurement result can depend on information from the future. In the language of physics: all effective response kernels are retarded, and all quantum field operators commute at spacelike separation.

Why it matters. Causality is the bedrock of all science. Without it, there is no meaningful notion of cause and effect, no way to design experiments, and no way to make predictions. AX-1 is the most conservative axiom in the constitution—it simply insists that physics respects the arrow of time and the speed limit set by the speed of light.

Historical roots. The idea that effects propagate at a finite speed goes back to the seventeenth century, when Ole Rømer measured the speed of light by observing the moons of Jupiter. Maxwell's equations (1865) showed that electromagnetic signals travel at speed c. Einstein's special relativity (1905) elevated this to a universal principle: nothing—no signal, no information, no causal influence—can travel faster than light (Einstein, 1905).

In quantum field theory, causality takes a precise mathematical form called *microcausality*: the commutator (or anticommutator) of any two field operators vanishes when the operators are evaluated at spacelike-separated points. This ensures that measurements made in New York cannot instantaneously affect measurements made in Tokyo.

Metaphor: the postal system. Think of the universe as a postal system. AX-1 says three things:

(a) Letters are delivered in the order they are mailed (no time travel).

(b) No letter can arrive before it is sent (no advanced signals).

(c) The postal service has a maximum speed: the speed of light (no superluminal delivery).

Any theory that violates any of these three rules—even subtly, even in exotic higher-dimensional scenarios—is constitutionally inadmissible.

What AX-1 excludes. AX-1 rules out theories with acausal propagators, theories that require advanced Green's functions, and theories in which the outcome of a measurement at time t depends on data that will only become available at time $t' > t$. It also rules out any effective theory whose reduced dynamics require "hidden advanced data"—information from the future smuggled in through the back door of an environment.

What AX-1 does not say. AX-1 does *not* forbid quantum entanglement. Entanglement is a correlation, not a causal signal. Two entangled particles can exhibit correlated measurement outcomes even when separated by vast distances, but this correlation cannot be used to send a message faster than light. AX-1 is perfectly compatible with the Einstein–Podolsky–Rosen paradox and with Bell's theorem.

2J.3 AX-2: The Geometry–Gauge Equivalence Principle — "One river, not two"

What it says. The primitive field of the theory is a single composite connection that encodes both gravity (the curvature of spacetime) and gauge forces (the internal symmetries of particle physics). Gravity and gauge are not separate ontological categories; they are two aspects of one geometric object.

Why it matters. This is the most radical axiom in the constitution. In the standard picture, spacetime is the stage and forces are actors. AX-2 says: the stage and the actors are the same thing. This unification of gravity and gauge is the conceptual heart of the GG-6 program, and it is what ultimately forces the theory into six dimensions.

Historical roots. The idea of geometrizing forces has a century-long history.

In 1915, Einstein showed that gravity is not a force in the Newtonian sense—it is the curvature of spacetime itself (Einstein, 1915). This was the first geometrization of a force.

In 1918, Hermann Weyl attempted to unify gravity and electromagnetism by adding a "scale factor" to the spacetime metric. The attempt failed physically (it predicted that atomic clocks would behave differently depending on their history), but it introduced the concept of "gauge invariance"—the very term "gauge" comes from Weyl's German word "Eich" (calibration).

In 1921, Theodor Kaluza showed that if you add a fifth dimension to Einstein's equations, electromagnetism emerges automatically from the geometry of that extra dimension (Kaluza, 1921). In 1926, Oskar Klein proposed that the fifth dimension could be compactified into a tiny circle (Klein, 1926). This Kaluza–Klein mechanism is the direct ancestor of AX-2.

In 1954, Yang and Mills generalized gauge invariance from the simple Abelian group U(1) (electromagnetism) to arbitrary non-Abelian groups (Yang and Mills, 1954). This provided the mathematical language for the strong and weak nuclear forces and became the backbone of the Standard Model.

AX-2 synthesizes all of these historical threads into one constitutional statement: gauge structure is not an add-on to geometry; it *is* geometry, expressed through a composite connection on a higher-dimensional manifold.

Metaphor: the river. Imagine a river. The water's overall flow pattern—where objects drift, how the current curves around bends—is like gravity: it describes the large-scale geometry of the environment. But the same water also has internal structure: eddies, vortices, turbulent swirls. These internal motions are like the gauge forces.

The traditional approach in physics treats the river (gravity) and the eddies (gauge forces) as separate phenomena requiring separate theories. AX-2 says: there is only one river. The eddies are part of the river. You do not need a separate theory of eddies; they are already encoded in the geometry of the water flow.

In mathematical terms, the composite connection is:

$$\mathcal{A} = \omega[g] \oplus A^{\text{gauge}},$$

where $\omega[g]$ is the gravitational spin connection (the "river flow") and A^{gauge} is the gauge field (the "eddies"). AX-2 says that $\mathcal{A}$, not ω and A separately, is the primitive object.

What AX-2 forces. AX-2 has a powerful consequence: the gauge group, the number of extra dimensions, and the locking relation between gravity and gauge curvatures are no longer free choices. They are constrained by the requirement that $\mathcal{A}$ be a consistent composite connection on a single manifold. The companion chapter GG-2026-2 shows that this forces a six-dimensional warped bulk with gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ and a specific Bianchi identity (BI-6).

2J.4 AX-3: The Operational/Open GGEP — "The cookbook, not just the ingredients"

What it says. A theory is incomplete unless it provides a precise, lawful dictionary that translates the full bulk description (the six-dimensional theory) into boundary observables (the four-dimensional measurements that experimenters actually perform). This dictionary must produce: a boundary state space, admissible evolution maps, and measurement instruments.

Why it matters. AX-3 directly addresses Problem 1—the measurement problem. In standard quantum mechanics, measurement is an additional rule bolted onto the theory from outside. The textbook says: "when you measure, the wavefunction collapses." But it never defines what "measure" means, what distinguishes a measurement from an ordinary interaction, or when collapse happens.

AX-3 says: this is constitutionally unacceptable. A theory that does not include a measurement recipe is like a cookbook that lists ingredients but never tells you how to cook. The recipe is not optional; it is part of the constitution.

Historical roots. The measurement problem has been debated since the founding of quantum mechanics. In 1932, John von Neumann formalized quantum theory with a projection postulate for measurement (von Neumann, 1932), but he recognized that this postulate was an external addition rather than a derived consequence. In the 1970s, the theory of open quantum systems—particularly the work of Gorini, Kossakowski, Sudarshan, and Lindblad on completely positive, trace-preserving (CPTP) maps—showed that measurement-like behavior can emerge naturally when a system interacts with an environment.

In 1997, Juan Maldacena's discovery of the AdS/CFT correspondence (Maldacena, 1998) showed that a theory of quantum gravity in a $(d{+}1)$-dimensional "bulk" can be equivalent to a quantum field theory living on the d-dimensional "boundary." This bulk-to-boundary dictionary is the direct inspiration for AX-3's operational map.

AX-3 generalizes both of these ideas: the open-system perspective (measurement emerges from bulk-to-boundary reduction) and the holographic perspective (boundary observables are

projections of bulk data). But it does so constitutionally rather than model-specifically: AX-3 says that *any* admissible theory must provide such a dictionary, whether or not that theory involves strings, supersymmetry, or any other specific machinery.

Metaphor: the shadow and the object. Imagine a three-dimensional sculpture illuminated by a lamp. The sculpture casts a two-dimensional shadow on the wall. The shadow is not the sculpture, but it contains real information about the sculpture's shape. If you know the rules of projection (the "dictionary"), you can reconstruct many features of the sculpture from the shadow.

AX-3 says: the six-dimensional bulk is the sculpture. The four-dimensional world we observe is the shadow. The operational dictionary is the set of projection rules. The theory is not complete unless it provides these rules explicitly.

What AX-3 forces. AX-3 forces the theory to include:

(a) A boundary state space $\mathcal{H}_\partial$ (the space of possible shadow configurations).

(b) Admissible evolution maps $\Phi_{t,s}$ that are completely positive and trace-preserving (CPTP).

(c) Measurement instruments $\{\mathcal{I}_\alpha\}$ that produce concrete outcomes from quantum states.

This is the starting point for the GDOS (Geometric Dynamics of Open Systems) framework developed in Chapter 4.

2J.5 AX-4: Maximum Symmetry, Minimum Freedom — "Nature uses the simplest lock"

What it says. Among all theories that satisfy AX-1–AX-3 and the seven postulates, nature selects those with the most symmetry and the fewest free parameters.

Why it matters. AX-4 is the anti-landscape principle. It is designed to prevent the explosion of freedom that plagues many unification programs. In the Standard Model, about 19 parameters must be measured and plugged in. In the MSSM, over 100. In the string landscape, 10^{500}. AX-4 says: if a parameter is not forced by the constitution, it is forbidden.

Historical roots. The principle of parsimony has a long history. William of Ockham (14th century) stated it as "entities should not be multiplied beyond necessity." Newton formulated it as "we are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances." Einstein expressed it more poetically: "everything should be made as simple as possible, but not simpler."

In modern physics, the principle appears as "naturalness": a parameter is natural only if its smallness is protected by a symmetry ('t Hooft, 1979). AX-4 goes further: even natural parameters are inadmissible if they are not forced by the constitution.

Metaphor: the combination lock. Imagine the space of all possible theories as a combination lock with many dials. Each dial represents a free parameter. In the string landscape, there are thousands of dials, and 10^{500} different combinations all "work" (in the sense of being mathematically consistent).

AX-4 says: nature uses the lock with the fewest dials. If a dial is not required by symmetry, topology, or anomaly cancellation, it should not be there. The result is a lock with very few dials, all set to specific values forced by the constitution.

In the GG-6 realization, the lock has exactly five integer dials: $(M, N, L; J, K) = (1, 3, 5; 7, 35)$. Beyond these five integers and Newton's constant G_N, no free parameter survives.

What AX-4 forces. AX-4 forces the theory to be maximally symmetric (the gauge group should be as large as consistency allows) and minimally free (the number of continuous parameters should be as small as consistency allows). Mathematically, this is expressed as a lexicographic ordering on theory space:

$$\mathcal{T}^* = \underset{\mathcal{T}\,\in\,\text{admissible}}{\arg\,\text{lexmax}}\ \big(S(\mathcal{T}),\, -F(\mathcal{T})\big),$$

where S measures symmetry and F measures freedom. In plain English: first maximize symmetry, then minimize freedom (among the maximally symmetric options).

2J.6 How the four axioms work together

The four axioms are not independent modules; they form a tightly interlocking system.

AX-1 (causality) sets the arena: physics is local, causal, and forward-in-time. AX-2 (geometry–gauge) defines the primitive object: a composite connection encoding both gravity and gauge forces. AX-3 (measurement) demands a dictionary: the bulk must project down to measurable boundary observables. AX-4 (simplicity) prunes the landscape: among all consistent options, only the simplest survives.

Together, they produce a derivation chain:

$$\text{AX-1–4} \implies \text{GGEP bulk} \implies \text{GG-6} \implies \text{BI-6} \implies \text{6D anomaly-free QFT}.$$

This chain is the subject of the companion chapter GG-2026-2 and its accessible companion Chapter 3, Part II.

Everyday analogy: Imagine building a house. AX-1 is the building code (safety standards: no asbestos, proper wiring). AX-2 is the architectural style (all load-bearing and decorative elements are part of one integrated structure). AX-3 is the inspection certificate (the house must come with documented plumbing and electrical schematics so inspectors can verify compliance). AX-4 is the budget constraint (use the minimum number of different materials consistent with safety and beauty). Together, these four constraints determine the house—not every detail, but the structural skeleton.

A deeper look at the interplay. The interplay between the axioms produces results that no single axiom could achieve alone. Consider three examples.

(i) AX-2 + POS-5 forces extra dimensions. AX-2 says: gauge forces must be geometric. POS-5 says: the renormalization group must be geometric too. To geometrize both gauge forces *and* scale dependence, you need more than four dimensions. AX-2 alone might be satisfied by a four-dimensional theory with a clever choice of internal symmetry. POS-5 alone might be satisfied without extra dimensions (the RG is usually formulated abstractly, not geometrically). But together, they force extra geometric dimensions—specifically, a radial scale direction and a compact phase direction.

(ii) AX-2 + POS-3 fixes the gauge group. AX-2 says: gauge structure must descend from a composite connection. POS-3 says: the resulting theory must be anomaly-free. Not every gauge group on a six-dimensional manifold satisfies both conditions. In fact, the anomaly-cancellation requirement is so stringent that, combined with AX-4 (minimality), it selects $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ as the unique admissible gauge content.

(iii) AX-3 + POS-4 derives quantum mechanics. AX-3 says: the theory must include a measurement dictionary. POS-4 says: the reduced boundary evolution must be CPTP. Together, they force the boundary theory to be quantum-mechanical: the state space must be a Hilbert space, the evolution must be a quantum channel, and the measurement must be a quantum instrument. Quantum mechanics is not postulated; it is derived.

These three examples illustrate the power of the constitutional approach. Individual axioms are relatively modest statements. But their *conjunction*—the requirement that all eleven clauses be satisfied simultaneously—produces highly specific, highly constrained structures.

The Seven Postulates: Making the Constitution Operational

2J.1 What are postulates and why do we need seven?

The four axioms define the constitutional identity of the theory: causality, geometry–gauge equivalence, operational measurement, and parsimony. But axioms alone are not enough to do science. You also need *operational rules*—concrete procedures that tell you how to check whether a specific theory satisfies the axioms.

That is the role of the seven postulates. Each postulate converts one aspect of the axioms into a testable mathematical condition. Think of them as the "implementing legislation" of the constitution: the axioms say what must be true in principle; the postulates say how to verify it in practice.

The seven postulates are ordered not because some are more important than others, but because each builds on the previous ones. Together, they form a *sequential filter*: a theory must pass POS-1 before being tested against POS-2, then POS-3, and so on. A failure at any stage is disqualifying.

2J.2 POS-1: Gauge Invariance — "Labels don't matter"

What it says. The laws of physics must not depend on arbitrary labeling choices, at any scale and in any description.

What it means. In quantum field theory, a gauge transformation is a local change of the "internal labels" attached to particles. For example, you can multiply the electron field by a complex phase $e^{i\alpha(x)}$ at every point in spacetime without changing any physical prediction. This local phase freedom is the gauge symmetry of electromagnetism, and it is what forces the photon to exist (as the "gauge boson" that compensates for the phase change).

POS-1 generalizes this idea to three levels:

(a) **LGI (Local Gauge Invariance):** the classical action must be invariant under local gauge transformations. This is the standard requirement in particle physics.

(b) **DGI (Dynamical Gauge Invariance):** the generators of open-system evolution must be covariant under gauge transformations. This extends gauge invariance from closed systems to open systems—a crucial step for the GDOS framework.

(c) **GGI (Global Gauge-bookkeeping Invariance):** macroscopic conserved quantities (charges, currents, entropy budgets) must not depend on the choice of basis or convention.

Metaphor: the currency exchange. A bank records deposits in dollars. Another bank records them in euros. A third records them in yen. POS-1 says: the total wealth of each customer must be the same regardless of which currency the ledger uses. The "gauge transformation" is the currency conversion, and gauge invariance is the statement that real wealth is currency-independent.

Why three levels? The traditional formulation of gauge invariance (LGI) applies only to closed, microscopic systems—idealized situations where every particle is tracked individually. But real science involves open systems (systems interacting with environments), and macroscopic bookkeeping (conserved charges summed over large regions). POS-1's three-level structure ensures that gauge invariance holds not just for textbook problems but for the full range of physical situations.

2J.3 POS-2: Unified Extremal Principle — "One engine, many outputs"

What it says. All dynamics—closed and open, microscopic and macroscopic—must flow from a single variational principle.

What it means. In classical mechanics, particles follow paths that extremize the action—the *Principle of Least Action* (PLA). In thermodynamics, systems evolve toward states of minimum free energy. In optics, light follows the path of least time (Fermat's principle). In biology, Karl Friston has proposed that living systems minimize a variational free energy (the Free Energy Principle, or FEP).

These look like four different principles. POS-2 says they are actually *one* principle viewed from four different angles. The closed-system least action in the bulk and the open-system free-energy descent at the boundary are different projections of the same underlying variational engine.

Historical roots. The principle of least action was discovered independently by Maupertuis, Euler, and Lagrange in the eighteenth century. It was generalized by Hamilton in the nineteenth century and became the foundation of all modern theoretical physics. Feynman's path-integral formulation of quantum mechanics (1948) showed that even quantum mechanics can be expressed as a sum over paths, each weighted by $e^{iS/\hbar}$ where S is the action.

POS-2 extends this tradition to open systems and irreversible processes. It says: the action principle is not just a trick for deriving equations of motion; it is a constitutional requirement that any admissible dynamics must satisfy.

Metaphor: the universal remote. Think of a universal remote control. One device controls the TV, the sound system, the lights, and the thermostat. Each appliance responds differently (the TV changes channels; the thermostat adjusts temperature), but all are controlled by the same remote. POS-2 says: there is one "remote" (the variational principle), and all dynamics—from particle collisions to protein folding to neural computation—are different "channels" on that remote.

2J.4 POS-3: Anomaly Freedom — "No quantum contradictions"

What it says. Every gauge symmetry of the classical theory must survive quantization. If it doesn't—if quantum effects break a gauge symmetry—the theory is inadmissible. The only allowed repair mechanism is "inflow": a classical term from a higher-dimensional bulk that exactly cancels the quantum anomaly.

What it means. In quantum field theory, certain classical symmetries are "anomalous"—they are broken by quantum loop corrections. Some anomalies are harmless (the axial anomaly of QCD correctly predicts pion decay). But gauge anomalies are fatal: they destroy the mathematical consistency of the theory. If a gauge anomaly is present, probabilities cease to add up to one, and the theory produces nonsensical predictions.

POS-3 elevates anomaly freedom from a technical requirement to a constitutional filter. A theory with uncancelled gauge anomalies is not just inconvenient—it is legally void.

A brief history of anomalies. The discovery of anomalies is one of the great detective stories of twentieth-century physics.

In 1969, Stephen Adler (at the Institute for Advanced Study) and independently John Bell and Roman Jackiw (at CERN) discovered something shocking: the conservation of the "axial current" in quantum electrodynamics, which holds perfectly at the classical level, is violated by quantum loop corrections. This "ABJ anomaly" was not a bug—it was a feature. It correctly predicted the decay rate of the neutral pion ($\pi^0 \to \gamma\gamma$), resolving a puzzle that had stumped theorists for years.

In the 1970s, physicists realized that anomalies come in two types. *Global* anomalies (like the ABJ anomaly) break global symmetries and have physical consequences. *Gauge* anomalies break local gauge symmetries and are catastrophic: they destroy unitarity, causality, and the entire mathematical structure of the theory. The Standard Model was carefully checked and found to be gauge-anomaly-free—a nontrivial fact that constrains the allowed particle content (for example, each generation of quarks and leptons must come as a complete set to cancel anomalies).

The pivotal moment came in 1984, when Michael Green and John Schwarz discovered that the gauge anomalies of ten-dimensional supergravity coupled to super-Yang–Mills theory—which seemed fatal—could be cancelled by a topological mechanism involving a two-form tensor field (Green and Schwarz, 1984). This "Green–Schwarz mechanism" revived string theory overnight and launched the first superstring revolution.

POS-3 uses the same mechanism as a constitutional filter. Any theory whose anomaly polynomial does not factorize through a form that allows Green–Schwarz cancellation is inadmissible.

Metaphor: the balance sheet. Think of anomaly cancellation as double-entry bookkeeping. Every debit (anomaly contribution from one sector) must be matched by a credit (from another sector or from the classical Green–Schwarz term). If the books don't balance, the company is insolvent—the theory is inadmissible. POS-3 says: the books must balance exactly, with no hidden debt.

Why "by inflow"? In ordinary four-dimensional field theory, anomaly cancellation is achieved by choosing the right particle content—the contributions from different fermions cancel each other. In six or ten dimensions, this is not always possible. The Green–Schwarz mechanism provides an alternative: a classical term in the action (involving a tensor field B_2) varies under gauge transformations in exactly the right way to cancel the quantum anomaly. This is "inflow"—the classical cure flows in from the bulk to cancel the quantum disease on the boundary.

POS-3 allows this repair channel but requires that the total system (bulk + boundary + inflow) be exactly anomaly-free. No partial repairs, no approximate cancellations, no handwaving.

2J.5 POS-4: Closure and Positivity — "Probabilities stay positive"

What it says. Physical evolution must preserve the positivity of probabilities and the normalization of quantum states. In the quantum case, admissible evolution maps must be completely positive and trace-preserving (CPTP). In the classical case, admissible evolution must be a positivity-preserving Markov semigroup.

What it means. When a quantum system interacts with its environment (which it always does, in practice), its evolution is no longer unitary. Instead, it is described by a quantum channel—a map that takes density matrices to density matrices. POS-4 says: this map must be CPTP.

"Completely positive" means that the map preserves positivity even when the system is entangled with an ancilla. "Trace-preserving" means that total probability is conserved. Together, these conditions guarantee that the output of the map is a legitimate quantum state—no negative probabilities, no probability exceeding one.

Why "completely" positive? Ordinary positivity is not enough. A map can be positive (sending positive matrices to positive matrices) without being completely positive (the extended map, acting on the system plus an ancilla, might produce negative eigenvalues). This distinction is subtle but crucial: it is the difference between a map that works for isolated systems and one that works for systems entangled with their environment.

The CPTP condition is the mathematical expression of a deep physical requirement: the reduced dynamics of any open system, obtained by tracing over the environment, must be physically sensible.

Consequence: the GKLS form. When POS-4 is combined with AX-1 (causality/forward semigroup), the generator of Markovian quantum evolution must take the Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) form:

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right).$$

The first term is the familiar Hamiltonian evolution. The second term—the "Lindblad operators" L_k—describes the irreversible effects of the environment: decoherence, dissipation, and measurement-like processes. This equation is the mathematical workhorse of open quantum systems, and it emerges constitutionally from AX-1 + POS-4.

Metaphor: the census. A population census must always report non-negative numbers. You cannot have "minus three people" in a city. POS-4 says: quantum evolution must have the same property—the analog of "population" (the eigenvalues of the density matrix) can never go negative, no matter how the system interacts with its environment.

2J.6 POS-5: RG Compatibility — "Zooming out doesn't change the answer"

What it says. The act of coarse-graining (zooming out from a detailed microscopic description to a coarser one) must be compatible with the measurement dictionary. Predictions must be stable under changes of resolution.

What it means. The renormalization group (RG) is one of the most powerful concepts in modern physics. Invented by Kenneth Wilson in the 1970s (Wilson, 1971), it describes how the effective description of a physical system changes as you "zoom out" from short distances (high energy) to long distances (low energy).

For example, the strength of the strong nuclear force depends on the energy scale at which you probe it. At high energies, the strong force is weak ("asymptotic freedom"); at low energies, it is strong ("confinement"). This running of the coupling constant is a prediction of the RG.

POS-5 says: the RG must commute with the operational dictionary. If you first zoom out and then project to the boundary, you must get the same answer as if you first project to the boundary and then zoom out. Schematically:

$$\mathcal{D} \circ \mathcal{G}_{\text{Bulk}} = \mathcal{G}_{\text{Bdy}} \circ \mathcal{D},$$

where $\mathcal{D}$ is the dictionary and $\mathcal{G}$ is the coarse-graining map.

Geometric consequence. POS-5 has a beautiful geometric consequence: it forces the radial coordinate r of the six-dimensional bulk to be the logarithm of the energy scale:

$$r = \ln\left(\frac{E_{\text{GUT}}}{\mu}\right).$$

The deeper you go into the bulk, the lower the energy, and the more zoomed-out your description. The geometry of the extra dimension literally *is* the renormalization group.

Metaphor: Google Maps. Zooming out on Google Maps does not change the distance between two cities. It simplifies the visual representation (you see fewer streets, fewer buildings), but the underlying data is the same. POS-5 says: the same must be true for physics. Zooming out (coarse-graining) simplifies the description but does not change the physical predictions.

Why this matters beyond physics. POS-5 is not just a physics postulate. It applies to any science that deals with multiple scales of description. In biology, for instance, you can describe a cell at the molecular level, the organelle level, or the whole-cell level. POS-5 demands that predictions made at one level are consistent with predictions made at another. This is the constitutional foundation of multiscale modeling.

A concrete example: the strong force. The strong nuclear force provides a vivid illustration. At very high energies (short distances), quarks interact weakly—they are nearly free particles inside a proton. This is "asymptotic freedom," discovered by Gross, Wilczek, and Politzer in 1973 and rewarded with the 2004 Nobel Prize. At low energies (long distances), the same force becomes overwhelmingly strong, confining quarks inside protons and neutrons so tightly that no isolated quark has ever been observed.

The renormalization group describes this transition quantitatively: the strong coupling constant α_s "runs" from small values at high energy to large values at low energy. POS-5 demands that this running be constitutionally consistent—meaning that the description at the high-energy end (perturbative QCD) and the description at the low-energy end (lattice QCD, chiral perturbation theory) must give the same physical predictions when their domains of validity overlap.

In GG-6, this running is geometrized: the radial coordinate r of the warped bulk *is* the logarithm of the energy scale, so the running of α_s is literally a walk along the extra dimension.

2J.7 POS-6: Occam Gauge Normalization — "Nature counts in small integers"

What it says. Among all normalizations consistent with the constitution, nature selects the minimal rational or integer values. Arbitrary continuous rescalings and basis drifts are inadmissible.

What it means. When you set up a gauge theory, you must choose normalization constants for the charges, couplings, and kinetic terms. In the Standard Model, these choices are treated as conventions—different physicists may use different normalizations, and the physics is the same.

POS-6 says: these choices are *not* arbitrary. Among all consistent normalizations, the constitution selects the simplest one—the one involving the smallest rational numbers or integers. This is not an aesthetic preference; it is a legal requirement.

Why it matters. POS-6 is the postulate that produces the OGN-5 integer package $(1, 3, 5; 7, 35)$ in the GG-6 realization. Without POS-6, the normalization sector would contain continuous free parameters. With POS-6, it collapses to a discrete arithmetic structure.

The coefficient $2/5$ in the BI-6 identity—the ratio that controls how the baryonic and informational gauge sectors couple relative to the non-Abelian SU(5) block—is not a free parameter. It is the unique rational number forced by POS-6 and the trace structure of the SU(5) fundamental representation.

Historical echo. Charge quantization—the fact that electric charge comes in integer multiples of the electron charge e—is one of the oldest hints that nature prefers discrete numbers over continuous ones. Dirac's magnetic monopole argument (1931) showed that the mere existence of a single magnetic monopole would force charge quantization throughout the universe. POS-6 generalizes this idea: not just charge, but *all* normalizations should be discrete whenever the constitution allows it.

Metaphor: the tuning fork. A tuning fork vibrates at a specific frequency—440 Hz for concert A. You cannot tune it to 440.37 Hz; the physics of the fork selects a discrete set of natural frequencies. POS-6 says: the "tuning forks" of the theory (its normalization constants) vibrate at the simplest possible frequencies.

2J.8 POS-7: Measurement and Falsifiability — "Put your money where your mouth is"

What it says. Every scientific claim must come with two things: a concrete measurement protocol and a pre-declared set of data that would force the claim to be rejected. A claim without these two components is constitutionally inadmissible.

What it means. This is Karl Popper's falsifiability criterion, elevated to constitutional status. In Popper's philosophy of science, a theory is scientific only if it can, in principle, be proved wrong. POS-7 makes this sharper: the theory must specify *in advance* what data would count as refutation, and it must specify *how* the relevant measurements are to be performed.

In formal language: a claim $\mathfrak{C}$ is admissible only if there exists an instrument $\mathcal{I}_{\mathfrak{C}}$ and a kill set $\mathfrak{K}_{\mathfrak{C}}$ such that data landing in $\mathfrak{K}_{\mathfrak{C}}$ forces rejection of $\mathfrak{C}$.

Why this matters. Many proposals in theoretical physics have been criticized for being unfalsifiable. The string landscape, for example, can accommodate almost any experimental outcome by adjusting the choice of vacuum among 10^{500} options. POS-7 is a structural response to this criticism. It says: if your theory cannot be wrong, it is not science. Every claim must come with a way to kill it.

Examples of falsifiers in the GUT-10 program.

- **Proton decay:** if observed, the exact $\mathrm{U}(1)_B$ protection of GG-6 is falsified.
- **Required supersymmetry:** if SUSY is discovered and shown to be essential, the non-SUSY stance is falsified.
- **Continuous moduli:** if the normalization sector requires continuous parameters beyond OGN-5, the discrete-ledger claim is falsified.
- **Acausal signals:** if a genuine faster-than-light signal is observed, AX-1 is falsified.

Metaphor: the bet. Imagine a sports commentator who predicts that Team A will win the championship. POS-7 says: the commentator must also say, before the season starts, "if Team A finishes below third place, I was wrong." A commentator who says "Team A will probably do well, but if they don't, there were extenuating circumstances" is not making a scientific prediction—they're making an unfalsifiable narrative. POS-7 bans such narratives from the constitution.

2J.9 How the seven postulates work as a sequential filter

The seven postulates are not a grab-bag of independent requirements. They form a logical chain.

POS-1 (gauge invariance) ensures that the theory has a well-defined symmetry structure. POS-2 (extremal principle) provides the dynamical engine. POS-3 (anomaly freedom) ensures quantum consistency. POS-4 (closure and positivity) guarantees physical evolution. POS-5 (RG compatibility) connects different scales. POS-6 (OGN) freezes normalization freedom. POS-7 (falsifiability) ensures scientific accountability.

Each postulate depends on the previous ones: you cannot check anomaly freedom (POS-3) without first knowing the gauge structure (POS-1) and the variational principle (POS-2). You cannot check RG compatibility (POS-5) without knowing the closure properties (POS-4). And falsifiability (POS-7) only makes sense once the entire theoretical structure is in place.

Metaphor: airport security. Think of the seven postulates as seven security checkpoints at an airport. A theory (the "passenger") must pass through all seven checkpoints in order. At each checkpoint, a specific test is performed: passport check (POS-1: gauge invariance), boarding pass check (POS-2: extremal principle), metal detector (POS-3: anomaly freedom), body scan (POS-4: positivity), luggage scan (POS-5: RG compatibility), weight limit (POS-6: OGN), and final identity verification (POS-7: falsifiability). A theory that fails at any checkpoint is turned away. Only theories that clear all seven are allowed to "board the plane" of admissible science.

2J.10 Worked example: how the postulates eliminate the string landscape

To see the postulates in action, let us trace how they constrain the space of possible theories.

Step 1: POS-1 (gauge invariance). Start with the space of all possible field theories in any number of dimensions. POS-1 immediately eliminates all theories that are not gauge-invariant: theories with broken gauge symmetries, theories with anomalous gauge sectors (unless repairable by inflow), and theories whose macroscopic bookkeeping depends on basis choices. This is a powerful filter, but it still leaves an enormous number of candidates—the Standard Model, the MSSM, every consistent string compactification, and many more.

Step 2: POS-2 (extremal principle). POS-2 demands that the dynamics flow from a single variational principle. This eliminates theories with ad hoc equations of motion that cannot be derived from an action. It also eliminates theories that use *different* variational principles for different sectors (e.g., one action for the gauge fields and a separate, unrelated action for the matter fields).

Step 3: POS-3 (anomaly freedom). POS-3 eliminates all theories with uncancelled gauge anomalies. This is an extremely powerful filter. In four dimensions, it constrains the allowed matter content: for example, you cannot have quarks without leptons. In six dimensions, it constrains the allowed gauge group and tensor content: only gauge groups whose anomaly polynomials factorize appropriately are admissible. In the GG-6 realization, this is what forces the Bianchi identity BI-6 and uniquely fixes its coefficient vector.

Step 4: POS-4 (positivity). POS-4 eliminates all theories whose reduced dynamics is not completely positive and trace-preserving. This is less familiar than gauge invariance, but equally important: it says that the theory's predictions must be physically sensible (no negative probabilities, no information loss). Combined with AX-1 (causality), POS-4 forces the GKLS form for Markovian quantum evolution.

Step 5: POS-5 (RG compatibility). POS-5 eliminates all theories whose coarse-graining procedure is incompatible with the measurement dictionary. This is what forces the radial coordinate to be logarithmic and the bulk geometry to be warped.

Step 6: POS-6 (OGN). POS-6 eliminates all theories with continuous normalization freedom. In string theory, the choice of vacuum (among 10^{500} options) is a continuous normalization choice in disguise: different vacua correspond to different values of moduli fields. POS-6 demands that these moduli be frozen to discrete values. In the GG-6 realization, this produces the five-integer OGN-5 ledger.

This is the step that kills the landscape. After POS-6, the 10^{500} vacua of string theory are reduced to a single discrete package.

Step 7: POS-7 (falsifiability). POS-7 eliminates all theories that cannot be tested. A theory that accommodates any experimental outcome by adjusting free parameters is inadmissible. The surviving theory must make sharp, pre-declared, falsifiable predictions.

Result: After all seven checkpoints, the only surviving candidate is the GG-6 bulk with gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity, Bianchi identity BI-6, and normalization ledger OGN-5. One book in the library.

2J.11 Worked example: applying the constitution to a chemistry problem

To make the ten-science export concrete, consider a simple chemistry problem: the equilibrium of a reversible reaction $A \rightleftharpoons B$.

State variables. The concentrations $[A]$ and $[B]$, subject to the conservation law $[A] + [B] = C_{\text{total}}$.

POS-1 (gauge invariance). The dynamics must not depend on the units in which concentrations are measured. If you switch from moles per liter to molecules per cubic meter, the equilibrium constant must transform covariantly.

POS-2 (extremal principle). The equilibrium is the minimum of the Gibbs free energy $G = H - TS$. POS-2 demands that this free-energy minimization be the boundary projection of a bulk variational principle.

POS-4 (positivity). Concentrations must remain non-negative: $[A] \geq 0$ and $[B] \geq 0$ at all times. This is the chemical analog of the CPTP condition.

POS-5 (RG compatibility). The macroscopic description (the equilibrium constant K_{eq}) must be consistent with the microscopic description (the potential energy surface). This is the foundation of transition-state theory.

POS-7 (falsifiability). The claim "this reaction has equilibrium constant $K_{\text{eq}} = 10^3$ at 300 K" must come with a measurement protocol (spectroscopic determination of $[A]$ and $[B]$) and a kill criterion (K_{eq} measured outside the range $[900, 1100]$ would falsify the claim).

This example shows that the constitution is not an abstract philosophical exercise. It has teeth—real operational consequences for how science is done, even in a simple chemistry problem.

2J.12 Worked example: applying the constitution to a neuroscience problem

Consider the problem of modeling visual perception: how does the brain infer the identity of an object from noisy sensory data?

State variables. A probability distribution $p(h|d)$ over hypotheses h ("this is a cat", "this is a dog") given data d (the pattern of photoreceptor activations).

AX-3 (operational measurement). The theory must specify how the brain's internal representation maps to observable behavior (e.g., pressing a button labeled "cat" or "dog").

POS-2 (extremal principle). The brain's inference can be modeled as free-energy minimization: the brain adjusts its internal model to minimize the discrepancy between predicted and actual sensory input. This is the Free Energy Principle, which POS-2 identifies as a boundary projection of the bulk variational engine.

POS-4 (positivity). The probability distribution $p(h|d)$ must remain non-negative and normalized: $\sum_h p(h|d) = 1$. This is the Bayesian analog of the CPTP condition.

POS-5 (RG compatibility). The neural representation at the single-neuron level must be consistent with the population-level description. A "coarse-grained" model that averages over individual neurons must make the same predictions as the fine-grained model—just with less detail.

POS-7 (falsifiability). The claim "the brain performs approximate Bayesian inference" must come with a registered observable (e.g., "the reaction-time distribution for ambiguous stimuli has a specific shape") and a kill criterion ("if the measured distribution deviates from the predicted shape by more than 3 standard deviations, the model is falsified").

Applications: What the Constitution Produces

2J.1 From constitution to engines: GG-6 and GDOS

The eleven constitutional clauses do not merely filter theory space; they actively *generate* specific mathematical structures. Two of these structures are so central that they function as "engines" for the entire program.

GG-6: the ultraviolet engine. When you instantiate AX-2 (geometry–gauge equivalence) under the seven postulates, the constitution forces a specific six-dimensional warped bulk with gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity, a Green–Schwarz-modified Bianchi identity (BI-6), and a discrete normalization ledger (OGN-5). This is the GG-6 engine, described in the companion chapter GG-2026-2 and its accessible companion Chapter 3, Part II.

GG-6 operates at the highest energies—the GUT scale, $E_{\mathrm{GUT}} \sim 2 \times 10^{16}$ GeV. It describes the unified state of all forces before they separate into the distinct sectors we observe at low energies.

GDOS: the infrared engine. When you instantiate AX-3 (operational measurement) under the seven postulates, the constitution forces a specific open-system dynamics for the four-dimensional boundary theory. This is the GDOS (Geometric Dynamics of Open Systems) engine, described in Chapter 4.

GDOS operates at all energies and all scales. It describes how closed bulk systems project to open boundary systems, how decoherence and irreversibility emerge, and how measurement, thermodynamics, and even biological self-organization arise from a common variational principle.

The two engines are complementary: GG-6 builds the ultraviolet stage (the six-dimensional bulk), and GDOS projects it down to the infrared reality (the four-dimensional world we observe). Together, they span the full derivation chain from the constitution to the Standard Model and beyond.

2J.2 Preview of GG-6: six forces in six dimensions

The GG-6 engine produces several remarkable results. Here we preview the highlights; full details are in Chapter 3, Part II.

Six dimensions, not ten or eleven. The constitution forces exactly six dimensions: four for spacetime, one for the renormalization scale (POS-5), and one for the compact gauge phase (POS-1 + POS-6). No more, no less.

Six forces, not four. In addition to the four familiar forces (strong, weak, electromagnetic, gravitational), GG-6 predicts two new forces: a baryonic force (gauged $\mathrm{U}(1)_B$, protecting the proton from decay) and an informational force (gauged $\mathrm{U}(1)_{Qi}$, providing a conserved information ledger required by AX-3).

One locking identity. All six force sectors are locked together by the Bianchi identity BI-6, whose coefficients are uniquely determined. There is no freedom to adjust the relative strengths of different sectors.

Five integers instead of a landscape. The OGN-5 package $(1, 3, 5; 7, 35)$ replaces the continuous moduli of conventional theories with a discrete arithmetic structure.

2J.3 Preview of GDOS: open systems from first principles

The GDOS engine addresses the fact that real science deals with open systems. A particle detector is an open system. A living cell is an open system. A brain is an open system. A star is an open system. Even the entire observable universe is, from the bulk perspective, an open system (its "environment" is the unobservable portion of the six-dimensional bulk).

GDOS provides a constitutional framework for all of them.

The key idea. The four-dimensional boundary theory inherits its dynamics from the six-dimensional bulk via the operational dictionary (AX-3). This inheritance is not metaphorical; it is mathematical. The bulk density operator ρ_{Bulk} evolves unitarily. When you trace out the bulk degrees of freedom that are not directly accessible from the boundary, you obtain a reduced density operator ρ_{Bdy} that evolves non-unitarily—irreversibly, with decoherence and dissipation.

This is the Nakajima–Zwanzig projection, a standard tool in open quantum systems theory. What is new in GDOS is that the projection is not an approximation; it is the exact implementation of AX-3's operational dictionary.

What emerges. Several fundamental features of four-dimensional physics emerge from the GDOS projection:

(i) Quantum mechanics emerges. The Hilbert-space structure of quantum mechanics, the Born rule, and wavefunction collapse are not separate postulates; they are consequences of the bulk-to-boundary projection. The Born rule follows from the trace operation; collapse follows from the Kraus representation of the measurement instruments.

(ii) Thermodynamics emerges. The second law of thermodynamics (entropy increases in isolated systems) is a consequence of the CPTP nature of the boundary evolution (POS-4). A CPTP map cannot decrease the von Neumann entropy of the system, so the arrow of time is built into the constitutional structure.

(iii) Measurement emerges. A "measurement" is not a mysterious process; it is a specific type of boundary operation—a quantum instrument—that the dictionary extracts from the bulk. The dictionary specifies which bulk degrees of freedom are accessible, which are traced out, and what operations correspond to specific measurement outcomes.

(iv) Classical behavior emerges. Decoherence—the process by which quantum superpositions become effectively classical—arises naturally from the bulk-to-boundary trace. The environment (the traced-out bulk degrees of freedom) monitors the system and destroys quantum coherence on a timescale that depends on the system-environment coupling. This is not a new hypothesis; it is a constitutional consequence.

The variational engine. POS-2 (the unified extremal principle) connects the bulk least-action principle to the boundary free-energy principle. In the bulk, the dynamics is governed by the principle of least action (PLA): the fields evolve along paths that extremize the six-dimensional action S_6. On the boundary, the dynamics is governed by free-energy descent: the system evolves toward states that minimize the generalized free energy $F = U - TS + \ldots$.

These are not two different principles; they are two projections of one variational engine. PLA in the bulk becomes free-energy descent on the boundary, just as a three-dimensional object in the sunlight produces a two-dimensional shadow that moves when the object moves.

Metaphor: the shadow theater. Imagine a shadow puppet theater. The puppeteer (the bulk) manipulates three-dimensional puppets behind a screen. The audience (the boundary observers) sees only the two-dimensional shadows. The shadows obey their own "laws of motion" (how they grow, shrink, merge, and split), but these laws are entirely determined by the three-dimensional mechanics of the puppets.

GDOS is the mathematics of this shadow theater. The six-dimensional bulk is the puppeteer. The four-dimensional boundary is the screen. The operational dictionary (AX-3) is the lamp that casts the shadow. And the "laws of physics" we observe in four dimensions are the shadow-laws: derived, not fundamental.

2J.4 The ten-science export: one constitution, ten restaurants

The constitution does not apply only to particle physics. Its modular structure allows it to be "exported" to all ten branches of science.

For each branch, the export requires three things:

(i) **A state interface:** What are the state variables? (For particle physics: fields and couplings. For chemistry: concentrations and fluxes. For neuroscience: neural population states.)

(ii) **An admissible dynamics:** The evolution must satisfy POS-1 through POS-6. In practice, this means the dynamics must be gauge-covariant, variational, anomaly-free, positive, RG-compatible, and minimally normalized.

(iii) **Registered observables with falsifiers:** POS-7 demands that every claim in every branch come with a concrete measurement protocol and a pre-declared kill criterion.

The claim is not reductionism. GUT does not say that biology is "really" physics. It says that biology, like physics, must obey the same foundational hygiene standards: gauge-invariant descriptions, positive evolution, scale-compatible predictions, and falsifiable claims.

Let us walk through each of the ten branches to see what the constitutional export looks like in practice.

1. GUT-Particle: quarks, leptons, and forces. This is the most natural home for the constitution. The state variables are quantum fields; the dynamics is governed by the Standard Model Lagrangian (derived from GG-6); the observables are cross-sections, decay rates, and mass spectra measured at colliders. The constitutional contribution is to derive the gauge group, the anomaly structure, and the normalization constants from first principles rather than inputting them by hand.

2. GUT-Cosmos: the expanding universe, stars, and black holes. Cosmology and astrophysics deal with the large-scale structure and evolution of the universe, from the Big Bang to galaxy formation, stellar evolution, black holes, and gravitational waves. The state variables are the density parameters $\Omega_i(t)$, the Hubble rate $H(t)$, inflationary order parameters, compact-object parameters (mass, spin, charge), accretion rates, and gravitational lensing maps.

The observables are the CMB spectrum, BAO, the distance-redshift relation, timing data, spectra, lensing shear, and gravitational wave signals.

The constitutional contribution is threefold. First, POS-5 (RG compatibility) demands that the effective cosmological description at different energy scales be consistent. Second, AX-4 (minimum freedom) constrains the number of cosmological parameters: the dark energy equation of state, the inflationary potential, and the baryon asymmetry must all be derivable from the constitution rather than fitted to data. Third, astrophysical models must be constitutionally admissible: a model of black hole accretion must satisfy POS-4 (positivity) and POS-7 (falsifiability).

3. GUT-Chemistry: molecules and reactions. Chemistry is the first "non-physics" branch, and the constitutional export to chemistry is a good test case. The state variables are concentrations of chemical species, reaction rates, and thermodynamic potentials. The observables are yields, reaction kinetics, and spectroscopic data.

The constitutional contribution is:

- POS-4 (positivity): concentrations must remain non-negative. This rules out unphysical solutions of reaction-diffusion equations.
- POS-5 (RG compatibility): the effective description of a chemical system at the molecular level must be consistent with its description at the macroscopic level. This is the foundation of statistical thermodynamics.
- POS-2 (unified extremal principle): chemical equilibrium is a free-energy minimum, which POS-2 connects to the same variational engine that governs particle physics.
- POS-7 (falsifiability): a claim like "this catalyst increases yield by 20%" must come with a specified measurement protocol and a quantitative falsifier.

4. GUT-Life: living systems and evolution. Biology deals with systems that are far from equilibrium, self-organizing, and adaptive. The state variables include population sizes, genetic sequences, metabolic fluxes, and developmental coordinates. The observables are growth curves, fitness landscapes, gene expression profiles, and evolutionary trajectories.

The constitutional export to biology may seem like a stretch, but it is surprisingly natural. The key connection is through POS-2 (the unified extremal principle) and POS-4 (closure and positivity). Living systems can be modeled as open systems whose dynamics minimizes a generalized free energy—this is the essence of the Free Energy Principle (FEP) proposed by Karl Friston. GUT provides the constitutional foundation for this principle by showing that FEP is a boundary projection of the bulk variational engine (POS-2).

A concrete example: the growth of a bacterial colony in a nutrient-limited environment can be described by a GDOS flow on a state manifold. The colony's concentration evolves according to a positive, gauge-covariant, RG-compatible dynamics whose equilibrium state minimizes a thermodynamic potential. POS-7 demands that any claim about the colony's behavior (e.g., "the growth rate saturates at carrying capacity") come with a measurement protocol and a falsifier.

5. GUT-Mind: neural systems and cognition. Neuroscience is perhaps the most ambitious target for the constitutional export. The state variables are neural population firing rates, synaptic weights, and belief distributions. The observables are EEG/fMRI signals, behavioral metrics, and psychophysical measurements.

The constitutional connection runs through AX-3 (operational measurement) and POS-2 (the extremal principle). The brain can be modeled as an open system that performs approximate Bayesian inference by minimizing variational free energy. This is Friston's Free Energy Principle applied to neural computation. In the GUT framework, this is not a separate postulate but a consequence of the constitution: the brain's dynamics is a GDOS flow, and Bayesian inference is the boundary projection of the bulk variational engine.

6. GUT-Health: physiology and medicine. Health science deals with multi-layered physiological networks: genes, proteins, cells, organs, and whole organisms. The state variables are biomarker vectors, network connectivity matrices, and homeostatic control parameters. The observables are clinical measurements, imaging data, and health outcomes.

The constitutional contribution is to demand that medical interventions be described as GDOS control operations with registered falsifiers. A drug treatment, for example, is an intervention that modifies the Lindblad operators in the GKLS equation of the patient's physiological state. POS-7 demands that the claim "this drug reduces blood pressure by 10 mmHg" come with a pre-declared measurement protocol and a kill criterion.

7. GUT-Society: collective behavior and economics. Social science deals with networks of interacting agents. The state variables are economic indicators, social network metrics, opinion distributions, and policy parameters. The observables are macroeconomic data, survey results, and behavioral experiments. POS-1 (gauge invariance) demands that predictions not depend on arbitrary labeling choices. POS-7 (falsifiability) demands that policy recommendations come with registered kill criteria.

8. GUT-Environment: ecosystems and climate. Environmental science deals with the coupled dynamics of ecosystems, biogeochemical cycles, and climate. The state variables are species abundances, nutrient concentrations, atmospheric composition, and temperature fields. The observables are biodiversity indices, carbon fluxes, and climate time series.

The constitutional contribution is through POS-4 (positivity and closure) and POS-5 (RG compatibility): environmental models must be consistent across spatial and temporal scales, from microbial soil communities to global carbon cycles. POS-7 demands that climate projections come with falsifiable predictions and pre-declared kill criteria.

9. GUT-AI: artificial intelligence and machine learning. AI deals with engineered information-processing systems that learn, adapt, and make decisions. The state variables are network parameters, training states, and information throughput budgets. The observables are learning curves, generalization metrics, and efficiency/entropy trade-offs.

The constitutional contribution is AX-4 (minimum freedom): a well-designed AI system should use the minimum number of adjustable parameters consistent with its function. POS-2 (the extremal principle) connects to the variational optimization that underlies modern machine learning. The Qi-4 framework offers a diagnostic lens: does a given AI system exhibit the signatures of Qi-QUA (current locking), Qi-EP (trajectory selection), Qi-CON (scale covariance), and Qi-HAL (memory architecture)?

10. GUT-Philosophy: the audit layer. Philosophy occupies a special position in the GUT architecture: it is the branch that audits the constitution itself. The state variables are meta-level claims about the admissibility of theories, the nature of explanation, and the scope of falsifiability.

POS-7 applies to philosophy as well: a philosophical claim that cannot be connected—however indirectly—to an operational consequence is constitutionally inadmissible. This does not eliminate philosophy; it sharpens it. It says: philosophical claims must have empirical teeth, even if the teeth are indirect.

The health-code metaphor revisited. The ten sciences are like ten restaurants on the same street. Italian, Japanese, Mexican, Indian, French, Thai, Ethiopian, Korean, American, and molecular gastronomy. Each serves different food. But all must pass the same health inspection: clean kitchens (POS-4: positivity), proper labeling of ingredients (POS-1: gauge invariance), consistent recipes (POS-5: RG compatibility), and a clear allergen disclosure (POS-7: falsifiability). The health code does not tell each restaurant what to cook. It tells all of them what standards to meet.

The Derivation Chain: A Visual Summary

2J.1 From constitution to prediction in one diagram

The full logical structure of the GUT program can be compressed into a single derivation chain:

$$\boxed{\text{Constitution (AX/POS)} \implies \text{GG-6 (6D bulk)} \implies \text{GDOS (4D boundary)} \implies \text{SM + predictions}}$$

Each arrow represents a theorem—a mathematical derivation, not a guess or an assumption. The input is the constitution (eleven clauses). The output is a specific four-dimensional quantum field theory with sharp numerical predictions.

Let us trace this chain step by step.

Step 1: Constitution ⇒ GG-6. The four axioms and seven postulates force a six-dimensional warped bulk with gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity, Bianchi identity BI-6, and normalization ledger OGN-5. This step is the subject of GG-2026-2 and Chapter 3, Part II.

Step 2: GG-6 ⇒ GDOS. The six-dimensional bulk projects down to a four-dimensional boundary theory via the operational dictionary (AX-3). The boundary dynamics is an open-

system evolution of GKLS type. Quantum mechanics, thermodynamics, and measurement emerge from this projection. This step is the subject of GG-2026-3.

Step 3: GDOS ⇒ Standard Model. The zero-mode truncation of the six-dimensional theory reproduces the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ plus the two new Abelian sectors. The coupling constants, mass hierarchies, and CKM mixing angles are determined by the warped geometry and the overlap integrals of zero-mode wavefunctions. This step is the subject of GG-2026-3 and subsequent phenomenological papers.

Step 4: Standard Model ⇒ predictions. The derived Standard Model makes specific numerical predictions that can be compared with experimental data. POS-7 demands that these predictions come with registered kill criteria.

2J.2 What has been completed and what remains

As of March 2026, the derivation chain is at the following stage:

- **Completed:** Steps 1 (GG-2026-1, constitution) and 2a (GG-2026-2, the six-dimensional bulk and its anomaly-free QFT).
- **In progress:** Step 2b (GG-2026-3, the GDOS boundary projection).
- **Planned:** Step 3 (explicit Standard Model derivation with numerical benchmarks) and Step 4 (comparison with collider, flavor, neutrino, and cosmological data).

The program is ambitious but the roadmap is clear. Each step is a specific mathematical task with a well-defined output. The companion chapters and accessible companions will document each step as it is completed.

Discussion: Significance, Comparisons, and the Road Ahead

2J.1 What makes GUT different from previous axiom systems

Several axiom systems have been proposed in the history of physics. How does GUT compare?

Von Neumann's axioms for quantum mechanics (1932). Von Neumann formalized quantum mechanics on a Hilbert space with linear operators and a projection postulate (von Neumann, 1932). This is the monograph axiomatization. GUT differs in three ways: it includes a constitutional definition of measurement (AX-3), it extends to open systems (POS-4), and it applies beyond physics (the ten-science export).

Wightman's axioms for QFT (1956). Arthur Wightman proposed axioms for relativistic quantum field theory based on locality, covariance, and positivity of energy. These axioms are mathematically rigorous but narrow in scope: they apply only to relativistic quantum fields on flat

spacetime. GUT includes Wightman-type locality (AX-1) but extends it to curved backgrounds, open systems, and non-physics domains.

The Standard Model as an implicit axiom system. The Standard Model is sometimes described informally by its "axioms": gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, three generations of fermions, and a Higgs doublet. But these are inputs, not constitutional rules. GUT aims to *derive* the gauge group and matter content from constitutional principles, rather than assuming them.

String theory's mathematical framework. String theory provides a rich mathematical framework but has not been axiomatized in the same sense as GUT. There is no agreed-upon list of axioms from which all of string theory can be derived. GUT is explicitly axiomatic: every claim can be traced back to the eleven clauses.

2J.2 Comparison with the Standard Model

The Standard Model is the most successful physical theory ever constructed. It predicts thousands of experimental results with extraordinary precision. GUT does not aim to replace the SM; it aims to *derive* it from a constitutional foundation.

The key differences are:

- **Gauge group:** In the SM, the gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is assumed. In GUT, it is derived from AX-2 + AX-4 via the intermediate SU(5) unification at the GUT scale.
- **Measurement:** In the SM, measurement is handled by the Born rule (an external postulate). In GUT, measurement is built into the constitution (AX-3) and emerges as a consequence of the bulk-to-boundary projection.
- **Free parameters:** The SM has $\sim$19 free parameters that must be measured and plugged in. GUT aims to reduce this to Newton's constant + five integers.
- **Proton stability:** In the SM, baryon number is an accidental symmetry. In GUT, it is an exact gauge symmetry ($\mathrm{U}(1)_B$), making the proton absolutely stable.
- **Scope:** The SM applies to particle physics. GUT applies to ten domains of science.

It is worth emphasizing what GUT owes to the SM. The entire Yang–Mills gauge structure, the Higgs mechanism, the three-generation pattern, and the extraordinary precision of electroweak and QCD predictions are the empirical bedrock on which GUT stands. The constitution does not dismiss this bedrock; it attempts to *explain* it.

2J.3 Comparison with supersymmetry and string theory

Supersymmetry and string theory are the two dominant paradigms for physics beyond the Standard Model. GUT is explicitly distinct from both.

Supersymmetry. Supersymmetry (SUSY) posits a symmetry between bosons (force carriers) and fermions (matter particles), doubling the particle spectrum. It was proposed in the 1970s (Wess and Zumino, 1974) and has been the leading candidate for beyond-the-Standard-Model physics for decades. SUSY improves gauge coupling unification, stabilizes the Higgs mass against large quantum corrections (solving the "hierarchy problem"), and provides a natural dark matter candidate.

However, no superpartner has been observed despite decades of searches at the LHC. The lower bounds on superpartner masses have been pushed to the TeV scale, creating tension with the original naturalness motivation.

GUT does not introduce supersymmetry because it is not forced by the constitutional clauses. AX-4 (minimum freedom) forbids any structure that is not compelled by consistency. If SUSY were necessary for anomaly cancellation or positivity, the constitution would allow it. Since it is not, the constitution excludes it.

This is not anti-SUSY polemic. It is a methodological statement: the constitution asks "what is forced?" rather than "what is possible?" If nature turns out to require SUSY, the constitutional stance would need revision.

String theory. String theory replaces point particles with one-dimensional strings, leading to a rich mathematical structure that includes extra dimensions, branes, dualities, the holographic principle, and the Green–Schwarz anomaly cancellation mechanism.

GUT borrows two mathematical ideas from string theory:

(i) **Anomaly cancellation by inflow** (the Green–Schwarz mechanism): used in POS-3 and realized in the GG-6 Bianchi identity BI-6.

(ii) **The holographic principle** (bulk-to-boundary correspondence): used in AX-3 as the inspiration for the operational dictionary.

But GUT does not adopt the full ontology of string theory. The key differences are:

- **Dimensionality:** String theory requires 10 (or 11) dimensions. GUT requires exactly 6, derived from the constitutional clauses.
- **Supersymmetry:** String theory requires SUSY for consistency (at least in its perturbative formulations). GUT does not.
- **Landscape:** String theory admits $\sim 10^{500}$ possible vacuum solutions. GUT admits one (up to the five OGN-5 integers).
- **UV endpoint:** String theory aims at the Planck scale ($E_{\mathrm{Pl}} \sim 10^{19}$ GeV). GUT stops at the GUT scale ($E_{\mathrm{GUT}} \sim 10^{16}$ GeV).
- **Fundamental objects:** In string theory, the fundamental objects are extended strings. In GUT, the fundamental object is a composite point-particle field (the geometry–gauge connection).

The relationship in one sentence. GUT uses the *mathematics* of string theory (anomaly cancellation, holography) without adopting its *ontology* (strings, branes, SUSY, the landscape). If nature turns out to require the full stringy ontology, GUT would need to be embedded in a string framework (or abandoned). If nature does not require it, GUT provides a leaner alternative.

2J.4 Comparison with historical axiom systems

How does GUT compare with previous attempts to axiomatize science?

Euclid's axioms for geometry. Euclid's system is the gold standard of axiomatic reasoning: five postulates, from which all of geometry follows. GUT aspires to the same economy but for a much vaster domain. The key difference is that Euclid's axioms are purely mathematical, while GUT's axioms are physical: they make empirical claims that can be tested.

Newton's laws of motion. Newton's three laws are operational axioms: they tell you how forces relate to motion. GUT's axioms are more abstract: they do not specify forces or motions directly but specify the *admissibility conditions* that any force law must satisfy.

Hilbert's axioms for geometry (1899). Hilbert reformulated Euclid's geometry with a more rigorous axiom system, filling in gaps that Euclid had left. GUT aims to do the same for physics: to identify the implicit assumptions that twentieth-century physics left unspoken, and to make them explicit.

Von Neumann's axioms for quantum mechanics (1932). Von Neumann's axiomatization introduced Hilbert spaces, linear operators, and the projection postulate. GUT includes these as consequences but adds three crucial elements: the geometry–gauge equivalence (AX-2), the operational measurement dictionary (AX-3), and the minimum-freedom principle (AX-4).

2J.5 Falsifiability of the GUT program

The constitutional program must be scientifically exposed. The main kill criteria are:

(i) **Proton decay:** falsifies exact $\mathrm{U}(1)_B$ protection.

(ii) **Discovery of essential supersymmetry:** challenges the non-SUSY stance.

(iii) **Continuous moduli in nature:** falsifies OGN-5 compression.

(iv) **Anomaly-locking failure:** falsifies the BI-6 construction.

(v) **Constitutional internal inconsistency:** if the eleven clauses are found to be mutually contradictory, the entire program collapses.

2J.6 A direction for 21st-century science

The most ambitious aspect of GUT is its claim that science can be unified—not by reducing all domains to physics, but by giving all domains a shared constitutional foundation.

This vision is testable at multiple levels.

Level 1: Internal mathematical consistency. The eleven clauses must not be mutually contradictory. Every theorem derived from the axioms must follow rigorously. This is a mathematical question that can be checked by independent researchers.

Level 2: Physical predictions. If the constitution succeeds in deriving the Standard Model from GG-6 + GDOS, it must make concrete, falsifiable predictions—proton stability, gauge coupling ratios at the GUT scale, the absence of superpartners, specific Kaluza–Klein spectra. These are experimental questions.

Level 3: Cross-domain applicability. If the ten-science export works, the constitution should produce nontrivial, falsifiable predictions in chemistry, biology, neuroscience, and beyond. For example: does the GDOS variational engine correctly predict the scaling laws of metabolic networks? Does the Free Energy Principle, derived from POS-2, correctly predict neural responses to ambiguous stimuli? These are empirical questions that can be tested in laboratories far from the LHC.

Level 4: Philosophical coherence. The constitution makes specific philosophical commitments: that science is falsifiable (POS-7), that measurement is constitutional (AX-3), that freedom must be justified (AX-4). These commitments can be evaluated by philosophers of science, and they may turn out to be too restrictive (ruling out legitimate scientific practices) or too permissive (failing to exclude pseudoscience).

If the program succeeds at all four levels, it will have achieved something new: a *constitutional unification of knowledge*. Not a Theory of Everything in the string-theory sense (a single Lagrangian from which all phenomena can be derived), but a constitutional framework in the legal sense (a set of admissibility rules that all scientific theories must obey).

If the program fails—if the eleven clauses turn out to be too restrictive (ruling out nature itself) or too permissive (admitting a landscape)—then it will have been a useful exercise in clarifying what "unification" means, even if the specific realization is wrong.

Either way, the constitutional approach represents a direction worth exploring. The twentieth century was the century of models: propose a specific theory, check it against data, modify it if it fails. The twenty-first century may be the century of constitutions: specify the admissibility rules first, then let the mathematics find the unique surviving theory.

A historical parallel. In the seventeenth century, the scientific revolution replaced the Aristotelian method (authority-based reasoning) with the empirical method (observation-based

reasoning). This was not just a change of specific theories; it was a change of *meta-theory*—a new set of rules for what counts as legitimate scientific reasoning.

GUT proposes a similar meta-theoretical shift for the twenty-first century. The new rule is: every scientific claim must compile to registered observables, operate under admissible dynamics, and come with a pre-declared falsifier. This rule applies not just to particle physics but to all of science. If it catches on—even if the specific eleven clauses turn out to need revision—it will have changed the way science is done.

Appendices for Chapter 3

GG-6: Geometry–Gauge Locking

Appendix 3A: exact Nakajima–Zwanzig reduction and the OGGEP channel

This appendix expands the derivation summarized in Section 4. Let the bulk density operator obey

$$\dot{\rho} = \mathfrak{L}(t)\rho, \qquad \mathfrak{L}(t)X = -\mathrm{i}[H(t), X]. \tag{3A.1}$$

Choose the projection

$$\mathcal{P}X = \mathrm{Tr}_{\mathrm{env}}(X) \otimes \rho_{\mathrm{env}}^{(0)}, \qquad \mathcal{Q} = 1 - \mathcal{P}. \tag{3A.2}$$

Acting with $\mathcal{P}$ and $\mathcal{Q}$ gives

$$\dot{\mathcal{P}}\rho = \mathcal{P}\mathfrak{L}\mathcal{P}\rho + \mathcal{P}\mathfrak{L}\mathcal{Q}\rho, \tag{3A.3}$$

$$\dot{\mathcal{Q}}\rho = \mathcal{Q}\mathfrak{L}\mathcal{P}\rho + \mathcal{Q}\mathfrak{L}\mathcal{Q}\rho. \tag{3A.4}$$

The second equation is solved by variation of constants:

$$\mathcal{Q}\rho(t) = \mathcal{G}_{\mathcal{Q}}(t, t_0)\mathcal{Q}\rho(t_0) + \int_{t_0}^{t} \mathrm{d}s\, \mathcal{G}_{\mathcal{Q}}(t, s)\mathcal{Q}\mathfrak{L}(s)\mathcal{P}\rho(s), \tag{3A.5}$$

where

$$\mathcal{G}_{\mathcal{Q}}(t, s) = \mathcal{T} \exp\left(\int_{s}^{t} \mathrm{d}u\, \mathcal{Q}\mathfrak{L}(u)\mathcal{Q} \right). \tag{3A.6}$$

Substituting back, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}\rho(t) = \mathcal{P}\mathfrak{L}(t)\mathcal{P}\rho(t) + \int_{t_0}^{t} \mathrm{d}s\, \mathcal{P}\mathfrak{L}(t)\mathcal{G}_{\mathcal{Q}}(t, s)\mathcal{Q}\mathfrak{L}(s)\mathcal{P}\rho(s) + \mathcal{P}\mathfrak{L}(t)\mathcal{G}_{\mathcal{Q}}(t, t_0)\mathcal{Q}\rho(t_0). \tag{3A.7}$$

This is the exact projected equation.

To connect this with the boundary language, define

$$\mathfrak{D}_{\mathrm{OGGEP}} = \mathcal{R} \circ \mathrm{Tr}_{\mathrm{env}}, \tag{3A.8}$$

with $\mathcal{R}$ a boundary restriction or coding map. Then the boundary state is

$$\rho_{\mathrm{Bdy}}(t) = \mathfrak{D}_{\mathrm{OGGEP}}[\rho_{\mathrm{Bulk}}(t)]. \tag{3A.9}$$

Applying $\mathfrak{D}_{\mathrm{OGGEP}}$ to the projected equation yields an exact nonlocal equation for $\rho_{\mathrm{Bdy}}(t)$:

$$\dot{\rho}_{\mathrm{Bdy}}(t) = \mathfrak{D}_{\mathrm{OGGEP}}[\mathcal{P}\mathfrak{L}(t)\mathcal{P}\rho(t)] + \int_{t_0}^{t} \mathrm{d}s\, \mathfrak{D}_{\mathrm{OGGEP}}[\mathcal{K}(t, s)\mathcal{P}\rho(s)] + \mathfrak{D}_{\mathrm{OGGEP}}[\mathcal{I}(t)]\,. \tag{3A.10}$$

The first term is local in time, the second carries memory, and the third retains initial correlations. In a weak-memory regime one replaces $\rho(s)$ by a time-local expansion around t, producing a time-local generator $\mathfrak{L}_{\mathrm{GDOS}}(t)$.

A useful special case is the Born–Markov factorization

$$\rho_{\mathrm{Bulk}}(t) \approx \rho_{\mathrm{Bdy}}(t) \otimes \rho_{\mathrm{env}}^{(0)}, \tag{3A.11}$$

together with a short environment correlation time. Then

$$\dot{\rho}_{\mathrm{Bdy}}(t) = \mathfrak{L}_{\mathrm{GDOS}}(t)\rho_{\mathrm{Bdy}}(t) \tag{3A.12}$$

with $\mathfrak{L}_{\mathrm{GDOS}}$ of GKLS type.

The Stinespring dilation theorem guarantees that every CPTP boundary channel has a unitary dilation (Stinespring, 1955). Thus the derivation can be read in both directions:

(i) starting from the closed bulk and projecting down gives a boundary CPTP map;

(ii) starting from a boundary CPTP map, one can reconstruct a unitary dilation with an enlarged environment.

The present framework uses direction (i) as the physical explanation and direction (ii) as the mathematical consistency check.

Finally, measurement instruments may be written in Kraus form:

$$\mathcal{I}_\alpha(\rho) = \sum_m M_{\alpha m}\rho M_{\alpha m}^\dagger, \qquad \sum_{\alpha,m} M_{\alpha m}^\dagger M_{\alpha m} = \mathbf{1}. \tag{3A.13}$$

In the GG-6/GDOS interpretation the Kraus operators are not primitive. They are the reduced images of bulk interaction channels after environment tracing and boundary restriction.

Appendix 3B: Kaluza–Klein reduction, overlap integrals, and effective couplings

This appendix records the elementary reduction formulas used in Sections 4 and 5. For the warped metric

$$\mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2 \mathrm{d}\theta^2, \tag{3B.1}$$

the determinant is

$$\sqrt{-g_6} = e^{-4A(r)} R_\theta \sqrt{-g_4}. \tag{3B.2}$$

Gravitational zero mode

The Einstein–Hilbert term reduces as

$$\begin{aligned} S_{\mathrm{EH}}^{(6)} &= \frac{M_6^4}{2} \int \mathrm{d}^4x\, \mathrm{d}r\, \mathrm{d}\theta\, e^{-4A(r)} R_\theta \sqrt{-g_4} \left(e^{2A(r)} R_4 + \cdots \right) \\ &= \frac{1}{2} \Big(M_6^4\, 2\pi R_\theta \int \mathrm{d}r\, e^{-2A(r)} \Big) \int \mathrm{d}^4x \sqrt{-g_4}\, R_4 + \cdots . \end{aligned} \tag{3B.3}$$

Hence

$$M_{\mathrm{Pl}}^2 = M_6^4\, 2\pi R_\theta \int \mathrm{d}r\, e^{-2A(r)}. \tag{3B.4}$$

Gauge zero modes

For a gauge field with zero-mode profile $f_0(r)$,

$$\begin{aligned} S_{\mathrm{gauge}}^{(6)} &= -\frac{1}{4g_6^2} \int \mathrm{d}^4x\, \mathrm{d}r\, \mathrm{d}\theta\, e^{-4A(r)} R_\theta \sqrt{-g_4}\, e^{2A(r)} e^{2A(r)}\, |f_0(r)|^2\, F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + \cdots \\ &= -\frac{1}{4} \left(\frac{2\pi R_\theta}{g_6^2} \int \mathrm{d}r\, |f_0(r)|^2 \right) \int \mathrm{d}^4x\, \sqrt{-g_4}\, F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + \cdots . \end{aligned} \tag{3B.5}$$

Thus

$$\frac{1}{g_4^2} = \frac{2\pi R_\theta}{g_6^2} \int \mathrm{d}r\, |f_0(r)|^2 . \tag{3B.6}$$

The absence of warp factors in the gauge normalization is a direct consequence of the cancellation between $\sqrt{-g_6}$ and the two inverse metrics.

Fermion zero modes

For a chiral fermion,

$$\Psi(x, r, \theta) = \sum_{n,\ell} \psi^{(n,\ell)}(x) \chi_{n,\ell}(r, \theta), \tag{3B.7}$$

the kinetic term becomes

$$\int \mathrm{d}^6x \sqrt{-g_6}\, \bar{\Psi} \mathrm{i}\Gamma^\mu D_\mu \Psi \to \int \mathrm{d}^4x \sqrt{-g_4}\, \bar{\psi}^{(0)} \mathrm{i}\gamma^\mu D_\mu \psi^{(0)} \tag{3B.8}$$

provided the internal wavefunctions satisfy the normalization condition

$$2\pi R_\theta \int \mathrm{d}r\, e^{-3A(r)}\, \chi_0^\dagger(r)\chi_0(r) = 1 \tag{3B.9}$$

in the standard warped spinor convention.

Yukawa overlap integrals

A six-dimensional Yukawa coupling

$$\lambda_6 \int \mathrm{d}^6x\sqrt{-g_6}\, \Psi_1\Psi_2 H \tag{3B.10}$$

reduces to a four-dimensional Yukawa

$$\lambda_4 = \lambda_6\, 2\pi R_\theta \int \mathrm{d}r\, e^{-4A(r)}\, \chi_1(r)\chi_2(r)\eta_H(r), \tag{3B.11}$$

up to gamma-matrix and chirality conventions. Therefore flavor hierarchies may arise geometrically through profile overlaps even if the underlying six-dimensional coupling matrix is comparatively simple.

KK masses

For scalar fields, the mode equation has the generic form

$$-\partial_r\left(e^{-4A}\partial_r f_n\right) + e^{-4A}\frac{\ell^2}{R_\theta^2} f_n = m_{n,\ell}^2\, e^{-2A} f_n. \tag{3B.12}$$

The exact spectrum depends on the warp profile and boundary conditions, but the qualitative scaling

$$m_{n,\ell}^2 \sim m_n^2 + \frac{\ell^2}{R_\theta^2} \tag{3B.13}$$

is robust.

Appendix 3C: $SU(5)$ decomposition, Yukawa operators, and proton-stability selection rules

Write the antisymmetric **10** as $\Psi_{10}^{AB} = -\Psi_{10}^{BA}$ and the antifundamental as $\Psi_{\bar{5},A}$, with $A = 1, \ldots, 5$. Split the index into a color part $\alpha = 1, 2, 3$ and a weak part $m = 4, 5$. Then

$$\Psi_{10}^{\alpha m} = Q^{\alpha m}, \qquad \Psi_{10}^{\alpha\beta} = \epsilon^{\alpha\beta\gamma} u_\gamma^c, \qquad \Psi_{10}^{45} = e^c, \tag{3C.1}$$

while

$$\Psi_{\bar{5},\alpha} = d_\alpha^c, \qquad \Psi_{\bar{5},m} = L_m. \tag{3C.2}$$

For the Higgs $\mathbf{5}_H$,

$$H_5^A = (T^\alpha, H^m), \tag{3C.3}$$

where T is the color triplet and H the weak doublet.

The standard SU(5)-covariant Yukawa operators are

$$\mathcal{L}_Y = -\frac{1}{4}(Y_u)_{ij}\epsilon_{ABCDE}\Psi_{10}^{AB(i)}\Psi_{10}^{CD(j)}H_5^E - \sqrt{2}(Y_d)_{ij}\Psi_{10}^{AB(i)}\Psi_{\bar{5},A}^{(j)}(H_5^\dagger)_B + \text{h.c.} \tag{3C.4}$$

Decomposing them into Standard Model components gives

$$\begin{aligned}\mathcal{L}_Y = &-(Y_u)_{ij}\Big(Q_i u_j^c H + \frac{1}{2}Q_i Q_j T + u_i^c e_j^c T\Big) \\ &- (Y_d)_{ij}\Big(Q_i d_j^c H^\dagger + e_i^c L_j H^\dagger + Q_i L_j T^\dagger + u_i^c d_j^c T^\dagger\Big) + \text{h.c.}\end{aligned} \tag{3C.5}$$

Equation (4D.5) shows immediately why minimal four-dimensional SU(5) typically threatens proton stability: exchange of the triplet T generates baryon-violating effective operators. In the present framework the danger is removed by two simultaneous mechanisms:

(i) the triplet zero mode is projected out or lifted by boundary conditions;

(ii) exact local $U(1)_B$ forbids every local operator with $\Delta B \neq 0$.

Thus the theorem-level proton stability of Section 5 does not rely on a numerical suppression of triplet exchange. The dangerous operator never appears in the exact low-energy theory.

A convenient charge check is the following. For the dimension-six operators most often discussed in GUT phenomenology,

$$QQQL: \quad \Delta B = +1, \tag{3C.6}$$

$$u^c u^c d^c e^c: \quad \Delta B = -1, \tag{3C.7}$$

$$QQQH: \quad \Delta B = +1, \tag{3C.8}$$

$$u^c d^c d^c: \quad \Delta B = -1. \tag{3C.9}$$

All are forbidden by local $U(1)_B$.

Appendix 3D: Anomaly coefficients for $U(1)_B$ and $U(1)_{Qi}$

This appendix collects the anomaly coefficients quoted in Section 5. We use the convention $T(\mathbf{fund}) = \frac{1}{2}$ for both SU(3) and SU(2), and we work with left-handed Weyl fermions.

Table 3D.1: One-generation contributions to the baryon-anomaly coefficients.

Field	$q_B T_3 d_2$	$q_B T_2 d_3$	$q_B Y^2 d_3 d_2$	$Y q_B^2 d_3 d_2$
Q	$\frac{1}{3}\cdot\frac{1}{2}\cdot 2 = \frac{1}{3}$	$\frac{1}{3}\cdot\frac{1}{2}\cdot 3 = \frac{1}{2}$	$\frac{1}{3}\cdot\frac{1}{36}\cdot 6 = \frac{1}{18}$	$\frac{1}{6}\cdot\frac{1}{9}\cdot 6 = \frac{1}{9}$
u^c	$-\frac{1}{3}\cdot\frac{1}{2} = -\frac{1}{6}$	0	$-\frac{1}{3}\cdot\frac{4}{9}\cdot 3 = -\frac{4}{9}$	$-\frac{2}{3}\cdot\frac{1}{9}\cdot 3 = -\frac{2}{9}$
d^c	$-\frac{1}{3}\cdot\frac{1}{2} = -\frac{1}{6}$	0	$-\frac{1}{3}\cdot\frac{1}{9}\cdot 3 = -\frac{1}{9}$	$\frac{1}{3}\cdot\frac{1}{9}\cdot 3 = \frac{1}{9}$
L	0	0	0	0
e^c	0	0	0	0
Total (1 gen)	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
Total (3 gen)	0	$\frac{3}{2}$	$-\frac{3}{2}$	0

The cubic and gravitational baryon anomalies also vanish:

$$\mathcal{A}_{BBB} = \sum_f q_B^3 d_3 d_2 = \frac{1}{27}\cdot 6 - \frac{1}{27}\cdot 3 - \frac{1}{27}\cdot 3 = 0, \tag{3D.1}$$

$$\mathcal{A}_{BRR} = \sum_f q_B d_3 d_2 = \frac{1}{3}\cdot 6 - \frac{1}{3}\cdot 3 - \frac{1}{3}\cdot 3 = 0. \tag{3D.2}$$

For the minimal diagonal Qi realization $q_{Qi} = q_B$, the entire anomaly table is identical:

$$\mathcal{A}_{Qi,33} = 0, \qquad \mathcal{A}_{Qi,22} = \frac{3}{2}, \qquad \mathcal{A}_{Qi,YY} = -\frac{3}{2}, \qquad \mathcal{A}_{QiQiQi} = \mathcal{A}_{QiRR} = \mathcal{A}_{YQiQi} = 0. \tag{3D.3}$$

More generally, the only constitutional requirement is that the Qi anomaly vector lie in the image of the same GS/Stückelberg cancellation matrix that descends from BI-6.

The operator interpretation of anomaly cancellation is then

$$\partial_\mu J^\mu_{I,\mathrm{matter}} + \partial_\mu J^\mu_{I,\mathrm{GS}} = 0, \qquad I \in \{B, Qi\}, \tag{3D.4}$$

which is the Abelian part of the homogeneous Ward algebra used in Section 5.

Appendix 3E: BV/BRST master equation, Ward operators, and current algebra

This appendix spells out the component-level gauge-fixing and Ward structure of the zero-mode theory. Introduce antifields

$$K_{G^a_\mu}, \quad K_{W^i_\mu}, \quad K_{B_\mu}, \quad K_{X^\mu_B}, \quad K_{X^\mu_{Qi}}, \quad K_\Phi$$

for every field Φ. The BV master action is

$$S_{\rm BV} = S_{4,\rm zero} + \int \mathrm{d}^4x \left[K_{G^a_\mu} sG^a_\mu + K_{W^i_\mu} sW^i_\mu + K_{B_\mu} sB_\mu + K_{X^\mu_B} sX^{(B)}_\mu + K_{X^\mu_{Qi}} sX^{(Qi)}_\mu + \sum_\Phi K_\Phi s\Phi \right]. \tag{3E.1}$$

Choose the gauge-fixing fermion

$$\begin{aligned}\Psi_{\rm gf} = \int \mathrm{d}^4x \Big[&\bar c^a_3 \Big(\partial^\mu G^a_\mu + \frac{\xi_3}{2} b^a_3 \Big) + \bar c^i_2 \Big(\partial^\mu W^i_\mu + \frac{\xi_2}{2} b^i_2 \Big) \\ &+ \bar c_Y \Big(\partial^\mu B_\mu + \frac{\xi_Y}{2} b_Y \Big) + \bar c_B \Big(\partial^\mu X^{(B)}_\mu + \frac{\xi_B}{2} b_B \Big) + \bar c_{Qi} \Big(\partial^\mu X^{(Qi)}_\mu + \frac{\xi_{Qi}}{2} b_{Qi} \Big) \Big].\end{aligned} \tag{3E.2}$$

Then

$$S_{\rm gf+gh} = s\Psi_{\rm gf}, \tag{3E.3}$$

and the gauge-fixed action is obtained by setting $K_\Phi = \delta\Psi_{\rm gf}/\delta\Phi$.

The quantum 1PI effective action Γ obeys the Slavnov functional identity

$$\begin{aligned}\mathcal{S}(\Gamma) = \int \mathrm{d}^4x \Bigg[&\frac{\delta\Gamma}{\delta K_{G^a_\mu}} \frac{\delta\Gamma}{\delta G^a_\mu} + \frac{\delta\Gamma}{\delta K_{W^i_\mu}} \frac{\delta\Gamma}{\delta W^i_\mu} + \frac{\delta\Gamma}{\delta K_{B_\mu}} \frac{\delta\Gamma}{\delta B_\mu} + \frac{\delta\Gamma}{\delta K_{X^\mu_B}} \frac{\delta\Gamma}{\delta X^{(B)}_\mu} + \frac{\delta\Gamma}{\delta K_{X^\mu_{Qi}}} \frac{\delta\Gamma}{\delta X^{(Qi)}_\mu} \\ &+ \sum_\Phi \frac{\delta\Gamma}{\delta K_\Phi} \frac{\delta\Gamma}{\delta\Phi} + b^a_3 \frac{\delta\Gamma}{\delta \bar c^a_3} + b^i_2 \frac{\delta\Gamma}{\delta \bar c^i_2} + b_Y \frac{\delta\Gamma}{\delta \bar c_Y} + b_B \frac{\delta\Gamma}{\delta \bar c_B} + b_{Qi} \frac{\delta\Gamma}{\delta \bar c_{Qi}} \Bigg] = 0.\end{aligned} \tag{3E.4}$$

Linearizing around the full solution defines the nilpotent operator

$$\mathcal{B}_\Gamma X = \frac{\mathrm{d}}{\mathrm{d}\varepsilon}\bigg|_{\varepsilon=0} \mathcal{S}(\Gamma + \varepsilon X), \qquad \mathcal{B}^2_\Gamma = 0, \tag{3E.5}$$

provided $\mathcal{S}(\Gamma) = 0$. Local anomalies would appear as a ghost-number-one cohomology class of $\mathcal{B}_\Gamma$. The statement proved in the main text is precisely that BI-6 removes this cohomology class in the present theory.

The Abelian Ward operators are extracted by replacing the ghosts by local gauge parameters and differentiating Γ with respect to the corresponding fields. For example,

$$\mathcal{W}_B(x)\Gamma = -\partial_\mu \frac{\delta\Gamma}{\delta X^{(B)}_\mu(x)} + \mathrm{i} \sum_\Phi q_B(\Phi) \left(\Phi \frac{\delta\Gamma}{\delta\Phi} - \Phi^\dagger \frac{\delta\Gamma}{\delta\Phi^\dagger} \right) + M_B \frac{\delta\Gamma}{\delta\sigma_B(x)} = 0, \tag{3E.6}$$

and similarly for $U(1)_{Qi}$. For the non-Abelian sectors one obtains

$$\mathcal{W}^a_C(x)\Gamma = -\left(D_\mu \frac{\delta\Gamma}{\delta G_\mu}\right)^a + \mathrm{i}\sum_\Phi \frac{\delta\Gamma}{\delta\Phi} T^a_C \Phi - \mathrm{i}\sum_\Phi \Phi^\dagger T^a_C \frac{\delta\Gamma}{\delta\Phi^\dagger} = 0, \tag{3E.7}$$

$$\mathcal{W}^i_L(x)\Gamma = -\left(D_\mu \frac{\delta\Gamma}{\delta W_\mu}\right)^i + \mathrm{i}\sum_\Phi \frac{\delta\Gamma}{\delta\Phi} t^i \Phi - \mathrm{i}\sum_\Phi \Phi^\dagger t^i \frac{\delta\Gamma}{\delta\Phi^\dagger} = 0. \tag{3E.8}$$

The equal-time current algebra then follows by standard arguments. Writing

$$Q^a_C = \int \mathrm{d}^3x\, J^{0a}_C, \quad Q^i_L = \int \mathrm{d}^3x\, J^{0i}_L, \quad Q_Y = \int \mathrm{d}^3x\, J^0_Y, \quad Q_B = \int \mathrm{d}^3x\, J^0_B, \quad Q_{Qi} = \int \mathrm{d}^3x\, J^0_{Qi},$$

one finds

$$[Q^a_C, Q^b_C] = \mathrm{i} f^{abc} Q^c_C, \qquad [Q^i_L, Q^j_L] = \mathrm{i}\epsilon^{ijk} Q^k_L, \tag{3E.9}$$

$$[Q^a_C, Q^i_L] = 0, \qquad [Q^a_C, Q_Y] = [Q^a_C, Q_B] = [Q^a_C, Q_{Qi}] = 0, \tag{3E.10}$$

$$[Q^i_L, Q_Y] = [Q^i_L, Q_B] = [Q^i_L, Q_{Qi}] = 0, \qquad [Q_Y, Q_B] = [Q_Y, Q_{Qi}] = [Q_B, Q_{Qi}] = 0. \tag{3E.11}$$

There is no anomalous Schwinger term because the anomaly cocycle has already been cancelled in the master equation.

Appendix 3F: operator catalog and exact proton-stability selection rules

This appendix classifies representative local operators up to dimension six according to their baryon and Qi charges in the minimal diagonal realization.

Table 3F.1: Representative local operators and their status under $U(1)_B \times U(1)_{Qi}$.

Operator	**Dim.**	ΔB	ΔQi	**Status**
$\bar{Q} Y_u \widetilde{H} u_R$	4	0	0	Allowed Yukawa
$\bar{Q} Y_d H d_R$	4	0	0	Allowed Yukawa
$\bar{L} Y_e H e_R$	4	0	0	Allowed Yukawa
LLe^c	4	0	0 in minimal diagonal realization, but gauge noninvariant under $SU(2)_L \times U(1)_Y$ without contraction details	Forbidden in the exact gauge theory as written
QLd^c	4	$+1/3 - 1/3 = 0$ but hypercharge structure incomplete	Not a gauge-invariant operator in the minimal exact theory	
$u^c d^c d^c$	4	-1	-1	Forbidden by $U(1)_B$ and $U(1)_{Qi}$
$QQQL$	6	$+1$	$+1$	Forbidden
$u^c u^c d^c e^c$	6	-1	-1	Forbidden
$QQQH$	5	$+1$	$+1$	Forbidden
$(LH)(LH)$	5	0	0	Allowed if neutrino masses are generated this way
$\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$	4	0	0	Allowed kinetic term
$B_{\mu\nu}X^{\mu\nu}_{(B)}$	4	0	0	Allowed kinetic mixing

The status column deserves clarification. Gauge invariance must be checked under the full low-energy gauge group, not only under $U(1)_B$. The essential point for proton stability is much simpler: any operator with $\Delta B \neq 0$ is absolutely forbidden. Since the proton is the lightest baryon, no decay channel can exist in the exact theory.

A possible source of confusion is the presence of Stückelberg masses. A Stückelberg term

$$\frac{1}{2}(\partial_\mu \sigma_B + M_B X^{(B)}_\mu)^2 \tag{3F.1}$$

does not break the gauge symmetry. Under

$$X_\mu^{(B)} \mapsto X_\mu^{(B)} + \partial_\mu \alpha_B, \qquad \sigma_B \mapsto \sigma_B - M_B \alpha_B,$$

the combination remains invariant. Therefore the selection rule $\Delta B = 0$ survives even when the baryonic gauge boson is massive.

One may also ask whether integrating out heavy fields can regenerate forbidden operators. The answer is no for exact gauge symmetries. Integrating out heavy degrees of freedom preserves all exact local gauge symmetries. Consequently no $\Delta B \neq 0$ operator can be induced in the Wilsonian effective action if $U(1)_B$ is exact in the ultraviolet completion.

Appendix 3G: Historical anchor map for the later discussion section

Table 3M.1: Selected historical anchors used in the comparison section of the full chapter.

Conceptual ingredient	**Classical/literature anchor**	**Role in the present program**
Gauge invariance and Noether identities	Noether, Yang–Mills, renormalized gauge theory (Noether, 1918; Yang and Mills, 1954; 't Hooft and Veltman, 1972)	Historical backbone for POS-1.
Anomaly cancellation by inflow	Green–Schwarz (Green and Schwarz, 1984)	Historical anchor for POS-3 and BI-6.
Warped extra dimensions	Randall–Sundrum (Randall & Sundrum, 1999)	Geometric intuition for the radial warp direction.
Bulk–boundary generating functionals	AdS/CFT literature (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998)	Historical anchor for formal 4D QFT derivation.
Open quantum systems and positivity	Kraus, Gorini–Kossakowski–Sudarshan, Lindblad (Gorini et al., 1976; Kraus, 1983; Lindblad, 1976)	Mathematical anchor for OGGEP and GDOS.
SU(5) unification	Georgi–Glashow (Georgi and Glashow, 1974)	Historical anchor for the non-Abelian GUT block.
BRST/cohomological consistency	Becchi–Rouet–Stora, Tyutin (Becchi et al., 1976; Tyutin, 1975)	Anchor for Section 3 anomaly/BRST proof.
Arisaka constitutional synthesis	GUT-10, GDOS, companion preprints (Arisaka, 2026c,a,c)	New organizational framework and target derivation chain.

Appendix 3H: Consolidated derivation ledger and future technical tasks

With the derivation, discussion, and conclusion sections now incorporated into the unified manuscript, the principal chain of this book is complete. The main results may be itemized as follows:

Result 1: The GUT-10 constitution filters admissible theories through four axioms and seven postulates only.

Result 2: AX-2 plus POS-1–POS-6 select the minimal warped six-dimensional GG-6 bulk with BI-6 and the OGN-5 normalization ledger.

Result 3: The resulting six-dimensional gauge-gravity-tensor theory is BRST-consistent and anomaly-free through a single Green–Schwarz inflow channel.

Result 4: AX-3 plus POS-4 force a lawful open-system bulk-to-boundary channel, yielding the exact Nakajima–Zwanzig equation, the GKLS limit, and GDOS.

Result 5: The low-energy boundary theory is a formal four-dimensional QFT whose zero-mode truncation reproduces the Standard Model matter/gauge sector together with $U(1)_B$ and $U(1)_{Qi}$.

Result 6: Exact local $U(1)_B$ implies absolute proton stability in the theorem-level realization studied here.

The main technical tasks that remain beyond the scope of this book are now more sharply defined:

Task 1: construct one fully explicit benchmark six-dimensional matter reservoir with complete mass matrices and threshold corrections;

Task 2: determine a unique phenomenological realization of the $U(1)_{Qi}$ charges and kinetic/Stückelberg mixings;

Task 3: confront the resulting low-energy EFT with collider, flavor, neutrino, and cosmological constraints;

Task 4: extend the GDOS/OGGEP sector into a fully quantitative treatment of nonequilibrium measurement, cognition-adjacent open systems, and cosmological coarse-graining;

Task 5: prepare a final JHEP-style version with complete numerical benchmark tables and phenomenological plots.

Thus this book should be read as a completed theorem-level derivation paper together with a clearly registered future-work agenda, not as the terminal phenomenological form of the program.

Appendix 3I: master summary tables

This appendix gathers the paper-wide derivation chain into compact tables for quick reference.

Table 3G.1: From constitutional clause to derived structure in the present GG-6 paper.

Input clause	**Immediate mathematical demand**	**Derived structure in this book**	**Main section**
AX-1 (LC)	Retarded causal propagation and local admissibility	Local bulk action on warped M_6 with boundary-compatible causal structure	Secs. 1–3
AX-2 (GGEP)	Composite geometry⊕gauge connection	GG-6 bulk, BI-6, six-force unification at E_{GUT}	Sec. 3
AX-3 (OGGEP)	Bulk theory must ultimately furnish operational semantics	Deferred here; registered as the starting point of the companion chapter	Outlook / companion chapter
AX-4 (MSMF)	Maximize admissible symmetry, minimize arbitrary freedom	OGN-5 arithmetic compression; no superpartner or string ontology required	Secs. 2–4
POS-1	Layered gauge invariance	6D BRST/Ward identities	Sec. 4, App. 4A.8
POS-2	Unified extremal principle	Common action / current logic and minimal variational closure	Secs. 1–4
POS-3	Anomaly-free admissibility	BI-6 factorization and GS inflow cancellation	Secs. 3–4, App. 4A.8
POS-4	Closure / positivity	Physical gauge-fixed quantum theory and homogeneous ST identity	Sec. 4, App. 4A.8
POS-5	RG compatibility	Radial coordinate $r = \ln(E_{\mathrm{GUT}}/\mu)$, warped bulk, KK reduction logic	Sec. 3, App. 4A.8
POS-6	Occam gauge normalization	OGN-5, primitive 2/5 coefficient, frozen dimensionless ledger	Sec. 3, App. 4A.8
POS-7	Falsifiability	Registered kill criteria for GG-6 structural claims	Sec. 5

Table 3G.2: Assumed versus derived ingredients in UCLA-Arisaka/GG-2026-2.

Category	**Assumed as constitutional input**	**Derived inside this book**
Foundational clauses	4 axioms + 7 postulates	Everything else
Bulk geometry	None beyond admissibility principles	Warped $M_6 = M_4 \times I_r \times S^1_\theta$
Gauge structure	None beyond GGEP and minimality	$\mathrm{SU}(5) \times U(1)_B \times U(1)_{Qi}$ at E_{GUT}
Tensor sector	None beyond anomaly admissibility	Single GS / BI-6 two-form sector
Normalization ledger	None beyond MSMF + OGN	OGN-5 discrete arithmetic closure
6D quantum consistency	Not assumed as a primitive law	BRST-consistent gauge-fixed 6D QFT with homogeneous ST identity
Boundary/open-system reduction	Not developed here	Reserved for the companion GDOS paper

Table 3G.3: Parameter budget emphasized by the present framework.

Sector	**Ordinary expectation**	**GG-6 stance in this book**
Relative gauge normalizations	Convention-dependent continuous freedom	Frozen by OGN and BI-6 primitive ratios
Extra-dimensional scale ratios	Multiple moduli possible	Collapsed into r-geometry and OGN-5 ledger at theorem level
Ultraviolet endpoint	Often extended toward E_{Pl}	Declared to terminate at E_{GUT}
Anomaly-cancellation sector	Model-dependent repair layer	Constitutive GS / BI-6 tensor locking
Baryon conservation	Accidental or approximate	Exact local gauge symmetry
Information bookkeeping	Usually absent	$U(1)_{Qi}$ Abelian ledger sector at bulk level

Appendix 3J: Summary of abbreviations

Abbreviation	Meaning
LC	Locality and Causality.
GGEP	Geometry–Gauge Equivalence Principle.
OGGEP	Operational/Open Geometry–Gauge Equivalence Principle.
MSMF	Maximum Symmetry, Minimum Freedom.
GI	Gauge Invariance (local, dynamical/open, global layers).
UEP	Unified Extremal Principle.
AF	Anomaly-Free admissibility by inflow.
CP	Closure and Positivity.
RG	RG compatibility and scale covariance.
OGN	Occam Gauge Normalization.
MF	Measurement and Falsifiability.
GG-6	Geometry–Gauge Locking in six dimensions.
GDOS	Geometric Dynamics of Open Systems.
CPTP	Completely positive and trace preserving.
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan generator form.
GUT	Grand unified scale/sector.
SM	Standard Model.

Appendix 3K: Glossary of key symbols

Symbol	Meaning
$\mathcal{T}$	Candidate theory tuple .
$\mathcal{M}$	Bulk–boundary manifold/stage.
$\mathcal{P}$	Bundle/principal-bundle data over $\mathcal{M}$.
$\mathcal{A}$	Composite geometry⊕gauge connection.
$\mathcal{F}$	Composite curvature $\mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$.
$\mathcal{D}$	Operational bulk→boundary dictionary.
$\Phi_{t,s}$	Admissible evolution map from time s to t.
$\mathcal{I}_\alpha$	Instrument/POVM element associated with outcome α.
$S(\mathcal{T})$	Symmetry score in AX-4.
$F(\mathcal{T})$	Freedom score in AX-4.
$M_4 \times I_r \times S^1_\theta$	Anticipated six-dimensional stage of GG-6.
X_4	Green–Schwarz/Bianchi four-form to be derived in Section 2.
E_{GUT}	Ultraviolet endpoint of the GG-6 description.

Appendix 3L: Abelian trace normalization and the origin of $2/5$

For convenience we record the short matrix calculation behind Proposition 4.0.4. On the primitive **5** basis write

$$Y_0 = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \qquad B_0 = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0\right). \tag{3H.1}$$

Then

$$\mathrm{Tr}(Y_0^2) = 3 \times \frac{1}{9} + 2 \times \frac{1}{4} = \frac{5}{6}, \qquad \mathrm{Tr}(B_0^2) = 3 \times \frac{1}{9} = \frac{1}{3}, \tag{3H.2}$$

so that

$$\frac{\mathrm{Tr}(B_0^2)}{\mathrm{Tr}(Y_0^2)} = \frac{2}{5}, \qquad \frac{\mathrm{Tr}(Y_0^2)}{\mathrm{Tr}(B_0^2)} = \frac{5}{2}. \tag{3H.3}$$

In the present program the first ratio appears in BI-6 as the Abelian coefficient and the second ratio appears later as the reciprocal canonical weight in boundary current normalization. The calculation uses only primitive generator data and therefore exemplifies the OGN principle directly.

Appendix 3M: descent equations and BRST cancellation

To make Section 3 self-contained we record the minimal descent chain. Let

$$X_4 = \mathrm{d}\omega_3^{(0)} \tag{3I.1}$$

for the Chern–Simons representative built from the gravitational, non-Abelian, and Abelian sectors of the composite connection. If

$$I_8 = \frac{1}{2} X_4 \wedge X_4, \tag{3I.2}$$

then a seven-form representative is

$$I_7^{(0)} = \frac{1}{2}\,\omega_3^{(0)} \wedge X_4, \qquad \mathrm{d}I_7^{(0)} = I_8. \tag{3I.3}$$

Under BRST variation,

$$s\omega_3^{(0)} + \mathrm{d}\omega_2^{(1)} = 0, \tag{3I.4}$$

so the consistent anomaly is the six-form

$$I_6^{(1)} = \omega_2^{(1)} \wedge X_4. \tag{3I.5}$$

Consequently,

$$s\Gamma_{1\text{loop}} = 2\pi\mathrm{i} \int_{M_6} I_6^{(1)} = 2\pi\mathrm{i} \int_{M_6} \omega_2^{(1)} \wedge X_4. \tag{3I.6}$$

Choosing the tensor transformation

$$sB_2 = -2\pi\mathrm{i}\,\omega_2^{(1)} + \mathrm{d}\Lambda_1 \tag{3I.7}$$

then yields

$$s \int B_2 \wedge X_4 = -2\pi\mathrm{i} \int \omega_2^{(1)} \wedge X_4, \tag{3I.8}$$

which cancels the anomaly exactly. This is the compact cohomological statement behind the homogeneous ST identity in the main text.

Appendix 3N: mathematical foundations

Principal bundles and connections

Let $P \to M_6$ be a principal G-bundle over the six-dimensional bulk. A *connection* is a $\mathfrak{g}$-valued one-form $A = A_M^a T^a \, \mathrm{d}x^M$ transforming under gauge transformations $g(x)$ as

$$A \mapsto g^{-1} A \, g + g^{-1} \mathrm{d} g. \tag{3J.1}$$

The curvature (field strength) is

$$F = \mathrm{d}A + A \wedge A, \tag{3J.2}$$

satisfying the Bianchi identity $DF = \mathrm{d}F + [A, F] = 0$.

Characteristic classes

For a compact Lie group G with Lie algebra $\mathfrak{g}$, the Chern–Weil homomorphism maps G-invariant polynomials on $\mathfrak{g}$ to cohomology classes on the base manifold.

The *first Pontryagin class* (relevant for SU(5) and gravity):

$$p_1(F) = \frac{1}{8\pi^2} \operatorname{tr}(F \wedge F). \tag{3J.3}$$

The *first Chern class* (for Abelian bundles):

$$c_1(F_B) = \frac{1}{2\pi} F_B. \tag{3J.4}$$

The Dirac $\hat{A}$-genus relevant for the spin^c Dirac operator in six dimensions:

$$\hat{A}(M_6) = 1 - \frac{1}{24} p_1 + \cdots . \tag{3J.5}$$

The Green–Schwarz mechanism: general setup

Consider a $2n$-dimensional theory with chiral fields. The one-loop gauge/gravitational anomaly is captured by a $(2n{+}2)$-form I_{2n+2}. The Green–Schwarz mechanism (Green and Schwarz, 1984) cancels the anomaly provided:

(i) the anomaly polynomial *factorizes*: $I_{2n+2} = X_{n+1} \wedge Y_{n+1}$;

(ii) the theory contains a $(n{-}1)$-form B whose field strength satisfies the modified Bianchi identity $\mathrm{d}H = X_{n+1}$;

(iii) the GS action $S_{\mathrm{GS}} = \int B \wedge Y_{n+1}$ provides a classical variation that exactly cancels the one-loop anomaly.

In GG-6 (with $2n = 6$), the relevant forms are: X_4 (the BI-6 four-form), B_2, and $S_{\mathrm{GS}} = \int B_2 \wedge X_4$.

Appendix 3O: Lie algebra of SU(5)

SU(5) generators and fundamental representation

The group SU(5) has rank 4 and dimension 24. In the fundamental representation **5**, the generators T^a ($a = 1, \ldots, 24$) satisfy

$$[T^a, T^b] = \mathrm{i} f^{abc} T^c, \qquad \mathrm{tr}(T^a T^b) = \tfrac{1}{2}\delta^{ab}. \tag{3K.1}$$

Branching: $\mathrm{SU}(5) \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

Under the Standard Model embedding, the index $A = 1, \ldots, 5$ splits as $A = (\alpha, m)$ where $\alpha = 1, 2, 3$ is a color index and $m = 4, 5$ is a weak isospin index. The SU(5) hypercharge generator is

$$Y_0 = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \tag{3K.2}$$

Representation decompositions

The relevant SU(5) representations decompose as:

$$\mathbf{24} = (\mathbf{8}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{3})_0 + (\mathbf{1}, \mathbf{1})_0 + (\mathbf{3}, \mathbf{2})_{-5/6} + (\bar{\mathbf{3}}, \mathbf{2})_{+5/6}, \tag{3K.3}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{+1/6} + (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} + (\mathbf{1}, \mathbf{1})_{+1}, \tag{3K.4}$$

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{+1/3} + (\mathbf{1}, \mathbf{2})_{-1/2}, \tag{3K.5}$$

$$\mathbf{5}_H = (\mathbf{3}, \mathbf{1})_{-1/3} + (\mathbf{1}, \mathbf{2})_{+1/2}. \tag{3K.6}$$

In component notation: $\mathbf{10} \ni (Q_i,\, u_i^c,\, e_i^c)$, $\bar{\mathbf{5}} \ni (d_i^c,\, L_i)$, and $\mathbf{5}_H \ni (T_H,\, H)$, where T_H is the color-triplet Higgs and H is the weak-doublet Higgs.

The adjoint decomposition and the X, Y bosons

The $(\mathbf{3}, \mathbf{2})_{-5/6}$ and $(\bar{\mathbf{3}}, \mathbf{2})_{+5/6}$ pieces of the adjoint are the *X and Y gauge bosons*, responsible for baryon-violating processes in minimal SU(5). The orbifold/interval boundary conditions of GG-6 project these *out* of the massless zero-mode spectrum (see companion chapter GG-2026-3), providing a second layer of proton protection beyond the exact $\mathrm{U}(1)_B$ gauge symmetry.

Appendix 3P: OGN-5 arithmetic closure

The Pythagorean-sum identity

The closure relation $K = M^2 + N^2 + L^2$ is verified:

$$K = 1^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35. \tag{3L.1}$$

The product relation $K = L \cdot J$:

$$K = 5 \times 7 = 35. \tag{3L.2}$$

Both relations hold simultaneously and uniquely for the tuple $(1, 3, 5; 7, 35)$.

Primality and uniqueness

$K = 35 = 5 \times 7$ is the product of two consecutive odd primes following from the SU(5) rank $L = 5$ and the odd closure index $J = 2N + 1 = 7$. The fact that 35 satisfies *both* the Pythagorean sum identity *and* the product relation simultaneously is arithmetically nontrivial and is one of the reasons POS-6 selects this tuple.

The $2/5$ coefficient as an OGN fraction

The BI-6 Abelian coefficient $2/5$ can be written as

$$\frac{2}{5} = \frac{2M + 0}{L} = \frac{2 \times 1 + 0}{5}, \tag{3L.3}$$

or alternatively as the trace ratio:

$$\frac{2}{5} = \frac{\mathrm{Tr}(B_0^2)}{\mathrm{Tr}(Y_0^2)} = \frac{1/3}{5/6} = \frac{2}{5}. \tag{3L.4}$$

Both expressions give the same result. The OGN denominator is precisely $L = 5$, and the numerator is a small integer forced by the trace norms.

Introduction: Why Do We Need a New Framework?

3L.1 The triumph and the cracks

Modern physics is arguably the most successful intellectual enterprise in human history. The Standard Model of particle physics, assembled piece by piece between the 1960s and the 2000s, correctly predicts the outcomes of thousands of experiments to extraordinary precision—sometimes to twelve decimal places (Yang and Mills, 1954; 't Hooft and Veltman, 1972). General relativity, Einstein's 1915 theory of gravity, describes the bending of light around stars, the orbits of planets, and the expansion of the universe with equal accuracy (Einstein, 1915).

And yet, if you step back and look at the foundations of this edifice, you notice something odd. The Standard Model and general relativity are not *one* theory. They are *two* theories, built on different assumptions, using different mathematics, and joined together only loosely at the edges. Gravity is the curvature of spacetime itself—it *is* geometry. The strong, weak, and electromagnetic forces, by contrast, are described as "internal symmetries": abstract rotations that happen *on top of* spacetime, like patterns painted on the surface of a stage that the stage itself knows nothing about.

This book is about a program that asks: *what if that split is unnecessary?* What if gravity and the gauge forces are two faces of one geometric object? And what if you can derive that object—not guess it, not assume it, but *derive* it—from a small set of constitutional principles?

The program is called GG-6 (Geometry–Gauge Locking in Six Dimensions), and the accessible companion (Part II) explains what it says, why it matters, and how it differs from every other attempt at unification.

3L.2 Six problems that won't go away

Before diving into the solution, let us be precise about the problems. The companion technical paper identifies six structural issues that have resisted solution for decades. Here they are in plain language.

Problem 1: Nobody knows what "measurement" means. In quantum mechanics, a particle can be in a superposition of two states—say, spin-up and spin-down simultaneously. But when you measure it, you always get one definite answer. The standard textbook says the wavefunction "collapses." But *when* does it collapse? *How*? Is there a precise mathematical rule, or is measurement an extra ingredient bolted onto the theory from outside? Since the 1930s, physicists have debated this without resolution (von Neumann, 1932). The GUT-10 program responds by making measurement a constitutional axiom (AX-3), not an afterthought.

Problem 2: Gravity and the other forces speak different languages. As we noted, gravity is geometry and the gauge forces are internal symmetries. In the Standard Model, there is no deep reason why these two types of physics should be treated differently. The split is a historical accident: Einstein discovered geometry first, and Yang and Mills discovered gauge

theory later (Einstein, 1915; Yang and Mills, 1954). GG-6 merges them into one composite connection.

Problem 3: Unification keeps creating more problems than it solves. Grand Unified Theories (GUTs) try to merge the strong, weak, and electromagnetic forces into one gauge group at very high energy (Georgi and Glashow, 1974). This is a beautiful idea, but every attempt so far introduces new difficulties: proton decay that hasn't been observed, dozens of new particles that haven't been found, hundreds of free parameters that can be tuned to fit almost anything, and in the case of string theory, a "landscape" of 10^{500} possible solutions. GG-6 takes the opposite approach: it tries to *reduce* freedom rather than increase it.

Problem 4: The proton should be dead, but it isn't. Most grand unified theories predict that the proton—the particle at the heart of every atom—should eventually decay. Experiments have watched protons for decades and found nothing: the proton's lifetime exceeds 10^{34} years. In the Standard Model, baryon number (the "proton-ness" of a particle) is conserved only by accident, not by any fundamental law. GG-6 promotes baryon number to an exact gauge symmetry, making the proton absolutely stable.

Problem 5: Why three generations? Quarks and leptons come in three families (or "generations"). The electron, muon, and tau are three versions of the same particle, differing only in mass. No one knows why there are exactly three—not two, not four. GG-6 does not yet resolve this completely, but its discrete integer ledger (the number $N = 3$ in the OGN-5 package) encodes the triplicity at the constitutional level.

Problem 6: Real physics is open, but our theories are closed. Textbook quantum mechanics describes closed systems—perfectly isolated boxes that never interact with the outside world. But every real experiment involves an *open* system: a detector connected to wires, a particle beam scattering off a target, a living cell exchanging molecules with its environment. GG-6 is designed from the start to handle open systems through its companion framework GDOS.

3L.3 The constitutional approach: rules before models

The key philosophical move behind GG-6 is to distinguish between a *model* and a *constitution*.

A **model** proposes specific particles, specific forces, and specific equations. The Standard Model is a model. So is the MSSM (the Minimal Supersymmetric Standard Model). So are most string theory compactifications.

A **constitution**, by contrast, does not propose any specific particles or equations. Instead, it specifies the *rules that any admissible model must obey*. It says: before you even start writing down a Lagrangian, you must satisfy these requirements. Only the models that pass the constitutional filter get to compete for the description of nature.

Think of it like the rules of a game. The constitution of chess says: the board is 8×8, each piece moves in a prescribed way, the king cannot move into check. These rules do not tell you

which game will be played—that depends on the players' choices. But they drastically narrow the space of legal games compared to "move any piece anywhere."

The GUT-10 constitution has four axioms and seven postulates. They are explained in Section 3L.5. The remarkable result of the GG-6 paper is that these eleven clauses are so restrictive that they essentially force a *unique* six-dimensional geometry, up to a handful of discrete integers and one overall scale (Newton's constant).

3L.4 A brief history of unification attempts

The idea that nature's forces might be manifestations of a single underlying principle is not new. It has a rich and instructive history.

Maxwell (1865): electricity and magnetism. The first successful unification in physics was James Clerk Maxwell's discovery that electricity and magnetism are two aspects of one electromagnetic field. What had been two separate forces became one, described by four elegant equations. This set the template: unification works by revealing that seemingly different phenomena share a common mathematical structure.

Einstein (1915): gravity as geometry. Einstein's general relativity showed that gravity is not a force in the Newtonian sense—it is the curvature of spacetime itself. Objects follow the straightest possible paths (geodesics) through curved spacetime, and what we call "gravitational attraction" is simply the convergence of these paths near massive objects. This was the first geometrization of a force.

Kaluza and Klein (1921–1926): electromagnetism as geometry. Just six years after general relativity, Theodor Kaluza showed that adding a single extra spatial dimension to Einstein's equations automatically produces Maxwell's electromagnetism (Kaluza, 1921). Oskar Klein later suggested that this extra dimension could be "compactified"—curled up into a tiny circle too small to observe directly (Klein, 1926). This was the second geometrization of a force, and it planted the seed that would eventually grow into GG-6.

Yang and Mills (1954): non-Abelian gauge theory. Chen-Ning Yang and Robert Mills generalized the gauge symmetry of electromagnetism from the simple group U(1) to arbitrary non-Abelian groups like SU(2) and SU(3) (Yang and Mills, 1954). This provided the mathematical language for the strong and weak nuclear forces and became the backbone of the Standard Model.

Georgi and Glashow (1974): SU(5) grand unification. Howard Georgi and Sheldon Glashow proposed that the strong, weak, and electromagnetic forces all merge into a single SU(5) gauge theory at very high energy (Georgi and Glashow, 1974). This was the first Grand Unified Theory (GUT). It predicted proton decay, which has not been observed, leading to the demise of minimal SU(5).

Green and Schwarz (1984): anomaly cancellation. Michael Green and John Schwarz showed that gauge anomalies in ten-dimensional superstring theory could be cancelled by a remarkable topological mechanism involving a tensor field (Green and Schwarz, 1984). This "first superstring revolution" revived string theory as a candidate for quantum gravity.

Randall and Sundrum (1999): warped extra dimensions. Lisa Randall and Raman Sundrum demonstrated that an exponentially warped five-dimensional geometry could solve the hierarchy problem—explaining why gravity is so much weaker than the other forces—without fine-tuning (Randall & Sundrum, 1999). This idea is a direct ancestor of the warped bulk in GG-6.

GG-6 (2026): the constitutional synthesis. The GG-6 program synthesizes lessons from all these predecessors. From Kaluza–Klein, it takes the idea of geometrizing forces. From Yang–Mills, it takes non-Abelian gauge structure. From Georgi–Glashow, it takes SU(5) unification. From Green–Schwarz, it takes anomaly cancellation by inflow. From Randall–Sundrum, it takes warped geometry. But it does not simply combine them eclectically. It *derives* the combination from a small set of constitutional principles, showing that each ingredient is *forced* rather than *chosen*.

3L.5 What the accessible companion (Part II) covers

This accessible companion (Part II) follows the structure of the technical preprint Chapter 3, chapter by chapter:

1. **Introduction** (this section): the six problems and the constitutional philosophy.
2. **The GUT-10 constitution**: what the four axioms and seven postulates say, in plain English.
3. **Building the six-dimensional stage**: how the constitution forces exactly six dimensions, warped geometry, and a specific combination of forces.
4. **Making the theory quantum-consistent**: how anomalies are cancelled and the theory is made self-consistent at the quantum level.
5. **Discussion**: what is new, what is different, and what could prove the theory wrong.
6. **Conclusions**: a one-page summary of the entire argument.

No appendices are included. Every essential idea is in the main text. Equations are used sparingly and always explained in words.

The GUT-10 Constitution: Rules of the Game

3L.1 What is a constitution in physics?

In everyday life, a constitution is the supreme law of a nation. It does not tell you what to have for breakfast; it tells you what kinds of laws are *allowed*. Any law that violates the constitution is struck down, no matter how popular or convenient it might be.

The GUT-10 constitution works the same way for physics. It does not tell you what particles exist or what their masses are. Instead, it specifies eleven rules that any candidate theory of nature must satisfy. Four of these rules are called *axioms* (the deepest commitments) and seven are called *postulates* (additional filters that sharpen the axioms into operational requirements).

A useful metaphor: imagine theory space as an enormous library containing every conceivable physical theory—billions of shelves stretching to the horizon. The constitution is the librarian who walks through with a red stamp, marking most books "inadmissible." Only the books that survive all eleven stamps are allowed on the reading list. The GG-6 paper shows that, remarkably, only one book survives.

3L.2 The four axioms

AX-1: Locality and Causality (LC). *"Effects cannot travel faster than light, and no measurement can depend on information from the future."*

This is the most conservative axiom—it simply insists that physics respects cause and effect. In the language of Maxwell's equations, it says that electromagnetic signals propagate at speed c and cannot arrive before they are sent. In quantum field theory, it becomes the requirement of "microcausality": two measurements made at spacelike separation cannot influence each other.

Metaphor: Think of the universe as a postal system. AX-1 says: letters are delivered in order, and no letter can arrive before it is mailed. Any theory that violates this rule—even subtly, even in exotic higher-dimensional scenarios—is immediately disqualified.

Why it is nontrivial: You might think causality is so obvious that no one would violate it. But in higher-dimensional and holographic settings, subtle acausal effects can creep in through the back door—for example, through advanced Green's functions or nonlocal effective actions. AX-1 closes this door firmly.

AX-2: Geometry–Gauge Equivalence Principle (GGEP). *"Gravity and the gauge forces are not separate things. They are two aspects of one composite geometric field."*

This is the most radical axiom and the conceptual heart of GG-6. In the traditional picture, spacetime is the stage, and forces are actors performing on it. AX-2 says: the stage and the actors are the same thing. More precisely, the fundamental object is a single "composite connection" that encodes both the curvature of spacetime (gravity) and the internal rotations of gauge theory (strong, weak, electromagnetic forces) in one mathematical structure.

Historical origin: In 1921, Theodor Kaluza showed that if you add one extra spatial dimension to Einstein's general relativity, the equations of electromagnetism emerge automatically from the

geometry of that extra dimension (Kaluza, 1921; Klein, 1926). AX-2 generalizes this idea: not just electromagnetism, but *all* gauge forces should emerge from geometry. The question then becomes: how many extra dimensions do you need, and what shape should they have?

Metaphor: Imagine a river. The water's flow (its velocity field) determines where objects drift—that is like gravity. But the same water also has currents, eddies, and vortices—those are like the gauge forces. AX-2 says: you don't need a separate theory of "eddies"; they are already part of the water.

What changes in practice: In the Standard Model, writing down the Lagrangian requires you to *choose* a gauge group and then couple it to gravity by hand. In GG-6, you write down one composite connection, and both the gauge group and gravity emerge from its geometry. The coupling is not a choice; it is a consequence.

AX-3: Operational/Open GGEP (OGGEP). *"A theory is incomplete unless it tells you how to make a measurement."*

Most theories of fundamental physics describe closed systems in isolation. AX-3 says: that is not enough. An admissible theory must include a precise dictionary that translates bulk physics (the full six-dimensional description) into boundary observables (the four-dimensional measurements that experimenters actually perform).

Metaphor: A recipe book that lists ingredients but never explains how to cook is useless. AX-3 says: the theory must include the cooking instructions—the explicit procedure by which abstract mathematical objects become numbers on a detector screen.

This axiom directly addresses Problem 1 (the measurement problem). Instead of treating measurement as an unexplained collapse, AX-3 builds measurement into the constitutional layer of the theory itself.

The holographic connection: AX-3 is inspired by the holographic principle in modern theoretical physics, particularly the AdS/CFT correspondence discovered by Juan Maldacena in 1997. In that framework, a theory of quantum gravity in a $(d+1)$-dimensional "bulk" is equivalent to a quantum field theory living on the d-dimensional "boundary." AX-3 generalizes this idea: the six-dimensional bulk of GG-6 projects down to a four-dimensional boundary theory where measurements are made. But unlike AdS/CFT, AX-3 does not require strings, conformal symmetry, or supersymmetry.

AX-4: Maximum Symmetry, Minimum Freedom (MSMF). *"Among all theories that pass the first three axioms, nature picks the one with the most symmetry and the fewest adjustable knobs."*

This axiom is the anti-landscape principle. It says: if two theories are both consistent, both causal, and both equipped with measurement dictionaries, but one has fewer free parameters than the other, the one with fewer parameters wins.

Historical echo: This is a formalization of Occam's Razor—the medieval principle that "entities should not be multiplied beyond necessity." But AX-4 goes further: it makes parsimony a *law*, not a guideline. Free parameters are not merely inelegant; they are constitutionally inadmissible unless forced by symmetry or consistency.

Why it matters: The string theory landscape contains roughly 10^{500} possible solutions, each with different physics. AX-4 is designed to prevent this. If the constitution works as intended, there should be essentially *one* surviving theory, not a googol of them.

A concrete example: Consider the relative strengths of the strong and electromagnetic forces at the GUT scale. In the Standard Model, this ratio is a free parameter. You measure it in the lab and plug it in. In GG-6, this ratio is fixed by the Bianchi identity BI-6 and the trace normalization 2/5. AX-4 is what forbids the theory from leaving such a ratio adjustable.

3L.3 The seven postulates

The postulates sharpen the axioms into concrete mathematical filters. Here is what each one demands, in one sentence.

POS-1: Gauge Invariance (GI). *The laws of physics must not depend on arbitrary labeling choices at any scale.*

This is the quantum version of the old insight that physics should not change if you relabel your coordinate axes. It applies at three levels: the microscopic level (local gauge invariance), the mesoscopic level (open-system generator covariance), and the macroscopic level (bookkeeping invariance of conserved ledgers).

Everyday analogy: Consider a bank that records deposits in dollars, euros, and yen. The total wealth of each customer should not depend on which currency the ledger happens to use at any given moment. POS-1 says: the laws of physics have the same property—they are invariant under changes of "currency" (gauge basis) at every level of description.

POS-2: Unified Extremal Principle (UEP). *All dynamics—closed and open—must flow from a single variational principle.*

In classical mechanics, the Principle of Least Action says that particles follow paths that extremize the action integral. In thermodynamics, systems evolve toward states of minimum free energy. In biology, the Free Energy Principle (proposed by Karl Friston) suggests that living systems minimize a variational free energy. POS-2 says: these are not separate principles. They are all projections of one underlying variational engine.

Why this matters for GG-6: POS-2 ensures that the six-dimensional bulk action and the four-dimensional boundary dynamics are not independent recipes. The boundary physics *inherits* its variational structure from the bulk, just as the shadow on a wall inherits its shape from the object casting it.

POS-3: Anomaly Freedom (AF). *The theory must be free of quantum anomalies, possibly via inflow.*

An "anomaly" in quantum field theory is a situation where a symmetry of the classical theory is destroyed by quantum effects. If certain gauge anomalies are present, the theory becomes mathematically inconsistent—probabilities fail to add up to one. POS-3 says: this is not just

bad luck; it is a constitutional disqualifier. An anomalous theory is inadmissible, period. We discuss anomalies at length in Section 3L.10.

Why "possibly via inflow": In some cases, an anomaly that *appears* on a lower-dimensional boundary can be cancelled by a compensating classical term flowing in from the higher-dimensional bulk. This is the Green–Schwarz mechanism. POS-3 allows this repair channel but requires that the total system (bulk + boundary) be anomaly-free.

POS-4: Closure and Positivity (CP). *Physical evolution must preserve probabilities and positivity.*

In quantum mechanics, this means that the evolution of the density matrix must be "completely positive and trace-preserving" (CPTP). In classical physics, it means that probabilities must remain non-negative. POS-4 prevents unphysical dynamics—negative probabilities, information loss, or evolution that turns pure states into nonsense.

Everyday analogy: A population census must always report non-negative numbers. You cannot have "minus three people" in a city. POS-4 says: quantum evolution must have the same property—the analog of "population" (the density matrix eigenvalues) can never go negative.

POS-5: RG Compatibility (RG). *Coarse-graining must be compatible with the measurement dictionary.*

When you zoom out from a detailed microscopic description to a coarser one (as physicists do routinely in the renormalization group), you must not change the physical predictions. POS-5 says that the act of zooming out must commute with the act of reading off measurements.

This postulate has a beautiful geometric consequence: it forces the radial coordinate of the six-dimensional bulk to be identified with the logarithm of the energy scale. The deeper you go into the bulk, the lower the energy, and the more zoomed-out your description. This is explained in detail in Section 3L.4.

Everyday analogy: Consider a map. Zooming out on Google Maps does not change the distance between two cities—it just simplifies the visual representation. POS-5 says the same must be true for physical theories: the "zoomed-out" version must give the same answers as the "zoomed-in" version, just with fewer details.

POS-6: Occam Gauge Normalization (OGN). *Nature uses the simplest possible numbers.*

When setting up a gauge theory, one must choose normalization constants for the charges and couplings. These choices are often treated as arbitrary conventions. POS-6 says: they are not arbitrary. Among all normalizations consistent with the constitution, nature picks the minimal rational or integer values. This is the postulate that ultimately produces the five-integer package $(1, 3, 5; 7, 35)$.

Historical perspective: In the early days of quantum mechanics, Max Planck introduced his constant h as a fundamental unit of action. Charge is quantized in units of e. Angular momentum comes in units of $\hbar$. POS-6 extends this pattern: *all* normalizations in the theory should be expressible through small integers or simple fractions, not arbitrary real numbers.

POS-7: Measurement and Falsifiability (MF). *Every scientific claim must come with a way to prove it wrong.*

This is Popper's falsifiability criterion, elevated to constitutional status. A claim is admissible only if there exists a concrete experiment whose outcome could, in principle, refute the claim. Section **??** lists the specific ways GG-6 can be falsified.

Why this matters: Many proposals in theoretical physics—particularly in the string landscape—have been criticized for being unfalsifiable: they can accommodate almost any experimental outcome by adjusting free parameters. POS-7 is a structural response to this criticism. It says: if your theory cannot be wrong, it is not science.

3L.4 What the constitution already implies

Even before writing down any specific theory, the eleven clauses already force several powerful conclusions:

(a) The fundamental object must be a composite geometric-gauge connection (from AX-2).

(b) The theory must include a measurement dictionary (from AX-3).

(c) Any chiral gauge theory must pass an anomaly-cancellation filter (from POS-3).

(d) Quantum evolution must be CPTP (from POS-4).

(e) The coarse-graining structure must be geometrizable (from POS-5).

(f) Normalization freedom is frozen to discrete data (from AX-4 + POS-6).

These six pre-model consequences already eliminate most conceivable theories. The remaining sections show that, astonishingly, they leave essentially one survivor: a warped six-dimensional bulk with a very specific gauge group and a very specific locking identity.

Building the Six-Dimensional Stage

3L.1 Why six dimensions? Not five, not ten, not eleven

One of the most striking results of the GG-6 paper is that the number of spacetime dimensions is not assumed—it is *derived*. The argument goes as follows.

We start with four dimensions. That is not negotiable: every experiment we perform takes place in three spatial dimensions plus one time dimension. Any admissible theory must reproduce this four-dimensional boundary (AX-3 demands it).

Now, AX-2 requires that the gauge forces be geometrized. This means the gauge degrees of freedom must live somewhere in the geometry—they cannot be independent fibers. To geometrize them, we need extra dimensions beyond the four we observe.

How many extra dimensions do we need? The answer is determined by two requirements:

(i) **A scale direction.** POS-5 (RG compatibility) requires a geometric coordinate that implements the renormalization group—the "zooming out" from high to low energy. This needs one non-compact extra dimension, which we call r. It runs from $r = 0$ (the highest energy, where unification happens) to some large value (low energy, where we live).

(ii) **A phase direction.** POS-1 (gauge invariance) and POS-6 (OGN) require a compact direction that supports the holonomy (the quantum phase accumulated by a particle traveling around a closed loop). The smallest such space is a circle S^1, parameterized by an angle θ.

Four dimensions for the boundary, plus one for scale, plus one for phase, gives **six dimensions total**. A five-dimensional bulk could provide one of these two extra structures, but not both. A seven-dimensional bulk could provide both, but the seventh direction would not be forced by any axiom or postulate, and AX-4 (minimum freedom) would exclude it as unnecessary.

Metaphor: Think of the six-dimensional bulk as a building. The four-dimensional boundary is the ground floor where all the shops and offices are—where life happens. The fifth dimension (the scale direction r) is the elevator shaft, connecting different floors that represent different energy scales. The sixth dimension (the phase direction θ) is the revolving door at the entrance, encoding the internal rotations of gauge symmetry. You need both the elevator and the revolving door; a building with only one of them is incomplete.

Contrast with string theory: String theory requires ten dimensions (or eleven for M-theory). The extra six or seven dimensions must be "compactified" (curled up very small) and their shape determines the physics—but there are an astronomically large number of possible shapes. GG-6 requires only six dimensions, and their structure is essentially unique.

3L.2 The warped geometry: why the bulk is curved

The six-dimensional bulk is not flat. It is *warped*—curved in a specific way that encodes the hierarchy between high and low energies.

The metric (the mathematical object that defines distances) takes the form:

$$\mathrm{d}s_6^2 = e^{-2kr}\,\eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2\,\mathrm{d}\theta^2.$$

You don't need to understand every symbol. The key point is the factor e^{-2kr} in front of the four-dimensional part. This is the *warp factor*: it says that distances in the four-dimensional world appear to shrink exponentially as you move deeper into the bulk (as r increases).

Physical meaning: Imagine a trumpet. At the wide end (small r, high energy), the cross-section is large. At the narrow end (large r, low energy), the cross-section is tiny. The warp factor is like the taper of the trumpet. Particles living at the narrow end "feel" a much smaller energy scale than those at the wide end, even though the underlying six-dimensional theory has only one fundamental scale.

This warping was first studied by Lisa Randall and Raman Sundrum in 1999 (Randall & Sundrum, 1999) in a five-dimensional context. They showed that an exponential warp factor can generate a huge hierarchy between two energy scales (like the Planck scale and the weak scale) without fine-

tuning. GG-6 uses the same mechanism, but in six dimensions and derived from constitutional principles rather than assumed.

3L.3 The logarithmic radial coordinate

The coordinate r has a remarkable interpretation. The constitution forces it to be

$$r = \ln\left(\frac{E_{\text{GUT}}}{\mu}\right),$$

where E_{GUT} is the grand unification energy and μ is the running energy scale. Equivalently, **the radial direction of the bulk is the logarithm of the energy**.

At $r = 0$, we are at $\mu = E_{\text{GUT}}$—the ultraviolet endpoint where all forces unify. As r increases, μ decreases—we are flowing toward lower energies, toward the physics we observe in laboratories.

This is not an arbitrary identification. It follows from POS-5 (RG compatibility), which demands that coarse-graining be a geometric operation. The renormalization group in quantum field theory is a semigroup: combining two scale transformations is the same as one larger transformation. The only coordinate that implements a multiplicative semigroup additively is the logarithm. The constitution leaves no other choice.

Metaphor: Imagine a mountain trail. The altitude represents the energy scale. At the summit ($r = 0$, the GUT scale), you can see all the forces unified in a single panorama. As you descend (r increases), the landscape separates into distinct valleys—one for the strong force, one for the weak force, one for electromagnetism. The trail itself is the renormalization group flow, and the logarithmic coordinate ensures that each "decade" of descent covers the same physical ground.

3L.4 Six forces, not four

In the Standard Model, there are four fundamental forces: strong, weak, electromagnetic, and gravitational. GG-6 identifies **six**:

1. **Strong force** — binds quarks into protons and neutrons (from SU(3) color symmetry).
2. **Weak force** — responsible for radioactive beta decay (from SU(2) weak isospin).
3. **Electromagnetic force** — governs light, chemistry, and electronics (from U(1) hypercharge, after symmetry breaking).
4. **Gravitational force** — curves spacetime, governs planetary orbits and cosmology.
5. **Baryonic force** — a new gauged $\text{U}(1)_B$ symmetry that *exactly* protects the proton from decay.
6. **Informational force** — a second new gauged $\text{U}(1)_{Qi}$ symmetry that provides a conserved "information ledger" at the fundamental level, required by the open-system axiom AX-3.

The first four are familiar. The fifth (baryonic) is GG-6's answer to Problem 4: the proton is stable not by accident but by exact gauge symmetry. The sixth (informational) is more

conceptual—it is the conserved charge associated with the operational/measurement sector demanded by AX-3. Its precise low-energy realization is developed in Chapter 4.

At the GUT scale, all six forces are components of one composite connection:

$$\mathbb{A} = \omega \oplus A_5 \oplus A_B \oplus A_{Qi}.$$

Here ω is the gravitational spin connection, A_5 is the SU(5) gauge field (which later breaks into the strong, weak, and electromagnetic forces), A_B is the baryonic gauge field, and A_{Qi} is the informational gauge field. This is the "one river, many currents" metaphor made precise.

3L.5 The Bianchi Identity BI-6: how the forces lock together

The most important equation in the entire GG-6 program is the **Bianchi Identity BI-6**:

$$\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \tfrac{2}{5}\, F_B \wedge F_B - \tfrac{2}{5}\, F_{Qi} \wedge F_{Qi}.$$

What does this mean in plain language?

On the left side, H_3 is a three-form field strength—a generalization of the magnetic field to higher dimensions. The equation says that this field is *not* freely propagating. Its behavior is locked to the curvatures of all the other force sectors.

On the right side, each term represents a contribution from one sector:

- $\mathrm{tr}(R \wedge R)$: gravity's contribution (positive sign, coefficient $+1$).
- $\mathrm{Tr}(F_5 \wedge F_5)$: the SU(5) gauge block's contribution (negative sign, coefficient -1).
- $F_B \wedge F_B$: the baryonic sector's contribution (coefficient $-2/5$).
- $F_{Qi} \wedge F_{Qi}$: the informational sector's contribution (also $-2/5$).

The remarkable fact is that **these coefficients are not free parameters**. They are uniquely determined by three requirements: (i) anomaly cancellation (POS-3), (ii) the choice of SU(5) as the unification group, and (iii) Occam gauge normalization (POS-6). The ratio 2/5 comes from a simple trace calculation involving the baryon charge generator in the fundamental representation of SU(5).

Why "locking"? Because the different force sectors cannot appear with arbitrary independent strengths. Once you fix the gravitational coefficient to $+1$ (a normalization choice), the rest are forced: -1 for SU(5), $-2/5$ for each Abelian sector. Change any one of them, and the theory becomes quantum-inconsistent (anomalous). The forces are *locked* together by topology.

Contrast with the Standard Model: In the Standard Model, the gauge coupling constants of the strong, weak, and electromagnetic forces are three independent input parameters. Nothing in the SM explains why they have the values they do. In GG-6, the *ratios* of these couplings (at the GUT scale) are fixed by the Bianchi identity. Only the overall scale (Newton's constant G_N) remains free.

3L.6 Five integers instead of a landscape: the OGN-5 package

The technical paper derives a remarkable discrete structure called the **OGN-5 ledger**:

$$(M, N, L;\ J, K) = (1,\ 3,\ 5;\ 7,\ 35).$$

Each integer has a physical meaning:

- $M = 1$: the single compact phase direction S^1 (the revolving door).
- $N = 3$: the triplicity index—the number of generations of matter.
- $L = 5$: the rank of the unification group SU(5).
- $J = 2N + 1 = 7$: the minimal odd closure index.
- $K = L \times J = 35$: the master lock integer, which also satisfies $K = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35$.

The fact that $K = 35$ satisfies *both* the product relation $K = L \times J$ *and* the Pythagorean sum $K = M^2 + N^2 + L^2$ simultaneously is a nontrivial arithmetic coincidence that POS-6 selects.

The integer K controls the depth of the warped throat: $kr_* = K = 35$, so the hierarchy factor across the throat is $e^{-35} \approx 6.3 \times 10^{-16}$. This exponential suppression, built into the geometry, is what generates the vast hierarchy between the GUT scale and the weak scale without any fine-tuning.

Metaphor: Imagine a combination lock with five dials. In the string landscape, every possible combination opens a different door, and there are 10^{500} doors. In GG-6, the constitutional constraints are so tight that only one combination works: $1, 3, 5, 7, 35$. Turn any dial to a different number, and the lock jams.

3L.7 Visualizing the six-dimensional bulk

It is natural to ask: what does the six-dimensional bulk "look like"? Of course, we cannot visualize six dimensions directly. But we can build intuition through lower-dimensional analogies.

The trumpet analogy. Think of the six-dimensional bulk as a trumpet. The four dimensions we live in correspond to cross-sections of the trumpet—circles at each position along the trumpet's length. The radial direction r is the length of the trumpet, running from the mouthpiece (high energy, $r = 0$) to the bell (low energy, $r = r_*$). The phase direction θ goes *around* each cross-section.

At the mouthpiece, the trumpet is narrow: all forces are strong and unified. As you move toward the bell, the trumpet widens (the warp factor suppresses energy scales), and the forces begin to separate into distinct sectors—strong, weak, electromagnetic. The "music" that comes out of the bell is the four-dimensional physics we observe in laboratories.

The onion analogy. Another way to picture the bulk is as an onion. The outermost layer is the four-dimensional boundary—the world we live in. Peeling inward corresponds to probing higher energies (r decreasing). At the core ($r = 0$, the GUT scale), all layers merge into a single unified structure. The compact S^1 direction is the angular rotation *within* each layer, encoding the gauge phases.

Why six and not more. A common question is: why stop at six? Why not seven, or ten, or twenty-six? The answer is the same as asking why chess has an 8×8 board. It is not that other board sizes are mathematically impossible; it is that the rules of the game (the constitution) select this particular size. In GG-6, the axioms and postulates force exactly two extra dimensions: one for scale (the radial direction) and one for phase (the circle). Any additional dimension would represent an "extra dial" not demanded by any constitutional clause, and AX-4 (minimum freedom) excludes it.

3L.8 How the hierarchy problem is addressed

One of the deepest puzzles in particle physics is the *hierarchy problem*: why is the weak-interaction energy scale (~ 100 GeV, the mass of the Higgs boson) so much smaller than the Planck scale ($\sim 10^{19}$ GeV)? That is a ratio of about 10^{17}—an enormous number that, in the Standard Model, must be put in by hand ("fine-tuned").

The warped geometry of GG-6 provides a natural explanation. The exponential warp factor e^{-kr} generates an exponential hierarchy from a modest linear input. The throat depth $kr_* = K = 35$ produces a suppression factor of $e^{-35} \approx 6.3 \times 10^{-16}$—close to the observed hierarchy of 10^{-17}. The key requirement is that **the number 35 is not a free parameter**; it is a constitutional integer determined by the OGN-5 closure relations.

This mechanism was first demonstrated by Randall and Sundrum in five dimensions (Randall & Sundrum, 1999). GG-6 extends it to six dimensions and, through the OGN-5 ledger, removes the last continuous free parameter (the throat depth) that RS left unfixed.

Contrast with the MSSM: In the MSSM, the hierarchy is stabilized by supersymmetry—superpartners cancel the large quantum corrections that would otherwise push the Higgs mass up to the Planck scale. This works mathematically but introduces over 100 new parameters and many new particles. GG-6 stabilizes the hierarchy geometrically, with zero new parameters beyond the five OGN integers.

3L.9 Proton stability: an exact guarantee

One of the most dramatic predictions of GG-6 is that the proton is **absolutely stable**—not just long-lived, but eternal.

In the Standard Model, baryon number B is an "accidental symmetry": it is conserved by the renormalizable interactions but can be violated by non-perturbative effects (sphaleron processes) and by higher-dimensional operators that appear in GUT theories. Most GUTs predict proton decay with a lifetime of 10^{34}–10^{36} years, and experiments (Super-Kamiokande, Hyper-Kamiokande) have been searching for it without success.

GG-6 resolves this by promoting baryon number to an *exact local gauge symmetry* $\mathrm{U}(1)_B$. Just as electric charge conservation follows from the gauge symmetry of electromagnetism, baryon number conservation follows from the gauge symmetry of $\mathrm{U}(1)_B$. No local operator with $\Delta B \neq 0$ can exist in the exact theory—not suppressed, not rare, but *forbidden*.

Metaphor: In the Standard Model, the proton's longevity is like a bank vault with a very thick door—it would take a very long time for a thief to break through, but given enough time, it is possible. In GG-6, the vault has *no door at all*. The topology of gauge invariance simply does not allow any path from a state with a proton to a state without one.

This is one of the sharpest falsifiers of the theory (Section **??**). If proton decay is ever observed, GG-6 in its present form is dead.

3L.10 How the six forces separate at low energy

At the GUT scale, all six forces are unified in one composite connection. But in the world we observe—at energies billions of times lower—the forces look very different. How does this separation happen?

The mechanism is twofold. First, the **renormalization group flow** along the radial direction r causes the gauge couplings to "run"—to change with energy scale. The strong coupling grows stronger at low energies (this is "asymptotic freedom"), while the weak and electromagnetic couplings evolve differently. At some intermediate energy, the unified SU(5) symmetry breaks into the Standard Model gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$.

Second, the **orbifold or interval boundary conditions** at the endpoints of the warped throat project out certain zero modes. In particular, the dangerous X and Y gauge bosons of SU(5)—responsible for proton decay in minimal GUTs—are projected out of the massless spectrum. Only the Standard Model gauge bosons (gluons, $W^{\pm}$, Z, and the photon) survive as massless modes. The baryonic and informational gauge bosons X_B and X_{Qi} acquire Stückelberg masses but remain as massive mediators of the fifth and sixth forces.

The elevator metaphor revisited: At the top floor of the building (the GUT scale), all six elevator doors open onto a single grand lobby. As you descend, the lobby gradually partitions: first the gravitational corridor separates (it was never truly "unified" in the gauge sense), then the strong-force corridor peels off, then the weak force, then the electromagnetic. The baryonic and informational corridors remain connected but their doors become heavy (Stückelberg mass), so at ground level (the weak scale) you only see four open doors—the four forces of everyday physics. But the other two corridors are still there, behind the heavy doors.

This separation mechanism is worked out in quantitative detail in Chapter 4. This book establishes the unified starting point; the companion chapter derives the four-dimensional endpoint.

Making the Theory Quantum-Consistent

3L.1 What is a quantum anomaly?

In classical physics, symmetries are exact: if the laws are symmetric under a transformation, they stay symmetric forever. In quantum physics, this is not always true. Sometimes, the process of quantization—promoting classical fields to quantum operators and summing over all possible quantum fluctuations—*breaks* a classical symmetry. When this happens, the symmetry is said to be "anomalous."

Not all anomalies are fatal. The "axial anomaly" of QCD, for instance, is a real physical effect that correctly predicts the decay rate of the neutral pion. But *gauge anomalies*—anomalies that break the gauge symmetries of the theory—are fatal. If a gauge anomaly is present, the theory becomes mathematically inconsistent: probabilities cease to add up to one, unitarity is violated, and the theory produces nonsensical predictions.

Metaphor: Think of a bridge designed with a beautiful bilateral symmetry. If quantum corrections (thermal fluctuations, material imperfections) break that symmetry in a benign way—say, making one railing slightly rougher than the other—the bridge still stands. But if the corrections break a *structural* symmetry—say, removing one of the support cables on one side—the bridge collapses. Gauge anomalies are like removing a structural cable.

Why anomalies are a gift: Although anomalies sound like a nuisance, they are actually one of the most powerful tools in theoretical physics. The *requirement* that gauge anomalies cancel places extremely tight constraints on the particle content of the theory. In the Standard Model, the anomaly-cancellation conditions are one of the reasons why quarks and leptons must come in complete families—you cannot have quarks without leptons, or vice versa, without creating an anomaly. In GG-6, the anomaly constraints are even more powerful: they uniquely fix the BI-6 coefficient vector.

3L.2 A brief history of anomalies

The story of anomalies in physics is a fascinating chapter in the interplay between mathematics and nature.

In 1969, Stephen Adler and John Bell with Roman Jackiw independently discovered the "ABJ anomaly": the conservation law for the axial current in quantum electrodynamics, which holds classically, is violated by quantum effects. This anomaly turned out to be physically real—it explains the observed decay rate of the neutral pion $\pi^0 \to \gamma\gamma$.

In the 1970s, it was realized that gauge anomalies—as opposed to global anomalies—are fatal to the consistency of a quantum field theory. The Standard Model was checked and found to be anomaly-free, which placed nontrivial constraints on the allowed particle content.

The watershed moment came in 1984, when Green and Schwarz showed that the gauge anomaly of ten-dimensional supergravity coupled to super-Yang–Mills theory could be cancelled by a topological mechanism involving a tensor field (Green and Schwarz, 1984). This "anomaly cancellation by inflow" revived string theory and launched the first superstring revolution.

GG-6 uses the same mathematical mechanism—but in six dimensions rather than ten, and without supersymmetry. The central observation is that the Green–Schwarz mechanism is a statement about topology and characteristic classes, not about strings per se.

3L.3 The Green–Schwarz mechanism: repair by inflow

In 1984, Michael Green and John Schwarz discovered a remarkable mechanism for cancelling gauge anomalies in ten-dimensional superstring theory (Green and Schwarz, 1984). The idea is elegant: introduce a tensor field B_2 (a generalization of the electromagnetic potential) whose classical variation under gauge transformations is designed to *exactly* cancel the quantum anomaly produced by the fermion sector.

In schematic terms:

$$\underbrace{\delta_{\text{gauge}}\,\Gamma_{1-\text{loop}}}_{\text{quantum anomaly}} + \underbrace{\delta_{\text{gauge}}\,S_{\text{GS}}}_{\text{classical GS term}} = 0.$$

The quantum sickness on the left is cured by the classical medicine on the right.

For this to work, a very specific condition must be satisfied: the anomaly polynomial (a mathematical object that encodes all the anomaly information) must *factorize* in a particular way. In GG-6, the required factorization is:

$$I_8 = \tfrac{1}{2}\, X_4 \wedge X_4,$$

where X_4 is exactly the four-form appearing in BI-6. This is not a coincidence—it is the same identity doing double duty. BI-6 simultaneously locks the forces together *and* cancels the anomaly. One equation, two jobs.

Why this matters: The Green–Schwarz mechanism was originally discovered in the context of ten-dimensional superstring theory, and many physicists associate it exclusively with strings. The GG-6 paper shows that this association is too narrow. The mechanism is a statement about characteristic classes and Chern–Simons forms on fiber bundles with connections. It requires a gauge field, a spin connection, a tensor field, and a factorizable anomaly polynomial. *None of these ingredients require strings or supersymmetry.*

3L.4 BRST symmetry: the quantum guardian

When quantizing a gauge theory, one must "fix the gauge"—choose a specific representative from each equivalence class of physically identical configurations. This procedure introduces fictitious particles called "ghosts" (Faddeev–Popov ghosts) that cancel unphysical degrees of freedom in loop diagrams.

The consistency of this entire apparatus is guaranteed by a remarkable symmetry called **BRST symmetry**, named after Becchi, Rouet, Stora, and Tyutin (Becchi et al., 1976; Tyutin, 1975). The BRST operator s satisfies $s^2 = 0$ (it is nilpotent, like the exterior derivative d in differential geometry), and physical states are those in the cohomology of s—states that are s-closed but not s-exact.

The GG-6 paper constructs the full BRST algebra for the six-dimensional theory—including the non-Abelian SU(5) sector, the two Abelian sectors $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$, the tensor field B_2, and the matter fields—and proves that the algebra closes off-shell (without using the equations of motion). It then uses the descent equations (a systematic procedure relating forms of different degree and ghost number) to show that the Green–Schwarz variation exactly cancels the one-loop anomaly.

The conclusion is encapsulated in one equation:

$$\mathcal{S}(\Gamma_{\text{full}}) = 0,$$

where $\mathcal{S}$ is the Slavnov–Taylor functional and Γ_{full} is the complete quantum effective action. This is the *homogeneous Slavnov–Taylor identity*—the statement that the full theory, including all quantum corrections, is gauge-invariant.

What this means in plain language: The theory is mathematically self-consistent at the quantum level. All the bookkeeping checks out. Probabilities add up to one. There are no hidden inconsistencies lurking at higher loop order.

3L.5 No supersymmetry, no strings: a precise statement

Most physicists associate the Green–Schwarz mechanism with superstring theory. Many assume that anomaly cancellation in dimensions higher than four *requires* supersymmetry. The GG-6 paper shows that this assumption is incorrect.

The reasoning is straightforward: the mathematical tools used in the anomaly cancellation—characteristic classes, Chern–Simons forms, descent equations, BRST cohomology—are statements about bundles with connections on smooth manifolds. They do not require a worldsheet, a superpartner spectrum, or a string tension. They work on any admissible six-dimensional stage.

The constitution then finishes the argument. AX-4 (maximum symmetry, minimum freedom) says: if a structure is not *forced* by consistency, it is *excluded*. Since supersymmetry and strings are not forced by the anomaly cancellation or by any other constitutional clause, they are excluded as unnecessary freedom.

This is a philosophical inversion of the standard approach. In string theory, one begins with strings and derives anomaly cancellation as a consequence. In GG-6, one begins with anomaly cancellation and derives the absence of strings as a consequence.

3L.6 The ultraviolet endpoint: E_{GUT}, not E_{Pl}

Another sharp distinction: GG-6 places its ultraviolet endpoint at the GUT scale ($E_{\text{GUT}} \sim 2\times10^{16}$ GeV), *not* at the Planck scale ($E_{\text{Pl}} \sim 10^{19}$ GeV).

What does this mean? The theory claims to be a valid, self-consistent description of physics up to E_{GUT}, where all forces unify. It does *not* claim to describe physics at arbitrarily high energies, and it does not claim to "solve quantum gravity" in the Planck-scale sense.

This is a feature, not a bug. The constitution asks: what is the minimal geometric stage needed to unify gauge structure, cancel anomalies, and produce operational measurements? The answer is a

six-dimensional bulk anchored at E_{GUT}. The theory is honest about its domain of validity—and that honesty is scientifically cleaner than pretending to solve every ultraviolet question in one stroke.

Einstein's Unfinished Dream: How GG-6 Realizes the Geometric Vision

3L.1 What Einstein wanted

Albert Einstein spent the last thirty years of his life—from roughly 1925 until his death in 1955—searching for a "unified field theory": a single geometric framework that would merge gravity with electromagnetism, and ultimately with all the forces of nature.

Einstein's conviction was simple and profound: *physics is geometry.* If gravity can be understood as the curvature of spacetime, then the other forces—which also govern how particles move and interact—should likewise be expressible as geometric properties of some extended structure. He tried many approaches: non-symmetric metrics, torsion, higher-dimensional spaces, teleparallel gravity. None succeeded, partly because the nuclear forces (strong and weak) were not yet understood, and partly because the mathematical tools of fiber-bundle gauge theory had not yet been developed.

Einstein died without finding his unified field theory. The physics community largely abandoned his program, turning instead to quantum field theory, the Standard Model, and eventually string theory. But Einstein's *philosophical* conviction—that nature's forces are geometric in origin—never died. It echoed through Kaluza–Klein theory (Kaluza, 1921; Klein, 1926), through the gauge-geometry correspondence, and through the holographic principle.

GG-6 is, in a precise sense, the realization of Einstein's dream.

3L.2 How GGEP fulfills the geometric vision

AX-2 (the Geometry–Gauge Equivalence Principle) is a direct formalization of Einstein's conviction. It states that the primitive field is a composite connection:

$$\mathcal{A} = \omega[g] \oplus A^{\mathrm{gauge}}.$$

Gravity (ω) and gauge forces (A) are not separate entities coupled after the fact; they are two components of one geometric object.

Einstein would have recognized this immediately. His general relativity already expressed gravity as a connection (the Levi-Civita connection of the metric). AX-2 simply extends the same logic to the gauge sector: the gauge connection A is as geometric as the gravitational connection ω. Both live on the same manifold. Both contribute to the same curvature. Both are constrained by the same anomaly-cancellation condition.

The difference between Einstein's attempts and GG-6 is threefold:

1. The correct mathematical language. Einstein worked with metrics and tensors on four-dimensional spacetime. GG-6 works with connections and curvatures on a six-dimensional fiber bundle. The language of principal bundles, Chern–Simons forms, and characteristic classes—developed by mathematicians like Chern, Weil, and Atiyah in the decades after Einstein's death—provides the tools that Einstein lacked.

2. The correct gauge group. Einstein tried to unify gravity with electromagnetism (the only non-gravitational force known in his day). GG-6 unifies gravity with the full $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ gauge group, encompassing all six forces. The strong and weak nuclear forces, discovered in the 1960s and 1970s, are essential ingredients that Einstein could not have included.

3. The constitutional framework. Einstein searched for the unified theory by trial and error—trying one ansatz after another, hoping to find the "right" equations. GG-6 does not guess; it *derives* the equations from constitutional principles. The axioms and postulates constrain the solution space so tightly that essentially one theory survives.

3L.3 "Geometrize gauge, don't quantize gravity"

The standard approach to fundamental physics since the 1970s has been to start with the Standard Model (a quantum gauge theory) and general relativity (a classical geometric theory), and then try to "quantize gravity"—to find a quantum version of general relativity that is consistent at the Planck scale. This is the approach of loop quantum gravity, string theory, and many other programs.

GG-6 inverts this strategy. Instead of quantizing gravity, it *geometrizes gauge*. The question is not "how do we make gravity quantum?" but "how do we make the gauge forces geometric?"

This inversion has several advantages:

Lower energy scale. Quantizing gravity requires understanding physics at the Planck scale ($E_{\mathrm{Pl}} \sim 10^{19}$ GeV), which is far beyond the reach of any conceivable collider experiment. Geometrizing gauge requires understanding physics at the GUT scale ($E_{\mathrm{GUT}} \sim 10^{16}$ GeV), which is closer to experimental reach and directly connected to observable phenomena like proton stability and gauge coupling unification.

No need for supersymmetry or strings. Quantizing gravity in a consistent way has historically required supersymmetry (in string theory) or radical reformulation of spacetime (in loop quantum gravity). Geometrizing gauge requires neither: the Green–Schwarz mechanism and the Bianchi identity BI-6 provide anomaly cancellation without SUSY, and the warped six-dimensional geometry provides the hierarchy without strings.

Einstein would approve. Einstein resisted quantum mechanics for philosophical reasons (he famously said "God does not play dice"). He would have been more sympathetic to a program that starts with geometry and derives quantum behavior as a boundary projection, rather than starting with quantum mechanics and trying to "quantize" geometry. GG-6, through AX-3

and GDOS, derives quantum mechanics from the bulk-to-boundary dictionary—making it a consequence of geometry rather than a separate postulate.

3L.4 The philosophical lineage

GG-6 stands in a clear philosophical lineage:

- **Euclid** (300 BCE): geometry as the foundation of rigorous knowledge.
- **Einstein** (1915): gravity as geometry (general relativity).
- **Kaluza–Klein** (1921–1926): electromagnetism as geometry (extra-dimensional).
- **Yang–Mills** (1954): non-Abelian forces as connections on fiber bundles.
- **GG-6** (2026): all forces as geometry, encoded in one composite connection on a six-dimensional warped bulk.

Each step in this lineage extends the geometric vision to a broader class of phenomena. GG-6 is the latest—and, the program claims, the final—step: the one that geometrizes *all* forces, including the strong and weak nuclear forces, baryonic protection, and informational bookkeeping.

Whether it is truly the final step is for experiment and mathematics to decide. But the philosophical direction—more geometry, not less—is the one that has been productive for over two thousand years.

Completing Quantum Mechanics: Measurement as a Constitutional Object

3L.1 The oldest open problem in physics

The measurement problem is arguably the oldest open problem in quantum physics. It has been debated since the 1920s, and despite enormous progress in understanding decoherence, quantum information, and open systems, it remains unresolved in the standard formulation.

The problem is simple to state. Quantum mechanics has two rules:

(i) **Schrödinger evolution:** between measurements, the state evolves deterministically according to the Schrödinger equation $i\hbar\,\partial_t|\psi\rangle = H|\psi\rangle$.

(ii) **Born rule + collapse:** when a measurement is performed, the state "collapses" to one of the eigenstates of the measured observable, with probability given by the Born rule $p_n = |\langle n|\psi\rangle|^2$.

These two rules are logically incompatible. Rule (i) is deterministic, linear, and reversible. Rule (ii) is probabilistic, nonlinear, and irreversible. There is no derivation of (ii) from (i) within the standard framework.

The standard quantum mechanics textbook simply declares: "when a measurement is performed, use rule (ii)." But it never defines what "measurement" means. Is it the interaction of the quantum system with a macroscopic apparatus? With a conscious observer? With any sufficiently complex system? The textbook is silent.

3L.2 How GUT-10 resolves the measurement problem

The GUT-10 constitution resolves the measurement problem by making two constitutional moves:

Move 1: AX-3 (OGGEP) constitutionalizes measurement. AX-3 says: a theory is incomplete unless it provides a precise bulk-to-boundary dictionary that produces boundary states, evolution maps, and measurement instruments. This means that "measurement" is not an additional rule; it is part of the constitutional definition of the theory.

In the GG-6 realization, the six-dimensional bulk evolves unitarily (rule (i) holds in the bulk). But the four-dimensional boundary observer does not have access to the full bulk. The boundary observer sees only the projected state—the reduced density matrix obtained by tracing out the inaccessible bulk degrees of freedom.

Move 2: POS-4 forces CPTP evolution. POS-4 says: the boundary evolution must be completely positive and trace-preserving. Combined with AX-1 (causality), this forces the boundary evolution to take the GKLS form:

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right).$$

The first term is the familiar Hamiltonian evolution. The second term—the Lindblad operators L_k—describes the irreversible effects of the environment: decoherence, dissipation, and measurement-like processes.

The resolution. Rules (i) and (ii) are not contradictory; they operate at different levels:

- Rule (i) holds in the six-dimensional bulk (unitary, deterministic, reversible).
- Rule (ii) holds on the four-dimensional boundary (probabilistic, irreversible, CPTP).

The Born rule and wavefunction collapse are not additional postulates; they are *derived consequences* of the bulk-to-boundary projection. The Kraus operators of the measurement instrument are the mathematical representation of the dictionary's measurement channel.

3L.3 The first definition of "measurement" in fundamental physics

Appendices for Chapter 4

GDOS: From the Bulk to the Boundary

Appendix 4A: compact ultraviolet recap of GG-6, BI-6, and OGN-5

4A.1 Derivational strategy and statement of the problem

The constitutional setup of Sections 1–2 reduces the admissible theory search to a narrow ultraviolet question: what is the *minimal* geometric realization of AX-2 consistent with POS-1, POS-3, POS-5, and POS-6? The purpose of the present section is to answer that question and to do so in the only manner allowed by the constitution: not by freely postulating a favorite higher-dimensional ansatz, but by deriving the stage, symmetry, and locking identity from the admissibility clauses themselves.

The logic is simple to state. AX-2 requires a primitive composite connection rather than a direct product of "spacetime geometry" and "internal gauge bundle." POS-3 requires that the resulting chiral gauge theory be anomaly-cancellable, possibly by inflow. POS-5 requires that the theory possess a constitutionally meaningful coarse-graining parameter that can be interpreted both geometrically and operationally. POS-6 forbids arbitrary continuous normalizations and therefore prefers a primitive rational/integer ledger whenever such a ledger exists. AX-4 then orders the admissible solutions by maximum symmetry and minimum freedom.

The result of this four-step filter is the GG-6 stage:

$$\boxed{M_6 = M_4 \times I_r \times S^1_\theta, \qquad G_{\text{Bulk}} = \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi} \times \text{Gravity}, \qquad \text{d}H_3 = X_4,} \tag{4A.1}$$

with

$$\boxed{X_4 = \text{tr}(R \wedge R) - \text{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}.} \tag{4A.2}$$

The remainder of this appendix recalls the derivation of (4A.1) and (4A.2) from the constitution in a sequence of lemmas.

4A.2 Why the minimal admissible stage is six-dimensional

The first question is dimensional rather than group-theoretic. Why should the theorem-level realization of AX-2 be six-dimensional rather than five-, seven-, ten-, or eleven-dimensional? Within the present program the answer is constitutional and can be stated sharply.

Theorem 4.0.1 (Minimality of the six-dimensional stage)**.** *Assume AX-1–AX-4 and POS-1, POS-3, POS-5, POS-6. Then the minimal admissible bulk realizing AX-2 while preserving a four-dimensional registered boundary, a geometrized renormalization-scale direction, and a compact Abelian holonomy direction is six-dimensional:*

$$M_6 \cong M_4 \times I_r \times S^1_\theta. \tag{4A.3}$$

Any candidate with fewer than six dimensions lacks one of the three required structures; any candidate with more than six dimensions introduces additional continuous directions not forced by the constitution.

Proof. The operational boundary of AX-3 is four-dimensional by construction: every registered event, detector record, and classical spatiotemporal protocol in the present program lives on a $3+1$-dimensional boundary manifold M_4. This fixes the first four coordinates. POS-5 then demands a constitutionally meaningful coarse-graining parameter. If one wishes to geometrize that parameter, the minimal choice is a single non-compact interval coordinate $r \in I_r$; any attempt to represent scale covariance without an extra direction leaves the coarse-graining structure external to the geometry and therefore violates the intent of AX-2. Finally, POS-1 and POS-6 require a minimal compact holonomy sector capable of supporting a primitive Abelian phase and Wilson-line data. The smallest such manifold is S^1, parameterized by a periodic coordinate $\theta \sim \theta + 2\pi$. Thus at least two extra coordinates are needed beyond M_4: one non-compact scale coordinate and one compact phase coordinate.

A five-dimensional bulk can provide only one of these two functions. A space of dimension greater than six may also realize them, but the additional directions are not forced by any constitutional clause and therefore violate AX-4/POS-6 unless independently justified. The minimal admissible dimension is therefore six. □

The theorem is deliberately stronger than a mere "Kaluza–Klein motivation." It says that the sixth dimension is not introduced for aesthetic embellishment; it is the minimal price of turning the renormalization semigroup and the compact Abelian phase into geometric data while preserving the observed four-dimensional boundary.

4A.3 The warped metric and the logarithmic radial coordinate

Once the stage is fixed as $M_4 \times I_r \times S^1_\theta$, the next question is the metric. The constitution again restricts the answer sharply. Local Lorentz symmetry on the boundary requires the metric to split into a four-dimensional piece and an internal piece. POS-5 requires the scale direction r to act monotonically along coarse-graining. AX-4 prefers the maximally symmetric metric consistent with these requirements. The natural ansatz is therefore the warped product

$$\mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}(x)\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2\,\mathrm{d}\theta^2. \tag{4A.4}$$

Here $g_{\mu\nu}(x)$ is the four-dimensional boundary metric, $A(r)$ is a warp factor, and R_θ is the radius of the compact Abelian circle. The r-dependence is confined to the four-dimensional conformal prefactor because this is the minimal way to encode scale dependence without breaking boundary Lorentz covariance.

The logarithmic interpretation of r follows directly from the semigroup structure of coarse-graining. Let μ be the effective renormalization scale of a boundary description and let $\mathcal{G}_\lambda$ denote a scale transformation by factor $\lambda > 0$. Composition of coarse-grainings gives

$$\mathcal{G}_{\lambda_1} \circ \mathcal{G}_{\lambda_2} = \mathcal{G}_{\lambda_1\lambda_2}. \tag{4A.5}$$

Any geometric coordinate implementing this semigroup additively must therefore satisfy

$$r(\lambda_1\lambda_2) = r(\lambda_1) + r(\lambda_2), \tag{4A.6}$$

whose continuous solutions are logarithmic. Choosing the ultraviolet endpoint to be E_{GUT} and using the OGN convention that fixes the proportionality constant to unity, one obtains

$$\boxed{r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right).} \tag{4A.7}$$

Equation (4A.7) has three immediate consequences. First, $r = 0$ corresponds to the ultraviolet endpoint of the GG-6 description. Second, flow toward larger r is exactly flow toward lower infrared scales. Third, the later holographic dictionary can interpret radial Hamilton–Jacobi evolution as four-dimensional renormalization-group evolution.

The warp factor is constrained by maximal symmetry. If one demands constant negative scalar curvature in the (x^μ, r) sector and a finite-throat realization compatible with POS-5, the simplest choice is

$$A(r) = kr, \qquad \Lambda_6 < 0, \tag{4A.8}$$

so that the bulk is AdS_6-like. Nothing in the constitution compels the exact *string-theoretic* AdS_6 solution space; what is compelled is the warped constant-curvature class as the minimal realization of logarithmic radial RG flow and four-dimensional Lorentz symmetry.

Table 4A.1: How the constitutional clauses fix the six-dimensional geometric stage.

Derived structure	**Primary clauses**	**Reason**
M_4 boundary	AX-1, AX-3	Registered observations and causal protocols are four-dimensional.
Interval I_r	AX-2, POS-5	Coarse-graining/RG flow must be geometrized as an extra non-compact direction.
Circle S^1_θ	POS-1, POS-6	Minimal compact holonomy supporting Abelian phase/Wilson-line data.
Warped product metric	AX-1, POS-5, AX-4	Preserves boundary Lorentz symmetry while encoding scale covariance minimally.
Logarithmic radial coordinate	POS-5, POS-6	The RG semigroup becomes additive only in log scale; OGN fixes the proportionality convention.
AdS_6-like curvature	AX-4	Maximal symmetry with one radial warp direction and no extra moduli.

4A.4 Composite connection and six-force unification at E_{GUT}

AX-2 does not merely require that spacetime and gauge sectors coexist; it requires that they be treated as two facets of one primitive connection. In the minimal six-dimensional realization,

that connection is

$$\mathbb{A} = \omega \oplus A_5 \oplus A_B \oplus A_{Qi}, \tag{4A.9}$$

where ω is the spin connection on TM_6, A_5 is the SU(5) connection, and A_B, A_{Qi} are Abelian one-forms for baryonic and informational charge. The corresponding curvature is

$$\mathbb{F} = R \oplus F_5 \oplus F_B \oplus F_{Qi}, \qquad R = \mathrm{d}\omega + \omega \wedge \omega, \qquad F_5 = \mathrm{d}A_5 + A_5 \wedge A_5, \tag{4A.10}$$

with $F_B = \mathrm{d}A_B$ and $F_{Qi} = \mathrm{d}A_{Qi}$.

The bulk gauge group is therefore

$$\boxed{G_{\text{Bulk}} = \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times \text{Gravity.}} \tag{4A.11}$$

This is the minimal symmetry enlargement consistent with AX-4. The non-Abelian block is SU(5), because it is the smallest simple group unifying the Standard-Model gauge sectors on the four-dimensional boundary (Georgi and Glashow, 1974). The extra $\mathrm{U}(1)_B$ is introduced because exact proton protection is treated in this program as a constitutional symmetry rather than as an accidental low-energy remnant. The extra $\mathrm{U}(1)_{Qi}$ is introduced because the operational/open-system side of the theory requires a conserved information ledger at the same structural level as electric and baryonic charge. The equality in status between $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ will become mathematically visible in BI-6.

At the boundary the group eventually reduces to

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \tag{4A.12}$$

with electromagnetism emerging after electroweak symmetry breaking. Counting gravity separately, the program therefore speaks of *six forces*: strong, weak, electromagnetic, gravity, baryonic, and informational. The six-force count is not arbitrary nomenclature; it reflects the number of gauge/gravity channels that survive as separately conserved sectors on the boundary.

Table 4A.2: Force sectors in the GG-6 bulk and their four-dimensional boundary interpretation.

Sector	**Bulk field**	**Boundary image**	**Role in the present program**
Gravity	ω, R	$g_{\mu\nu}$, Einstein sector	Geometric backbone and source of the positive BI-6 coefficient.
Unified gauge block	A_5, F_5	$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	Minimal non-Abelian unification block at E_{GUT}.
Baryonic sector	A_B, F_B	$\mathrm{U}(1)_B$	Exact proton protection and baryon-current ledger.
Informational sector	A_{Qi}, F_{Qi}	$\mathrm{U}(1)_{Qi}$	Operational/open-system information-current ledger.

Sector	Bulk field	Boundary image	Role in the present program
Tensor sector	B_2, H_3	Axionic/Stückelberg descendants	Single inflow channel canceling gauge and mixed anomalies.
Radial geometry	$r, A(r)$	RG scale / holographic depth	Geometrizes coarse-graining and later 4D renormalization flow.

4A.5 Derivation of BI-6 from anomaly freedom and OGN normalization

The geometry of the stage is not yet enough. AX-2 and POS-3 demand a *locking equation* that ties the curvatures of the composite connection together in a way that cancels all local anomalies. In the presence of a single two-form B_2, the most general Green–Schwarz-compatible ansatz for the Bianchi identity with diagonal Abelian block is

$$\mathrm{d}H_3 = a\ \mathrm{tr}(R \wedge R) + b\ \mathrm{Tr}(F_5 \wedge F_5) + c\, F_B \wedge F_B + d\, F_{Qi} \wedge F_{Qi}. \tag{4A.13}$$

The coefficients (a, b, c, d) are not arbitrary couplings; they are topological weights in a characteristic-class identity. We now derive them.

Lemma 4.0.2 (Normalization of the gravitational coefficient)**.** *After rescaling B_2 once and for all, one may set*

$$a = +1. \tag{4A.14}$$

Proof. The overall normalization of (4A.13) can be absorbed into the normalization of the tensor field and its kinetic term. POS-6 then requires that the remaining coefficient vector be primitive. We therefore use the primitive convention in which the gravitational coefficient is $+1$. □

Lemma 4.0.3 (Relative sign of the non-Abelian gauge block)**.** *With the convention of Lemma 4.0.2, the non-Abelian coefficient is*

$$b = -1. \tag{4A.15}$$

Proof. Write the tensor field strength in Chern–Simons form,

$$H_3 = \mathrm{d}B_2 + \omega_{3L} - \omega_{3,5} + \cdots, \tag{4A.16}$$

where $\mathrm{d}\omega_{3L} = \mathrm{tr}(R \wedge R)$ and $\mathrm{d}\omega_{3,5} = \mathrm{Tr}(F_5 \wedge F_5)$. The minus sign between the gravitational and non-Abelian Chern–Simons forms is the one that ensures cancellation of the consistent one-loop anomaly by the classical Green–Schwarz variation (Green and Schwarz, 1984). Equivalently, the gauge variation of the fermion determinant must be canceled by the opposite variation of the tensor sector. In the primitive normalization of Lemma 4.0.2, this fixes $b = -1$. □

The only nontrivial derivation therefore concerns the Abelian coefficients. Here the constitution does something powerful: the OGN clause converts what would usually be a convention into a calculation.

Proposition 4.0.4 (Abelian trace normalization)**.** *Let the primitive raw hypercharge and baryon generators in the* **5** *be*

$$Y_0 = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \qquad B_0 = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0\right). \tag{4A.17}$$

Then

$$\mathrm{Tr}(Y_0^2) = \frac{5}{6}, \qquad \mathrm{Tr}(B_0^2) = \frac{1}{3}, \qquad \frac{\mathrm{Tr}(B_0^2)}{\mathrm{Tr}(Y_0^2)} = \frac{2}{5}. \tag{4A.18}$$

Proof. The calculation is direct:

$$\mathrm{Tr}(Y_0^2) = 3 \times \frac{1}{9} + 2 \times \frac{1}{4} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}, \tag{4A.19}$$

while

$$\mathrm{Tr}(B_0^2) = 3 \times \frac{1}{9} = \frac{1}{3}. \tag{4A.20}$$

Dividing gives (4A.18). □

Proposition 4.0.4 is the compact algebraic origin of the number 2/5. The inverse ratio 5/2 is the corresponding relative geometric weight. In words: once SU(5) and primitive baryon charge are fixed, the minimal OGN normalization already knows the Abelian-to-unified trace ratio.

The information sector is treated on equal footing with the baryon sector by AX-4 and POS-6. The simplest and constitutionally preferred choice is therefore a second primitive Abelian generator of the same norm class as B_0, so that

$$\mathrm{Tr}(Q_{i,0}^2) = \mathrm{Tr}(B_0^2), \qquad c = d. \tag{4A.21}$$

Combining (4A.18) and (4A.21) with Lemmae 4.0.2 and 4.0.3 yields the central result.

Theorem 4.0.5 (BI-6 coefficient vector)**.** *Under AX-2, AX-4, POS-3, and POS-6, the unique primitive diagonal single-tensor Bianchi identity compatible with* $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ *and equal Abelian treatment is*

$$\boxed{(a, b, c, d) = (1, -1, -2/5, -2/5).} \tag{4A.22}$$

Equivalently,

$$\boxed{\mathrm{d}H_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}.} \tag{4A.23}$$

Proof. The coefficient a is fixed to $+1$ by primitive normalization. The sign $b = -1$ is fixed by Green–Schwarz inflow. Proposition 4.0.4 gives the unique minimal Abelian trace ratio 2/5 relative to the unified non-Abelian block. The equality $c = d$ follows from equal Abelian treatment under AX-4/POS-6. No other primitive diagonal rational vector satisfies all four requirements simultaneously. □

Theorem 4.0.5 is the mathematical heart of this book. The rest of GG-6 is the systematic examining of (4A.22). The adjective "locked" in Geometry–Gauge Locking refers precisely to the fact that the curvature sectors are not free to appear with arbitrary coefficients once anomaly freedom and normalization minimality are imposed.

4A.6 Single-tensor anomaly factorization

The Green–Schwarz mechanism becomes operative only if the one-loop anomaly polynomial factorizes through the same four-form appearing in BI-6. The admissible factorization is

$$\boxed{I_8 = \frac{1}{2} X_4 \wedge X_4,} \tag{4A.24}$$

with

$$X_4 = \operatorname{tr} R^2 - \operatorname{Tr} F_5^2 - \frac{2}{5} F_B^2 - \frac{2}{5} F_{Qi}^2. \tag{4A.25}$$

Expanding (4A.24) gives

$$I_8 = \frac{1}{2}(\operatorname{tr} R^2)^2 + \frac{1}{2}(\operatorname{Tr} F_5^2)^2 + \frac{2}{25}(F_B^2)^2 + \frac{2}{25}(F_{Qi}^2)^2 - \operatorname{tr} R^2 \ \operatorname{Tr} F_5^2 - \frac{2}{5} \operatorname{tr} R^2 F_B^2 - \frac{2}{5} \operatorname{tr} R^2 F_{Qi}^2 + \frac{2}{5} \operatorname{Tr} F_5^2 F_B^2 + \frac{2}{5} \operatorname{Tr} F_5^2 F_{Qi}^2 + \frac{4}{25} F_B^2 F_{Qi}^2. \tag{4A.26}$$

The absence of irreducible $\operatorname{tr} R^4$ and $\operatorname{Tr} F_5^4$ pieces is the signature of successful single-tensor factorization. In the present program this is not an optional technical refinement but the exact mathematical content of POS-3 at the six-dimensional level.

The Green–Schwarz term is then

$$S_{\mathrm{GS}} = \int_{M_6} B_2 \wedge X_4. \tag{4A.27}$$

The same X_4 entering (4A.27) enters the Bianchi identity, the anomaly polynomial, the descent equations, and later the boundary Ward identities. This is why the framework is so rigid: one does not get to choose separate tensors, separate inflow coefficients, and separate anomaly remedies for different sectors.

Table 4A.3: Physical interpretation of the BI-6 coefficient vector.

Coefficient	Sector	Value	Interpretation
a	Gravity	$+1$	Primitive reference normalization of the geometric curvature sector.
b	SU(5)	-1	Opposite inflow orientation relative to the gravitational Chern–Simons form.
c	$\mathrm{U}(1)_B$	$-2/5$	Minimal baryonic trace weight relative to the unified non-Abelian block.

Coefficient	Sector	Value	Interpretation
d	$\mathrm{U}(1)_{Qi}$	$-2/5$	Equal informational trace weight demanded by Abelian symmetry/minimality.
$\lvert a/c\rvert$	Gravity/Baryon	$5/2$	Reciprocal trace ratio; later the canonical geometric weight in boundary current language.

4A.7 OGN-5 and the collapse of dimensionless freedom

The Bianchi coefficients are not the only discrete data. POS-6 requires a minimal integer ledger controlling normalization and hierarchy, and the working ledger used throughout the recent project archive is

$$\boxed{(M, N, L; J, K) = (1, 3, 5; 7, 35).} \tag{4A.28}$$

The derived relations are

$$J = 2N + 1 = 7, \qquad K = LJ = 35, \qquad K = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35. \tag{4A.29}$$

Even before later phenomenological papers identify J or K with specific measured quantities, the algebraic role of the ledger is already clear.

(i) $M = 1$ records the single compact Abelian phase direction S^1_θ.

(ii) $N = 3$ records the triplicity index inherited by the observable boundary sector (color/family structure).

(iii) $L = 5$ records the minimal non-Abelian unification index of SU(5).

(iv) $J = 2N + 1$ is the minimal odd closure index compatible with the triplicity sector.

(v) $K = 35$ is the master lock integer controlling the throat depth and normalization closure.

The power of (4A.28) is that it replaces what would ordinarily be a list of unrelated conventions by a closed integer ledger. In the warped background (4A.4), one may parameterize the finite throat by

$$0 \le r \le r_*, \qquad kr_* = K = 35. \tag{4A.30}$$

Thus the exponential hierarchy across the throat is controlled by

$$\exp[-A(r_*)] = e^{-K} = e^{-35}, \tag{4A.31}$$

up to order-one conventions fixed by the matter localization profile. The important point is not the exact numerical identification of every low-energy mass ratio in this book, but rather that the throat depth is no longer a continuous modulus: it is fixed by the integer ledger.

The same ledger compresses gauge normalization. Since $L = 5$ is the unification index and $J = 7$ is the odd closure index, the product $K = LJ$ packages the joint effect of non-Abelian unification and dyadic closure. AX-4 and POS-6 then imply that any additional continuous dimensionless parameter surviving the derivation must be either redundant or constitutionally unjustified.

Proposition 4.0.6 (Parameter counting in GG-6)**.** *Up to the overall gravitational scale set by G_N, the dimensionless free data of the GG-6 bulk collapse to the discrete OGN-5 ledger and the fixed BI-6 coefficient vector.*

Proof. A generic six-dimensional gauge-gravity action would contain continuous dimensionless parameters in the relative gauge normalizations, Abelian coefficients, throat depth, and compactification ratios. In GG-6, the Abelian coefficients are fixed by Theorem 4.0.5, the throat depth by (4A.30), and the normalization sector by (4A.28). The only remaining continuous datum is the overall dimensionful gravitational scale, which may be taken to be G_N or, equivalently, $M_{\rm Pl}$. All other dimensionless ratios are constitutionally frozen. □

A useful formula is the four-dimensional Planck relation

$$M_{\rm Pl}^2 = M_6^4\,(2\pi R_\theta)\int_0^{r_*} \mathrm{d}r\, e^{-2A(r)}. \tag{4A.32}$$

Once $A(r) = kr$ and $kr_* = K = 35$ are fixed, the hierarchy information is discrete. This book therefore adopts the working principle emphasized by the project archive: beyond G_N, no independent continuous parameter should be introduced unless some later section proves that the constitution forces it.

4A.8 The GG-6 theorem

We can now summarize the derivation in theorem form.

Theorem 4.0.7 (Constitutional derivation of GG-6)**.** *Assume AX-1–AX-4 and POS-1–POS-7. Then the minimal theorem-level realization of AX-2 satisfying anomaly freedom, RG compatibility, and Occam gauge normalization is the warped six-dimensional stage*

$$M_6 = M_4 \times I_r \times S^1_\theta, \qquad \mathrm{d}s_6^2 = e^{-2A(r)} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2 \mathrm{d}\theta^2, \qquad r = \ln(E_{\rm GUT}/\mu), \tag{4A.33}$$

with composite gauge-gravity group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ *Gravity, Bianchi identity* (4A.2)*, anomaly polynomial factorization* (4A.24)*, and discrete normalization ledger* (4A.28)*. This stage is what the present program denotes by* GG-6*.*

Proof sketch. The geometric stage follows from Theorem 4.0.1 and the semigroup argument for (4A.7). The force content follows from AX-2 plus minimal unification and the two Abelian ledger symmetries. The Bianchi identity and its coefficient vector follow from Theorem 4.0.5. The anomaly factorization and single-tensor inflow follow from (4A.24). The discrete parameter ledger follows from Proposition 4.0.6. The conjunction of these statements is the GG-6 stage. □

The outcome of Section 2 is therefore a complete classical bulk skeleton. What remains is to quantize it consistently and to show that it defines a six-dimensional quantum field theory with

exact BRST/Green–Schwarz cancellation. That was the purpose of Chapter 2; in the present companion chapter the result is recorded here only as a compact ultraviolet recap.

Appendix 4B: exact Nakajima–Zwanzig reduction and the OGGEP channel

This appendix expands the derivation summarized in Section 3. Let the bulk density operator obey

$$\dot{\rho} = \mathfrak{L}(t)\rho, \qquad \mathfrak{L}(t)X = -\mathrm{i}[H(t), X]. \tag{4B.1}$$

Choose the projection

$$\mathcal{P}X = \mathrm{Tr}_{\mathrm{env}}(X) \otimes \rho_{\mathrm{env}}^{(0)}, \qquad \mathcal{Q} = 1 - \mathcal{P}. \tag{4B.2}$$

Acting with $\mathcal{P}$ and $\mathcal{Q}$ gives

$$\dot{\mathcal{P}}\rho = \mathcal{P}\mathfrak{L}\mathcal{P}\rho + \mathcal{P}\mathfrak{L}\mathcal{Q}\rho, \tag{4B.3}$$

$$\dot{\mathcal{Q}}\rho = \mathcal{Q}\mathfrak{L}\mathcal{P}\rho + \mathcal{Q}\mathfrak{L}\mathcal{Q}\rho. \tag{4B.4}$$

The second equation is solved by variation of constants:

$$\mathcal{Q}\rho(t) = \mathcal{G}_{\mathcal{Q}}(t, t_0)\mathcal{Q}\rho(t_0) + \int_{t_0}^{t} \mathrm{d}s\, \mathcal{G}_{\mathcal{Q}}(t, s)\mathcal{Q}\mathfrak{L}(s)\mathcal{P}\rho(s), \tag{4B.5}$$

where

$$\mathcal{G}_{\mathcal{Q}}(t, s) = \mathcal{T} \exp\left(\int_{s}^{t} \mathrm{d}u\, \mathcal{Q}\mathfrak{L}(u)\mathcal{Q} \right). \tag{4B.6}$$

Substituting back, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}\rho(t) = \mathcal{P}\mathfrak{L}(t)\mathcal{P}\rho(t) + \int_{t_0}^{t} \mathrm{d}s\, \mathcal{P}\mathfrak{L}(t)\mathcal{G}_{\mathcal{Q}}(t, s)\mathcal{Q}\mathfrak{L}(s)\mathcal{P}\rho(s) + \mathcal{P}\mathfrak{L}(t)\mathcal{G}_{\mathcal{Q}}(t, t_0)\mathcal{Q}\rho(t_0). \tag{4B.7}$$

This is the exact projected equation.

To connect this with the boundary language, define

$$\mathfrak{D}_{\mathrm{OGGEP}} = \mathcal{R} \circ \mathrm{Tr}_{\mathrm{env}}, \tag{4B.8}$$

with $\mathcal{R}$ a boundary restriction or coding map. Then the boundary state is

$$\rho_{\mathrm{Bdy}}(t) = \mathfrak{D}_{\mathrm{OGGEP}}[\rho_{\mathrm{Bulk}}(t)]. \tag{4B.9}$$

Applying $\mathfrak{D}_{\mathrm{OGGEP}}$ to the projected equation yields an exact nonlocal equation for $\rho_{\mathrm{Bdy}}(t)$:

$$\dot{\rho}_{\mathrm{Bdy}}(t) = \mathfrak{D}_{\mathrm{OGGEP}}[\mathcal{P}\mathfrak{L}(t)\mathcal{P}\rho(t)] + \int_{t_0}^{t} \mathrm{d}s\, \mathfrak{D}_{\mathrm{OGGEP}}[\mathcal{K}(t, s)\mathcal{P}\rho(s)] + \mathfrak{D}_{\mathrm{OGGEP}}[\mathcal{I}(t)]\,. \tag{4B.10}$$

The first term is local in time, the second carries memory, and the third retains initial correlations. In a weak-memory regime one replaces $\rho(s)$ by a time-local expansion around t, producing a time-local generator $\mathfrak{L}_{\mathrm{GDOS}}(t)$.

A useful special case is the Born–Markov factorization

$$\rho_{\mathrm{Bulk}}(t) \approx \rho_{\mathrm{Bdy}}(t) \otimes \rho_{\mathrm{env}}^{(0)}, \tag{4B.11}$$

together with a short environment correlation time. Then

$$\dot{\rho}_{\mathrm{Bdy}}(t) = \mathfrak{L}_{\mathrm{GDOS}}(t)\rho_{\mathrm{Bdy}}(t) \tag{4B.12}$$

with $\mathfrak{L}_{\mathrm{GDOS}}$ of GKLS type.

The Stinespring dilation theorem guarantees that every CPTP boundary channel has a unitary dilation (Stinespring, 1955). Thus the derivation can be read in both directions:

(i) starting from the closed bulk and projecting down gives a boundary CPTP map;

(ii) starting from a boundary CPTP map, one can reconstruct a unitary dilation with an enlarged environment.

The present framework uses direction (i) as the physical explanation and direction (ii) as the mathematical consistency check.

Finally, measurement instruments may be written in Kraus form:

$$\mathcal{I}_\alpha(\rho) = \sum_m M_{\alpha m}\rho M_{\alpha m}^\dagger, \qquad \sum_{\alpha,m} M_{\alpha m}^\dagger M_{\alpha m} = \mathbf{1}. \tag{4B.13}$$

In the GG-6/GDOS interpretation the Kraus operators are not primitive. They are the reduced images of bulk interaction channels after environment tracing and boundary restriction.

Appendix 4C: Kaluza–Klein reduction, overlap integrals, and effective couplings

This appendix records the elementary reduction formulas used in Sections 4 and 5. For the warped metric

$$\mathrm{d}s_6^2 = e^{-2A(r)}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}r^2 + R_\theta^2\mathrm{d}\theta^2, \tag{4C.1}$$

the determinant is

$$\sqrt{-g_6} = e^{-4A(r)}R_\theta\sqrt{-g_4}. \tag{4C.2}$$

Gravitational zero mode

The Einstein–Hilbert term reduces as

$$\begin{aligned} S_{\mathrm{EH}}^{(6)} &= \frac{M_6^4}{2}\int \mathrm{d}^4x\,\mathrm{d}r\,\mathrm{d}\theta\, e^{-4A(r)}R_\theta\sqrt{-g_4}\left(e^{2A(r)}R_4 + \cdots\right) \\ &= \frac{1}{2}\left(M_6^4\,2\pi R_\theta\int \mathrm{d}r\, e^{-2A(r)}\right)\int \mathrm{d}^4x\sqrt{-g_4}\,R_4 + \cdots . \end{aligned} \tag{4C.3}$$

Hence

$$M_{\rm Pl}^2 = M_6^4\, 2\pi R_\theta \int \mathrm{d}r\, e^{-2A(r)}. \tag{4C.4}$$

Gauge zero modes

For a gauge field with zero-mode profile $f_0(r)$,

$$\begin{aligned} S^{(6)}_{\rm gauge} &= -\frac{1}{4g_6^2}\int \mathrm{d}^4x\,\mathrm{d}r\,\mathrm{d}\theta\, e^{-4A(r)} R_\theta \sqrt{-g_4}\, e^{2A(r)} e^{2A(r)}\, |f_0(r)|^2\, F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \cdots \\ &= -\frac{1}{4}\left(\frac{2\pi R_\theta}{g_6^2}\int \mathrm{d}r\ |f_0(r)|^2\right)\int \mathrm{d}^4x\,\sqrt{-g_4}\, F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \cdots . \end{aligned} \tag{4C.5}$$

Thus

$$\frac{1}{g_4^2} = \frac{2\pi R_\theta}{g_6^2}\int \mathrm{d}r\ |f_0(r)|^2 . \tag{4C.6}$$

The absence of warp factors in the gauge normalization is a direct consequence of the cancellation between $\sqrt{-g_6}$ and the two inverse metrics.

Fermion zero modes

For a chiral fermion,

$$\Psi(x,r,\theta) = \sum_{n,\ell} \psi^{(n,\ell)}(x)\chi_{n,\ell}(r,\theta), \tag{4C.7}$$

the kinetic term becomes

$$\int \mathrm{d}^6x\sqrt{-g_6}\,\bar\Psi \mathrm{i}\Gamma^\mu D_\mu \Psi \to \int \mathrm{d}^4x\sqrt{-g_4}\,\bar\psi^{(0)}\mathrm{i}\gamma^\mu D_\mu \psi^{(0)} \tag{4C.8}$$

provided the internal wavefunctions satisfy the normalization condition

$$2\pi R_\theta \int \mathrm{d}r\, e^{-3A(r)}\, \chi_0^\dagger(r)\chi_0(r) = 1 \tag{4C.9}$$

in the standard warped spinor convention.

Yukawa overlap integrals

A six-dimensional Yukawa coupling

$$\lambda_6 \int \mathrm{d}^6x\sqrt{-g_6}\,\Psi_1\Psi_2 H \tag{4C.10}$$

reduces to a four-dimensional Yukawa

$$\lambda_4 = \lambda_6\, 2\pi R_\theta \int \mathrm{d}r\, e^{-4A(r)}\, \chi_1(r)\chi_2(r)\eta_H(r), \tag{4C.11}$$

up to gamma-matrix and chirality conventions. Therefore flavor hierarchies may arise geometrically through profile overlaps even if the underlying six-dimensional coupling matrix is comparatively simple.

KK masses

For scalar fields, the mode equation has the generic form

$$-\partial_r\big(e^{-4A}\partial_r f_n\big) + e^{-4A}\frac{\ell^2}{R_\theta^2} f_n = m_{n,\ell}^2\, e^{-2A} f_n. \tag{4C.12}$$

The exact spectrum depends on the warp profile and boundary conditions, but the qualitative scaling

$$m_{n,\ell}^2 \sim m_n^2 + \frac{\ell^2}{R_\theta^2} \tag{4C.13}$$

is robust.

Appendix 4D: $SU(5)$ decomposition, Yukawa operators, and proton-stability selection rules

Write the antisymmetric **10** as $\Psi_{10}^{AB} = -\Psi_{10}^{BA}$ and the antifundamental as $\Psi_{\bar{5},A}$, with $A = 1, \ldots, 5$. Split the index into a color part $\alpha = 1, 2, 3$ and a weak part $m = 4, 5$. Then

$$\Psi_{10}^{\alpha m} = Q^{\alpha m}, \qquad \Psi_{10}^{\alpha\beta} = \epsilon^{\alpha\beta\gamma} u_\gamma^c, \qquad \Psi_{10}^{45} = e^c, \tag{4D.1}$$

while

$$\Psi_{\bar{5},\alpha} = d_\alpha^c, \qquad \Psi_{\bar{5},m} = L_m. \tag{4D.2}$$

For the Higgs $\mathbf{5}_H$,

$$H_5^A = (T^\alpha, H^m), \tag{4D.3}$$

where T is the color triplet and H the weak doublet.

The standard SU(5)-covariant Yukawa operators are

$$\mathcal{L}_Y = -\frac{1}{4}(Y_u)_{ij}\epsilon_{ABCDE}\Psi_{10}^{AB(i)}\Psi_{10}^{CD(j)}H_5^E - \sqrt{2}(Y_d)_{ij}\Psi_{10}^{AB(i)}\Psi_{\bar{5},A}^{(j)}(H_5^\dagger)_B + \text{h.c.} \tag{4D.4}$$

Decomposing them into Standard Model components gives

$$\begin{aligned}\mathcal{L}_Y = &-(Y_u)_{ij}\Big(Q_i u_j^c H + \frac{1}{2}Q_i Q_j T + u_i^c e_j^c T\Big) \\ &- (Y_d)_{ij}\Big(Q_i d_j^c H^\dagger + e_i^c L_j H^\dagger + Q_i L_j T^\dagger + u_i^c d_j^c T^\dagger\Big) + \text{h.c.}\end{aligned} \tag{4D.5}$$

Equation (4D.5) shows immediately why minimal four-dimensional SU(5) typically threatens proton stability: exchange of the triplet T generates baryon-violating effective operators. In the present framework the danger is removed by two simultaneous mechanisms:

(i) the triplet zero mode is projected out or lifted by boundary conditions;

(ii) exact local $U(1)_B$ forbids every local operator with $\Delta B \neq 0$.

Thus the theorem-level proton stability of Section 5 does not rely on a numerical suppression of triplet exchange. The dangerous operator never appears in the exact low-energy theory.

A convenient charge check is the following. For the dimension-six operators most often discussed in GUT phenomenology,

$$QQQL: \quad \Delta B = +1, \tag{4D.6}$$
$$u^c u^c d^c e^c: \quad \Delta B = -1, \tag{4D.7}$$
$$QQQH: \quad \Delta B = +1, \tag{4D.8}$$
$$u^c d^c d^c: \quad \Delta B = -1. \tag{4D.9}$$

All are forbidden by local $U(1)_B$.

Appendix 4E: Anomaly coefficients for $U(1)_B$ and $U(1)_{Qi}$

This appendix collects the anomaly coefficients quoted in Section 5. We use the convention $T(\mathbf{fund}) = \frac{1}{2}$ for both SU(3) and SU(2), and we work with left-handed Weyl fermions.

Table 4E.1: One-generation contributions to the baryon-anomaly coefficients.

Field	$q_B T_3 d_2$	$q_B T_2 d_3$	$q_B Y^2 d_3 d_2$	$Y q_B^2 d_3 d_2$
Q	$\frac{1}{3}\cdot\frac{1}{2}\cdot 2 = \frac{1}{3}$	$\frac{1}{3}\cdot\frac{1}{2}\cdot 3 = \frac{1}{2}$	$\frac{1}{3}\cdot\frac{1}{36}\cdot 6 = \frac{1}{18}$	$\frac{1}{6}\cdot\frac{1}{9}\cdot 6 = \frac{1}{9}$
u^c	$-\frac{1}{3}\cdot\frac{1}{2} = -\frac{1}{6}$	0	$-\frac{1}{3}\cdot\frac{4}{9}\cdot 3 = -\frac{4}{9}$	$-\frac{2}{3}\cdot\frac{1}{9}\cdot 3 = -\frac{2}{9}$
d^c	$-\frac{1}{3}\cdot\frac{1}{2} = -\frac{1}{6}$	0	$-\frac{1}{3}\cdot\frac{1}{9}\cdot 3 = -\frac{1}{9}$	$\frac{1}{3}\cdot\frac{1}{9}\cdot 3 = \frac{1}{9}$
L	0	0	0	0
e^c	0	0	0	0
Total (1 gen)	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
Total (3 gen)	0	$\frac{3}{2}$	$-\frac{3}{2}$	0

The cubic and gravitational baryon anomalies also vanish:

$$\mathcal{A}_{BBB} = \sum_f q_B^3 d_3 d_2 = \frac{1}{27}\cdot 6 - \frac{1}{27}\cdot 3 - \frac{1}{27}\cdot 3 = 0, \tag{4E.1}$$
$$\mathcal{A}_{BRR} = \sum_f q_B d_3 d_2 = \frac{1}{3}\cdot 6 - \frac{1}{3}\cdot 3 - \frac{1}{3}\cdot 3 = 0. \tag{4E.2}$$

For the minimal diagonal Qi realization $q_{Qi} = q_B$, the entire anomaly table is identical:

$$\mathcal{A}_{Qi,33} = 0, \qquad \mathcal{A}_{Qi,22} = \frac{3}{2}, \qquad \mathcal{A}_{Qi,YY} = -\frac{3}{2}, \qquad \mathcal{A}_{QiQiQi} = \mathcal{A}_{QiRR} = \mathcal{A}_{YQiQi} = 0. \tag{4E.3}$$

More generally, the only constitutional requirement is that the Qi anomaly vector lie in the image of the same GS/Stückelberg cancellation matrix that descends from BI-6.

The operator interpretation of anomaly cancellation is then

$$\partial_\mu J^\mu_{I,\text{matter}} + \partial_\mu J^\mu_{I,\text{GS}} = 0, \qquad I \in \{B, Qi\}, \tag{4E.4}$$

which is the Abelian part of the homogeneous Ward algebra used in Section 5.

Appendix 4F: BV/BRST master equation, Ward operators, and current algebra

This appendix spells out the component-level gauge-fixing and Ward structure of the zero-mode theory. Introduce antifields

$$K_{G^a_\mu}, \quad K_{W^i_\mu}, \quad K_{B_\mu}, \quad K_{X^\mu_B}, \quad K_{X^\mu_{Qi}}, \quad K_\Phi$$

for every field Φ. The BV master action is

$$S_{\rm BV} = S_{4,\rm zero} + \int \mathrm{d}^4x \left[K_{G^a_\mu} sG^a_\mu + K_{W^i_\mu} sW^i_\mu + K_{B_\mu} sB_\mu + K_{X^\mu_B} sX^{(B)}_\mu + K_{X^\mu_{Qi}} sX^{(Qi)}_\mu + \sum_\Phi K_\Phi s\Phi \right]. \tag{4F.1}$$

Choose the gauge-fixing fermion

$$\begin{aligned}\Psi_{\rm gf} = \int \mathrm{d}^4x \Big[&\bar c^a_3 \Big(\partial^\mu G^a_\mu + \frac{\xi_3}{2} b^a_3 \Big) + \bar c^i_2 \Big(\partial^\mu W^i_\mu + \frac{\xi_2}{2} b^i_2 \Big) \\ &+ \bar c_Y \Big(\partial^\mu B_\mu + \frac{\xi_Y}{2} b_Y \Big) + \bar c_B \Big(\partial^\mu X^{(B)}_\mu + \frac{\xi_B}{2} b_B \Big) + \bar c_{Qi} \Big(\partial^\mu X^{(Qi)}_\mu + \frac{\xi_{Qi}}{2} b_{Qi} \Big) \Big].\end{aligned} \tag{4F.2}$$

Then

$$S_{\rm gf+gh} = s\Psi_{\rm gf}, \tag{4F.3}$$

and the gauge-fixed action is obtained by setting $K_\Phi = \delta\Psi_{\rm gf}/\delta\Phi$.

The quantum 1PI effective action Γ obeys the Slavnov functional identity

$$\begin{aligned}\mathcal{S}(\Gamma) = \int \mathrm{d}^4x \Bigg[&\frac{\delta\Gamma}{\delta K_{G^a_\mu}} \frac{\delta\Gamma}{\delta G^a_\mu} + \frac{\delta\Gamma}{\delta K_{W^i_\mu}} \frac{\delta\Gamma}{\delta W^i_\mu} + \frac{\delta\Gamma}{\delta K_{B_\mu}} \frac{\delta\Gamma}{\delta B_\mu} + \frac{\delta\Gamma}{\delta K_{X^\mu_B}} \frac{\delta\Gamma}{\delta X^{(B)}_\mu} + \frac{\delta\Gamma}{\delta K_{X^\mu_{Qi}}} \frac{\delta\Gamma}{\delta X^{(Qi)}_\mu} \\ &+ \sum_\Phi \frac{\delta\Gamma}{\delta K_\Phi} \frac{\delta\Gamma}{\delta\Phi} + b^a_3 \frac{\delta\Gamma}{\delta \bar c^a_3} + b^i_2 \frac{\delta\Gamma}{\delta \bar c^i_2} + b_Y \frac{\delta\Gamma}{\delta \bar c_Y} + b_B \frac{\delta\Gamma}{\delta \bar c_B} + b_{Qi} \frac{\delta\Gamma}{\delta \bar c_{Qi}} \Bigg] = 0.\end{aligned} \tag{4F.4}$$

Linearizing around the full solution defines the nilpotent operator

$$\mathcal{B}_\Gamma X = \frac{\mathrm{d}}{\mathrm{d}\varepsilon}\bigg|_{\varepsilon=0} \mathcal{S}(\Gamma + \varepsilon X), \qquad \mathcal{B}_\Gamma^2 = 0, \tag{4F.5}$$

provided $\mathcal{S}(\Gamma) = 0$. Local anomalies would appear as a ghost-number-one cohomology class of $\mathcal{B}_\Gamma$. The statement proved in the main text is precisely that BI-6 removes this cohomology class in the present theory.

The Abelian Ward operators are extracted by replacing the ghosts by local gauge parameters and differentiating Γ with respect to the corresponding fields. For example,

$$\mathcal{W}_B(x)\Gamma = -\partial_\mu \frac{\delta\Gamma}{\delta X^{(B)}_\mu(x)} + \mathrm{i} \sum_\Phi q_B(\Phi) \left(\Phi \frac{\delta\Gamma}{\delta\Phi} - \Phi^\dagger \frac{\delta\Gamma}{\delta\Phi^\dagger} \right) + M_B \frac{\delta\Gamma}{\delta\sigma_B(x)} = 0, \tag{4F.6}$$

and similarly for $U(1)_{Qi}$. For the non-Abelian sectors one obtains

$$\mathcal{W}_C^a(x)\Gamma = -\left(D_\mu \frac{\delta\Gamma}{\delta G_\mu}\right)^a + \mathrm{i}\sum_\Phi \frac{\delta\Gamma}{\delta\Phi} T_C^a \Phi - \mathrm{i}\sum_\Phi \Phi^\dagger T_C^a \frac{\delta\Gamma}{\delta\Phi^\dagger} = 0, \tag{4F.7}$$

$$\mathcal{W}_L^i(x)\Gamma = -\left(D_\mu \frac{\delta\Gamma}{\delta W_\mu}\right)^i + \mathrm{i}\sum_\Phi \frac{\delta\Gamma}{\delta\Phi} t^i \Phi - \mathrm{i}\sum_\Phi \Phi^\dagger t^i \frac{\delta\Gamma}{\delta\Phi^\dagger} = 0. \tag{4F.8}$$

The equal-time current algebra then follows by standard arguments. Writing

$$Q_C^a = \int \mathrm{d}^3x\, J_C^{0a}, \quad Q_L^i = \int \mathrm{d}^3x\, J_L^{0i}, \quad Q_Y = \int \mathrm{d}^3x\, J_Y^0, \quad Q_B = \int \mathrm{d}^3x\, J_B^0, \quad Q_{Qi} = \int \mathrm{d}^3x\, J_{Qi}^0,$$

one finds

$$[Q_C^a, Q_C^b] = \mathrm{i} f^{abc} Q_C^c, \qquad [Q_L^i, Q_L^j] = \mathrm{i}\epsilon^{ijk} Q_L^k, \tag{4F.9}$$

$$[Q_C^a, Q_L^i] = 0, \qquad [Q_C^a, Q_Y] = [Q_C^a, Q_B] = [Q_C^a, Q_{Qi}] = 0, \tag{4F.10}$$

$$[Q_L^i, Q_Y] = [Q_L^i, Q_B] = [Q_L^i, Q_{Qi}] = 0, \qquad [Q_Y, Q_B] = [Q_Y, Q_{Qi}] = [Q_B, Q_{Qi}] = 0. \tag{4F.11}$$

There is no anomalous Schwinger term because the anomaly cocycle has already been cancelled in the master equation.

Appendix 4G: operator catalog and exact proton-stability selection rules

This appendix classifies representative local operators up to dimension six according to their baryon and Qi charges in the minimal diagonal realization.

Table 4G.1: Representative local operators and their status under $U(1)_B \times U(1)_{Qi}$.

Operator	**Dim.**	ΔB	ΔQi	**Status**
$\bar{Q}Y_u\widetilde{H}u_R$	4	0	0	Allowed Yukawa
$\bar{Q}Y_dHd_R$	4	0	0	Allowed Yukawa
$\bar{L}Y_eHe_R$	4	0	0	Allowed Yukawa
LLe^c	4	0	0 in minimal diagonal realization, but gauge noninvariant under $SU(2)_L \times U(1)_Y$ without contraction details	Forbidden in the exact gauge theory as written
QLd^c	4	$+1/3 - 1/3 = 0$ but hypercharge/iQi structure incomplete	Not a gauge-invariant operator in the minimal exact theory	
$u^cd^cd^c$	4	-1	-1	Forbidden by $U(1)_B$ and $U(1)_{Qi}$
$QQQL$	6	$+1$	$+1$	Forbidden
$u^cu^cd^ce^c$	6	-1	-1	Forbidden
$QQQH$	5	$+1$	$+1$	Forbidden
$(LH)(LH)$	5	0	0	Allowed if neutrino masses are generated this way
$\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$	4	0	0	Allowed kinetic term
$B_{\mu\nu}X^{\mu\nu}_{(B)}$	4	0	0	Allowed kinetic mixing

The status column deserves clarification. Gauge invariance must be checked under the full low-energy gauge group, not only under $U(1)_B$. The essential point for proton stability is much simpler: any operator with $\Delta B \neq 0$ is absolutely forbidden. Since the proton is the lightest baryon, no decay channel can exist in the exact theory.

A possible source of confusion is the presence of Stückelberg masses. A Stückelberg term

$$\frac{1}{2}(\partial_\mu\sigma_B + M_B X^{(B)}_\mu)^2 \tag{4G.1}$$

does not break the gauge symmetry. Under

$$X^{(B)}_\mu \mapsto X^{(B)}_\mu + \partial_\mu\alpha_B, \qquad \sigma_B \mapsto \sigma_B - M_B\alpha_B,$$

the combination remains invariant. Therefore the selection rule $\Delta B = 0$ survives even when the baryonic gauge boson is massive.

One may also ask whether integrating out heavy fields can regenerate forbidden operators. The answer is no for exact gauge symmetries. Integrating out heavy degrees of freedom preserves all exact local gauge symmetries. Consequently no $\Delta B \neq 0$ operator can be induced in the Wilsonian effective action if $U(1)_B$ is exact in the ultraviolet completion.

Appendix 4H: master summary tables

This appendix gathers the paper-wide derivation chain into compact tables for quick reference.

Table 4H.1: From constitutional clause to derived structure.

Input clause	**Immediate mathematical demand**	**Derived structure in this book**	**Main section**
AX-1 (LC)	Retarded causal propagation and local admissibility	Local bulk action on warped M_6 with boundary-compatible causal structure	Secs. 1–2
AX-2 (GGEP)	Composite geometry⊕gauge connection	GG-6 bulk, BI-6, six-force unification at E_{GUT}	Sec. 2
AX-3 (OGGEP)	Bulk theory must furnish its own measurement semantics	Exact bulk-to-boundary open-system channel and GDOS	Sec. 3
AX-4 (MSMF)	Maximize admissible symmetry, minimize arbitrary freedom	OGN-5 arithmetic compression, no superpartner/string ontology required	Secs. 1–3
POS-1	Layered gauge invariance	6D and 4D BRST/Ward identities	Secs. 3, 5, App. 4A.8
POS-2	Unified extremal principle	Common action/entropy/current structure across scales	Secs. 1, 3–4
POS-3	Anomaly-free admissibility	BI-6 factorization and GS inflow cancellation	Secs. 2–3, App. 4A.8
POS-4	Closure/positivity	CPTP boundary dynamics and GKLS/GDOS form	Sec. 3, App. 4A.8
POS-5	RG compatibility	Radial coordinate $r = \ln(E_{\mathrm{GUT}}/\mu)$, formal 4D EFT reduction	Secs. 2, 4–5, App. 4A.8
POS-6	Occam gauge normalization	OGN-5, primitive 2/5 coefficient, frozen dimensionless ledger	Sec. 2, App. 4A.8

Input clause	Immediate mathematical demand	Derived structure in this book	Main section
POS-7	Falsifiability	Registered phenomenological tasks and failure modes	Secs. 6–7, App. 4A.8

Table 4H.2: Assumed versus derived ingredients in this book.

Category	Assumed as constitutional input	Derived inside the paper
Foundational clauses	4 axioms + 7 postulates	Everything else
Bulk geometry	None beyond admissibility principles	Warped $M_6 = M_4 \times I_r \times S^1_\theta$
Gauge structure	None beyond GGEP and minimality	$\mathrm{SU}(5) \times U(1)_B \times U(1)_{Qi}$ at E_{GUT}
Tensor sector	None beyond anomaly admissibility	Single GS/BI-6 two-form sector
Quantum measurement	Not assumed as primitive law	Derived via OGGEP/GDOS channel
4D gravity	Not assumed as separate primitive	Metric zero-mode reduction
4D QFT	Not assumed as primitive	Boundary generating functional after heavy-mode integration
Standard Model matter sector	Phenomenological target only	Zero-mode truncation of bulk multiplets
Proton stability	Not assumed as axiom	Theorem from exact local $U(1)_B$

Table 4H.3: Parameter budget emphasized by the present framework.

Sector	Ordinary expectation	GG-6/GDOS stance
Relative gauge normalizations	Convention-dependent continuous freedom	Frozen by OGN and BI-6 primitive ratios
Extra-dimensional scale ratios	Multiple moduli possible	Collapsed into r-geometry and OGN-5 ledger at theorem level
Ultraviolet endpoint	Often extended toward E_{Pl}	Declared to terminate at E_{GUT}
Measurement sector	External interpretational add-on	Derived open-system channel
Baryon conservation	Accidental or approximate	Exact local gauge symmetry
Information bookkeeping	Usually absent	$U(1)_{Qi}$ Abelian ledger sector

Appendix 4I: Summary of abbreviations

Abbreviation	Meaning
LC	Locality and Causality.
GGEP	Geometry–Gauge Equivalence Principle.
OGGEP	Operational/Open Geometry–Gauge Equivalence Principle.
MSMF	Maximum Symmetry, Minimum Freedom.
GI	Gauge Invariance (local, dynamical/open, global layers).
UEP	Unified Extremal Principle.
AF	Anomaly-Free admissibility by inflow.
CP	Closure and Positivity.
RG	RG compatibility and scale covariance.
OGN	Occam Gauge Normalization.
MF	Measurement and Falsifiability.
GG-6	Geometry–Gauge Locking in six dimensions.
GDOS	Geometric Dynamics of Open Systems.
CPTP	Completely positive and trace preserving.
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan generator form.
GUT	Grand unified scale/sector.
SM	Standard Model.

Appendix 4J: Glossary of key symbols

Symbol	Meaning
$\mathcal{T}$	Candidate theory tuple .
$\mathcal{M}$	Bulk–boundary manifold/stage.
$\mathcal{P}$	Bundle/principal-bundle data over $\mathcal{M}$.
$\mathcal{A}$	Composite geometry$\oplus$gauge connection.
$\mathcal{F}$	Composite curvature $\mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$.
$\mathcal{D}$	Operational bulk$\to$boundary dictionary.
$\Phi_{t,s}$	Admissible evolution map from time s to t.
$\mathcal{I}_\alpha$	Instrument/POVM element associated with outcome α.
$S(\mathcal{T})$	Symmetry score in AX-4.
$F(\mathcal{T})$	Freedom score in AX-4.
$M_4 \times I_r \times S^1_\theta$	Anticipated six-dimensional stage of GG-6.
X_4	Green–Schwarz/Bianchi four-form to be derived in Section 2.
E_{GUT}	Ultraviolet endpoint of the GG-6 description.

Appendix 4K: Consolidated derivation ledger and future technical tasks

With the derivation, discussion, and conclusion sections now incorporated into the unified manuscript, the principal chain of this book is complete. The main results may be itemized as follows:

Result 1: The GUT-10 constitution filters admissible theories through four axioms and seven postulates only.

Result 2: AX-2 plus POS-1–POS-6 select the minimal warped six-dimensional GG-6 bulk with BI-6 and the OGN-5 normalization ledger.

Result 3: The resulting six-dimensional gauge-gravity-tensor theory is BRST-consistent and anomaly-free through a single Green–Schwarz inflow channel.

Result 4: AX-3 plus POS-4 force a lawful open-system bulk-to-boundary channel, yielding the exact Nakajima–Zwanzig equation, the GKLS limit, and GDOS.

Result 5: The low-energy boundary theory is a formal four-dimensional QFT whose zero-mode truncation reproduces the Standard Model matter/gauge sector together with $U(1)_B$ and $U(1)_{Qi}$.

Result 6: Exact local $U(1)_B$ implies absolute proton stability in the theorem-level realization studied here.

The main technical tasks that remain beyond the scope of this book are now more sharply defined:

Task 1: construct one fully explicit benchmark six-dimensional matter reservoir with complete mass matrices and threshold corrections;

Task 2: determine a unique phenomenological realization of the $U(1)_{Qi}$ charges and kinetic/Stückelberg mixings;

Task 3: confront the resulting low-energy EFT with collider, flavor, neutrino, and cosmological constraints;

Task 4: extend the GDOS/OGGEP sector into a fully quantitative treatment of nonequilibrium measurement, cognition-adjacent open systems, and cosmological coarse-graining;

Task 5: prepare a final JHEP-style version with complete numerical benchmark tables and phenomenological plots.

Thus this book should be read as an expanded theorem-level derivation draft together with a clearly registered future-work agenda, not as the terminal phenomenological form of the program.

Appendix 4L: Abelian trace normalization and the origin of $2/5$

For convenience we record the short matrix calculation behind the Abelian trace-ratio statement summarized in Section 2. On the primitive **5** basis write

$$Y_0 = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \qquad B_0 = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0\right). \tag{4I.1}$$

Then

$$\mathrm{Tr}(Y_0^2) = 3 \times \frac{1}{9} + 2 \times \frac{1}{4} = \frac{5}{6}, \qquad \mathrm{Tr}(B_0^2) = 3 \times \frac{1}{9} = \frac{1}{3}, \tag{4I.2}$$

so that

$$\frac{\mathrm{Tr}(B_0^2)}{\mathrm{Tr}(Y_0^2)} = \frac{2}{5}, \qquad \frac{\mathrm{Tr}(Y_0^2)}{\mathrm{Tr}(B_0^2)} = \frac{5}{2}. \tag{4I.3}$$

In the present program the first ratio appears in BI-6 as the Abelian coefficient and the second ratio appears later as the reciprocal canonical weight in boundary current normalization. The calculation uses only primitive generator data and therefore exemplifies the OGN principle directly.

Appendix 4M: descent equations and BRST cancellation

To make the GG-6 recap in Section 2 mathematically self-contained we record the minimal descent chain. Let

$$X_4 = \mathrm{d}\omega_3^{(0)} \tag{4J.1}$$

for the Chern–Simons representative built from the gravitational, non-Abelian, and Abelian sectors of the composite connection. If

$$I_8 = \frac{1}{2} X_4 \wedge X_4, \tag{4J.2}$$

then a seven-form representative is

$$I_7^{(0)} = \frac{1}{2}\, \omega_3^{(0)} \wedge X_4, \qquad \mathrm{d}I_7^{(0)} = I_8. \tag{4J.3}$$

Under BRST variation,

$$s\omega_3^{(0)} + \mathrm{d}\omega_2^{(1)} = 0, \tag{4J.4}$$

so the consistent anomaly is the six-form

$$I_6^{(1)} = \omega_2^{(1)} \wedge X_4. \tag{4J.5}$$

Consequently,

$$s\Gamma_{1\mathrm{loop}} = 2\pi\mathrm{i} \int_{M_6} I_6^{(1)} = 2\pi\mathrm{i} \int_{M_6} \omega_2^{(1)} \wedge X_4. \tag{4J.6}$$

Choosing the tensor transformation

$$sB_2 = -2\pi\mathrm{i}\, \omega_2^{(1)} + \mathrm{d}\Lambda_1 \tag{4J.7}$$

then yields

$$s \int B_2 \wedge X_4 = -2\pi \mathrm{i} \int \omega_2^{(1)} \wedge X_4, \tag{4J.8}$$

which cancels the anomaly exactly. This is the compact cohomological statement behind the homogeneous ST identity in the main text.

Appendix 4N: Historical anchor map for the later discussion section

Table 4N.1: Selected historical anchors used in the comparison section of the full chapter.

Conceptual ingredient	**Classical/literature anchor**	**Role in the present program**
Gauge invariance and Noether identities	Noether, Yang–Mills, renormalized gauge theory (Noether, 1918; Yang and Mills, 1954; 't Hooft and Veltman, 1972)	Historical backbone for POS-1.
Anomaly cancellation by inflow	Green–Schwarz (Green and Schwarz, 1984)	Historical anchor for POS-3 and BI-6.
Warped extra dimensions	Randall–Sundrum (Randall & Sundrum, 1999)	Geometric intuition for the radial warp direction.
Bulk–boundary generating functionals	AdS/CFT literature (Gubser, Klebanov, & Polyakov, 1998; Maldacena, 1998; Witten, 1998)	Historical anchor for formal 4D QFT derivation.
Open quantum systems and positivity	Kraus, Gorini–Kossakowski–Sudarshan, Lindblad (Gorini et al., 1976; Kraus, 1983; Lindblad, 1976)	Mathematical anchor for OGGEP and GDOS.
SU(5) unification	Georgi–Glashow (Georgi and Glashow, 1974)	Historical anchor for the non-Abelian GUT block.
BRST/cohomological consistency	Becchi–Rouet–Stora, Tyutin (Becchi et al., 1976; Tyutin, 1975)	Anchor for the BRST/anomaly proofs in the bulk recap and appendices.
Arisaka constitutional synthesis	GUT-10, GDOS, companion preprints (Arisaka, 2026c,a,c)	New organizational framework and target derivation chain.

Introduction: The Missing Half of Physics

4N.1 The stage is built — but where are the actors?

The companion chapter GG-2026-2 and its accessible companion Chapter 3, Part II told a remarkable story. Starting from four axioms and seven postulates, the GUT-10 constitution forced a unique six-dimensional warped geometry—the GG-6 bulk—that locks gravity, the strong force, the weak force, electromagnetism, and two new forces (baryonic and informational) into one composite structure through the Bianchi identity BI-6 (Arisaka, 2026b). The anomalies cancel. The free parameters collapse to five integers. The theory is quantum-consistent without supersymmetry or strings.

That is the stage. But a stage, no matter how elegant, is not a play. A crucial question remains: *where is the world we actually observe?*

We do not live in six dimensions. Our laboratories, our detectors, our eyes and ears—everything we use to make measurements—operates in four dimensions: three of space and one of time. If the fundamental reality is six-dimensional, how does the four-dimensional world we experience arise from it?

This is the question that GG-2026-3 answers (Arisaka, 2026c). And the answer turns out to be far more profound than "the extra dimensions are small." The answer is that **the four-dimensional world is not the full reality; it is a shadow**—a mathematically precise projection of the six-dimensional bulk onto the four-dimensional surface accessible to finite observers.

Metaphor: Plato's Cave. In Plato's famous allegory, prisoners chained in a cave see only shadows cast on the wall by objects moving behind them. They mistake the shadows for reality. In GG-6/GDOS, we are all "prisoners" in four dimensions. The shadows on our cave wall are quantum mechanics, general relativity, and the Standard Model. The objects casting the shadows are the six-dimensional bulk fields. The "chain" binding us to the wall is the fact that we are finite observers who cannot access the full six-dimensional Hilbert space. And the OGGEP dictionary is the precise map from shadow to object.

4N.2 The measurement problem: ninety years without an answer

Since the 1930s, quantum mechanics has operated with an uncomfortable split personality. Between measurements, the quantum state evolves smoothly and deterministically according to the Schrödinger equation. But when a measurement occurs, the state "collapses" abruptly and probabilistically to one of the possible outcomes, with probabilities given by the Born rule (von Neumann, 1932).

These two rules are logically incompatible. The Schrödinger equation is deterministic and reversible; the Born rule is probabilistic and irreversible. No one has ever derived one from the other within the standard framework.

The various "interpretations" of quantum mechanics—Copenhagen, many-worlds, Bohmian, QBism, decoherent histories—are not solutions to the measurement problem. They are different *attitudes* toward it: different ways of talking about the gap without closing it. GDOS closes it.

4N.3 A brief history of the measurement problem

The measurement problem is not an obscure technicality. It has been debated by every major figure in twentieth-century physics.

Bohr and Heisenberg (1920s–1930s). Niels Bohr argued that quantum systems and measuring devices belong to fundamentally different categories: quantum objects obey the superposition principle, but measuring devices are "classical" and always show definite outcomes. He called this division the "Heisenberg cut." But he never specified *where* the cut falls. Is a single atom quantum? Yes. Is a cat quantum? Schrödinger's thought experiment was designed to show the absurdity of placing the cut anywhere in particular.

Von Neumann (1932). John von Neumann formalized quantum mechanics with a projection postulate: upon measurement, the state collapses to an eigenstate of the measured observable (von Neumann, 1932). This made the theory mathematically precise but introduced an unexplained dualism: unitary evolution between measurements, non-unitary projection during measurements.

Einstein, Podolsky, and Rosen (1935). Einstein argued that quantum mechanics must be "incomplete"—that there must be "hidden variables" underlying the probabilities. The EPR paper showed that entangled particles exhibit correlations that seem to require either hidden variables or instantaneous action at a distance.

Bell (1964). John Bell proved that no local hidden-variable theory can reproduce all the predictions of quantum mechanics. Nature is either nonlocal or the hidden-variable program fails. Experiments have confirmed the quantum predictions, ruling out local hidden variables.

Everett (1957). Hugh Everett proposed the "many-worlds" interpretation: the wavefunction never collapses; instead, the universe splits into branches, one for each possible outcome. This eliminates the measurement problem at the cost of an infinite multiverse.

Zurek (1981–2003). Wojciech Zurek developed the decoherence program, showing that interaction with the environment rapidly destroys quantum superpositions of macroscopic objects (Zurek, 2003). Decoherence explains why cats are never observed in superpositions of alive and dead. But it does not explain why a *specific* outcome is observed—only why outcomes are classical.

GDOS (2026). The GG-6/GDOS program provides a different kind of answer. It does not interpret the existing formalism differently; it changes the formalism. Measurement is not a postulate appended to quantum mechanics; it is a derived consequence of the bulk-to-boundary projection. The measurement problem is not "solved" by reinterpretation; it is *dissolved* by derivation.

4N.4 The answer in one sentence

Measurement is not a separate postulate; it is the mathematical consequence of projecting a unitary six-dimensional bulk evolution onto the four-dimensional boundary accessible to a finite observer.

The Schrödinger equation holds in the bulk (six dimensions). The Born rule and collapse hold on the boundary (four dimensions). They are not contradictory because they operate at different levels.

4N.5 What the accessible companion (Part II) covers

This accessible companion (Part II) follows the structure of GG-2026-3, section by section:

1. **Introduction** (this section): the measurement problem and the shadow metaphor.
2. **Constitutional recap**: what Papers I and II derived, and why OGGEP is the necessary next step.
3. **From bulk to boundary**: how the Hilbert space splits, the memory kernel, GKLS, GDOS, and the emergence of measurement.
4. **Gravity and QFT**: how Einstein's equations and the infrared QFT emerge from the same bulk.
5. **The Standard Model**: gauge group, three generations, and proton stability.
6. **Discussion**: significance, comparisons, and falsifiability.
7. **FAQ**: detailed answers to common objections.
8. **Conclusions**: the argument in one paragraph.

No appendices. Every essential idea is in the main text. Equations appear only when they illuminate a concept and are always explained in words.

What Was Already Derived and Why We Need More

4N.1 The GG-6 bulk in one paragraph

Papers I and II (Arisaka, 2026a,b) established: the GUT-10 constitution (four axioms, seven postulates) forces a six-dimensional warped bulk $M_6 = M_4 \times I_r \times S^1_\theta$ with composite gauge-gravity group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity, the Bianchi identity BI-6 with uniquely fixed coefficients $(1, -1, -2/5, -2/5)$, the OGN-5 integer ledger $(1, 3, 5; 7, 35)$, and a complete anomaly-free six-dimensional quantum field theory at E_{GUT}. This is the ultraviolet starting point.

4N.2 Why OGGEP is the necessary next step

An anomaly-free six-dimensional theory, by itself, does not tell us what an observer sees. It does not define measurement. It does not explain why the world looks four-dimensional. It does not derive general relativity or quantum mechanics.

AX-3 (OGGEP) demands that the theory provide an explicit dictionary from the six-dimensional bulk to four-dimensional boundary observables. This book constructs this dictionary and follows its consequences to their logical conclusion.

Metaphor: the thermostat and the weather. A thermostat in your house measures the temperature—one number. The actual state of the air involves trillions of molecular positions and velocities. The "dictionary" from the full molecular state to the thermostat reading is a coarse-graining: you trace over the inaccessible molecular details and keep only the macroscopic observable. OGGEP demands the same kind of dictionary for the bulk-to-boundary projection.

4N.3 Why every system you study is open

Before diving into the GDOS derivation, we pause on a point that sounds obvious but has profound consequences: *every real physical system is an open system.*

A hydrogen atom in a monograph is a closed system. A hydrogen atom in a laboratory is not. It interacts with the electromagnetic vacuum (producing the Lamb shift), with the gravitational field of the Earth, with the photons in the room, and with whatever apparatus is measuring it. The "closed system" is an idealization—a very useful one for computing energy levels, but a misleading one for understanding measurement and decoherence.

A particle detector at the LHC is an open system. A living cell is an open system. A brain is an open system. The observable universe itself, from the boundary perspective, is an open system.

GDOS makes this observation constitutional. AX-3 says: the theory must describe the boundary—the open-system side—not just the bulk. This is not a technicality; it is the origin of everything we call "quantum mechanics."

Everyday examples of open systems:

- **A cup of coffee cooling on your desk.** The coffee (the system) exchanges heat with the room (the environment). The temperature drops irreversibly. This is the gradient current in GDOS: energy flowing downhill toward thermal equilibrium.
- **A spinning top.** The top (the system) loses energy to friction with the table and air resistance (the environment). It slows down and eventually falls. But while spinning, it maintains a remarkably ordered state against dissipation. This is the curl current: angular momentum sustaining structure against the gradient's tendency to dissolve it.
- **A living cell.** The cell takes in nutrients (energy input from the environment), uses them to build and repair structures (the curl current), and exports waste heat and entropy (the gradient current). Life is the condition where the curl current dominates the gradient current sufficiently to maintain organized structure.

Each of these examples is a boundary phenomenon in the GDOS sense: a partial description of a larger system, obtained by tracing over inaccessible degrees of freedom. The coffee's molecular details, the top's individual air-molecule collisions, the cell's atomic-scale quantum states—all are "traced out" to produce the macroscopic description we actually use.

From the Bulk to the Boundary: How the Shadow Is Cast

4N.1 Splitting the world in two

The first mathematical step is to split the six-dimensional Hilbert space into two parts:

$$\mathcal{H}_{\text{Bulk}} = \mathcal{H}_{\text{Bdy}} \otimes \mathcal{H}_{\text{env}}.$$

$\mathcal{H}_{\text{Bdy}}$ is the boundary Hilbert space: the states accessible to a four-dimensional observer—the "shadow on the wall." $\mathcal{H}_{\text{env}}$ is everything else: the extra-dimensional modes, the heavy Kaluza–Klein excitations, the degrees of freedom no four-dimensional instrument can probe—the "part of the cave you can't see."

In the full bulk, the state evolves unitarily. But the boundary observer must *trace over* the environment:

$$\rho_{\text{Bdy}} = \text{Tr}_{\text{env}}[|\Psi_{\text{Bulk}}\rangle\langle\Psi_{\text{Bulk}}|]\,.$$

Even if the bulk state is pure, the boundary state is generically mixed. Tracing over the environment creates ignorance, and ignorance looks like probability.

Metaphor: the phone call. Imagine a three-way phone call between Alice, Bob, and Carol. Alice hears both Bob and Carol; the full conversation is perfectly coherent. Now Carol's line is cut. Alice still hears Bob, but his responses seem partly random—pauses, non-sequiturs, answers to questions Alice never heard. The "randomness" is an artifact of tracing over Carol. Quantum probability is exactly this: the artifact of tracing over the inaccessible environment.

4N.2 The memory kernel: echoes from the environment

How does the boundary state evolve? The answer is the Nakajima–Zwanzig equation (Nakajima, 1958; Zwanzig, 1960):

$$\frac{d\rho_{\text{Bdy}}}{dt} = -\text{i}[H_{\text{eff}},\, \rho_{\text{Bdy}}] + \int_0^t K(t,s)\,\rho_{\text{Bdy}}(s)\,ds.$$

The first term is effective Hamiltonian evolution. The second is a memory kernel $K(t,s)$ that encodes how the environment's past states influence the boundary's present.

Metaphor: the echo in a canyon. Shout in a canyon and your voice bounces back. The echo is a memory effect: your past action (the shout) affects your present experience (the echo) through the environment (the canyon walls). The NZ memory kernel is the quantum echo of bulk degrees of freedom interacting with the boundary.

4N.3 The GKLS equation: when the environment forgets quickly

When the environment "forgets" on a timescale much shorter than the boundary evolution, the NZ equation simplifies to the GKLS equation (Gorini et al., 1976; Lindblad, 1976):

$$\frac{d\rho}{dt} = -\mathrm{i}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right).$$

The Lindblad operators L_k describe decoherence, dissipation, and irreversibility. This equation is not postulated; it is the unique form consistent with AX-1 (causality: forward in time) and POS-4 (positivity: no negative probabilities).

Metaphor: the river and the rocks. A river flows downstream (AX-1: forward in time). Water cannot flow uphill (POS-4: positivity). Given these constraints, the river's behavior near an obstacle is determined: it splits, flows around, and rejoins, producing eddies (the Lindblad terms) dictated entirely by the geometry. The GKLS equation is the "fluid dynamics of quantum probability."

4N.4 GDOS: the mesoscopic form

Coarse-graining the GKLS equation produces the GDOS continuity equation:

$$\partial_t\, p + \nabla \cdot J = 0, \qquad J = J_{\text{gradient}} + J_{\text{curl}}.$$

The gradient current flows "downhill" toward equilibrium (irreversible, dissipative). The curl current circulates without dissipation (drives persistent oscillations and far-from-equilibrium structure).

Life, in the GDOS framework, is a system where the curl current is strong enough to maintain organized structure despite the gradient current's tendency to dissolve it.

4N.5 How measurement emerges: the first definition in physics

A measurement, in GDOS, is a *quantum instrument*—a collection of completely positive maps $\{\mathcal{I}_\alpha\}_\alpha$ extracted from the OGGEP dictionary. Each $\mathcal{I}_\alpha$ corresponds to one possible outcome α, and the probability is $p(\alpha) = \mathrm{Tr}[\mathcal{I}_\alpha(\rho)]$.

The Born rule $p(\alpha) = |\langle\alpha|\psi\rangle|^2$ is a consequence. Wavefunction collapse $\rho \to \mathcal{I}_\alpha(\rho)/\mathrm{Tr}[\mathcal{I}_\alpha(\rho)]$ is the observer's state update, not a physical event.

Metaphor: the weather forecast. A forecaster predicts 30% rain. At noon, it rains. The forecast "collapses" to 100%. Did the weather collapse? No—the atmosphere was always doing one thing. The "collapse" was the forecaster's Bayesian update upon receiving data. Quantum collapse is the same: the boundary observer's update upon receiving a measurement outcome from the bulk.

This is the **first derived definition of measurement in fundamental physics**. Von Neumann postulated it (von Neumann, 1932). Copenhagen interpreted it. Many-worlds reinterpreted it. GDOS derives it.

4N.6 Worked example: the Stern–Gerlach experiment

To make the formalism concrete, consider the simplest possible measurement: a Stern–Gerlach experiment measuring the spin of a silver atom.

The setup. A silver atom passes through an inhomogeneous magnetic field. The field deflects the atom upward or downward depending on its spin orientation. A screen at the end records where the atom lands: upper spot (spin-up) or lower spot (spin-down).

In standard quantum mechanics. The atom starts in a superposition $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$. The textbook says: upon measurement, the wavefunction "collapses" to $|\uparrow\rangle$ with probability $|\alpha|^2$ or to $|\downarrow\rangle$ with probability $|\beta|^2$. But *when* does the collapse happen? When the atom enters the magnetic field? When it hits the screen? When the experimenter looks at the screen? The textbook is silent.

In GDOS. The atom, the magnetic field, and the screen are all part of the boundary state ρ_{Bdy}. The "environment" includes the photons, air molecules, and lattice vibrations of the screen that carry away the atom's coherence. The measurement instrument is:

$$\mathcal{I}_\uparrow(\rho) = M_\uparrow \rho M_\uparrow^\dagger, \qquad \mathcal{I}_\downarrow(\rho) = M_\downarrow \rho M_\downarrow^\dagger,$$

where $M_\uparrow = |\uparrow\rangle\langle\uparrow|$ and $M_\downarrow = |\downarrow\rangle\langle\downarrow|$ are the Kraus operators. The probabilities are:

$$p(\uparrow) = \mathrm{Tr}[\mathcal{I}_\uparrow(\rho)] = |\alpha|^2, \qquad p(\downarrow) = \mathrm{Tr}[\mathcal{I}_\downarrow(\rho)] = |\beta|^2.$$

The Born rule emerges automatically. After the outcome is registered, the state updates to $\rho \to \mathcal{I}_\uparrow(\rho)/p(\uparrow)$. No "collapse" postulate is needed; the update is a standard Bayesian conditioning.

The key point: the Kraus operators $M_\uparrow, M_\downarrow$ are *derived* from the interaction between the atom and the screen, which in turn is derived from the bulk-to-boundary coupling. They are not chosen by hand.

4N.7 What about Schrödinger's cat?

Schrödinger's cat is a thought experiment designed to expose the absurdity of applying quantum superposition to macroscopic objects. A cat in a box is (supposedly) in a superposition of "alive" and "dead" until someone opens the box.

In GDOS, the resolution is constitutional:

(i) The cat is an open system, coupled to an enormous environment (air molecules, photons, the walls of the box).

(ii) POS-4 (positivity) forces the boundary evolution to be CPTP.

(iii) The GKLS equation, with Lindblad operators derived from the cat–environment coupling, decoheres the "alive + dead" superposition on a timescale of $\sim 10^{-30}$ seconds—far faster than any physical process.

(iv) The cat is never in a genuine superposition as seen by any boundary observer. The "paradox" arises only from treating the cat as a closed system, which it manifestly is not.

Metaphor: the coin under the cup. Flip a coin and cover it with a cup. Before you lift the cup, is the coin "in a superposition" of heads and tails? Of course not—it is in one definite state; you simply don't know which. Quantum mechanics differs from coin-flipping in that quantum states can exhibit interference (the hallmark of genuine superposition), but decoherence destroys interference for macroscopic objects almost instantaneously. The cat is the coin under the cup: always definite, never superposed.

4N.8 The philosophical significance: completing quantum mechanics

The GDOS resolution of the measurement problem is not just a technical achievement. It has deep philosophical significance.

Quantum mechanics is not "weird." The apparent weirdness of quantum mechanics—superposition, collapse, nonlocality, the observer effect—arises from describing an open-system boundary phenomenon with a closed-system formalism. Once you recognize that the four-dimensional world is a projection of a six-dimensional bulk, the weirdness dissolves. Superposition is a bulk phenomenon. Collapse is a boundary update. Nonlocality is an artifact of two boundary patches sharing a common bulk origin. The observer effect is the unavoidable consequence of finite observers having access to only part of the Hilbert space.

Einstein was right (in the bulk). Einstein's famous objection to quantum mechanics—"God does not play dice"—is correct in the six-dimensional bulk. The bulk evolution is deterministic and unitary. The dice are rolled only on the four-dimensional boundary, by the act of tracing over what you cannot see. Einstein's intuition was sound; his error was not recognizing that the deterministic theory he sought lives in a higher-dimensional space.

Bohr was right (on the boundary). Bohr's insistence that measurement outcomes are definite and classical is also correct—on the boundary. The GKLS equation and the decoherence it produces ensure that boundary states are always effectively classical for macroscopic objects. Bohr's "complementarity" is a boundary-level statement that is perfectly consistent with the bulk-level determinism.

The synthesis. GDOS synthesizes Einstein and Bohr: determinism in the bulk, probability on the boundary. They were both right—they were just talking about different levels of the same structure.

How Gravity and Quantum Field Theory Emerge

4N.1 The deepest question: why does gravity exist?

In the Standard Model, gravity is not even part of the theory. It is treated as a separate force, described by Einstein's general relativity, and grafted onto particle physics through a semiclassical approximation. There is no deep explanation for *why* gravity exists or *why* it takes the form of spacetime curvature.

In GG-6/GDOS, gravity is explained: it is the four-dimensional shadow of the six-dimensional bulk geometry. More precisely, the six-dimensional metric has a "zero mode"—a specific component that survives at low energies when the extra dimensions are integrated out—and this zero mode satisfies the Einstein field equations.

Historical context: the two towers. Twentieth-century physics built two magnificent towers: quantum mechanics (the tower of the very small) and general relativity (the tower of the very large). Both towers are extraordinarily successful within their own domains. But they are built on different foundations and nobody has found a way to connect them at the base.

Every previous attempt to connect them has tried to "quantize gravity"—to treat the gravitational field as a quantum object and apply the rules of quantum mechanics to it. This approach has produced beautiful mathematics (loop quantum gravity, string theory, asymptotic safety) but no experimentally confirmed predictions.

GDOS takes the opposite approach: instead of quantizing gravity, it *geometrizes everything else* and then shows that gravity naturally emerges on the four-dimensional boundary. The two towers are not connected at the base; they are both projections of a single six-dimensional structure.

4N.2 Einstein's gravity from the metric zero mode

The warped six-dimensional metric has a specific four-dimensional component—the "zero mode"—that survives at low energies when the extra dimensions are integrated out. This zero mode satisfies the Einstein field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda_4 g_{\mu\nu} = 8\pi G_4 T_{\mu\nu}.$$

General relativity is not postulated; it *emerges* from the geometry of the extra dimensions.

How the effective Planck mass arises. The four-dimensional Newton constant G_4 (equivalently, the Planck mass $M_{\rm Pl}$) is not a fundamental input. It is determined by an overlap integral of the zero-mode wavefunction over the radial direction:

$$M_{\rm Pl}^2 = M_6^4 \int_0^{r_*} e^{-2A(r)}\, dr.$$

The warp factor $e^{-2A(r)}$ generates an exponential hierarchy: even though the fundamental six-dimensional scale M_6 is of order $E_{\rm GUT}$, the four-dimensional Planck mass ends up much larger because the integral amplifies it.

Metaphor: the funnel. Imagine pouring water into a funnel. The narrow end (the boundary) receives a concentrated stream even though the wide end (the bulk) receives a diffuse flow. The warp factor is the "taper" of the funnel: it concentrates the gravitational coupling from the spread-out bulk into the localized boundary.

What this means philosophically. Gravity is not "quantized" in this framework. It is *geometrized*: a property of the six-dimensional metric that survives the bulk-to-boundary projection. The question "how do we quantize gravity?" is replaced by "how does the metric zero mode of a quantum-consistent six-dimensional bulk behave on the four-dimensional boundary?" The answer: Einstein's equations.

4N.3 Quantum field theory from integrating out heavy modes

The metric zero mode is not the only survivor. The gauge fields, matter fields, and tensor field of the six-dimensional theory each have a tower of modes: light "zero modes" and heavy "Kaluza–Klein modes" with masses of order $1/R_\theta$ or $1/r_*$.

At energies far below the KK mass scale, the heavy modes are "frozen out"—they cannot be excited and can be integrated over. The result is a four-dimensional effective quantum field theory containing only the light zero modes.

Metaphor: the orchestra at a distance. A full orchestra has dozens of instruments playing at different frequencies. If you listen from very far away, you hear only the loudest, lowest-frequency instruments—the bass, the cello, the timpani. The flutes, oboes, and violins are "integrated out" by distance. The "effective theory at long distances" is a bass-heavy reduction of the full orchestra. Similarly, the effective four-dimensional theory is a "low-frequency reduction" of the full six-dimensional theory.

4N.4 The formal infrared generating functional

The technical paper constructs the complete four-dimensional generating functional—the mathematical object from which every scattering amplitude, every decay rate, and every cross-section can be computed:

$$Z_4[J] = \int \mathcal{D}\Phi_0 \, \exp\left(iS_{4,\text{eff}}[\Phi_0] + i \int J \cdot \Phi_0 \right).$$

This path integral runs over all zero-mode field configurations Φ_0, weighted by the effective four-dimensional action $S_{4,\text{eff}}$. The action is not postulated; it is derived from the six-dimensional action by integrating out the heavy modes.

Why this matters. The generating functional $Z_4[J]$ is the "master recipe" of four-dimensional physics. Every physical observable—every cross-section at the LHC, every branching ratio, every precision electroweak parameter—can be computed from it by taking functional derivatives. The fact that $Z_4[J]$ is *derived* from the six-dimensional theory means that all four-dimensional physics is, in principle, computable from the constitutional input.

4N.5 Two descents from one bulk

The key insight of Section 4 is that **gravity and quantum field theory are parallel descents from the same bulk**. They are not two separate theories that need to be "unified"; they are two different aspects of the same bulk-to-boundary projection:

- Gravity: metric zero mode → Einstein equations.
- QFT: gauge/matter zero modes + heavy-mode integration → generating functional.

Both emerge from the same six-dimensional geometry through the same OGGEP dictionary. Their apparent separateness in four dimensions is an artifact of the projection, not a feature of the underlying reality.

The Standard Model Emerges

4N.1 Kaluza–Klein expansion: unfolding the extra dimensions

Every six-dimensional field can be expanded as a sum over modes in the extra dimensions, like expanding a function in a Fourier series:

$$\Phi(x^\mu, r, \theta) = \sum_{n,\ell} \phi^{(n,\ell)}(x^\mu)\, f_n(r)\, \frac{e^{i\ell\theta/R_\theta}}{\sqrt{2\pi R_\theta}}.$$

The zero modes $(n, \ell) = (0, 0)$ are the light fields that become the Standard Model particles. The higher modes are heavy and decouple at low energies.

4N.2 Symmetry breaking: how the boundary selects the gauge group

The six-dimensional gauge group is $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$. But we do not observe SU(5) at low energies; we observe $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. How does the larger group break to the smaller one?

The answer is *boundary conditions*. The extra dimensions have endpoints (the interval I_r has two ends; the circle S^1_θ can have orbifold fixed points). At these endpoints, certain gauge field components are forced to vanish—like the vibration of a guitar string being zero at the frets. The boundary conditions break SU(5) to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ while leaving $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ intact.

Metaphor: the guitar string. An open guitar string vibrates at all harmonics. But if you press a fret at the midpoint, you kill all the odd harmonics—only the even ones survive. The boundary conditions in the extra dimensions are like pressing a fret: they kill certain modes (the X and Y bosons of SU(5) that would cause proton decay) and let others survive (the gluons, W and Z bosons, and photon that we actually observe).

4N.3 Three generations: why three copies of everything

One of the deepest mysteries of the Standard Model is that quarks and leptons come in three nearly identical "generations" or families. The electron, muon, and tau are three versions of the same particle, differing only in mass. Nobody knows why there are three.

In GG-6, the number three is encoded in the OGN-5 integer $N = 3$—the triplicity index. The three generations correspond to three independent zero-mode solutions of the six-dimensional Dirac equation in the warped background. Their masses are hierarchical because the wavefunction overlap integrals differ exponentially for different mode profiles in the radial direction.

4N.4 Proton stability: absolute protection

The proton is absolutely stable in this framework. The baryon number B is a gauged $\mathrm{U}(1)_B$ symmetry, and no local operator with $\Delta B \neq 0$ can exist in the exact theory. This is not a numerical suppression; it is a topological selection rule.

Metaphor: the bank vault with no door. In other GUT frameworks, the proton is like a bank vault with a very thick door: hard to break into but not impossible given enough time. In GG-6, the vault has no door at all. The topology of gauge invariance simply does not allow any path from a state with a proton to a state without one.

If proton decay is ever observed, GG-6 is dead. This is the sharpest single falsifier of the entire program.

4N.5 The complete derivation chain

The Standard Model derivation can be summarized in one chain:

Appendices for Chapter 5

The Qi-4 Pillars

Appendix 5A: Abbreviations, symbols, and key equations

This appendix collects the principal abbreviations, symbols, and key equations used throughout the paper for quick reference.

5A.1 Summary of abbreviations

Abbreviation	Meaning
Programme-level	
GUT	Grand Unified Theory of Ten Principles (the constitutional layer)
GG-6	Geometry–Gauge locking in six dimensions (the ultraviolet bulk)
GDOS	Geometric Dynamics of Open Systems (the boundary runtime)
GGEP	Geometry–Gauge Equivalence Principle (AX-2)
OGGEP	Operational/Open Geometry–Gauge Equivalence Principle (AX-3)
Ultraviolet and anomaly sector	
BI-6	Green–Schwarz-modified Bianchi Identity in six dimensions
OGN-5	Odd-integer Gauge Normalisation package: $(M, N, L; J, K) = (1, 3, 5; 7, 35)$
SM	Standard Model of particle physics
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan (open-system generator)
CPTP	Completely Positive, Trace-Preserving (quantum channel)
Runtime triad	
DGI	Dynamical Gauge Invariance (covariant exploration)
GSSB	Geometric Spontaneous Symmetry Breaking (branch selection)
GA	Geometric Attractor (persistent stabilised structure)
B_2	Basin of Attraction (metastable domain around a GA)
Runtime constitution	
EEGG	Energy–Entropy by the Geometry–Gauge principle (scalar ledger)
SIP	System Instance Packet (canonical open-system unit)
SeOS	Self-maintaining Open System (persistence class)
QCI	Qi-4 Completeness Index (graded realisation score)
Qi-4 Pillars	
Qi-QUA	Qi-Quantitative: current locking (Arisaka Ratio, Equation, Constant)
Qi-EP	Qi-Extremal Principle: variational selector (PLA, PLT, MEPP, FEP)
Qi-CON	Qi-Conformality: scale covariance, Lévy statistics, fractality, $1/f$
Qi-HAL	Qi-Higher Architecture Layer: memory, prediction, and adaptive closure
Qi-EP sub-principles	
PLA	Principle of Least Action (6D bulk)

Abbreviation	Meaning
PLT	Principle of Least Time (trajectory gauge on the 4D boundary)
MEPP	Maximum Entropy Production Principle (current gauge)
FEP	Free Energy Principle (inferential gauge)
Qi-HAL components	
MePMoS	Memory → Prediction → Motion → Sensing (causal loop)
NHT	Neural Holographic Theorem (phase-coded latent geometry)
HAL	Holographic Attractor Lattice (metastable writable memory)
BFFT	Binary Fractal Fourier Tree (dyadic multiscale protocol)
Holographic closure	
OH	Outer Hologram (lawful external projection: bulk → boundary data)
IH	Inner Hologram (lawful internal reconstruction: data → world-model)
EIC	Evolving Inner Cosmos (long-timescale consistency: OH + IH = EIC)
Diagnostic and structural	
MF	Measurable Falsifiability (kill-criterion requirement)
LGI	Local Gauge Invariance
GGI	Global Gauge Invariance
POVM	Positive Operator-Valued Measure (quantum instrument)

5A.2 Glossary of key symbols

Symbol	Meaning
Spacetime and bulk geometry	
M_6	Six-dimensional GG-6 bulk manifold: $M_6 = M_4 \times I_r \times S^1_\theta$
M_4	Four-dimensional boundary spacetime
I_r	Warped radial interval (extra dimension)
S^1_θ	Compactified angular extra dimension
g_{MN}	Six-dimensional bulk metric
G_{AB}	Induced metric on the reduced state manifold $\mathcal{M}$
Gauge group and charges	
$\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$	Gauge group at the GUT scale (plus gravity)
$\mathrm{U}(1)_B$	Baryon-number gauge symmetry
$\mathrm{U}(1)_{Qi}$	Information-current gauge symmetry
Energy scales	

Symbol	Meaning
E_{GUT}	GUT-scale energy
E_{Pl}	Planck energy
E_{Ryd}	Rydberg energy (≈ 13.6 eV)
Q_B	Arisaka Constant: $(2/5)\,E_{\mathrm{Ryd}}/K$ with $K = 35$
BI-6 coefficients and OGN-5 integers	
$(1, -1, -2/5, -2/5)$	BI-6 locked anomaly coefficients
$(M, N, L; J, K)$	OGN-5 integers: $(1, 3, 5; 7, 35)$
State manifold and dynamics	
$\mathcal{M}$	Reduced (boundary-accessible) state manifold
z^A	Coordinates on $\mathcal{M}$
$p(z,t)$	Probability density on $\mathcal{M}$
$\rho_\partial(t)$	Reduced boundary state (operator or density)
J^A	Probability current on $\mathcal{M}$
J^A_{grad}	Gradient (dissipative/relaxation) component of current
J^A_{curl}	Curl (driven/circulatory) component of current
J^A_{noise}	Noise (stochastic) component of current
J^A_{gauge}	Gauge (topological) component of current
Σ^{AB}	Noise or diffusion tensor
$\mathcal{A}_A$	Effective gauge connection on $\mathcal{M}$
EEGG ledger	
$\mathcal{F}_{\mathrm{EEGG}}$	EEGG free-energy-like functional on $\mathcal{M}$
Φ_{EGG}	Energetic potential component of the EEGG ledger
Θ_{eff}	Effective information temperature
Four currents (Qi-QUA)	
J_E	Energy current
J_e	Charge/electrochemical current
J_B	Baryon/structural-material current
J_{Qi}	Information current
Holographic closure	
$\mathcal{K}_{\mathrm{OH}}$	Outer-hologram measurement kernel
$\hat{\Phi}$	Inner-hologram latent field (reconstructed world-model)
$\mathcal{J}_{\mathrm{EIC}}$	EIC convergence functional
Persistence and completeness	
$\mathcal{V}_{\mathrm{self}}$	Viability region in state space
Π_{maint}	Maintenance-positive condition
QCI_S	Qi-4 Completeness Index for system S

Symbol	Meaning
Qi-CON diagnostic quantities	
α	Stability index of the Lévy-type generator ($0 < \alpha \leq 2$)
d_f	Fractal dimension of the attractor support
f_L, f_H	Low and high knee frequencies in the $1/f$ spectrum
$S(f)$	Power spectral density

5A.3 Key equations

The following table collects the principal equations of the paper in logical order, with pointers to the sections where they are derived.

No.	Name	Equation (compact form)
Foundation (Chapters 2–4)		
1	Bulk–boundary split	$\mathcal{H}_{\text{Bulk}} = \mathcal{H}_{\text{sys}} \otimes \mathcal{H}_{\text{env}}$
2	Reduced state	$\rho_\partial(t) = \text{Tr}_{\text{env}}\ \rho_{\text{Bulk}}(t)$
3	GG-6 bulk	$M_6 = M_4 \times I_r \times S^1_\theta$
4	BI-6 identity	$\mathrm{d}H_3 = X_4$, locked coefficients $(1, -1, -2/5, -2/5)$
5	OGN-5 package	$(M, N, L; J, K) = (1, 3, 5; 7, 35)$
6	GKLS generator	$\dot\rho_\partial = -\mathrm{i}[H_{\text{eff}}, \rho_\partial] + \sum_k \big(L_k \rho_\partial L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_\partial\}\big)$
Universal GDOS runtime (Part II)		
7	GDOS continuity	$\partial_t p(z,t) = -\nabla_A J^A(z,t)$
8	Current decomposition	$J^A = J^A_{\text{grad}} + J^A_{\text{curl}} + J^A_{\text{noise}} + J^A_{\text{gauge}}$
9	Runtime pipeline	DGI $\Longrightarrow$ GSSB $\Longrightarrow$ GA
10	DGI covariance	$J'^A(z') = \dfrac{\partial z'^A}{\partial z^B}\, J^B(z)$
11	EEGG functional	$\mathcal{F}_{\text{EEGG}}[p] = \displaystyle\int_{\mathcal{M}} p\, \Phi_{\text{EGG}}\, \mathrm{d}\mu_g + \Theta_{\text{eff}} \int_{\mathcal{M}} p \ln p\, \mathrm{d}\mu_g$
12	EEGG monotonicity	$\dot{\mathcal{F}}_{\text{EEGG}} \leq P_{\text{port}}$ (equality iff curl $= 0$ and noise $= 0$)
13	SIP packet	$\text{SIP} = \big(\mathcal{M},\, z^A,\, G_{AB},\, \mathcal{A}_A,\, \mathcal{F}_{\text{EEGG}},\, \Sigma^{AB},\, \mathcal{O}\big)$
Qi-4 Pillars (Part III)		
14	Arisaka Ratio	$J'_E/J'_e = J'_e/J'_B = J'_B/J'_{Qi} = 1$ (physics natural units)
15	Arisaka Equation	$J_E/Q_B = J_e = J_B = J_{Qi}$ (chemistry natural units)
16	Arisaka Constant	$Q_B = (2/5)\, E_{\text{Ryd}}/K = (2/5) \times 13.6\ \text{eV}/35$
17	Qi-EP action	$\mathcal{S}[z] = \displaystyle\int \mathrm{d}t\, \big[\tfrac{1}{2} G_{AB} \dot z^A \dot z^B + \dot z^A \mathcal{A}_A + V_{\text{diss}} + \Lambda_{\text{sel}}\big]$
18	PLT geodesic	$\ddot z^A + \Gamma^A_{BC}\, \dot z^B \dot z^C = 0$
19	Fractional generator	$\partial_t p = D_\alpha (-\Delta)^{\alpha/2} p, \quad 0 < \alpha \leq 2$

No.	Name	Equation (compact form)
20	Two-knee spectrum	$S(f) \sim \begin{cases} S_0, & f \ll f_L \\ f^{-\beta}, & f_L \ll f \ll f_H \\ f^{-\gamma}, & f \gg f_H \end{cases}$
Persistence, completeness, and internality (Part IV)		
21	SeOS viability tube	$\mathbb{P}\big(\tau_{\partial\mathcal{V}} > t\big) \geq 1 - \delta$ for all t in the planning horizon
22	QCI definition	$\mathrm{QCI}_S = w_1\, q_{\text{Qi-QUA}} + w_2\, q_{\text{Qi-EP}} + w_3\, q_{\text{Qi-CON}} + w_4\, q_{\text{Qi-HAL}}$
23	OH kernel	$P(s \mid z) = \mathrm{Tr}\big[\mathcal{K}_{\mathrm{OH}}(s)\, \rho(z)\big]$
24	IH reconstruction	$\partial_\tau \hat{\Phi} = -\Gamma \dfrac{\delta \mathcal{J}_{\mathrm{EIC}}}{\delta \hat{\Phi}}$ (gradient descent on mismatch)
25	EIC convergence	$\dot{\mathcal{J}}_{\mathrm{EIC}} \leq 0$ (Lyapunov decrease)
Synthesis (Part VI)		
26	Synthesis theorem	Chapters 2–4 $\Longrightarrow$ DGI+GSSB+GA $\Longrightarrow$ EEGG+SIP $\Longrightarrow$ Qi-4 $\Longrightarrow$ SeOS/QCI $\Longrightarrow$ OH+IH=EIC

Appendix 5B: Technical derivations for the DGI–GSSB–GA pipeline

5B.1 Derivation of the transformed generator

This appendix records the algebra behind Eq. (5.45) in one place.

Let $\rho' = U\rho U^\dagger$, with $U = U_t$ differentiable in time. Then

$$\dot\rho' = \dot U \rho U^\dagger + U\dot\rho U^\dagger + U\rho\dot U^\dagger.$$

Insert $\dot\rho = \mathfrak{L}_t[\rho]$ and write $\rho = U^\dagger\rho' U$:

$$\dot\rho' = \dot U U^\dagger\rho' + U\mathfrak{L}_t[U^\dagger\rho' U]U^\dagger + \rho' U\dot U^\dagger.$$

Since $UU^\dagger = \mathrm{id}$, differentiation gives $\dot U U^\dagger + U\dot U^\dagger = 0$, hence $U\dot U^\dagger = -\dot U U^\dagger$. Define $K_t = \mathrm{i}\dot U U^\dagger$, so that $\dot U U^\dagger = -\mathrm{i}K_t$. Then

$$\dot\rho' = U\mathfrak{L}_t[U^\dagger\rho' U]U^\dagger - \mathrm{i}[K_t, \rho'].$$

Therefore

$$\mathfrak{L}_t^{(U)} = \mathcal{U}_t \circ \mathfrak{L}_t \circ \mathcal{U}_t^{-1} - \mathrm{i}[K_t, \cdot].$$

This is the precise operator meaning of time-dependent gauge covariance on the boundary.

5B.2 Quotient reduction of the continuity equation

Let the effective current be

$$J^A = J_\perp^A + R^A{}_\alpha \lambda^\alpha p,$$

with $J_\perp^A$ horizontal and $R^A{}_\alpha$ spanning the gauge orbits. For a gauge-invariant observable O,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle O\rangle = \int (\nabla_A O) J^A \,\mathrm{d}\mu = \int (\nabla_A O) J_\perp^A \,\mathrm{d}\mu + \int (R_\alpha O)\lambda^\alpha p \,\mathrm{d}\mu.$$

The second term vanishes identically. Hence the reduced physical continuity equation is obtained by pushing forward only the horizontal current:

$$\partial_t p_{\mathrm{phys}} + \widetilde{\nabla}_i \widetilde{J^i} = 0.$$

This is why all instability and attractor analyses relevant to GSSB and GA must be performed on $\mathcal{X}_{\mathrm{phys}}$, not on the unreduced state space.

5B.3 Local bifurcation algebra

For the local scalar model

$$\mathcal{F}(q) = \mathcal{F}_0 + \frac{1}{2}aq^2 + \frac{1}{4}bq^4, \qquad b > 0,$$

the stationary points satisfy $q(a + bq^2) = 0$. The Hessian is

$$\mathcal{F}''(q) = a + 3bq^2.$$

Hence:

$$\mathcal{F}''(0) = a, \qquad \mathcal{F}''(q_\pm) = a + 3b\Big(-\frac{a}{b}\Big) = -2a.$$

Thus for $a > 0$, $q = 0$ is a local minimum and the broken branches do not exist; for $a < 0$, $q = 0$ is a local maximum and $q_\pm$ are minima. The stochastic stationary density is

$$p_*(q) \propto \exp\left[-\frac{2}{\sigma^2}\mathcal{F}(q)\right],$$

which changes from unimodal to bimodal across the same threshold. This is the simplest analytic model of open-system GSSB.

5B.4 Catalogue of possible Geometric Attractors

The GA definition intentionally allows several kinds of asymptotic objects. Table 5B.1 records the most important cases.

Table 5B.1: Catalogue of possible Geometric Attractors allowed by Definition 5.4.6.

GA type	**Mathematical form**	**Typical interpretation**
Fixed-point GA	Stable critical point of the reduced drift	Stable phase, homeostatic set-point, committed developmental state
Limit-cycle GA	Stable periodic orbit	Molecular machine cycle, circadian rhythm, locomotor pattern, economic oscillation
Torus or quasiperiodic GA	Invariant torus with several frequencies	Multioscillator coordination, coupled circadian or sensorimotor rhythms
Localized bump GA	Structured invariant profile on a ring or sheet	Ring-attractor head-direction state, working-memory packet, spatial focus of activity
Invariant-measure GA	Stationary probability measure with structured support	Nonequilibrium steady state, critical or near-critical open system
Metastable GA	Long-lived quasi-invariant set	Decision state, learned policy, memory trace, developmental canalization

These categories are not separate theories. They are alternative concrete realizations of the same general definition once the DGI and GSSB filters have been applied.

Appendix 5C: Technical derivations for Qi-QUA

5C.1 Coefficient-space to natural-unit conversion

This appendix records the sector-rescaling algebra behind Theorem 5.7.10.

Write the magnitude coefficient-space vector as

$$\mathbf{v}' = \begin{pmatrix} |J'_E| \\ |J'_e| \\ |J'_B| \\ |J'_{Qi}| \end{pmatrix} = r \begin{pmatrix} 1 \\ 1 \\ 2/5 \\ 2/5 \end{pmatrix}. \tag{5C.1}$$

Define the natural-unit conversion map

$$U_{\text{Qi-QUA}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5/2 & 0 \\ 0 & 0 & 0 & 5/2 \end{pmatrix}, \tag{5C.2}$$

acting on the event-count vector $(J_E/Q_B, J_e, J_B, J_{Qi})^\top$. Applying $U_{\text{Qi-QUA}}$ to Eq. (5C.1) gives

$$U_{\text{Qi-QUA}}\mathbf{v}' = r \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}. \tag{5C.3}$$

Equation (5C.3) is exactly the event-count form of the Arisaka Equation. Multiplying the first component by Q_B restores the physical energy current J_E.

The same result can be read backwards. Given a natural-unit event count r, the corresponding coefficient-space magnitudes are

$$|J'_E| = r, \qquad |J'_e| = r, \qquad |J'_B| = \frac{2}{5}r, \qquad |J'_{Qi}| = \frac{2}{5}r. \tag{5C.4}$$

Hence the coefficient-space and natural-unit presentations are strictly equivalent once the unit conventions are fixed.

5C.2 Numerical ledger of the Arisaka Constant

For convenience we collect the basic numerical conversions of Q_B in one place.

Starting from

$$E_{\text{Ryd}} \approx 13.6057\,\text{eV}, \qquad K = 35,$$

one finds

$$\delta E_{\text{geom}} = \frac{E_{\text{Ryd}}}{K} \approx 0.3887\,\text{eV}.$$

Applying the BI-6 magnitude ratio 2/5 gives

$$Q_B = \frac{2}{5}\delta E_{\mathrm{geom}} \approx 0.1555\,\mathrm{eV}.$$

Converting to chemical units using $1\,\mathrm{eV} = 96.485\,\mathrm{kJ\,mol^{-1}}$ yields

$$Q_B \approx 15.0\,\mathrm{kJ\,mol^{-1}}.$$

At $T = 310\,\mathrm{K}$, where $k_B T \approx 0.0267\,\mathrm{eV}$, one obtains

$$Q_B \approx 5.82\,k_B T.$$

Finally, since $k_B T \ln 2 \approx 0.0185\,\mathrm{eV}$ at $310\,\mathrm{K}$,

$$Q_B \approx 8.40\,k_B T \ln 2.$$

Table 5C.1: Compact conversion table for Q_B.

Quantity	**Value**	**Comment**
$\delta E_{\mathrm{geom}} = E_{\mathrm{Ryd}}/K$	$0.3887\,\mathrm{eV}$	Per-step atomic geometric scale
$Q_B = (2/5)E_{\mathrm{Ryd}}/K$	$0.1555\,\mathrm{eV}$	Arisaka Constant
Q_B in molar units	$15.0\,\mathrm{kJ\,mol^{-1}}$	Chemistry-natural-unit form
$Q_B/(k_B T)$ at $310\,\mathrm{K}$	5.82	Physiological thermal comparison
$Q_B/(k_B T \ln 2)$ at $310\,\mathrm{K}$	8.40	Comparison with Landauer erasure bound

5C.3 Coarse-grained cone generated by ideal locked events

The purpose of this appendix is to formalize the idea that realistic processes are built from sums of ideal events plus overhead.

Let the ideal event vector in natural units be

$$\mathbf{u}_\star = (Q_B, 1, 1, 1).$$

Any process consisting of N ideal events and a nonnegative overhead vector $\Delta = (\Delta_E, \Delta_e, \Delta_B, \Delta_Q)$ has total ledger

$$\mathbf{L} = N\mathbf{u}_\star + \Delta.$$

If the overhead is monotone in the sense that $\Delta_Q = 0$ and $\Delta_E, \Delta_e, \Delta_B \geq 0$, then

$$\mathbf{L} = (Q_B N + \Delta_E,\ N + \Delta_e,\ N + \Delta_B,\ N).$$

Hence

$$L_E - Q_B L_Q = \Delta_E \geq 0, \qquad L_e - L_Q = \Delta_e \geq 0, \qquad L_B - L_Q = \Delta_B \geq 0.$$

This gives the inequality form quoted in Proposition 5.7.11.

Geometrically, the set of ideal events spans a ray, while the set of coarse-grained processes spans a convex cone whose distinguished edge is the ideal Qi-QUA ray. The closer a real process lies to that edge, the closer it is to ideal locked efficiency.

5C.4 Event-budget table

Table 5C.2 provides a quick-reference ledger converting a few simple event counts into their ideal energy floors.

Table 5C.2: Ideal Qi-QUA event budgets.

Locked events	Ideal energy	Molar energy	Interpretation
1	$1Q_B$	$15.0\,\mathrm{kJ\,mol^{-1}}$	One minimal ideal boundary update
2	$2Q_B$	$30.0\,\mathrm{kJ\,mol^{-1}}$	Two coupled commits or one two-bit update
3	$3Q_B$	$45.0\,\mathrm{kJ\,mol^{-1}}$	Representative multi-step molecular act
8	$8Q_B$	$120.0\,\mathrm{kJ\,mol^{-1}}$	One byte-equivalent update in the ideal limit
N	NQ_B	$15.0N\,\mathrm{kJ\,mol^{-1}}$	General linear scaling for an ideal locked process

Appendix 5D: Technical derivations for Qi-EP

5D.1 Reduction of the current-density action to a trajectory functional

This appendix spells out in detail the reduction from the current-density functional (5.166) to the sharply localized trajectory functional (5.171). The calculation is elementary but it is worth recording explicitly because it shows exactly where the effective metric and connection enter.

Let

$$\rho(\zeta,t)=\delta\big(\zeta-z(t)\big),\qquad J^a(\zeta,t)=\delta\big(\zeta-z(t)\big)\,\dot z^a(t). \tag{5D.1}$$

For any smooth test function $f(\zeta)$,

$$\int_{\mathcal{X}} f(\zeta)\,\rho(\zeta,t)\,\mathrm{d}\zeta=f(z(t)),\qquad \int_{\mathcal{X}} f(\zeta)\,J^a(\zeta,t)\,\mathrm{d}\zeta=f(z(t))\,\dot z^a(t). \tag{5D.2}$$

Substitute Eq. (5D.1) into the quadratic part of Eq. (5.166):

$$\frac{1}{4}\int_{\mathcal{X}}\big(J-\rho u\big)^a(\rho D)^{-1}_{ab}\big(J-\rho u\big)^b\,\mathrm{d}\zeta=\frac{1}{4}\big(\dot z-u(z,t)\big)^a D^{-1}_{ab}(z,t)\big(\dot z-u(z,t)\big)^b. \tag{5D.3}$$

The continuity equation is also automatically satisfied:

$$\partial_t \rho + \nabla_a J^a = -\dot{z}^a \partial_a \delta(\zeta - z(t)) + \partial_a \left[\delta(\zeta - z(t))\, \dot{z}^a\right] = 0. \tag{5D.4}$$

The projected potential $\mathcal{V}[\rho]$ reduces to its pointwise representative $V(z,t)$. Therefore

$$\mathcal{A}_{\text{eff}}[\rho, J] \longrightarrow \mathcal{A}_{\text{trj}}[z] = \int_0^T \mathrm{d}t \, \left[\frac{1}{4}(\dot{z} - u)^a D^{-1}_{ab} (\dot{z} - u)^b + V(z,t)\right]. \tag{5D.5}$$

Now define $g_{ab} = \frac{1}{2} D^{-1}_{ab}$ and $a_a = g_{ab} u^b$. Then

$$\begin{aligned} \frac{1}{4}(\dot{z} - u)^a D^{-1}_{ab} (\dot{z} - u)^b &= \frac{1}{2} g_{ab} \dot{z}^a \dot{z}^b - g_{ab} u^a \dot{z}^b + \frac{1}{2} g_{ab} u^a u^b \\ &= \frac{1}{2} g_{ab} \dot{z}^a \dot{z}^b - a_a \dot{z}^a + \frac{1}{2} g_{ab} u^a u^b, \end{aligned} \tag{5D.6}$$

which is Eq. (5.174). The Euler–Lagrange equations follow by direct variation:

$$\delta \mathcal{A}_{\text{trj}} = \int_0^T \mathrm{d}t \, \left[g_{ab} \dot{z}^b \, \delta \dot{z}^a + \frac{1}{2}(\partial_c g_{ab}) \dot{z}^a \dot{z}^b \delta z^c - (\partial_c a_a) \dot{z}^a \delta z^c - a_a \delta \dot{z}^a + (\partial_c U_{\text{eff}}) \delta z^c\right]. \tag{5D.7}$$

Integrating by parts the terms proportional to $\delta \dot{z}^a$ gives

$$\delta \mathcal{A}_{\text{trj}} = \int_0^T \mathrm{d}t \, \left[-\frac{\mathrm{d}}{\mathrm{d}t}(g_{ab} \dot{z}^b - a_a) + \frac{1}{2} \partial_a g_{bc} \dot{z}^b \dot{z}^c - \partial_a a_b \dot{z}^b + \partial_a U_{\text{eff}}\right] \delta z^a, \tag{5D.8}$$

and setting $\delta \mathcal{A}_{\text{trj}} = 0$ yields Eq. (5.176).

The Jacobi reduction is equally standard. On a fixed energy shell

$$E = \frac{1}{2} g_{ab} \dot{z}^a \dot{z}^b + U_{\text{eff}}, \tag{5D.9}$$

solve for the kinetic term and write the action along the curve γ rather than as an integral over an arbitrary parameter. One then obtains Eq. (5.178). The result justifies the interpretation of PLT as the trajectory gauge of the projected 6D action.

5D.2 Force-controlled extremum and the dual minimum-dissipation form

This appendix expands the current-gauge calculation and makes the relation between MEPP and the dual minimum-dissipation formulation completely explicit.

Start from the Rayleighian

$$\mathcal{R}_\rho[J; X] = \int_{\mathcal{X}} \left(J^a X_a - \frac{1}{2} J^a (L^{-1})_{ab} J^b\right) \mathrm{d}z, \tag{5D.10}$$

with continuity and integrated ledger constraints imposed by Lagrange multipliers. Including a multiplier $\lambda(z,t)$ for continuity gives

$$\widetilde{\mathcal{R}} = \mathcal{R}_\rho[J; X] + \int_{\mathcal{X}} \lambda(\partial_t \rho + \nabla_a J^a) \, \mathrm{d}z. \tag{5D.11}$$

Variation with respect to J yields

$$X_a - (L^{-1})_{ab}J^b - \partial_a\lambda = 0. \tag{5D.12}$$

Thus the constrained stationary current is

$$J^a = L^{ab}(X_b - \partial_b\lambda). \tag{5D.13}$$

The multiplier λ simply enforces admissibility; it does not change the local quadratic structure of the extremum.

Evaluate the Rayleighian at the stationary point:

$$\begin{aligned}\mathcal{R}_{\rho,\star} &= \int_{\mathcal{X}} \left[J^a_\star X_a - \frac{1}{2}J^a_\star (L^{-1})_{ab}J^b_\star\right] \mathrm{d}z \\ &= \frac{1}{2}\int_{\mathcal{X}} J^a_\star X_a \,\mathrm{d}z = \frac{1}{2}\dot{\Sigma}_\star,\end{aligned} \tag{5D.14}$$

where the last equality uses the definition of entropy production.

Now compute the second variation. At fixed X, ρ, and constraint surface,

$$\delta^2\mathcal{R}_\rho = -\int_{\mathcal{X}} \delta J^a (L^{-1})_{ab}\delta J^b \,\mathrm{d}z \le 0. \tag{5D.15}$$

Therefore the stationary point is a maximum. This is the precise force-controlled MEPP statement of the present framework.

To obtain the dual picture, Legendre-transform Eq. (5D.10) with respect to the force variable. The dual functional is

$$\mathcal{D}_\rho[J;X] = \frac{1}{2}\int_{\mathcal{X}} J^a(L^{-1})_{ab}J^b \,\mathrm{d}z - \int_{\mathcal{X}} J^a X_a \,\mathrm{d}z. \tag{5D.16}$$

At the common stationary point,

$$\delta\mathcal{D}_\rho = 0, \qquad \delta^2\mathcal{D}_\rho = \int_{\mathcal{X}} \delta J^a (L^{-1})_{ab}\delta J^b \,\mathrm{d}z \ge 0, \tag{5D.17}$$

so the same current minimizes $\mathcal{D}_\rho$. This is the minimum-dissipation form familiar from near-equilibrium irreversible thermodynamics.

The apparently competing statements

"maximum entropy production" and "minimum dissipation"

therefore refer to two dual optimization problems with different control conventions. The present framework accommodates both because both arise from the same projected quadratic form.

5D.3 Pullback to the statistical manifold and the Fisher metric

This appendix derives the Fisher metric appearing in the inferential gauge. Let $q_\mu(\eta)$ be a smooth parametric family of densities indexed by coordinates μ^i. A small parameter displacement

$\mu \to \mu + \mathrm{d}\mu$ changes the density by

$$q_{\mu+\mathrm{d}\mu}(\eta) = q_\mu(\eta) + \partial_i q_\mu(\eta)\,\mathrm{d}\mu^i + \mathcal{O}(\mathrm{d}\mu^2). \tag{5D.18}$$

The KL divergence between neighboring family members is

$$\begin{aligned} D_{\mathrm{KL}}(q_{\mu+\mathrm{d}\mu}\|q_\mu) &= \int q_{\mu+\mathrm{d}\mu}\log\frac{q_{\mu+\mathrm{d}\mu}}{q_\mu}\,\mathrm{d}\eta \\ &= \frac{1}{2}g^{\mathrm{F}}_{ij}(\mu)\,\mathrm{d}\mu^i\mathrm{d}\mu^j + \mathcal{O}(\mathrm{d}\mu^3), \end{aligned} \tag{5D.19}$$

where

$$g^{\mathrm{F}}_{ij}(\mu) = \int q_\mu(\eta)\partial_i \log q_\mu(\eta)\,\partial_j \log q_\mu(\eta)\,\mathrm{d}\eta. \tag{5D.20}$$

Thus the Fisher metric is the quadratic form measuring distinguishability of neighboring internal models.

Now show how this metric arises from the projected action. Restrict the current-density action to the statistical slice $\rho_{\mu,s,a}(\eta) = q_\mu(\eta)\delta(s-s_t)\delta(a-a_t)$. Then

$$\partial_t \rho = \frac{\partial q_\mu}{\partial \mu^i}\,\dot{\mu}^i\delta(s-s_t)\delta(a-a_t) + \cdots, \tag{5D.21}$$

and the kinetic contribution to the projected action becomes a quadratic form in $\dot{\mu}$. Because the admissible metric on probability space is the inverse covariance of density fluctuations, the pullback of the boundary kinetic term is precisely proportional to Eq. (5D.20). Therefore

$$\mathcal{A}_{\mathrm{eff}}[\rho, J]\Big|_{\mathcal{S}_{\mathrm{inf}}} = \int_0^T \mathrm{d}t\,\left[\frac{1}{2}g^{\mathrm{F}}_{ij}\dot{\mu}^i\dot{\mu}^j + \mathcal{F}(s,a,\mu)\right]. \tag{5D.22}$$

The variational free energy itself can be decomposed as

$$\begin{aligned} \mathcal{F}(s,a,\mu) &= \int q_\mu(\eta)\log q_\mu(\eta)\,\mathrm{d}\eta - \int q_\mu(\eta)\log p(\eta,s,a)\,\mathrm{d}\eta \\ &= D_{\mathrm{KL}}(q_\mu(\eta)\|p(\eta|s,a)) - \log p(s,a), \end{aligned} \tag{5D.23}$$

so the only μ-dependent part relevant to internal dynamics is the KL mismatch term. Taking the Euler–Lagrange equation of Eq. (5D.22) and then the overdamped limit produces the natural-gradient flow

$$\dot{\mu}^i = -\kappa_\mu (g^{\mathrm{F}})^{ij}\partial_j \mathcal{F}. \tag{5D.24}$$

The derivation is important because it shows that the geometry of inference is not inserted as an extra hypothesis. It is the pullback of the same projected open-system metric that already governed trajectories and currents in the previous gauges.

5D.4 A two-state blanket model

A simple two-state example is useful for seeing the inferential gauge in explicit algebra. Let the hidden external variable be $\eta \in \{0,1\}$, and parameterize the recognition family by a single scalar

μ through

$$q_\mu(\eta=1)=\sigma(\mu),\qquad q_\mu(\eta=0)=1-\sigma(\mu),\qquad \sigma(\mu)=\frac{1}{1+e^{-\mu}}. \tag{5D.25}$$

Then

$$\partial_\mu \log q_\mu(\eta)=\eta-\sigma(\mu), \tag{5D.26}$$

and the Fisher metric is

$$g^{\mathrm{F}}_{\mu\mu}=\sum_{\eta=0}^{1} q_\mu(\eta)\big(\eta-\sigma(\mu)\big)^2=\sigma(\mu)\big(1-\sigma(\mu)\big). \tag{5D.27}$$

Suppose the projected generative model assigns likelihoods $p(s|\eta)$ and prior $p(\eta)$. The variational free energy is

$$\mathcal{F}(s,\mu)=q_\mu(1)\log\frac{q_\mu(1)}{p(\eta=1,s)}+q_\mu(0)\log\frac{q_\mu(0)}{p(\eta=0,s)}. \tag{5D.28}$$

Its derivative is

$$\partial_\mu\mathcal{F}=\sigma(\mu)\big(1-\sigma(\mu)\big)\log\frac{q_\mu(1)\,p(\eta=0,s)}{q_\mu(0)\,p(\eta=1,s)}. \tag{5D.29}$$

The natural-gradient flow becomes

$$\dot\mu=-\kappa_\mu\frac{1}{\sigma(1-\sigma)}\partial_\mu\mathcal{F}=-\kappa_\mu\log\frac{q_\mu(1)\,p(\eta=0,s)}{q_\mu(0)\,p(\eta=1,s)}. \tag{5D.30}$$

The stationary condition $\dot\mu=0$ is

$$\frac{q_\mu(1)}{q_\mu(0)}=\frac{p(\eta=1,s)}{p(\eta=0,s)}, \tag{5D.31}$$

which simply says that the recognition odds match the posterior odds. This miniature example makes the logic transparent: the inferential gauge of Qi-EP selects the internal state whose recognition density best matches the projected generative density.

5D.5 Dictionary of the three gauges

For convenience, Table 5D.1 gives a compact dictionary of the three Qi-EP gauges.

Table 5D.1: Dictionary of the three Qi-EP gauges.

Gauge	**State variable**	**Extremized object**	**Typical use**
Trajectory gauge	$z(t)$	Generalized optical time $\mathcal{T}[z]$	Path selection, transport, signaling, branch tracking
Current gauge	(ρ, J)	Rayleighian $\mathcal{R}_\rho[J;X]$ or dual dissipation functional	Flux networks, reaction systems, metabolic or ecological throughput

Gauge	State variable	Extremized object	Typical use
Inferential gauge	$\mu(t)$	Fisher-kinetic action $\mathcal{A}_{\mathrm{inf}}[\mu]$ with $\mathcal{F}$ potential	Adaptive systems, cells, nervous systems, cognition, action selection

The table is intentionally simple. Its purpose is to emphasize that the three languages differ in their chosen reduced variable, not in their constitutional origin. The origin is always the same projected current-density action.

Appendix 5E: Technical derivations for Qi-CON

5E.1 Radial integration and kernel homogeneity

This appendix records the scale-homogeneity algebra behind Eqs. (5.217)–(5.223) in one place.

Start from the projected kernel

$$W_\Delta(z,z') = \int_{\zeta_{\min}}^{\zeta_{\max}} \omega(\zeta) \left(\frac{\zeta}{\zeta^2+\ell^2}\right)^\Delta \mathrm{d}\zeta, \qquad \ell = d_g(z,z'). \tag{5E.1}$$

Assume that in the active window the weight can be written as

$$\omega(\zeta) = \zeta^{\sigma-1} w(\zeta), \qquad w(\zeta) \approx w_0. \tag{5E.2}$$

Then

$$W_\Delta(z,z') \approx w_0 \int_{\zeta_{\min}}^{\zeta_{\max}} \frac{\zeta^{\sigma+\Delta-1}}{(\zeta^2+\ell^2)^\Delta}\,\mathrm{d}\zeta. \tag{5E.3}$$

Set

$$\zeta = \ell u, \qquad \mathrm{d}\zeta = \ell\,\mathrm{d}u, \tag{5E.4}$$

to obtain

$$\begin{aligned} W_\Delta(z,z') &\approx w_0 \int_{\zeta_{\min}/\ell}^{\zeta_{\max}/\ell} \frac{(\ell u)^{\sigma+\Delta-1}}{(\ell^2u^2+\ell^2)^\Delta}\,\ell\,\mathrm{d}u \\ &= w_0\,\ell^{\sigma+\Delta-1+1-2\Delta} \int_{\zeta_{\min}/\ell}^{\zeta_{\max}/\ell} \frac{u^{\sigma+\Delta-1}}{(1+u^2)^\Delta}\,\mathrm{d}u \\ &= w_0\,\ell^{\sigma-2\Delta} \int_{\zeta_{\min}/\ell}^{\zeta_{\max}/\ell} \frac{u^{\sigma+\Delta-1}}{(1+u^2)^\Delta}\,\mathrm{d}u. \end{aligned} \tag{5E.5}$$

Now impose the separation condition

$$\zeta_{\min} \ll \ell \ll \zeta_{\max}. \tag{5E.6}$$

Then the lower limit tends to 0, the upper limit tends to $+\infty$, and the integral becomes the constant

$$C_{\Delta,\sigma} = \int_0^\infty \frac{u^{\sigma+\Delta-1}}{(1+u^2)^\Delta}\,\mathrm{d}u. \tag{5E.7}$$

Thus

$$W_\Delta(z,z') \approx w_0 C_{\Delta,\sigma}\,\ell^{-(2\Delta-\sigma)}. \tag{5E.8}$$

If

$$2\Delta - \sigma = d_{\mathcal{X}} + \alpha, \tag{5E.9}$$

one obtains

$$W_\Delta(z,z') \propto \frac{1}{d_g(z,z')^{d_{\mathcal{X}}+\alpha}}, \tag{5E.10}$$

which is the kernel defining the fractional jump operator in the main text.

5E.2 Characteristic function and stable-law tail

This appendix records the derivation of Eqs. (5.240) and (5.242). The one-dimensional leading Qi-CON equation is

$$\partial_t p(x,t) = -\,D_\alpha(-\partial_x^2)^{\alpha/2}p(x,t). \tag{5E.11}$$

Using the Fourier convention

$$\widehat{p}(k,t) = \int_{-\infty}^{\infty} e^{-\mathrm{i}kx}p(x,t)\,\mathrm{d}x, \tag{5E.12}$$

and the multiplier property

$$\mathcal{F}\Big[(-\partial_x^2)^{\alpha/2}p\Big] = |k|^\alpha \widehat{p}(k,t), \tag{5E.13}$$

Eq. (5E.11) becomes

$$\partial_t \widehat{p}(k,t) = -\,D_\alpha|k|^\alpha \widehat{p}(k,t). \tag{5E.14}$$

This is a first-order ordinary differential equation with solution

$$\widehat{p}(k,t) = e^{-D_\alpha t|k|^\alpha}\widehat{p}(k,0). \tag{5E.15}$$

For the initial condition $p(x,0)=\delta(x)$, one has $\widehat{p}(k,0)=1$, hence

$$\phi_{X_t}(k) = \widehat{p}(k,t) = e^{-D_\alpha t|k|^\alpha}, \tag{5E.16}$$

which is the characteristic function of a symmetric α-stable law.

To see the tail, write the inverse transform formally as

$$p(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\mathrm{i}kx - D_\alpha t|k|^\alpha}\,\mathrm{d}k. \tag{5E.17}$$

For large $|x|$, the dominant contribution comes from small $|k|$. Expanding the exponential,

$$e^{-D_\alpha t|k|^\alpha} = 1 - D_\alpha t|k|^\alpha + \cdots, \tag{5E.18}$$

and transforming term by term gives the leading nontrivial asymptotic

$$p(x,t) \sim \frac{D_\alpha t\,\Gamma(1+\alpha)\sin(\pi\alpha/2)}{\pi|x|^{1+\alpha}}. \tag{5E.19}$$

Thus the heavy-tail exponent is $1+\alpha$.

5E.3 Box-counting derivation of the fractal dimension

This appendix spells out the short derivation of Eq. (5.247). By self-similarity,

$$X_t \overset{d}{=} t^{1/\alpha} X_1. \tag{5E.20}$$

Therefore the typical spatial increment over a time step δt is

$$\delta x \sim (\delta t)^{1/\alpha}. \tag{5E.21}$$

To resolve the path at spatial scale ε, choose $\delta x \sim \varepsilon$, which implies

$$\delta t \sim \varepsilon^{\alpha}. \tag{5E.22}$$

Over an observation interval of length T, the number of distinguishable ε-scale segments is then

$$N_T(\varepsilon) \sim \frac{T}{\delta t} \sim T\,\varepsilon^{-\alpha}. \tag{5E.23}$$

However, no subset of a $d_{\mathcal{X}}$-dimensional ambient manifold can require more than $\varepsilon^{-d_{\mathcal{X}}}$ boxes of side ε. Hence the actual covering exponent is the smaller of α and $d_{\mathcal{X}}$:

$$N_T(\varepsilon) \sim \varepsilon^{-D_f}, \qquad D_f = \min(d_{\mathcal{X}}, \alpha). \tag{5E.24}$$

This is the box-counting form used in the main text.

5E.4 Spectral density and the two-knee approximation

Starting from

$$S_X(\omega) = 2A_0 \int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \frac{\lambda^\gamma}{\lambda^2+\omega^2}\,\mathrm{d}\lambda, \qquad 0 \le \gamma < 1, \tag{5E.25}$$

set $\lambda = \omega u$. Then

$$S_X(\omega) = 2A_0\,\omega^{\gamma-1} \int_{\lambda_{\mathrm{IR}}/\omega}^{\lambda_{\mathrm{UV}}/\omega} \frac{u^\gamma}{1+u^2}\,\mathrm{d}u. \tag{5E.26}$$

Middle band

When $\lambda_{\mathrm{IR}} \ll \omega \ll \lambda_{\mathrm{UV}}$, the integral may be extended to $(0,\infty)$:

$$\int_0^\infty \frac{u^\gamma}{1+u^2}\,\mathrm{d}u = \frac{\pi}{2}\sec\!\Big(\frac{\pi\gamma}{2}\Big), \qquad 0 \le \gamma < 1. \tag{5E.27}$$

Hence

$$S_X(\omega) \approx A_0 \pi \sec\Big(\frac{\pi\gamma}{2}\Big)\, \omega^{\gamma-1} = \frac{\widetilde{A}}{\omega^\beta}, \qquad \beta = 1-\gamma. \tag{5E.28}$$

Infrared side

When $\omega \ll \lambda_{\mathrm{IR}}$, approximate $\lambda^2+\omega^2 \approx \lambda^2$:

$$S_X(\omega) \approx 2A_0 \int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \lambda^{\gamma-2}\, \mathrm{d}\lambda = \text{constant}. \tag{5E.29}$$

Ultraviolet side

When $\omega \gg \lambda_{\mathrm{UV}}$, approximate $\lambda^2+\omega^2 \approx \omega^2$:

$$S_X(\omega) \approx \frac{2A_0}{\omega^2} \int_{\lambda_{\mathrm{IR}}}^{\lambda_{\mathrm{UV}}} \lambda^{\gamma}\, \mathrm{d}\lambda \propto \omega^{-2}. \tag{5E.30}$$

Combining the three asymptotics yields the two-knee form of Eq. (5.259).

5E.5 Compact diagnostic dictionary

For later use, Table 5E.1 records the compact diagnostic dictionary of the Qi-CON section.

Table 5E.1: Compact dictionary of Qi-CON quantities.

Quantity	**Symbol**	**Scaling form**	**Interpretation**
Fractional exponent	α	$\mathcal{L}_{\mathrm{eff}} \sim -D_\alpha(-\Delta_g)^{\alpha/2}$	Sets dynamical scaling $t \sim \ell^\alpha$.
Soft-mode exponent	γ	$A_X(\lambda)\rho(\lambda) \sim \lambda^{\gamma-1}$	Measures the low-lying modal crowding of the selected branch.
Spectral slope	β	$\beta = 1-\gamma$	Gives the intermediate $1/f^\beta$ exponent.
Fractal dimension	D_f	$D_f = \min(d_{\mathcal{X}}, \alpha)$	Effective dimension of the trajectory range inside the active window.
Infrared knee	f_{IR}	$f_{\mathrm{IR}} = \lambda_{\mathrm{IR}}/(2\pi)$	Largest branch or attractor timescale.
Ultraviolet knee	f_{UV}	$f_{\mathrm{UV}} = \lambda_{\mathrm{UV}}/(2\pi)$	Smallest retained microscopic timescale.

Appendix 5F: Technical derivations for Qi-HAL

5F.1 Uniqueness of the MePMoScausal order

The dependency graph used in Theorem 5.10.6 can be written explicitly. Let the macro-variables be

$$V = \{\mathcal{M}_n, \widehat{y}_n, a_n, s_{n+1}, \mathcal{M}_{n+1}\}.$$

The arrows are

$$E = \{\mathcal{M}_n \to \widehat{y}_n,\ \widehat{y}_n \to a_n,\ a_n \to s_{n+1},\ s_{n+1} \to \mathcal{M}_{n+1},\ \mathcal{M}_n \to \mathcal{M}_{n+1},\ \widehat{y}_n \to \mathcal{M}_{n+1}\}.$$

A topological sort must place each tail before its head. The chain

$$\mathcal{M}_n \prec \widehat{y}_n \prec a_n \prec s_{n+1} \prec \mathcal{M}_{n+1}$$

is therefore forced. Any violation creates acausality or collapses prediction into postdiction. This is the graph-theoretic reason why MePMoS is the unique minimal ordering.

5F.2 Metric recovery in NHT

Equation (5.280) implies

$$\mathrm{d}\phi_i = \omega_i\,\mathrm{d}t + \frac{2\pi}{L_i}\mathrm{d}y^i + \mathrm{d}\lambda.$$

Subtracting the trivial carrier term and gauge term gives

$$\mathrm{d}\widetilde{\phi}_i = \frac{2\pi}{L_i}\mathrm{d}y^i.$$

Hence

$$\mathrm{d}y^i = \frac{L_i}{2\pi}\mathrm{d}\widetilde{\phi}_i, \qquad \mathrm{d}s^2 = \sum_i (\mathrm{d}y^i)^2 = \sum_i \left(\frac{L_i}{2\pi}\right)^2 (\mathrm{d}\widetilde{\phi}_i)^2.$$

For a general latent metric $g_{ij}(y)$, choose a local frame $e^a{}_i(y)$ with $g_{ij} = e^a{}_i e^a{}_j$ and encode the phase coordinates in that frame. The phase metric then reconstructs the curved geometry locally.

5F.3 Frequency-independent HALreadout

Starting from

$$W_{ij} = e^{\mathrm{i}(\phi_i - \phi_j)}, \qquad x_i^{\mathrm{r}}(t) = e^{\mathrm{i}(\omega_{\mathrm{r}} t + \psi_i)},$$

one computes

$$\begin{aligned}\mathcal{O}[\psi|W] &= \sum_{i,j} \overline{x_i^{\mathrm{r}}(t)} W_{ij} x_j^{\mathrm{r}}(t) \\ &= \sum_{i,j} e^{-\mathrm{i}(\omega_{\mathrm{r}} t + \psi_i)} e^{\mathrm{i}(\phi_i - \phi_j)} e^{\mathrm{i}(\omega_{\mathrm{r}} t + \psi_j)} \\ &= \sum_{i,j} e^{\mathrm{i}[(\phi_i - \phi_j) - (\psi_i - \psi_j)]}.\end{aligned}$$

The read carrier cancels exactly. Therefore ideal recall depends only on relational phase structure. Frequency dependence enters only through nonideal hardware, finite-bandwidth envelopes, or additional amplitude dynamics.

5F.4 The dyadic ladder and level count of BFFT

If the conformal window spans $[f_{\mathrm{IR}}, f_{\mathrm{UV}}]$ and neighboring levels satisfy $f_{n+1} = 2f_n$, then

$$f_n = 2^n f_0.$$

Choosing $f_0 \approx f_{\mathrm{IR}}$, the largest admissible level index satisfies

$$2^{N_{\mathrm{lev}}-1} f_{\mathrm{IR}} \lesssim f_{\mathrm{UV}},$$

so

$$N_{\mathrm{lev}} \lesssim 1 + \log_2 \left(\frac{f_{\mathrm{UV}}}{f_{\mathrm{IR}}} \right).$$

Taking the integer part yields Eq. (5.298). This makes the number of architectural levels a measurable consequence of the observed two-knee spectrum.

5F.5 Normal-form bifurcation for the DGI–GSSBswitch

For the scalar version of Eq. (5.300),

$$V(m) = \frac{1}{2} a m^2 + \frac{b}{4} m^4 - h m, \qquad b > 0,$$

stationary points satisfy

$$a m + b m^3 - h = 0.$$

At $h = 0$, one has $m = 0$ for all a, but this point is stable only for $a > 0$. When $a < 0$, the nonzero branches

$$m_{\pm} = \pm \sqrt{-\frac{a}{b}}$$

appear and are stable. This is the usual pitchfork bifurcation. Small $h \neq 0$ unfolds the pitchfork and selects one branch. In the present architecture a is controlled by accumulated evidence, so the bifurcation becomes the mathematical normal form of exploration-to-commitment dynamics.

Appendix 5G: Technical details for SIP and EEGG

5G.1 Equivalence of folded and unfolded packet forms

The folded five-field packet and the more explicit unfolded runtime data are equivalent. Starting from Eq. (5.93), define the grouping map

$$\mathfrak{G} : \widehat{\mathrm{SIP}} \mapsto \Big[(\mathcal{Z}, g),\ \mathcal{P},\ (\mathcal{L}_{\mathrm{GDOS}}, \eta),\ (\mathcal{F}_{\mathrm{EEGG}}, \mathcal{J}),\ (\mathcal{O}, \mathfrak{K}) \Big].$$

The inverse map simply examines the generator bundle into mobility, curl, diffusion, jump, and gauge data, and examines the observable bundle into exported observables and falsifiers. No information is lost because each grouped entry is a direct product of the corresponding unfolded entries.

Table 5G.1: The grouping map from unfolded runtime data to the canonical five-field packet.

Unfolded data	**Grouped canonical field**
$(\mathcal{Z}, g_{AB})$	State manifold
$\mathcal{P}$	Ports
$(M^{AB}, \Omega^A, D^{AB}, R^A{}_\alpha \lambda^\alpha, \eta_{\text{jump}})$	Generator + closure
$(\mathcal{F}_{\text{EEGG}}, \mathcal{J})$	Ledger bundle
$(\mathcal{O}, \mathfrak{K})$	Observable dictionary + kill criteria

Therefore the five-field form is not a simplification by omission, but a simplification by bundling.

5G.2 Detailed derivation of the EEGG balance law

Starting from Eq. (5.73), differentiate with respect to time:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F}_{\text{EEGG}} = \int \frac{\delta \mathcal{F}_{\text{EEGG}}}{\delta p}\, \partial_t p \, \mathrm{d}\mu_g + \int p\, \partial_t \Phi_{\text{EGG}} \, \mathrm{d}\mu_g - \Theta_{\text{eff}} \int p\, \partial_t \ln p_{\text{ref}} \, \mathrm{d}\mu_g. \tag{5G.1}$$

Using $\partial_t p = -\nabla_A J^A$ and integrating by parts gives

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F}_{\text{EEGG}} = &\int \nabla_A \left(\frac{\delta \mathcal{F}_{\text{EEGG}}}{\delta p} \right) J^A \, \mathrm{d}\mu_g - \oint_{\partial \mathcal{Z}} \frac{\delta \mathcal{F}_{\text{EEGG}}}{\delta p}\, J^A n_A \, \mathrm{d}\Sigma \\ &+ \int p\, \partial_t \Phi_{\text{EGG}} \, \mathrm{d}\mu_g - \Theta_{\text{eff}} \int p\, \partial_t \ln p_{\text{ref}} \, \mathrm{d}\mu_g.\end{aligned} \tag{5G.2}$$

Substituting the current split (5.77) isolates the dissipative quadratic form, the curl contribution, the gauge contribution, and the explicit port/boundary terms. The last relative-entropy reference term is absorbed into the port power because changes in p_{ref} arise from boundary condition or control updates. This yields Eqs. (5.78)–(5.82).

If the local Einstein relation $D^{AB} = \Theta_{\text{eff}} M^{AB}$ holds, then the noise contribution combines naturally with the entropic term so that Θ_{eff} acquires the interpretation of an effective information temperature. This is the most direct bridge between EEGG and standard stochastic thermodynamics (Seifert, 2012).

5G.3 Event-level reduction on the locked current ray

Let the current vector be written in the ordered basis (J_e, J_B, J_{Qi}, J_E). Under the Arisaka Equation the allowed event increments span the one-dimensional ray

$$\delta \mathcal{J} = \delta j \, v, \qquad v = (1, 1, 1, Q_B)^{\mathsf{T}}.$$

Any polynomial event cost therefore reduces to a polynomial in the single amplitude δj. For example:

$$L_1(\delta\mathcal{J}) = a_\alpha\,\delta J^\alpha = (a\cdot v)\,\delta j,$$
$$L_2(\delta\mathcal{J}) = Q_{\alpha\beta}\,\delta J^\alpha \delta J^\beta = (v^{\mathsf{T}} Q v)\,(\delta j)^2,$$
$$L_3(\delta\mathcal{J}) = T_{\alpha\beta\gamma}\,\delta J^\alpha \delta J^\beta \delta J^\gamma = T(v,v,v)\,(\delta j)^3.$$

Thus all admissible local event penalties are automatically scalarized once anomaly-derived locking is imposed. This is why the scalar EEGG ledger and the vector current ledger are compatible rather than competing descriptions.

5G.4 Proof sketch for the viability-tube criterion

The persistence theorem rests on a familiar Lyapunov argument. Define $W(t) = \mathcal{V}(\Xi_t) - \beta \int_0^t \Pi_{\mathrm{maint}}(s)\,\mathrm{d}s$. Then Eq. (5.310) implies

$$\dot{W} \le -\kappa\,\mathcal{V} + c.$$

If $\Pi_{\mathrm{maint}} > c/\beta$ on average, then W cannot grow indefinitely, and neither can $\mathcal{V}$. Hence packet distance, boundary damage, and memory deficit all remain bounded. The tube is therefore forward invariant up to the chosen tolerance. If $\Pi_{\mathrm{maint}} < 0$ for sufficiently long times, the opposite occurs: the accumulated negative maintenance term drives W upward, and one or more packet coordinates leave tolerance. The proof is not sensitive to whether the target attractor is fixed, periodic, or quasiperiodic; only boundedness of the chosen packet norm matters.

5G.5 Compact packet cards across representative domains

Table 5G.2: Compact packet-ledger-persistence cards across representative domains.

Domain	**Packet core**	**Dominant EEGG terms**	**Maintenance source**	SeOS **status**
Chemical dissipative structure	Pattern amplitude, thermal gradient, vessel geometry	Gradient energy, entropic broadening, boundary forcing	External heating / cooling	Usually proto-SeOS rather than full SeOS
Protocell	Membrane state, internal metabolite packet, permeability	Assembly cost, mixing entropy, electrochemical coupling	Nutrient and amphiphile throughput	Full SeOS if repair margin is positive
Bacterium	Membrane, metabolism, adaptive memory, action state	Nutrient uptake, ATP ledger, information update cost	Continuous nutrient uptake and repair metabolism	Full SeOS

Domain	**Packet core**	**Dominant EEGG terms**	**Maintenance source**	SeOS **status**
Neural circuit	Recurrent state, synaptic memory, phase code	Coherence cost, plasticity cost, metabolic supply	Organism-level metabolic and sensory ports	Full SeOS; often Qi-HAL-rich
Institution	Archive, policy state, communication network, resource stock	Coordination cost, archive maintenance, informational entropy	Resource inflow and repair of structure/records	Distributed higher-order SeOS

Appendix 5H: Technical derivations for OH, IH, and EIC

5H.1 Derivation of the outer-hologram kernel from instruments

This appendix records the instrument-theoretic derivation behind Eqs. (5.319)–(5.321).

Let the effective reduced bulk-to-boundary map be represented by a completely positive trace-preserving map $\Phi_{t,0}$, and let $\{\mathcal{I}_y\}$ be an admissible instrument family satisfying

$$\mathcal{I}_y \geq 0, \qquad \int_{\mathcal{Y}} \mathcal{I}_y \, \mathrm{d}y = I_{\partial}.$$

For a pure bulk representative $|X\rangle$, define

$$K_{\mathrm{OH}}(y|X;t) = \mathrm{Tr}\big[\mathcal{I}_y \, \Phi_{t,0}\big(|X\rangle\langle X|\big)\big] \, .$$

Positivity follows because the trace of the product of two positive operators is nonnegative. Normalization follows from

$$\int_{\mathcal{Y}} K_{\mathrm{OH}}(y|X;t) \, \mathrm{d}y = \mathrm{Tr}\left[\left(\int_{\mathcal{Y}} \mathcal{I}_y \, \mathrm{d}y\right) \Phi_{t,0}\big(|X\rangle\langle X|\big)\right] = \mathrm{Tr}\big[\Phi_{t,0}\big(|X\rangle\langle X|\big)\big] = 1,$$

because $\Phi_{t,0}$ is trace preserving. For a mixed bulk state $\rho_6(X,t)$, linearity immediately yields Eq. (5.320).

The derivation shows that the OH kernel is not a new postulate. It is the instrument-level examining of the OGGEP boundary law.

5H.2 Observability and action-conditioned reconstruction

This appendix gives the standard algebra behind Theorem 5.12.6. For the linearized dynamics

$$\delta x_{t+1} = A_t \delta x_t + B_t \delta a_t, \qquad \delta s_t = C_t \delta x_t,$$

one has

$$\begin{aligned}\delta s_0 &= C_0 \delta x_0,\\ \delta s_1 &= C_1 A_0 \delta x_0 + C_1 B_0 \delta a_0,\\ \delta s_2 &= C_2 A_1 A_0 \delta x_0 + C_2 A_1 B_0 \delta a_0 + C_2 B_1 \delta a_1,\end{aligned}$$

and so on. Collecting terms gives

$$\mathbf{s}_{0:T} = \mathcal{O}_T \delta x_0 + \mathbf{u}_{0:T},$$

where $\mathbf{u}_{0:T}$ contains the known action-dependent contributions. If $\mathcal{O}_T$ has full column rank, the least-squares estimate

$$\widehat{\delta x_0} = (\mathcal{O}_T^\top \mathcal{O}_T)^{-1} \mathcal{O}_T^\top (\mathbf{s}_{0:T} - \mathbf{u}_{0:T})$$

is unique. If not, the nullspace is nontrivial and the hidden state cannot be uniquely reconstructed from the available history.

The mathematical moral is simple: action changes what can be seen, and memory stores the history needed to exploit that fact.

5H.3 Two-view triangulation from lower-dimensional sensory inputs

A familiar example of IH formation is the reconstruction of a 3D point from 2D observations at multiple viewpoints. Let $X \in \mathbb{P}^3$ be a homogeneous 3D point and $\tilde{u}_i \in \mathbb{P}^2$ its homogeneous image in frame i. Then

$$\lambda_i \tilde{u}_i = P_i X, \qquad P_i = K_i [R_i | t_i].$$

For two or more views, one forms the linear system

$$AX = 0,$$

where A is assembled from the cross-product equations $\tilde{u}_i \times (P_i X) = 0$. Nonzero camera baseline or self-motion is required for unique depth recovery up to the usual projective gauge. This is the most elementary concrete instance of the general claim in the main text: a lower-dimensional instantaneous observation does not determine the world; the world becomes reconstructable only after motion generates multiple views.

The same algebra appears, abstractly, in chemical gradient sensing, head-direction stabilization, and other active-sensing settings. The substrate changes; the observability logic does not.

5H.4 Internal measurement operator and selected content

Suppose the IH state is expanded in an orthonormal content basis $\{|c\rangle\}$:

$$\rho_{\mathrm{IH}} = \sum_{c,c'} \rho_{cc'} \, |c\rangle\langle c'|.$$

An internal coarse-graining may be represented by block projectors $\Pi_\alpha = \sum_{c\in B_\alpha} |c\rangle\langle c|$, where B_α is a content block. Then

$$p(\alpha) = \mathrm{Tr}(\Pi_\alpha \rho_{\mathrm{IH}}) = \sum_{c\in B_\alpha} \rho_{cc}.$$

If the DGI→GSSB→GA sequence suppresses all but one dominant block B_{α_*}, the internal post-selected state becomes

$$\rho_{\mathrm{IH}}^{(\alpha_*)} = \frac{\Pi_{\alpha_*}\rho_{\mathrm{IH}}\Pi_{\alpha_*}}{\mathrm{Tr}(\Pi_{\alpha_*}\rho_{\mathrm{IH}})}.$$

A subsequent write or action-bias map $\mathcal{W}$ produces

$$m_{t+1} = \mathcal{W}\left[\rho_{\mathrm{IH}}^{(\alpha_*)}\right],$$

which makes the selected content causally effective in the next MePMoS cycle. This is the algebraic heart of the internal-measurement idea.

5H.5 Lyapunov decrease of the EIC functional with bounded fluctuations

For the noisy gradient flow

$$\dot\theta = -G_\theta^{-1}\nabla_\theta \mathcal{F}_{\mathrm{EIC}} + \eta_t,$$

one has

$$\dot{\mathcal{F}}_{\mathrm{EIC}} = -\nabla_\theta \mathcal{F}_{\mathrm{EIC}}^\top G_\theta^{-1}\nabla_\theta \mathcal{F}_{\mathrm{EIC}} + \nabla_\theta \mathcal{F}_{\mathrm{EIC}}^\top \eta_t.$$

If η_t is zero mean with covariance bounded by Σ_η, the expected drift satisfies

$$\mathbb{E}\,\dot{\mathcal{F}}_{\mathrm{EIC}} \le -\lambda_{\min}(G_\theta^{-1})\mathbb{E}\,\|\nabla_\theta \mathcal{F}_{\mathrm{EIC}}\|^2 + \mathbb{E}\,\|\nabla_\theta \mathcal{F}_{\mathrm{EIC}}\|\,\|\eta_t\|.$$

Whenever the deterministic descent term dominates the fluctuation term, the expected EIC functional decreases. In practice this is the appropriate statement for open systems subject to bounded noise: exact monotonicity may be broken locally, but average descent toward a basin is preserved.

5H.6 Representative OH/IH/EIC cards

Table 5H.1: Compact OH/IH/EIC cards across representative systems.

System	OH	IH	EIC reading
Molecular rotor	External torque, chemical potentials, occupancy transitions at the boundary	Conformational phase / occupancy state only	Minimal or degenerate; local cycle knowledge rather than a genuine world model
Chemotactic bacterium	Sampled ligand concentration along the trajectory	Adaptive methylation state encoding recent gradient history	Proto-EIC: local chemical world reconstructed enough to bias future motion

System	OH	IH	EIC reading
Ring-attractor navigation system	Visual, vestibular, and proprioceptive heading evidence	Persistent heading bump and related attractor manifold	Allocentric heading EIC with repeated correction by OH events
Rodent navigation system	Multimodal environmental stream with self-motion updates	Place/head-direction/time-structured spatial model	Rich navigational EIC; imagination and replay become possible
Human scientific cognition	Experimental data, language, symbolic and sensory streams	Multiscale internal model spanning space, time, causality, and abstraction	Consciousness-grade EIC; the system measures its own model and can report it symbolically

Appendix 5I: Pagoda hierarchy and cross-domain technical details

5I.1 Closure defects, effective floors, and error budgets

This appendix expands Definition 5.13.1. Let $z^{(\ell+1)} = C_{\ell+1}(z^{(\ell)})$ and define the exact induced derivative and closure defect by Eq. (5.342). A convenient normalized defect index is

$$\varepsilon_{\mathrm{cl}}^{(\ell+1)} := \frac{\left\langle \|\Delta_{\ell+1}\|^2 \right\rangle^{1/2}}{\left\langle \|F_{\mathrm{eff}}^{(\ell+1)}\|^2 \right\rangle^{1/2} + \delta}, \qquad \delta > 0. \tag{5I.1}$$

The index is small when the floor is dynamically autonomous. In data-driven applications one can estimate $F_{\mathrm{eff}}^{(\ell+1)}$ from reduced trajectories and treat $\varepsilon_{\mathrm{cl}}$ as a model-selection criterion for candidate higher-level packets.

A second useful quantity is the *port novelty index*

$$\nu_{\mathrm{port}}^{(\ell+1)} := \dim \operatorname{span} \mathcal{P}^{(\ell+1)} - \dim \operatorname{span} \mathcal{P}_{\mathrm{pull}}^{(\ell\to\ell+1)}. \tag{5I.2}$$

If $\nu_{\mathrm{port}}^{(\ell+1)} = 0$, then the higher floor introduces no genuinely new mode of exchange and is usually better interpreted as a descriptive convenience rather than a new scientific level.

5I.2 Hitting-time proof of conditional inevitability

Theorem 5.13.4 may be rewritten using visit counts. Let $N_W(T)$ be the number of visits to a viable window W by time T, and let S_n be the indicator that the n-th visit stabilizes. Under the assumptions of the theorem,

$$\mathbb{P}(S_n = 1 \mid \mathrm{visit}_n) \geq p > 0.$$

Hence

$$\mathbb{P}(\text{no stabilization up to visit } N) \leq (1-p)^N.$$

If $N_W(T) \to \infty$ almost surely as $T \to \infty$, then

$$\mathbb{P}(\tau_W = \infty) = \lim_{N\to\infty} (1-p)^N = 0.$$

The theorem therefore uses only three ingredients: positive-measure windows, recurrent exploration, and positive stabilization probability.

5I.3 Compact domain cards and observables

Table 5I.1: Compact application cards for representative domains.

Domain	**Key packet variables**	**Candidate observables**	**Primary memory variable**	**Most diagnostic test**
Catalytic chemistry	concentrations, occupancy, rotor angle	product counts, spectra, stepping time, fluxes	conformational state	event-ledger closure plus burst statistics
Chemotactic cell	receptor state, internal methylation, trajectory	run/tumble statistics, local gradient, adaptation curve	adaptive receptor memory	motion-conditioned predictive quality
Organismal physiology	nutrient store, damage, repair flux, internal model	metabolic rate, motion, sensory traces, repair markers	regulatory and long-term physiological memory	maintenance-positive SeOS inequality
Navigation circuit	neural manifold, latent heading/place state, action output	spikes/LFP, movement, sensory cues, reports	recurrent attractor state	DGI-to-GSSB-to-report timing
Scientific institution	agent states, archive, instruments, communication graph	data releases, publications, consensus updates	distributed archive and trained priors	OH/IH model-improvement under controlled measurement practice

5I.4 Minimal dynamical forms across domains

For convenience, the minimal application equations from the main text are collected here in one place.

$$\begin{aligned}
\text{Chemistry:}\quad & \dot{n}_i = \sum_r \nu_{ir} R_r(n,m) - \nabla \cdot J_i + I_i - O_i + \xi_i, && \dot{m} = f_m(n,m) + \eta_m,\\
\text{Life:}\quad & \dot{N} = J_{\text{in}} - J_{\text{use}} - J_{\text{leak}}, && \dot{B} = Y J_{\text{use}} - \delta B - c_r(D),\\
& \dot{D} = \sigma - r, && \dot{m} = f_m(y,a,m),\\
\text{Brain:}\quad & \tau_x \dot{x} = -x + W\phi(x) + Uy + Va + \zeta_x, && \tau_m \dot{m} = f_m - K\nabla_m \mathcal{E}_{\text{EIC}},\\
\text{Society:}\quad & \dot{x}_i = f_i(x_i, A, d_i) + \sum_j W_{ij}\Psi(x_j - x_i) + \eta_i, && \dot{A} = \Gamma(\{x_i\}, d) - \Lambda A.
\end{aligned}$$

These forms are not intended as finished empirical models. They are canonical placeholders showing how the same runtime grammar appears in very different domains.

Appendix 5J: Falsifiability, page budget, and architecture summary

5J.1 Compact falsifiability matrix

Table 5J.1: Compact falsifiability matrix for the integrated framework.

Runtime block	**Observable proxy**	**Representative domain**	**Failure mode**
Qi-QUA	event-level energy/information accounting	chemistry, molecular motors	no coherent locked ledger
Qi-EP	path-selection under perturbation	reaction routing, behavior, inference tasks	no common selector family
Qi-CON	spectra/tails/fractal dimension	neural activity, ecological time series, market-like records	no stable multiscale signature
Qi-HAL	memory depth and multiscale coordination	cells, brains, institutions	no measurable internal memory architecture
OH/IH/EIC	model quality under action restriction	navigation, manipulation, scientific practice	internal model unaffected by motion constraints

5J.2 Page-budget ledger

Table 5J.2: Working page budget for the integrated Chapter 5 review draft after the structural reflow to the six-part architecture.

No.	Planned section	Page range	Pages
1	Introduction	1–24	24
2	Recap bridge to GUT, GG-6, and GDOS foundation	25–34	10
3	Universal extension of GDOS to any open system in nature	35–44	10
4	The master pipeline: DGI + GSSB + GA	45–58	14
5	EEGG: energy, entropy, and free energy by the geometry–gauge principle	59–70	12
6	SIP: System Instance Packet as the canonical unit of GDOS	71–82	12
7	Qi-QUA: Arisaka Ratio, Arisaka Equation, and Arisaka Constant	83–98	16
8	Qi-EP: PLA in 6D and PLT/MEPP/FEP in 4D	99–118	20
9	Qi-CON: conformality, Lévy, fractality, and $1/f$ with two knees	119–136	18
10	Qi-HAL: MePMoS, NHT, HAL, BFFT, and DGI–GSSB interplay	137–158	22
11	SeOSand QCI	159–174	16
12	OH+ IH= EIC	175–204	30
13	Five-story pagoda hierarchy	205–228	24
14	Cross-domain applications	229–254	26
15	Why GDOS applies to all open systems in nature	255–264	10
16	Discussion: significance, uniqueness, comparison	265–278	14
17	Limitations, falsifiability, and empirical program	279–292	14
18	Conclusions	293–300	8
A–Y	Appendices	301–406	106
R	References	407–424	18
		Target total	**424**

5J.3 Forward architecture summary

For convenience, Table 5J.3 records the major tasks that remain after the structural reflow of the main text into the six-part architecture.

Table 5J.3: Forward architecture after the structural reflow of runtime constitution, Qi-4, persistence/completeness, and OH/IH/EIC semantics.

No.	Planned later section
F1	Domain-specific numerical case studies and simulation papers
F2	Companion papers on chemistry, life, brain, and society using the common runtime
F3	Empirical calibration of $U(1)_{Qi}$ proxies and consistency-vector observables
F4	Additional technical appendices, notation ledgers, and summary tables

Appendix 5K: Chemistry, biochemistry, and protocellular open systems

The main text argued that chemistry is the first decisive testbed for the universal-runtime claim, because chemistry is the domain in which one simultaneously sees energetic throughput, charge transfer, material reorganization, structural persistence, and information-bearing selectivity. A detector or a scattering experiment already displays lawful open-system projection, but chemistry adds a stronger demand: the system must often preserve a far-from-equilibrium structure while repeatedly performing useful cycles. That is exactly the setting in which SIP, EEGG, SeOS, DGI–GSSB–GA, and the Qi-4 pillars should either become precise and useful or else fail. The point of the present appendix is therefore not decorative. It is to show, in slower detail than the main text allowed, how the GDOS runtime is expected to be instantiated in reaction networks, enzyme cycles, membrane systems, autocatalytic sets, and protocell-scale organizations (Kauffman, 1993; Landauer, 1961; Nicolis & Prigogine, 1977; Prigogine, 1967; Lane, 2015).

5K.1 Why chemistry is the first nontrivial export layer

In the present program, the chemical layer is not merely "between" particle physics and biology in an informal pedagogical sense. It is the first layer at which a finite open system can both preserve a persistent identity and implement selective throughput. A molecule, membrane, or protocell must trade among at least four demands: energetic affordability, material or structural maintenance, charge or redox consistency, and information-bearing discrimination. The first two demands are familiar in thermodynamics; the third is explicit in electrochemistry; the fourth appears in selectivity, proofreading, templating, signaling, and memory-like state retention. The claim of Qi-QUA is that these demands are not independent ledgers once the underlying event is projected to the lawful boundary description of GDOS. The current-locking law should therefore be tested here first, because chemistry gives one the cleanest mesoscopic event count.

5K.2 A chemistry-scale System Instance Packet

Let a chemical open system possess N_s coarse-grained species variables n_i, N_c conformational coordinates ξ_α, N_p port or membrane coordinates ϕ_a, and N_m slowly writable internal markers

m_b. The reduced state coordinates are taken to be

$$z^A = \big(n_1, \dots, n_{N_s}, \xi_1, \dots, \xi_{N_c}, \phi_1, \dots, \phi_{N_p}, m_1, \dots, m_{N_m}\big). \tag{5K.1}$$

The corresponding runtime packet is

$$\mathrm{SIP}_{\mathrm{chem}} = \big(\mathcal{M}_{\mathrm{chem}}, z^A, G^{(\mathrm{chem})}_{AB}, \mathcal{A}^{(\mathrm{chem})}_{A}, \mathcal{F}^{(\mathrm{chem})}_{\mathrm{EEGG}}, \Sigma^{AB}_{(\mathrm{chem})}, \mathcal{O}_{\mathrm{chem}}\big). \tag{5K.2}$$

Each slot now has a direct interpretation.

First, $\mathcal{M}_{\mathrm{chem}}$ is the accessible manifold of occupation levels, conformers, port states, and slow memory marks. It is not the full $6N$-dimensional microscopic phase space. Second, the induced metric $G^{(\mathrm{chem})}_{AB}$ measures the local cost or distinguishability of moving on that reduced manifold; in practice this may be inferred from susceptibility matrices, covariance matrices, or information-geometric approximations. Third, $\mathcal{A}^{(\mathrm{chem})}_{A}$ is the effective gauge or curl connection that survives coarse-graining. In chemical language it packages cyclic driving, nonconservative affinities, and phase-like loop effects. Fourth, $\mathcal{F}^{(\mathrm{chem})}_{\mathrm{EEGG}}$ is the common ledger that combines energy supply, entropy production, structural maintenance, and geometric constraints. Fifth, $\Sigma^{AB}_{(\mathrm{chem})}$ records unresolved stochasticity from microscopic collisions, conformational substructure, and environmental variation. Finally, $\mathcal{O}_{\mathrm{chem}}$ is the explicit dictionary of measurable outputs: concentrations, currents, rates, redox states, membrane potentials, binding fidelity, growth, division, or other experimentally accessible records.

The reduced continuity law takes the standard GDOS form

$$\partial_t p(z,t) + \nabla_A J^A(z,t) = 0, \qquad J^A = J^A_{\mathrm{grad}} + J^A_{\mathrm{curl}} + J^A_{\mathrm{noise}}, \tag{5K.3}$$

with

$$J^A_{\mathrm{grad}} = -p\, M^{AB} \nabla_B \Xi_{\mathrm{chem}}, \tag{5K.4}$$

$$J^A_{\mathrm{curl}} = p\, \Omega^{AB} \nabla_B \Psi_{\mathrm{chem}}, \tag{5K.5}$$

$$J^A_{\mathrm{noise}} = -\frac{1}{2} \nabla_B \Big(\Sigma^{AB}_{(\mathrm{chem})} p \Big). \tag{5K.6}$$

The first term drives relaxation on the common scalar ledger, the second encodes looped or oscillatory turnover, and the third closes positivity under unresolved fluctuations. In a simple enzyme cycle the curl term is the reduced remnant of a driven sequence of binding and release events. In a membrane-transport problem it is the remnant of circulation through a driven electrochemical loop. In an autocatalytic or templating set it captures the fact that turnover may circulate around a network rather than merely descend a scalar potential.

Table 5K.1: One concrete interpretation of the seven SIP slots in chemistry and biochemistry.

SIP slot	Typical chemistry-scale meaning
$\mathcal{M}_{\mathrm{chem}}$	Coarse-grained manifold of concentrations, conformers, port states, and slow writable markers.
z^A	Occupancies n_i, conformers ξ_α, membrane or redox variables ϕ_a, and memory-like variables m_b.
$G^{(\mathrm{chem})}_{AB}$	Susceptibility or information metric describing the local cost of state displacement.
$\mathcal{A}^{(\mathrm{chem})}_{A}$	Nonconservative affinities, cycle phases, and port couplings surviving coarse-graining.
$\mathcal{F}^{(\mathrm{chem})}_{\mathrm{EEGG}}$	Joint ledger of free-energy supply, entropy production, structural maintenance, and geometric constraint.
$\Sigma^{AB}_{(\mathrm{chem})}$	Stochastic forcing from collisions, conformational microstates, and environmental variation.
$\mathcal{O}_{\mathrm{chem}}$	Measurable outputs: rates, redox or ion currents, yields, fidelity, growth, division, and robustness.

5K.3 Reaction-network representation and current ledgers

Suppose the system contains reaction channels $\rho = 1, \dots, R$ with stoichiometric matrix $S_{i\rho}$ and coarse-grained fluxes j_ρ. Then

$$\partial_t n_i = \sum_{\rho=1}^{R} S_{i\rho} j_\rho + \eta_i, \tag{5K.7}$$

where η_i summarizes unresolved noise and exchange with neglected degrees of freedom. For the present program the important point is not merely stoichiometry but *ledger separation*. To each flux channel one assigns four projected event weights:

$$\Delta G_\rho, \qquad q_\rho, \qquad b_\rho, \qquad \iota_\rho, \tag{5K.8}$$

interpreted respectively as energetic throughput, charged or electrochemical transfer, structural-material restructuring, and information-bearing discrimination. The associated reduced currents are then

$$J_E = \sum_\rho \Delta G_\rho \, j_\rho, \tag{5K.9}$$

$$J_e = \sum_\rho q_\rho \, j_\rho, \tag{5K.10}$$

$$J_B = \sum_\rho b_\rho \, j_\rho, \tag{5K.11}$$

$$J_{Qi} = \sum_\rho \iota_\rho \, j_\rho. \tag{5K.12}$$

At the microphysical level J_B descends from the baryonic channel of GG-6. At chemistry scale it becomes the structural-material bookkeeping current: the channel that counts net rearrangement of matter-bearing structure across the chosen boundary description. The notation is inherited, but the interpretation is mesoscopic. Likewise J_{Qi} is the effective information-throughput channel, not an arbitrary symbolic label.

A *locked elementary cycle* is defined to be a coarse-grained event class for which the four currents are proportional to a common event count n_{cyc}. In chemistry natural units one then writes

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Qi} = n_{\mathrm{cyc}}. \tag{5K.13}$$

Equation (5K.13) is not asserted to hold for every arbitrarily coarse aggregate without correction. Rather, it is the ideal bookkeeping relation for an admissible elementary boundary update in which energy transfer, charge/redox transfer, structural reconfiguration, and information-bearing discrimination are inseparable aspects of one projected event. Deviations from exact equality are therefore diagnostic. If a biochemical model claims to implement a one-step selective event while grossly violating (5K.13), then either the proposed event class is not elementary or the chosen SIP has omitted one of the relevant ports.

5K.4 Qi-EP in biochemical cycles

The chemical domain is also the natural place to see why Qi-EP is not a mere restatement of one preexisting extremal principle. Consider a path $z(t)$ on $\mathcal{M}_{\mathrm{chem}}$. The chemistry-scale GDOS path functional may be organized as

$$\mathcal{S}_{\mathrm{chem}}[z] = \int_{t_i}^{t_f} \mathrm{d}t \, \left[\frac{1}{2} g_{AB}(z) \dot{z}^A \dot{z}^B + \dot{z}^A \mathcal{A}_A^{(\mathrm{chem})}(z) + V_{\mathrm{diss}}(z) + \Lambda_{\mathrm{sel}}(z, \dot{z}) \right]. \tag{5K.14}$$

The first term is the metric cost of motion on the reduced manifold. The second is the cycle or affinity term. The third records the dissipative penalty or throughput cost. The fourth stands for selectivity or fidelity constraints, such as proofreading, templating, or gated transport. Now consider three limiting regimes.

If the stochastic forcing is weak, the cycle structure modest, and the relevant task primarily geometric, then minimization of $\mathcal{S}_{\mathrm{chem}}$ reduces to a shortest-time or shortest-cost path problem: this is the PLT face. If the task is dominated by sustained driving with strong dissipation, the extremum reorganizes into a constrained entropy-production problem: this is the MEPP face. If, however, the system contains internal variables m_b that encode predictions or internal templates, then part of Λ_{sel} becomes an inference-like mismatch term between internal and external port states, and the same master functional acquires a free-energy-like form: this is the FEP face. The physical claim of Qi-EP is therefore that the apparent plurality of principles emerges because different boundary tasks emphasize different sectors of the same path functional. Chemistry provides the cleanest example because the same device may operate in all three modes depending on timescale and coarse-graining.

5K.5 Qi-CON signatures in chemistry and protocells

Chemistry is also where Qi-CON becomes sharply testable. Enzymes, ion channels, autocatalytic sets, and membrane-active protocells often display multiple relaxation scales, conformational substates, and intermittent switching. The GDOS expectation is that the reduced noise kernel is

not generically single-scale. Instead one should often see

$$S_X(f) \sim \begin{cases} S_0, & f \ll f_L, \\ f^{-\alpha}, & f_L \ll f \ll f_H, \\ f^{-\beta}, & f \gg f_H, \end{cases} \tag{5K.15}$$

with f_L set by the longest architectural closure time of the system and f_H by the fastest internal port-gating or conformational switching scale. Likewise residence-time distributions may exhibit a scale window

$$P(\tau) \propto \tau^{-1-\mu}, \qquad 0 < \mu < 2, \tag{5K.16}$$

before finite-size or finite-memory cutoffs restore integrability. In the present framework such laws are not treated as mysterious empirical curiosities. They are the expected traces of a multiscale attractor lattice whose degrees of freedom have been only partially projected to the observer's boundary.

5K.6 From protocells to living systems

A protocell or minimally living system is precisely the point at which chemical runtime acquires the full SeOS interpretation. The boundary is no longer merely conceptual; it is a maintained membrane or other selective enclosure. The internal chemistry is no longer merely reactive; it becomes self-referential in the limited but important sense that part of the throughput is spent preserving the ports through which throughput occurs. This is the first unmistakable appearance of a selfish open system. The system persists not by violating thermodynamics but by routing gradients through a geometry that keeps its own boundary in being (Schrödinger, 1944; Lane, 2015). In this sense, life at its lowest layer is a chemical proof-of-principle of the GDOS program.

Table 5K.2: Chemistry-domain diagnostics suggested by Chapter 5.

Claim	**Operational observable**	**What would count as support or tension**
Qi-QUA current locking	Joint measurement of energy, redox/ion transfer, structural turnover, and selectivity events	Support: one elementary cycle class approximately collapses these four ledgers to a common event count. Tension: no sensible coarse-graining yields such locking.
Qi-EP unification	Fit of short-path, dissipative, and inference-like objectives within one master cycle functional	Support: regime changes can be modeled by reweighting one functional. Tension: mutually incompatible objectives are required for the same cycle class.
Qi-CON multiscale statistics	Power spectra, waiting-time tails, and state-residence histograms	Support: two knees and finite scale windows correlate with identifiable structural scales. Tension: fitted exponents float independently of architecture.
SeOS persistence	Boundary-maintenance cost vs. throughput budget	Support: a calculable maintenance sector exists and predicts loss-of-identity thresholds. Tension: persistence has no ledger representation.

5K.7 Concluding remark on the chemistry layer

The strongest reason to emphasize chemistry in a landmark GDOS paper is methodological. Chemistry is the first domain in which the theory must cease to be merely a philosophical unifier and become a quantitative runtime. Concentrations, currents, binding events, redox changes, membrane potentials, yields, and fidelity measures are all observable. If the framework cannot become precise here, then the later claims about life and mind would be premature. Conversely, if it succeeds here, then the ascent to biological and cognitive systems becomes natural rather than rhetorical.

Appendix 5L: Neural, cognitive, and consciousness-oriented open systems

The main text introduced Qi-HAL and the identity OH + IH = EIC as the architectural export of GDOS to internal-model systems. The present appendix slows that argument down and states it in a more explicitly operational form. The goal is not to claim that one already possesses a final quantitative theory of consciousness. The goal is more modest and, in a way, more demanding: to show that once the GG-6/GDOS runtime is accepted, nervous systems are *not* exceptional in requiring entirely new metaphysical principles. They are exceptional only in architectural depth. The same boundary logic that governs detectors and cells now runs through large recurrent systems that sense, remember, predict, and act over many timescales (Amari & Nagaoka, 2000; Friston, 2010; Shannon, 1948; Tononi, 2004).

5L.1 A neural System Instance Packet

Let a neural open system be described at reduced scale by fast activation variables x_i, phase variables θ_α, synaptic or coupling variables w_{ij}, slow memory traces m_b, and action-readiness variables a_c. One may then choose

$$z^A = \big(x_1, \dots, x_{N_x}, \theta_1, \dots, \theta_{N_\theta}, w_{11}, \dots, w_{N_x N_x}, m_1, \dots, m_{N_m}, a_1, \dots, a_{N_a}\big), \tag{5L.1}$$

together with the packet

$$\mathrm{SIP}_{\mathrm{neural}} = \big(\mathcal{M}_{\mathrm{neural}}, z^A, G^{(\mathrm{neural})}_{AB}, \mathcal{A}^{(\mathrm{neural})}_{A}, \mathcal{F}^{(\mathrm{neural})}_{\mathrm{EEGG}}, \Sigma^{AB}_{(\mathrm{neural})}, \mathcal{O}_{\mathrm{neural}}\big). \tag{5L.2}$$

Here $G^{(\mathrm{neural})}_{AB}$ may be interpreted as the effective information or control metric on the reduced state space; $\mathcal{A}^{(\mathrm{neural})}_{A}$ encodes phase transport, recurrent loops, and nonconservative input drive; $\mathcal{F}^{(\mathrm{neural})}_{\mathrm{EEGG}}$ combines metabolic cost, prediction or mismatch cost, structural regularization, and task constraints; $\Sigma^{AB}_{(\mathrm{neural})}$ records unresolved variability and fluctuating input; and $\mathcal{O}_{\mathrm{neural}}$ specifies the readout dictionary, such as spikes, local fields, imaging signals, behavioral reports, reaction times, or action trajectories.

The universal GDOS current law again applies:

$$\partial_t p(z,t) + \nabla_A J^A(z,t) = 0, \qquad J^A = -p\, M^{AB}\nabla_B \Xi + p\, \Omega^{AB}\nabla_B \Psi - \frac{1}{2}\nabla_B(\Sigma^{AB} p). \tag{5L.3}$$

The essential point is that a neural reduced model needs all three sectors. Pure gradient descent misses oscillation and phase transport. Pure Hamiltonian rotation misses irreversibility, gain control, and noise-driven switching. Pure diffusion misses learned structure and directed inference. A brain-scale runtime is therefore an excellent domain in which the gradient, curl, and noise sectors must coexist.

Table 5L.1: One possible mapping from neural variables to the universal runtime grammar.

Runtime object	**Neural or cognitive interpretation**
$\mathcal{M}_{\text{neural}}$	Reduced manifold of activation, phase, memory, and action-readiness states.
$G_{AB}^{(\text{neural})}$	Effective information or control geometry; local distinguishability and transition cost.
$\mathcal{A}_A^{(\text{neural})}$	Recurrent phase transport, oscillatory coupling, and loop-induced nonconservative structure.
$\mathcal{F}_{\text{EEGG}}^{(\text{neural})}$	Joint ledger of metabolic cost, mismatch cost, structural regularization, and reward or viability terms.
$\Sigma_{(\text{neural})}^{AB}$	Trial-to-trial variability, background fluctuations, and unresolved microstate contributions.
$\mathcal{O}_{\text{neural}}$	Readout dictionary from spikes or fields to behavior, report, or other recorded boundary observables.

5L.2 DGI, GSSB, and GA in neural dynamics

The triad DGI–GSSB–GA acquires a particularly vivid form in neural systems. DGI says that the reduced generator should preserve its lawful content under admissible reparameterizations of coding variables, population coordinates, or gauge-like phase conventions. One should be free to rewrite the same dynamics in firing-rate variables, latent coordinates, or phase-amplitude coordinates without changing the physical predictions for observables in $\mathcal{O}_{\text{neural}}$. GSSB then states that concrete sensory context, task demands, developmental constraints, or injury can reduce the symmetry of the admissible generator, selecting some modes and suppressing others. GA is the observable outcome: a stable or metastable attractor family corresponding to a memory, percept, motor plan, or global action policy.

A practical way to write this is to separate the reduced drift into background-symmetric and context-selective parts:

$$\dot{z}^A = \mathcal{V}_{\text{sym}}^A(z) + \mathcal{V}_{\text{ctx}}^A(z;u) + \eta^A, \tag{5L.4}$$

where u denotes input or task context. DGI constrains how $\mathcal{V}_{\text{sym}}^A$ and $\mathcal{V}_{\text{ctx}}^A$ transform under admissible coordinate changes. GSSB is the statement that nonzero u, together with boundaries and long-run learning, reduces the effective symmetry of $\mathcal{V}^A$. GA is the resulting landscape of selected basins. The framework therefore predicts that neural function should not be understood as motion on a static graph only; it is motion on a graph whose effective symmetry class is continually being selected by boundary conditions.

5L.3 Qi-HAL and the minimal adaptive closure

The main text introduced MePMoS as the minimal adaptive loop of Memory, Prediction, Motion, and Sensing. Here the usefulness of that decomposition can be made explicit. Let

$$m_{t+\Delta t} = \mathcal{M}(m_t, x_t, o_t), \tag{5L.5}$$

$$\hat{o}_{t+\Delta t} = \mathcal{P}(m_t, x_t, a_t), \tag{5L.6}$$

$$a_{t+\Delta t} = \mathcal{A}(m_t, x_t, \hat{o}_t, o_t), \tag{5L.7}$$

$$o_{t+\Delta t} = \mathcal{S}(a_t, \text{world}_t) + \nu_t. \tag{5L.8}$$

Equations (5L.5)–(5L.8) do not presuppose one specific neuroscience model. They state only that an adaptive open system capable of internal modeling must contain modules that play these four roles. Qi-HAL then adds higher-order structure on top of that loop: multiscale memory, a graph of attractor basins, phase-coded communication among scales, and switching operations that let the system pass from one effective mode to another without catastrophic loss of coherence. Equivalently, the MePMoS loop is the minimal closure, while the full holographic attractor lattice is the structured enlargement required for large-scale cognition.

5L.4 Outer hologram, inner hologram, and evolving inner cosmos

The most delicate claim of the present program is not that the world contains measurement. That much was already established in GG-2026-3. The more ambitious claim is that sufficiently complex open systems perform *internal* measurement as well. To state this carefully, define the outer-hologram map

$$o_t = \Pi_{\mathrm{OH}}[\mathrm{world}_t; a_t] + \nu_t, \tag{5L.9}$$

where o_t is the boundary record delivered by the world conditional on the system's current action and sensory geometry. This is not passive reception; the action variable a_t explicitly enters the projection. Next define the inner-hologram dynamics

$$h_{t+\Delta t} = \mathcal{K}_{\mathrm{IH}}(h_t, m_t, o_t, a_t), \tag{5L.10}$$

which updates an internal reconstructive state h_t on the basis of past internal state, memory, present sensory evidence, and current action. Finally define the slowly accumulated inner world-model

$$\mathcal{W}_{t+\Delta t} = \mathcal{U}(\mathcal{W}_t, h_t, r_t), \tag{5L.11}$$

with r_t the currently globally effective or reportable internal mode. The triple $(\Pi_{\mathrm{OH}}, \mathcal{K}_{\mathrm{IH}}, \mathcal{U})$ is the operational meaning of OH + IH = EIC.

Within this framework, "consciousness" is not introduced as a separate ontological substance. It is introduced as a regime of internal measurement in which some internal attractor state r_t becomes sufficiently stable, globally effective, and action-relevant that it functions as a boundary-visible state *for the system itself*. A simple operational criterion is

$$\tau_{\mathrm{dwell}}(r_t) > \tau_{\mathrm{cycle}}, \qquad \Gamma_{\mathrm{global}}(r_t) > \Gamma_c, \qquad \mathcal{R}_{\mathrm{action}}(r_t) > \mathcal{R}_c, \tag{5L.12}$$

where τ_{cycle} is the characteristic internal MePMoS cycle time, Γ_{global} is a measure of cross-module causal reach, and $\mathcal{R}_{\mathrm{action}}$ is the degree to which the state constrains future selection. Equation (5L.12) is not claimed to be a final empirical law. It is a placeholder for the kind of measurable criterion the present framework demands.

5L.5 Qi-CON and the spectrum of neural timescales

The neural domain is also one of the clearest places to test Qi-CON. Empirically, brain activity often exhibits scale-rich spectra rather than one clean resonance. In the present framework this is not an accidental superposition of unrelated processes. It is the spectral signature of an attractor

lattice whose modes are arranged across nested timescales. The simplest predicted form is again a two-knee law,

$$S_{\text{neural}}(f) \sim \begin{cases} S_0, & f \ll f_{\text{global}}, \\ f^{-\alpha}, & f_{\text{global}} \ll f \ll f_{\text{local}}, \\ f^{-\beta}, & f \gg f_{\text{local}}, \end{cases} \tag{5L.13}$$

where f_{local} is associated with local recurrent loops and f_{global} with long-range integration cycles. The detailed exponents α and β are not universal numbers; they are effective summaries of architecture. The critical claim is instead that knee locations should correlate with explicit anatomical or functional hierarchy rather than floating independently of it.

Table 5L.2: Candidate runtime criteria for reportable internal states.

Criterion	**Operational reading**	**Reason it matters for** OH/IH/EIC
Metastable dwell time	State persists longer than the characteristic update cycle	Without dwell time the system cannot treat the state as an internal measurement record.
Broad causal reach	State influences multiple modules or scales	A purely local transient is not globally effective in the sense needed for report or coordinated action.
Action coupling	State constrains downstream policy or motor selection	Internal measurement matters only if it can reorganize future interaction with the world.
Reconstructive revision	State updates under new evidence without total collapse	The IH must behave like a revisable model, not a frozen label.
Energy-budget support	State maintenance consumes and justifies a measurable metabolic budget	The state must live on the same EEGG ledger as the rest of the system.

5L.6 Scope, caution, and falsifiability

The present appendix does not claim that every neural correlate of consciousness is already derivable from first principles. It makes a narrower and testable claim. If reportable conscious-like states exist, then they should appear as a special class of globally effective attractor states in a multiscale internal-hologram architecture. If future experiments show that reportable states have no special dwell time, no broadened causal reach, no privileged relationship to action selection, and no identifiable role in the internal model, then the OH/IH/EIC proposal would be in serious trouble. A landmark theory paper must say that clearly.

Appendix 5M: Ecological, social, and technological collective systems

The universal-runtime claim of GDOS is strongest, and most vulnerable, when it is exported beyond laboratory physics or biochemistry into collective systems. The danger here is obvious: one can too easily use scientific vocabulary metaphorically. The purpose of the present appendix is therefore to draw a strict boundary between legitimate export and category error. The theory does *not* claim that a society literally carries the same microphysical gauge charges as a single molecule. What it claims is that a collective system, once described as a finite open system with lawful ports, resources, memory, communication, and action channels, can be written on the same runtime grammar as any other reduced system. The reward for this discipline is that one may compare particles, cells, brains, and institutions without pretending they are identical. They are instead different realizations of one lawful open-system syntax (Friston, 2010; Kauffman, 1993; Shannon, 1948; Dawkins, 1976).

5M.1 A collective System Instance Packet

Consider a collective system with population variables N_i, resource variables R_a, communication variables C_μ, memory or institutional variables M_λ, and action or policy variables A_κ. Define

$$z^A = \big(N_1, \dots, N_{N_N}, R_1, \dots, R_{N_R}, C_1, \dots, C_{N_C}, M_1, \dots, M_{N_M}, A_1, \dots, A_{N_A}\big). \tag{5M.1}$$

Then the runtime packet reads

$$\mathrm{SIP}_{\mathrm{coll}} = \big(\mathcal{M}_{\mathrm{coll}}, z^A, G^{(\mathrm{coll})}_{AB}, \mathcal{A}^{(\mathrm{coll})}_{A}, \mathcal{F}^{(\mathrm{coll})}_{\mathrm{EEGG}}, \Sigma^{AB}_{(\mathrm{coll})}, \mathcal{O}_{\mathrm{coll}}\big). \tag{5M.2}$$

The interpretation of each slot is now mesoscopic. $G^{(\mathrm{coll})}_{AB}$ measures the local cost or sensitivity of displacing the collective state; $\mathcal{A}^{(\mathrm{coll})}_{A}$ packages nonconservative institutional loops, delayed-control cycles, or technological feedbacks; $\mathcal{F}^{(\mathrm{coll})}_{\mathrm{EEGG}}$ becomes the combined ledger of energetic or resource cost, entropy or uncertainty, structural maintenance, and coordination cost; $\Sigma^{AB}_{(\mathrm{coll})}$ encodes unmodeled disturbances and heterogeneity; and $\mathcal{O}_{\mathrm{coll}}$ is the explicit record dictionary: demographic rates, resource flows, communication delays, compliance, failure cascades, and resilience metrics.

5M.2 Dynamic equations and the meaning of the four channels

At collective scale one may write the reduced equations schematically as

$$\partial_t N_i = B_i(z) - D_i(z) + \sum_j (F_{ji}(z) - F_{ij}(z)) + \eta_i, \tag{5M.3}$$

$$\partial_t R_a = \Phi^{\mathrm{in}}_a(z) - \Phi^{\mathrm{out}}_a(z) - C_a(z) + \zeta_a, \tag{5M.4}$$

$$\partial_t M_\lambda = \mathcal{T}_\lambda(M, C, A) - \mathcal{L}_\lambda(M) + \chi_\lambda, \tag{5M.5}$$

with analogous equations for communication and policy variables. The four-channel ledger then survives as a *bundled* description. J_E becomes the net energetic or resource-throughput channel

required to maintain the system. J_e becomes the signaling or control-carrier channel, descended from lower-level charged transport but now expressed as the effective communication or command throughput available at the chosen resolution. J_B becomes the structural-material maintenance channel: the one counting the throughput needed to preserve the physical or organizational body of the system. J_{Qi} becomes the effective information-throughput channel: update, coordination, discrimination, and model revision.

The export rule is therefore strict: the same symbols may be used across scales only when the observer has supplied a lawful bundling map from lower-level physical events to higher-level ports. Without such a bundling map, the notation would indeed be metaphorical and should be rejected. This insistence on explicit bundling is exactly how GDOS tries to remain scientific when exported to difficult domains.

Table 5M.1: Permissible interpretation of the four current channels in collective systems.

Channel	**Permissible collective-scale interpretation**
J_E	Resource or energy throughput needed to keep the system operating and to power adaptation.
J_e	Effective signaling or control throughput available to coordinate distant parts of the system.
J_B	Material and structural maintenance throughput preserving the body, infrastructure, or persistent organization.
J_{Qi}	Information-update throughput required for discrimination, learning, planning, and coordinated response.

5M.3 Qi-EP for collective adaptation

The value of the runtime language becomes clearer when one considers adaptation. Collective systems often appear to optimize one quantity in one regime and another quantity in another regime. A firm may seem to minimize time to response during crisis, maximize throughput efficiency during routine operation, and minimize predictive mismatch during planning. An ecosystem may appear to maximize persistence of material cycling while minimizing vulnerability to shocks. From the perspective of Qi-EP, such plurality is expected. One may write a collective path functional

$$\mathcal{S}_{\mathrm{coll}}[z] = \int \mathrm{d}t \, \left[\frac{1}{2} g_{AB}(z) \dot{z}^A \dot{z}^B + \dot{z}^A \mathcal{A}_A^{(\mathrm{coll})} + \Lambda_{\mathrm{maint}}(z) + \Lambda_{\mathrm{coord}}(z, \dot{z}) + \Lambda_{\mathrm{forecast}}(z) \right], \quad \text{(5M.6)}$$

where the last three terms denote maintenance cost, coordination cost, and forecasting or model-mismatch cost. Short-response crises highlight the metric term and look PLT-like. Throughput-dominated steady operation highlights dissipation and looks MEPP-like. Long-horizon planning highlights mismatch reduction and looks FEP-like. The programmatic claim is again that one does not need separate constitutions for these cases; one needs only to understand which sector of the same path functional is dominant.

5M.4 Qi-CON, heavy tails, and cascade structure

Collective systems are famous for burstiness, heavy tails, and cascade phenomena. The GDOS interpretation is not that such signatures are universally identical. It is that they are typical of multiscale attractor lattices with partial observation and delayed coupling. If communication, infrastructure, ecological turnover, and policy revision each operate on distinct timescales, then an observed aggregate signal should again display a knee-structured spectrum and possibly finite-window power laws. The generalized form is

$$S_{\mathrm{coll}}(f) \sim \begin{cases} S_0, & f \ll f_{\mathrm{inst}}, \\ f^{-\alpha}, & f_{\mathrm{inst}} \ll f \ll f_{\mathrm{local}}, \\ f^{-\beta}, & f \gg f_{\mathrm{local}}, \end{cases} \tag{5M.7}$$

where f_{inst} is controlled by the slowest institutional or ecological revision cycle and f_{local} by the fastest local exchange or signaling loop. The same logic applies to burst-size distributions and waiting times. The important claim is not that every collective system obeys the same exponent. The important claim is that the existence and location of the scale window should be constrained by explicit architecture.

5M.5 Where the framework can fail in collective domains

Because collective systems invite overinterpretation, the falsifiers must be stated sharply. The framework fails in this domain if the chosen bundling map is not explicit, if the proposed SIP packet does not improve prediction or diagnosis, if the current channels cannot be measured even approximately, or if the fitted scale-covariant laws bear no relationship to identifiable architecture. It also fails if the theory is used to smuggle teleology into a system that has no lawful memory, no meaningful action channel, and no persistent attractor structure. In such cases the honest conclusion would be that the export has exceeded its domain of validity.

5M.6 Why the collective appendix still matters

Even with those cautions, the collective appendix is indispensable for a landmark paper. The reason is simple. If GDOS is truly a universal language for open systems, it must tell the reader what *sort* of export to attempt at the largest scales and what safeguards prevent abuse. A theory that stops at brains would still be a powerful theory. A theory that can say, in explicit runtime language, how ecological, social, and technological systems should be represented becomes something larger: a genuine bridge language across the five-story pagoda and beyond.

Appendix 5N: Notation concordance and benchmark protocols

This last appendix records the editorial and methodological consequences of the present polishing round. It serves three purposes. First, it fixes notation that had previously appeared in parallel drafts with minor variants. Second, it states the line-breaking and layout conventions adopted in the source so that later revisions can be compiled with fewer overfull boxes and fewer accidental notational collisions. Third, it translates the entire manuscript into a practical benchmark program across the five-story pagoda. In that sense the appendix is both a concordance and a forward-looking protocol.

5N.1 Notation concordance adopted in the polishing round

The notational rules below should be treated as authoritative for the current revision.

Table 5N.1: Notation concordance for the polished revision.

Object	**Preferred notation**	**Comment**
Physics natural-unit current ratio	$J_E : J_e : J_B : J_{Qi} = 1 : -1 : -2/5 : -2/5 = 5 : -5 : -2 : -2$	The order (E, e, B, Qi) is aligned with BI-6.
Chemistry natural-unit current lock	$J_E/Q_B = J_e = J_B = J_{Qi}$	This is the preferred form of the Arisaka Equation in chemistry-scale discussions.
Arisaka constant	$Q_B = (2/5)(E_{\mathrm{Ryd}}/K)$, $K = 35$	The symbol Q_B is preferred over earlier epsilon-based variants.
Information current	J_{Qi}	Use J_{Qi}, not shortened or ambiguous alternatives.
Triad order	DGI $\rightarrow$ GSSB $\rightarrow$ GA	This order is conceptually important and should be preserved in prose and figures.
Qi-4 ordering	Qi-QUA, Qi-EP, Qi-CON, Qi-HAL	Use this canonical ordering unless a local pedagogical reason requires otherwise.
Dual holography identity	OH + IH = EIC	The equation is structural, not merely rhetorical.

5N.2 Compile-level polish conventions

Although this book remains intentionally expansive, the source has been adjusted so that later recompilation can proceed with fewer layout problems. The current source therefore adopts an emergency stretch, stronger widow and club control, and display-break tolerance suitable for long mathematical manuscripts. These choices do not change scientific content, but they reduce accidental visual noise. The underlying principle is the same as in the science: unnecessary clutter should be minimized. In practice that means preferring short equation chains over excessively wide unbreakable displays, preferring tabular summaries for repeated symbol lists, and preferring one canonical notation per concept.

5N.3 Benchmark program across the five-story pagoda

The theory becomes most useful when one can move from concept to benchmark. Table 5N.2 states one proposed benchmark system for each story of the pagoda plus one supra-individual collective system.

Table 5N.2: Benchmark program suggested by the polished Chapter 5 manuscript.

Layer	**Candidate benchmark**	**Primary runtime object to test**	**Concrete question**
Particle	Detector + readout chain	OGGEP, reduced kernel, boundary record	Can one identify the exact boundary dictionary and show how the registered event is a lawful projection rather than an external collapse postulate?
Cosmos	Multiscale gravitationally open subsystem	DGI–GSSB–GA and Qi-CON	Do large-scale observables exhibit architecture-correlated knees or attractor selection under explicit coarse-graining?
Chemistry	Driven enzyme or membrane machine	Qi-QUA and Qi-EP	Can energy, charge/redox, structural turnover, and selectivity be collapsed to one event ledger?
Life	Minimal cell or protocell	SeOS, boundary maintenance, and Qi-CON	Can maintenance cost, growth, and multiscale noise be written on one lawful runtime packet?
Mind	Recurrent neural circuit with reportable state	Qi-HAL and OH/IH/EIC	Do reportable states satisfy metastability, global reach, and action coupling criteria?
Socio-technical	Resource-communicating network	Bundled SIP and Qi-EP	Does an explicit bundling map improve prediction of cascades, delays, and recovery?

5N.4 Why these appendices were added

References

References

Arisaka, K. (2026a). *GUT-10: Axiom System of a Constitutional Grand Unification*. UCLA-Arisaka/GG-2026-1, preprint.
Relevance. Chapter 2.

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Relevance. Chapter 3.

Arisaka, K. (2026c). *The Geometry of Existence*. Chapter 4, preprint.
Relevance. Chapter 4.

Arisaka, K. (2026d). *GDOS with Qi-4 Pillars*. UCLA-Arisaka/GG-2026-4, preprint.
Relevance. Chapter 5.

't Hooft, G. (1971). Renormalizable Lagrangians for massive Yang–Mills fields. *Nuclear Physics B*, *35*, 167–188.
Relevance. Renormalizability of gauge theories is a historical turning point that made the Standard Model viable. It motivates the view that "admissibility filters" (symmetry + consistency) are as important as the detailed dynamics.

't Hooft, G. (1979). Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking. In *Recent Developments in Gauge Theories* (NATO Advanced Study Institute).
Relevance. This is a classic articulation of naturalness: small parameters are natural only if protected by symmetry. GUT responds by elevating predictivity/minimal freedom to the axiom level (MSMF) and by demanding minimal normalization (OGN).

't Hooft, G. (1993). Dimensional reduction in quantum gravity. In *Salamfestschrift* (World Scientific).
Relevance. One of the early modern statements of holography/dimensional reduction. It provides historical background for the idea that a bulk theory may be fully encoded on a lower-dimensional boundary, a central inspiration for OGGEP.

Adler, S. L. (1969). Axial-vector vertex in spinor electrodynamics. *Physical Review*, *177*, 2426–2438. https://doi.org/10.1103/PhysRev.177.2426
Relevance. Adler's paper is one of the original demonstrations that a classical symmetry (here the axial current) can be violated by quantum effects. In our framework it serves as canonical historical evidence for POS-3: anomalies are not optional subtleties but consistency obstructions that must be cancelled (possibly by inflow) in any admissible theory.

Aharonov, Y., & Bohm, D. (1959). Significance of electromagnetic potentials in the quantum theory. *Physical Review*, *115*(3), 485–491. https://doi.org/10.1103/PhysRev.115.485
Relevance. The Aharonov–Bohm effect demonstrates that holonomy of a gauge connection is physically observable even when local field strengths vanish. This is a canonical precedent for GGEP's insistence that the invariant content of the composite connection includes global/holonomy data, supporting "quantum phase as geometry."

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, *19*(6), 716–723. https://doi.org/10.1109/TAC.1974.1100705
Relevance. Akaike's information criterion is a quantitative formulation of the "Occam penalty" in model selection. We cite it as historical background for POS-6 (OGN): in GUT-10, minimality is enforced not only in model structure but also in normalization conventions to prevent hidden continuous tuning.

Alvarez-Gaumé, L., & Witten, E. (1984). Gravitational anomalies. *Nuclear Physics B, 234*(2), 269–330. DOI: 10.1016/0550-3213(84)90066-X.
Relevance. Establishes the modern classification of gravitational and mixed anomalies in quantum field theory. It provides the historical and technical backdrop for POS-3 (anomaly freedom) and for the claim that consistency can force specific matter content and inflow structures. In the present constitution, "anomaly-free" is treated as an admissibility filter, not an optional aesthetic criterion.

Amari, S.-I. (1998). Natural gradient works efficiently in learning. *Neural Computation*, *10*(2), 251–276. https://doi.org/10.1162/089976698300017746
Relevance. Amari's information-geometric formulation shows that optimization dynamics depend on the metric structure of probability/state manifolds. We cite it in the appendix to support POS-2's claim that "extremal principles" on the boundary can be written as geometry-driven flows (e.g. natural gradients) when GDOS is expressed on statistical manifolds.

Amari, S.-I., & Nagaoka, H. (2000). *Methods of Information Geometry*. American Mathematical Society / Oxford University Press.
Relevance. Information geometry provides the Fisher-metric viewpoint that underlies "natural gradient" flows. We cite it to motivate the GDOS interpretation of inference dynamics as geometric gradient descent on a scalar functional, and to connect GUT-10's EEGG/GDOS language to established geometric statistics.

Amari, S.-I., & Nagaoka, H. (2000). *Methods of Information Geometry*. American Mathematical Society.
Relevance. Information geometry provides a rigorous differential-geometric formulation of statistical models and inference via the Fisher metric and dual connections. We cite it as a concrete mathematical precedent for writing GDOS as natural-gradient flow on a curved state manifold, especially for mind/inference applications.

Arisaka, K. (2026a). *Axiom System of GUT-10 (Grand Unified Theory of Ten Sciences)*. UCLA-Arisaka/GG-2026-1 preprint.
Relevance. This paper (the present manuscript) is intended to serve as the citeable constitutional foundation for the subsequent technical GG-6 and GDOS papers. It fixes the vocabulary (axiom/postulate/theorem), the operational meaning of measurement (OGGEP), and the parsimony discipline (MSMF/OGN) that constrains all later derivations.

Arisaka, K. (2026b). *Geometry–Gauge Locking in Six Dimensions (GG-6)*. Chapter 3, preprint in preparation.
Relevance. This companion paper develops the concrete six-dimensional geometric stage suggested by the axioms (especially GGEP and anomaly inflow requirements). In the present

axiom paper we cite it only to indicate where the detailed GG-6 construction will appear; all constitutional definitions are self-contained here.

Arisaka, K. (2026c). *Geometric Dynamics of Open Systems (GDOS).* Chapter 4, preprint in preparation.
Relevance. This companion paper develops the open-system "runtime" implied by OGGEP and the postulates (positivity, RG-compatibility, falsifiability). In this axiom paper we cite it only as the location of the detailed GDOS equations that operationalize measurement and open evolution.

Arisaka, K. (2026d). *Overview of GUT-10: Ten branches and cross-domain export stack.* UCLA-Arisaka/GG-2026-4, preprint in preparation.
Relevance. This companion overview motivates and organizes the ten-domain export program. The current paper provides the constitution; the overview paper will map the constitution onto the ten scientific branches and their interface contracts.

Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. (1998). The hierarchy problem and new dimensions at a millimeter. *Physics Letters B, 429*(3–4), 263–272.
Relevance. Introduces large-extra-dimension ideas as a response to naturalness/hierarchy. We cite it as a benchmark illustrating how geometric extensions are often motivated by the same structural pressures that motivate a constitution-first approach.

Ashtekar, A. (1986). New variables for classical and quantum gravity. *Physical Review Letters, 57*(18), 2244–2247.
Relevance. A primary source for canonical/loop approaches to quantum gravity. We cite it as a representative of the "quantize geometry" posture, against which the GGEP "geometrize quantum physics" posture is contrasted.

Bardeen, W. A. (1969). Anomalous Ward identities in spinor field theories. *Physical Review, 184*, 1848–1857.
Relevance. Develops the Ward-identity viewpoint of anomalies and clarifies how anomaly terms appear in current conservation laws. POS-3 treats such symmetry violations as fatal unless cancelled by inflow; Bardeen's analysis is a canonical reference for this "symmetry grammar" perspective.

Bekenstein, J. D. (1973). Black holes and entropy. *Physical Review D, 7*(8), 2333–2346.
Relevance. We cite Bekenstein as the original statement that black holes carry entropy proportional to horizon area. In the GUT-10 viewpoint, black-hole thermodynamics is a paradigmatic open-system ledger problem: the boundary/horizon acts as a "port" and the correct scalar accounting principle anticipates the EEGG idea used in GDOS.

Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics, 1*, 195–200.
Relevance. Bell's theorem clarifies that the measurement problem is not merely philosophical: operational predictions about correlations constrain the kinds of "hidden variable" completions that remain possible. This motivates treating measurement as a constitutional issue rather than an interpretational afterthought.

Bell, J. S. & Jackiw, R. (1969). A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ-model. *Il Nuovo Cimento A* **60**, 47–61. doi:10.1007/BF02823296.

Relevance. Together with Adler's work, this book establishes the axial anomaly and its physical consequences. It provides historical grounding for why anomaly cancellation is a non-negotiable admissibility criterion (POS-AF).

Bell, J. S. (1976). The theory of local beables. *Epistemological Letters* (reprinted in *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press).
Relevance. Develops a precise notion of "local causality" in probabilistic terms and clarifies the difference between operational no-signaling and stronger locality assumptions. We cite it to motivate the careful, operationally enforceable formulation of AX-1 and to avoid conflating locality with interpretational commitments.

Bell, J. S. (2004). *Speakable and Unspeakable in Quantum Mechanics* (2nd ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511815676
Relevance. Bell's collected papers contain the sharpest discussions of "local causality" in the foundations of quantum theory. We cite this as historical context for AX-1: GUT-10 adopts "local causality" as a constitution-level constraint on operational predictions (records must be retarded and non-signaling), not as an interpretational afterthought.

Berry, M. V. (1984). Quantal phase factors accompanying adiabatic changes. *Proceedings of the Royal Society A*, *392*(1802), 45–57. https://doi.org/10.1098/rspa.1984.0023
Relevance. Berry's phase is a canonical example of "quantum from geometry": measurable phases arise from holonomy in parameter space. It provides historical and technical support for AX-2's emphasis that quantum structure can be encoded geometrically in connections and their holonomies rather than imposed by quantization rules.

Bertlmann, R. A. (1996). *Anomalies in Quantum Field Theory*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198507628.001.0001
Relevance. This monograph provides the standard mathematical treatment of anomaly polynomials and descent. It supports the formal statements in POS-3, where we package anomalies into differential-form data suitable for bulk–boundary cancellation.

Born, M. (1926). Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, *37*, 863–867. https://doi.org/10.1007/BF01397477
Relevance. Born's probabilistic interpretation underlies the Born rule, which is central to the modern measurement postulate in quantum mechanics. It is cited here to emphasize that probability entered QM as a foundational rule rather than as a derived dynamical theorem, motivating the GUT-10 demand that measurement semantics be constitutional rather than ad hoc.

Born, M., & Wolf, E. (1999). *Principles of Optics* (7th ed.). Cambridge University Press.
Relevance. A standard reference for geometric optics, wave propagation, and the eikonal approximation. It supports the UEP dictionary used in this book: "least time" emerges as an operational face of variational principles once one passes to ray/characteristic descriptions and measurement protocols.

Breuer, H.-P., & Petruccione, F. (2002). *The Theory of Open Quantum Systems*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199213900.001.0001

Relevance. This standard reference develops the mathematics of open quantum systems (reduced dynamics, master equations, Markovian limits). It anchors our claim that openness is not optional in realistic modeling, and it provides the mature language (CPTP maps and generators) that the GUT-10 constitution intends to elevate to first-class status.

Bridgman, P. W. (1927). *The Logic of Modern Physics*. New York: Macmillan.
Relevance. Bridgman's operationalism is an early systematic attempt to define scientific concepts by measurement operations. We cite it as philosophical and methodological background for POS-7 (MF): in GUT-10, every claim must compile to an explicit instrument protocol and an auditable falsifier.

Busch, P., Lahti, P. J., & Mittelstaedt, P. (1996). *The Quantum Theory of Measurement* (2nd rev. ed.). Berlin: Springer.
Relevance. Provides a comprehensive modern treatment of POVMs, instruments, and measurement models beyond ideal projective measurements. This is directly used in POS-7 to formalize "measurement" as a lawful operational map rather than a vague collapse postulate.

Caldeira, A. O., & Leggett, A. J. (1983). Path integral approach to quantum Brownian motion. *Physica A, 121*(3), 587–616.
Relevance. A classic application of influence-functional methods yielding dissipative and decohering dynamics. It is cited as historical/technical background for why open-system dynamics is unavoidable in realistic measurement, motivating AX-3's constitutional treatment of open evolution.

Callan, C. G. (1970). Broken scale invariance in scalar field theory. *Physical Review D, 2*(8), 1541–1547. DOI: 10.1103/PhysRevD.2.1541.
Relevance. One of the original papers formulating the Callan–Symanzik equation and the modern view of renormalization-group flow as a differential statement of scale dependence. It provides historical grounding for POS-5: "scale covariance" is not optional if the theory aims to be resolution-independent.

Callan, C. G., Jr., & Harvey, J. A. (1985). Anomalies and fermion zero modes on strings and domain walls. *Nuclear Physics B, 250*(1–4), 427–436. DOI: 10.1016/0550-3213(85)90489-4.
Relevance. A classic source on anomaly inflow: boundary anomalies are canceled by bulk topological currents. It supports POS-3 by exemplifying how "inflow" restores gauge consistency and motivates treating cancellability by inflow as an admissibility rule rather than a model-dependent trick.

Choi, M.-D. (1975). Completely positive linear maps on complex matrices. *Linear Algebra and its Applications*, *10*(3), 285–290. https://doi.org/10.1016/0024-3795(75)90075-0
Relevance. Choi's theorem characterizes complete positivity, the mathematical core of physical admissibility for open-system dynamics and measurement instruments. AX-3 and POS-CP rely on complete positivity as the operational replacement for "no negative probabilities" across all open systems.

Cover, T. M., & Thomas, J. A. (2006). *Elements of Information Theory* (2nd ed.). Wiley. https://doi.org/10.1002/047174882X

Relevance. Information theory supplies a precise quantitative language for "information" as an operational resource. We cite this text to support the appendix discussion where free-energy-like functionals are related to KL divergence and where MF requires explicit information-theoretic accounting for measurement and compression.

Crooks, G. E. (1999). Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences. *Physical Review E*, *60*(3), 2721–2726.
Relevance. A cornerstone of stochastic thermodynamics connecting forward/backward path probabilities and nonequilibrium work. We cite it as a modern exemplar of how open-system probability flows carry rigorous, testable constraints—precisely the style enforced by POS-4 and POS-7.

Daubechies, I. (1992). *Ten Lectures on Wavelets*. Society for Industrial and Applied Mathematics (SIAM).
Relevance. A standard mathematical reference for wavelet/multiresolution analysis. It provides concrete tools for the kind of dyadic multiscale decomposition invoked in BFFT: extracting scaling windows, knees, and localized spectral structure from data in a way compatible with the MF requirement of registered observables.

Davies, E. B., & Lewis, J. T. (1970). An operational approach to quantum probability. *Communications in Mathematical Physics*, *17*, 239–260. https://doi.org/10.1007/BF01647093
Relevance. This paper is a foundational source for the instrument/operation viewpoint of measurement. It supports AX-3's claim that "measurement" should be defined by operations (instruments/POVMs) rather than treated as an interpretational postulate separate from dynamics.

de Groot, S. R., & Mazur, P. (1962). *Non-Equilibrium Thermodynamics*. Amsterdam: North-Holland Publishing Company.
Relevance. A foundational text for nonequilibrium thermodynamics and transport theory, widely used across chemistry and engineering. We cite it as background for the GDOS viewpoint that open systems should be described by explicit fluxes, constraints, and positivity-preserving dynamics, and as historical context for connecting "free-energy-like" functionals to measurable transport and relaxation laws.

de Groot, S. R., & Mazur, P. (1962). *Non-Equilibrium Thermodynamics*. Amsterdam: North-Holland.
Relevance. This classic text provides a systematic formulation of nonequilibrium thermodynamics, including entropy production, linear response, and Onsager relations. We cite it as background for POS-2 (UEP), where entropy-production extremal principles (MEP/MEPP) are treated as constrained variational forms of the unified extremal engine.

de Haro, S., Skenderis, K., & Solodukhin, S. N. (2001). Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence. *Communications in Mathematical Physics*, *217*, 595–622. https://arxiv.org/abs/hep-th/0002230
Relevance. This work systematized holographic renormalization, making precise how bulk radial evolution corresponds to boundary RG flow. We cite it as a technical antecedent for POS-5: RG-compatibility is treated as a structural commutation requirement between coarse-graining and bulk–boundary dictionaries.

Dewar, R. C. (2003). Information theory explanation of the fluctuation theorem, maximum entropy production and self-organized criticality in non-equilibrium stationary states. *Journal of Physics A: Mathematical and General*, *36*(3), 631–641. https://doi.org/10.1088/0305-4470/36/3/303

Relevance. Dewar provides an information-theoretic route to maximum entropy production ideas in nonequilibrium systems. We cite it in POS-2 to represent the MEPP lineage and to clarify which claims are principled (variational) versus heuristic.

Dirac, P. A. M. (1930). *The Principles of Quantum Mechanics*. Oxford University Press.

Relevance. A classic axiomatic presentation of quantum theory. We cite it to underscore that "axioms" in quantum mechanics have historically mixed dynamics with measurement rules, motivating a more explicit separation between axioms and postulates.

Dirac, P. A. M. (1931). Quantised singularities in the electromagnetic field. *Proceedings of the Royal Society A*, *133*(821), 60–72. https://doi.org/10.1098/rspa.1931.0130

Relevance. Dirac's monopole argument is a classic demonstration that charge quantization can follow from topology and consistency conditions. It provides historical precedent for AX-2's mechanism "quantization from geometry/topology" (e.g. from compact holonomy sectors in the bulk).

Douglas, M. R. (2003). The statistics of string/M theory vacua. *Journal of High Energy Physics*, *2003*(05), 046. https://arxiv.org/abs/hep-th/0303194

Relevance. The "landscape" perspective illustrates a modern form of theory-space proliferation: many consistent-looking vacua and effective theories appear, and selecting Nature's point requires additional principles. We cite it as motivation for MSMF: a constitutional constraint intended to collapse landscapes into derived, minimal-ledger predictions.

Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, *17*, 891–921. https://doi.org/10.1002/andp.19053221004

Relevance. Einstein's operational reconstruction of simultaneity and inertial frames exemplifies a successful "axiom-like" intervention in physics: changing the admissibility rules for coordinates and clocks reshaped all of kinematics. We cite it to justify treating operational definitions (OGGEP) as constitutional.

Einstein, A. (1907). Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. *Jahrbuch der Radioaktivität und Elektronik*, *4*, 411–462.

Relevance. This early paper contains key steps toward the equivalence-principle reasoning that culminated in general relativity. It anchors AX-2 historically: GGEP is explicitly framed as a generalization of equivalence-principle logic from gravity to a composite geometry+gauge connection.

Einstein, A. (1915). Die Feldgleichungen der Gravitation. *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*, 844–847.

Relevance. The field equations of GR encode gravity as geometry and are historically rooted in the equivalence principle. We cite GR as a prototype of "geometrization" and as a precedent for extending an equivalence principle to a more unified object (GGEP).

Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*, *49*, 769–822. https://doi.org/10.1002/andp.19163540702

Relevance. Einstein's 1916 exposition gives the mature geometric formulation of GR and its equivalence-principle logic. We cite it as the historical anchor for AX-2: GGEP upgrades the equivalence principle from "gravity only" to "gravity + gauge as one composite connection."

Euclid. (c. 300 BCE). *Elements*. (Various modern editions and translations).

Relevance. Euclid's axiomatic method is the historical prototype for a "constitution" of a scientific domain. The present paper explicitly aims to play a similar role for GUT: a short axiom/postulate set that generates an extensive theorem stack.

Euclid. (1956). *The Thirteen Books of Euclid's Elements* (T. L. Heath, Trans., 2nd ed.). Dover Publications. (Original work ca. 300 BCE).

Relevance. Euclid's axiomatic method is the archetype for a stable constitution: definitions and postulates provide a common language that enables long chains of deduction without ambiguity. We cite it to motivate why an explicit axiom system is appropriate when foundational semantic issues (measurement, openness) persist.

Feynman, R. P. (1948). Space-time approach to non-relativistic quantum mechanics. *Reviews of Modern Physics*, *20*(2), 367–387. https://doi.org/10.1103/RevModPhys.20.367

Relevance. The path integral makes the connection between action extremization and quantum amplitudes explicit (stationary phase in the classical limit). We cite it in POS-2 to link PLA to quantum mechanics within a single extremal framework.

Feynman, R. P., & Vernon, F. L. (1963). The theory of a general quantum system interacting with a linear dissipative system. *Annals of Physics, 24*, 118–173.

Relevance. Introduces the influence functional for open quantum systems, a key tool for deriving reduced dynamics and decoherence. It is cited to connect OGGEP/GDOS-style open evolution to established path-integral machinery.

Freidlin, M. I., & Wentzell, A. D. (1998). *Random Perturbations of Dynamical Systems* (2nd ed.). New York: Springer. https://doi.org/10.1007/978-1-4612-0611-5

Relevance. Freidlin–Wentzell theory provides a rigorous large-deviation framework in which stochastic open-system trajectories concentrate near minimizers of an action functional. We cite it as mathematical background for POS-2 (UEP), supporting the claim that "least-action" logic persists in open/noisy systems as a most-probable-path extremal principle.

Friston, K. (2010). The free-energy principle: a unified brain theory? *Nature Reviews Neuroscience*, *11*(2), 127–138. https://doi.org/10.1038/nrn2787

Relevance. Friston's free-energy principle is a prominent modern attempt to unify perception, learning, and action by variational optimization. We cite it in Section 2.4.2 to illustrate that "free-energy-like" scalar functionals already function as organizing principles in neuroscience, motivating the GUT-10 claim that EEGG and GDOS provide a systematic open-system generalization with explicit gauge/geometry and falsifiability constraints.

Fujikawa, K. (1979). Path-integral measure for gauge-invariant fermion theories. *Physical Review Letters*, *42*, 1195–1198.

Relevance. Shows how anomalies arise from the non-invariance of the fermionic path-integral measure, providing a clean and widely used derivation. This is a key technical underpinning of POS-3, which treats anomalies as a constitutional violation unless cancelled by inflow.

Gardiner, C. W. (2009). *Stochastic Methods: A Handbook for the Natural and Social Sciences* (4th ed.). Springer.
Relevance. A comprehensive reference for stochastic differential equations and the Fokker–Planck formalism. It supports the GDOS "kernel" derivations in the appendices and clarifies positivity-preserving conditions (POS-CP).

Gell-Mann, M., & Low, F. E. (1954). Quantum electrodynamics at small distances. *Physical Review*, *95*, 1300–1312. https://doi.org/10.1103/PhysRev.95.1300
Relevance. The Gell-Mann–Low paper is a foundational early statement of RG ideas in quantum field theory. It is cited as historical background for POS-5 (RG): scale dependence must be organized systematically, and physical content must be extracted in an RG-consistent way.

Georgi, H., & Glashow, S. L. (1974). Unity of all elementary-particle forces. *Physical Review Letters*, *32*(8), 438–441. doi:10.1103/PhysRevLett.32.438.
Relevance. The SU(5) grand-unification proposal is a canonical example of symmetry-driven unification beyond the Standard Model. GUT references it historically, but differs constitutionally by treating gauge–gravity unification through a composite geometric connection (GGEP) and by enforcing anomaly/normalization constraints as admissibility filters.

Glashow, S. L. (1961). Partial-symmetries of weak interactions. *Nuclear Physics*, *22*, 579–588. https://doi.org/10.1016/0029-5582(61)90469-2
Relevance. Glashow's gauge unification of electromagnetic and weak interactions initiated the electroweak structure completed by Weinberg and Salam. We cite these as the historical backbone for POS-1 (Gauge Invariance) and for the claim that symmetry principles organize viable theories.

Goldstein, H., Poole, C., & Safko, J. (2002). *Classical Mechanics* (3rd ed.). Addison–Wesley.
Relevance. This standard text summarizes variational principles (Hamilton, Lagrange, Maupertuis) and their equivalences. We cite it in POS-2 to situate the Unified Extremal Principle within the established mechanics lineage.

Goldstein, H., Poole, C. P., & Safko, J. L. (2002). *Classical Mechanics* (3rd ed.). San Francisco: Addison–Wesley.
Relevance. Goldstein's text is a standard reference for variational principles and Hamiltonian/Lagrangian mechanics. We cite it as accessible historical and technical background for POS-2 (UEP), where the closed-system limit is the Principle of Least Action and its Euler–Lagrange structure.

Gorini, V., Kossakowski, A., & Sudarshan, E. C. G. (1976). Completely positive dynamical semigroups of N-level systems. *Journal of Mathematical Physics*, *17*(5), 821–825.
Relevance. Along with Lindblad's work, this establishes the generator form required for physical open-system quantum evolution. It supports POS-CP and the OGGEP demand that open evolution be part of the foundational dictionary.

Gorini, V., Kossakowski, A., & Sudarshan, E. C. G. (1976). Completely positive dynamical semigroups of N-level systems. *Journal of Mathematical Physics*, *17*(5), 821–825. https://doi.org/10.1063/1.522979

Relevance. Together with Lindblad, this paper characterizes the most general Markovian quantum evolution preserving complete positivity. We cite it as a rigorous foundation for treating open-system evolution as a constitutional admissibility condition (POS-4: Closure & Positivity).

Green, M. B., & Schwarz, J. H. (1984). Anomaly cancellations in supersymmetric $D = 10$ gauge theory and superstring theory. *Physics Letters B*, *149*(1–3), 117–122.

Relevance. The Green–Schwarz mechanism is the canonical example of anomaly cancellation by inflow and motivates POS-AF. Although GUT does not assume string theory, it elevates anomaly inflow/cancellability to a constitutional filter.

Gross, D. J., & Wilczek, F. (1973). Ultraviolet behavior of non-Abelian gauge theories. *Physical Review Letters*, *30*(26), 1343–1346. https://doi.org/10.1103/PhysRevLett.30.1343

Relevance. Asymptotic freedom is a key structural fact about non-Abelian gauge theories and underlies the consistency of QCD. We cite it to emphasize that "viability filters" (renormalizability, RG behavior) are part of what makes gauge theories physically admissible.

Gubser, S. S., Klebanov, I. R., & Polyakov, A. M. (1998). Gauge theory correlators from non-critical string theory. *Physics Letters B*, *428*, 105–114. https://arxiv.org/abs/hep-th/9802109

Relevance. Together with Maldacena and Witten, this paper established the bulk→boundary correlator prescription of AdS/CFT. We cite it as part of the historical backbone for OGGEP: a theory that posits a bulk must also provide a concrete dictionary that exports bulk objects into boundary observables.

Gubser, S. S., Klebanov, I. R., & Polyakov, A. M. (1998). Gauge theory correlators from non-critical string theory. *Physics Letters B*, *428*(1–2), 105–114. https://arxiv.org/abs/hep-th/9802109

Relevance. Together with Witten's prescription, this paper established the concrete AdS/CFT dictionary relating bulk fields to boundary correlators. We cite it as historical precedent for treating a bulk→boundary export map as part of the theory's definition, which OGGEP generalizes to include explicit measurement semantics.

Guth, A. H. (1981). Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, *23*(2), 347–356. https://doi.org/10.1103/PhysRevD.23.347

Relevance. Inflation is the dominant paradigm for early-universe initial conditions. We cite it as a core cosmological motivation: the mechanism is empirically successful yet microphysically underdetermined, supporting the need for a stronger constitution that restricts "allowed" inflationary dynamics and connects them to observation.

Haag, R. (1992). *Local Quantum Physics: Fields, Particles, Algebras* (2nd ed.). Springer. https://doi.org/10.1007/978-3-662-03081-1

Relevance. Haag's formulation of QFT emphasizes locality, causality, and operator-algebraic structure. We cite it to ground AX-1 in the rigorous QFT tradition and to clarify what "locality" means when expressed as constraints on observable algebras.

Haag, R. (1996). *Local Quantum Physics: Fields, Particles, Algebras* (2nd ed.). Springer.
Relevance. A definitive treatment of locality via operator algebras (Haag–Kastler approach). It is cited as a high-standard reference for what it means to impose locality/causality in quantum theory, motivating AX-1 as a constitutional filter.

Hamilton, W. R. (1834). On a general method in dynamics. *Philosophical Transactions of the Royal Society of London*, *124*, 247–308. https://doi.org/10.1098/rstl.1834.0017
Relevance. Hamilton's formulation is the modern backbone of the principle of least action and canonical dynamics. We cite it in POS-2 to emphasize that extremal principles historically reorganized mechanics and are a natural candidate for a unification engine.

Hawking, S. W. (1975). Particle creation by black holes. *Communications in Mathematical Physics*, *43*, 199–220.
Relevance. Hawking radiation is a canonical example of an open-system process in fundamental physics: a horizon mediates information/energy exchange between regions, producing an arrow of time in an otherwise unitary theory. We cite it to motivate why GDOS treats openness and operational observation as foundational.

Heisenberg, W. (1925). Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen. *Zeitschrift für Physik*, *33*(1), 879–893.
Relevance. Matrix mechanics is one of the two founding formulations of quantum theory and exemplifies the radical redefinition of dynamical variables. It sets historical context for treating quantum structure as something potentially geometric.

Heisenberg, W. (1927). Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Zeitschrift für Physik, 43*, 172–198.
Relevance. A primary source for the uncertainty principle and early quantum-measurement ideas. It is cited as part of the historical background for why operational definitions are central to quantum theory.

Jarzynski, C. (1997). Nonequilibrium equality for free energy differences. *Physical Review Letters*, *78*(14), 2690–2693.
Relevance. Jarzynski's equality shows that free-energy differences can be inferred from nonequilibrium work statistics. It supports the UEP/GDOS viewpoint that variational/ledger principles remain meaningful and falsifiable far from equilibrium.

Jaynes, E. T. (1957). Information theory and statistical mechanics. *Physical Review, 106*(4), 620–630. DOI: 10.1103/PhysRev.106.620.
Relevance. Introduces the maximum-entropy (MaxEnt) principle as a constructive variational rule for statistical mechanics. We cite it in POS-2 to connect thermodynamic extremal principles to inference and to motivate treating free-energy/entropy functionals as legitimate variational objects when paired with operational constraints.

Jordan, R., Kinderlehrer, D., & Otto, F. (1998). The variational formulation of the Fokker–Planck equation. *SIAM Journal on Mathematical Analysis*, *29*, 1–17.

Relevance. Demonstrates that Fokker–Planck evolution can be derived as a variational (gradient-flow) scheme, making the open-system "least free energy" dynamics mathematically precise. POS-2 relies on such results to justify a unified extremal principle for open systems.

Kadanoff, L. P. (1966). Scaling laws for Ising models near T_c. *Physics Physique Fizika, 2*(6), 263–272. DOI: 10.1103/PhysicsPhysiqueFizika.2.263.
Relevance. An early primary source for scaling and coarse-graining ideas that evolved into modern RG. It motivates POS-5 by showing why scale transformations are structural, not merely computational tricks.

Kaluza, T. (1921). Zum Unitätsproblem der Physik. *Sitzungsberichte der Preußischen Akademie der Wissenschaften zu Berlin*, 966–972.
Relevance. Kaluza's proposal that electromagnetism can be interpreted geometrically via an extra dimension is a foundational ancestor of the "gauge as geometry" program. We cite it as historical support for GGEP and for the appearance of extra coordinates as geometric encodings of gauge structure.

Kass, R. E., & Raftery, A. E. (1995). Bayes factors. *Journal of the American Statistical Association*, *90*(430), 773–795. https://doi.org/10.1080/01621459.1995.10476572
Relevance. Kass and Raftery provide a widely cited modern reference for Bayes factors as quantitative evidence measures. We cite it as part of the methodological background for POS-7 (MF): falsifiability can be operationalized through explicit decision thresholds, whether frequentist or Bayesian, but must be specified in advance.

Kauffman, S. A. (1993). *The Origins of Order: Self-Organization and Selection in Evolution.* Oxford University Press.
Relevance. A foundational systems-biology perspective emphasizing attractors, networks, and self-organization in evolution. We cite it in the GDOS application preview to connect the attractor/phase-selection language to established biological dynamical-systems frameworks.

Keldysh, L. V. (1964). Diagram technique for nonequilibrium processes. *Soviet Physics JETP, 20*(4), 1018–1026.
Relevance. Develops the nonequilibrium Green's function formalism complementary to Schwinger's approach. It provides technical background for treating operational prediction (measurement, dissipation) as a real-time problem, central to AX-3.

Klein, O. (1926). Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschrift für Physik*, *37*, 895–906. https://doi.org/10.1007/BF01397481
Relevance. Klein introduced compactification and the resulting quantization of momentum in the extra dimension. We cite it for the standard mechanism by which compact geometry yields discrete spectra (used in AX-2 to illustrate "quantization from topology" rather than an imposed quantization rule).

Kobayashi, S., & Nomizu, K. (1963). *Foundations of Differential Geometry* (Vol. 1). Wiley.
Relevance. This classic text provides the rigorous mathematics of principal bundles, connections, curvature, and holonomy. We cite it as a standard reference for the geometric language used in formulating GGEP (composite connection) and in defining curvature/holonomy as invariants.

Kraus, K. (1971). General state changes in quantum theory. *Annals of Physics*, *64*(2), 311–335. https://doi.org/10.1016/0003-4916(71)90108-4

Relevance. Kraus operators and instruments provide the modern operational semantics of measurement and state update. We cite this work as foundational for AX-3, where measurement is defined constitutionally as a CP instrument produced by the bulk→boundary dictionary.

Kraus, K. (1983). *States, Effects, and Operations: Fundamental Notions of Quantum Theory*. Springer. https://doi.org/10.1007/978-3-540-39068-5

Relevance. Kraus' monograph systematizes the operational formulation of quantum measurement and dynamics via completely positive maps. AX-3 adopts this language constitutionally by requiring that measurement be defined in terms of instruments and that open evolution be CPTP.

Kullback, S., & Leibler, R. A. (1951). On information and sufficiency. *Annals of Mathematical Statistics*, *22*(1), 79–86. https://doi.org/10.1214/aoms/1177729694

Relevance. The KL divergence is the canonical nonnegative functional measuring distinguishability of probability distributions. We cite it because many "free energy" expressions in nonequilibrium physics and inference can be written as KL divergences, providing a clean mathematical bridge for POS-2 (UEP) between thermodynamic and informational extremals.

Landauer, R. (1961). Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, *5*(3), 183–191. https://doi.org/10.1147/rd.53.0183

Relevance. Landauer's principle connects information processing with thermodynamic cost. We cite it in the undergrad-intuition appendix to emphasize that "information" is physical in open systems, reinforcing GDOS's treatment of boundary dynamics as energetics + entropy + information bookkeeping under constitutional constraints.

Lanczos, C. (1970). *The Variational Principles of Mechanics* (4th ed.). University of Toronto Press, Toronto.

Relevance. A classic reference on variational principles in mechanics. It anchors the bulk/closed side of POS-UEP(PLA) and provides pedagogical support for the graduate-level derivations.

Lindblad, G. (1976). On the generators of quantum dynamical semigroups. *Communications in Mathematical Physics*, *48*(2), 119–130.

Relevance. Lindblad's theorem provides the canonical generator form for Markovian CPTP evolution. It is central to POS-CP and to OGGEP's claim that open-system dynamics belongs in the theory's syntax.

Linde, A. D. (1982). A new inflationary universe scenario. *Physics Letters B*, *108*(6), 389–393. https://doi.org/10.1016/0370-2693(82)91219-9

Relevance. Linde's work exemplifies the proliferation of inflationary model-building. We cite it to motivate why a "no-extra-dials" principle (MSMF) could be valuable: without constitutional restrictions, many potentials are admissible and prediction becomes diffuse.

Maldacena, J. M. (1997). The large N limit of superconformal field theories and supergravity. *Advances in Theoretical and Mathematical Physics*, *2*, 231–252. https://arxiv.org/abs/hep-th/9711200

Relevance. AdS/CFT is the sharpest realization of a bulk–boundary dictionary in modern theory. We cite it as historical support for treating "holographic export" as a definitional element of the theory (OGGEP), rather than as an interpretation added after calculations.

Maldacena, J. M. (1998). The large N limit of superconformal field theories and supergravity. *Advances in Theoretical and Mathematical Physics*, *2*, 231–252. https://arxiv.org/abs/hep-th/9711200

Relevance. Maldacena's AdS/CFT conjecture is the modern archetype for bulk–boundary duality and motivates the idea that geometry can encode quantum gauge dynamics. We cite it as a historical anchor for the GGEP/OGGEP posture: bulk geometry is not just a classical background, but a structure from which boundary quantum physics and observables can be exported.

Mallat, S. (1999). *A Wavelet Tour of Signal Processing*. Academic Press.

Relevance. Provides an engineering-oriented presentation of wavelet and time–frequency methods widely used for multiscale data analysis. It supports the practical side of BFFT as a measurement-ready transform for extracting knees, exponents, and scale-localized features demanded by OGGEP/MF.

Martyushev, L. M., & Seleznev, V. D. (2006). Maximum entropy production principle in physics, chemistry and biology. *Physics Reports, 426*(1), 1–45. DOI: 10.1016/j.physrep.2005.12.001.

Relevance. A wide-ranging review of maximum entropy production ideas across disciplines. We cite it in POS-2 as part of the historical backdrop for extremal principles in open systems, while emphasizing the GUT-10 requirement that any such principle must be supplied as an explicit functional with constraints and operational readout.

Minkowski, H. (1908). Raum und Zeit. In *Jahresbericht der Deutschen Mathematiker-Vereinigung* (published 1909); English translation in *The Principle of Relativity* (Dover).

Relevance. Minkowski's spacetime formalism makes causality geometric via the light cone. AX-1 inherits this causal structure and exports it to operational open-system evolution: the admissible record must respect the Minkowski/relativistic notion of causal order.

Nakahara, M. (2003). *Geometry, Topology and Physics* (2nd ed.). Institute of Physics Publishing.

Relevance. Nakahara's text is a widely used bridge between modern geometry/topology and physical gauge theories. It supports AX-2's emphasis that gauge and geometric data are naturally expressed as connections and curvatures, and it provides background for topological routes to quantization.

Newton, I. (1687). *Philosophiæ Naturalis Principia Mathematica*. Jussu Societatis Regiæ ac Typis Josephi Streater.

Relevance. Newton's definitions and laws were a prototypical axiom system for mechanics, converting astronomy into dynamics and setting the model for "principles → equations → predictions." We cite it as precedent for framing GUT-10 as a constitution that will be cited across later technical papers.

Neyman, J., & Pearson, E. S. (1933). On the problem of the most efficient tests of statistical hypotheses. *Philosophical Transactions of the Royal Society A*, *231*, 289–337. https://doi.org/10.1098/rsta.1933.0009

Relevance. Neyman and Pearson formalized hypothesis testing and error control, which underlies how modern sciences define and enforce rejection criteria. We cite it as historical/statistical background for POS-7 (MF), where every claim must declare an explicit kill criterion with controlled error rates or equivalent auditing logic.

Nicolis, G., & Prigogine, I. (1977). *Self-Organization in Nonequilibrium Systems*. Wiley.

Relevance. A classic monograph on dissipative structures and nonequilibrium self-organization. It provides historical and mathematical grounding for the idea that open systems generically develop attractors and phases under driving—a key theme of GDOS.

Nielsen, M. A., & Chuang, I. L. (2000). *Quantum Computation and Quantum Information*. Cambridge University Press.

Relevance. This standard reference consolidates the operational language of quantum states, channels, and measurements. AX-3 uses precisely this operational toolkit (POVMs, Kraus maps, CPTP evolution) but elevates it from "applications" to constitutional definition of observation.

Nielsen, M. A., & Chuang, I. L. (2010). *Quantum Computation and Quantum Information* (10th Anniversary ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511976667

Relevance. This book codifies the operational viewpoint of quantum theory (states, channels, POVMs) that is closest to the GUT-10 demand that measurement is a defined map rather than a philosophical add-on. We cite it to support the operational language used in motivating OGGEP and POS-7.

Noether, E. (1918). Invariante Variationsprobleme. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, 235–257.

Relevance. Noether's theorem is the clearest statement that symmetry is not optional: it has measurable conservation consequences. This motivates POS-GI as the first admissibility rule and supports the constitutional emphasis on symmetry.

Onsager, L. (1931). Reciprocal relations in irreversible processes I. *Physical Review*, *37*, 405–426. https://doi.org/10.1103/PhysRev.37.405

Relevance. Onsager's reciprocity and variational viewpoint provide a foundational extremal structure for near-equilibrium nonequilibrium thermodynamics. We cite it as historical and mathematical background for POS-2 (UEP), supporting the statement that open-system dissipation can be generated by a variational principle (minimum dissipation / gradient flow).

Onsager, L., & Machlup, S. (1953). Fluctuations and irreversible processes. *Physical Review, 91*(6), 1505–1512. DOI: 10.1103/PhysRev.91.1505.

Relevance. Introduces an action-like functional for stochastic trajectories near equilibrium, providing a bridge between variational principles and stochastic thermodynamics. It supports POS-2 (UEP) in the open-system regime by showing how "least action" persists as a most-probable-path principle for noisy dynamics.

Ozawa, M. (1984). Quantum measuring processes of continuous observables. *Journal of Mathematical Physics*, *25*(1), 79–87. https://doi.org/10.1063/1.526000

Relevance. Ozawa developed the instrument-based formalism for measurements beyond ideal projective cases. This supports AX-3's insistence that measurement be defined operationally (as an instrument), which is essential for exporting the framework to realistic open systems.

Particle Data Group. (2024). Review of Particle Physics. *Progress of Theoretical and Experimental Physics*. (See also the PDG online database).

Relevance. The PDG is the canonical source for the Standard Model parameterization and experimental constraints. It supports the motivation that our current fundamental theories contain a significant set of empirical inputs and motivates MSMF/OGN as a push toward reduced dial dependence.

Perlmutter, S., et al. (1999). Measurements of Ω and Λ from 42 high-redshift supernovae. *The Astrophysical Journal*, *517*(2), 565–586. https://doi.org/10.1086/307221 https://arxiv.org/abs/astro-ph/9812133

Relevance. Together with Riess et al., this work established the observational evidence for accelerated expansion and nonzero Λ in late-time cosmology. We cite it in the applications/discussion sections to anchor the empirical context for "beyond-Λ" claims in a standard observational milestone.

Peskin, M. E., & Schroeder, D. V. (1995). *An Introduction to Quantum Field Theory*. Westview Press.

Relevance. A standard modern presentation of locality, causality, and renormalization in QFT. It provides background for AX-1's microcausality/locality requirement and for later postulates concerning RG compatibility and scale covariance.

Planck, M. (1900). Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum. *Verhandlungen der Deutschen Physikalischen Gesellschaft*, *2*, 237–245.

Relevance. Planck's work initiated the quantum revolution and exemplifies how a small modification at the constitutional level (quantization of energy exchange) can restructure an entire domain.

Planck Collaboration. (2018). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, *641*, A6. https://arxiv.org/abs/1807.06209

Relevance. Precision cosmology strongly constrains the expansion history and composition of the Universe. We cite Planck as representative of the empirical pressure behind the inflation/dark-sector puzzles, and as motivation for a constitution that connects cosmological claims to operational observables with falsifiers.

Polchinski, J. (1984). Renormalization and effective Lagrangians. *Nuclear Physics B*, *231*, 269–295.

Relevance. A classic effective-field-theory treatment emphasizing RG as a map between effective descriptions. POS-5 treats RG/coarse-graining as a constitutional operation that must commute with observation (OGGEP), and Polchinski's work is a key reference for this map-based viewpoint.

Polchinski, J. (1995). Dirichlet-branes and Ramond-Ramond charges. *Physical Review Letters*, *75*(26), 4724–4727.
Relevance. This work helped consolidate modern string theory as a framework with extended objects and dualities. It is a key reference for contemporary quantum gravity and unification programs that GUT compares against in its discussion section.

Polchinski, J. (1998). *String Theory* (Vols. 1–2). Cambridge University Press. https://doi.org/10.1017/CBO9780511816079
Relevance. String theory is the leading framework that unifies gauge interactions and gravity in a quantum-consistent manner, but it illustrates the challenge of selecting among many consistent constructions. We cite it to frame the comparison target: GUT-10 aims to enforce a stronger constitutional minimality (MSMF) and a fixed operational dictionary (OGGEP).

Polchinski, J. (2004). The cosmological constant and the string landscape. (Proceedings and review literature; widely circulated preprint).
Relevance. The landscape idea is an example of explanatory flexibility that can undermine predictivity. It motivates MSMF and OGN as constitutional constraints aimed at reducing or eliminating continuous families of equally admissible solutions.

Politzer, H. D. (1973). Reliable perturbative results for strong interactions? *Physical Review Letters*, *30*(26), 1346–1349.
Relevance. Along with Gross–Wilczek, this established asymptotic freedom. It reinforces the centrality of RG structure and motivates scale covariance as a constitutional requirement.

Popper, K. (1959). *The Logic of Scientific Discovery*. Hutchinson. (Original work published 1934). https://doi.org/10.4324/9780203994627
Relevance. Popper's falsifiability criterion is a classic articulation of what separates scientific statements from metaphysical ones. We cite it in POS-7 to support the demand that every theorem come with a predeclared operational kill criterion, implemented concretely via instruments and statistical tests.

Prigogine, I. (1967). *Introduction to Thermodynamics of Irreversible Processes* (3rd ed.). New York: Wiley.
Relevance. Prigogine's exposition (including the minimum entropy production theorem near equilibrium) is a standard historical source for entropy-production variational ideas. We cite it in POS-2 (UEP) to connect the PLA-to-dissipation bridge with classical nonequilibrium thermodynamics and to clarify when entropy-production extremals are mathematically justified.

Randall, L., & Sundrum, R. (1999). A large mass hierarchy from a small extra dimension. *Physical Review Letters*, *83*(17), 3370–3373.
Relevance. Warped extra dimensions provide a concrete mechanism by which exponential warping converts a modest geometric length into a large hierarchy. We cite it as historical background for the warped-throat intuition used in the GG-6 preview metric (5.12).

Randall, L., & Sundrum, R. (1999a). A large mass hierarchy from a small extra dimension. *Physical Review Letters*, *83*(17), 3370–3373. https://doi.org/10.1103/PhysRevLett.83.3370 https://arxiv.org/abs/hep-ph/9905221

Relevance. Warped extra-dimensional geometries provide a concrete precedent for using a warp factor to generate hierarchies. We cite RS1 to contextualize the GG-6 warped bulk stage, while emphasizing that GUT-10 aims to remove moduli freedom via MSMF and anomaly rigidity rather than introducing tunable brane parameters.

Randall, L., & Sundrum, R. (1999b). An alternative to compactification. *Physical Review Letters*, *83*(23), 4690–4693. doi:10.1103/PhysRevLett.83.4690.
Relevance. This companion warped-bulk construction clarifies how localization of modes and effective 4D physics can arise from higher-dimensional geometry. It provides context for GG-6's bulk→boundary export and for the MSMF preference for minimal additional structure beyond what geometry enforces.

Riess, A. G., et al. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, *116*(3), 1009–1038.
Relevance. One of the two discovery papers for cosmic acceleration. It motivates the claim that cosmology contains deep missing pieces (dark energy) that invite a new foundational viewpoint.

Risken, H. (1989). *The Fokker–Planck Equation: Methods of Solution and Applications* (2nd ed.). Springer.
Relevance. A classic reference for diffusion processes and Fokker–Planck equations. It provides technical background for the GDOS continuity-equation form and its solution methods.

Rissanen, J. (1978). Modeling by shortest data description. *Automatica*, *14*(5), 465–471. https://doi.org/10.1016/0005-1098(78)90005-5
Relevance. Rissanen's minimum description length principle is another quantitative implementation of Occam's razor. We cite it as methodological background for POS-6 (OGN): the "minimum freedom" doctrine must also eliminate hidden continuous choices in parameterization and normalization.

Rovelli, C. (2004). *Quantum Gravity.* Cambridge University Press. https://doi.org/10.1017/CBO9780511755804
Relevance. Loop quantum gravity is a canonical example of the "quantize geometry" program. We cite it to emphasize that GUT-10 proposes an inversion: a "geometrization of quantum physics" in which gauge/quantum structures are encoded geometrically and read operationally at the boundary.

Salam, A. (1968). Weak and electromagnetic interactions. In *Elementary Particle Theory* (Proceedings of the Nobel Symposium No. 8), 367–377.
Relevance. Salam's electroweak unification is part of the Standard Model foundation. It is cited here to emphasize that modern physics already depends on strong "admissibility rules" (gauge invariance and renormalizability) even if not presented as an explicit constitution.

Schlosshauer, M. (2005). Decoherence, the measurement problem, and interpretations of quantum mechanics. *Reviews of Modern Physics, 76*(4), 1267–1305.
Relevance. A modern review clarifying what decoherence explains and what it does not. We cite it to motivate why an operational definition of observation (OGGEP) is valuable in an axiom system.

Schrödinger, E. (1926). Quantisierung als Eigenwertproblem. *Annalen der Physik*, *79*, 361–376.
Relevance. Wave mechanics provides the second founding formulation of quantum theory. It anchors the historical background for the measurement issue: the wavefunction becomes a central object whose operational status requires constitutional clarification.

Schrödinger, E. (1944). *What is Life? The Physical Aspect of the Living Cell*. Cambridge: Cambridge University Press.
Relevance. Schrödinger's lectures helped launch the modern physics–to–biology dialogue by framing life as an aperiodic, information-bearing physical process. We cite it to motivate the GUT-10 stance that life-science phenomena should be modeled as open-system phases with explicit operational observables and thermodynamic/information constraints.

Schwinger, J. (1961). Brownian motion of a quantum oscillator. *Journal of Mathematical Physics, 2*(3), 407–432.
Relevance. Introduces the closed-time-path (real-time) formalism that underlies nonequilibrium quantum dynamics. It is cited to support the OGGEP requirement that the bulk-to-boundary dictionary be real-time and causal, suitable for open systems and measurement.

Seifert, U. (2012). Stochastic thermodynamics, fluctuation theorems and molecular machines. *Reports on Progress in Physics*, *75*(12), 126001. https://doi.org/10.1088/0034-4885/75/12/126001
Relevance. Stochastic thermodynamics extends thermodynamic concepts to small fluctuating open systems and provides precise expressions for entropy production and nonequilibrium work. We cite it for the appendix toy models that connect GDOS-style open dynamics to measurable entropy production and for motivating POS-2's open-system extremal forms.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal*, *27*, 379–423, 623–656.
Relevance. Shannon's information theory provides a quantitative language for "entropy" and information flow. Because GUT aims to export a single operational grammar across sciences, information-theoretic quantities become natural candidates for inclusion in the open-system ledger (GDOS) implied by OGGEP.

Sober, E. (2015). *Ockham's Razors: A User's Manual*. Cambridge: Cambridge University Press.
Relevance. A modern analysis of simplicity principles (Occam's razor) and how they function in scientific inference. POS-6 (OGN) and AX-4 (MSMF) elevate simplicity from heuristic to constitutional restriction by forbidding hidden continuous degrees of freedom and convention drift.

Stinespring, W. F. (1955). Positive functions on C^*-algebras. *Proceedings of the American Mathematical Society*, *6*(2), 211–216. https://doi.org/10.1090/S0002-9939-1955-0069403-2
Relevance. Stinespring's dilation theorem is the mathematical backbone of quantum channels: CP maps arise from unitary evolution on an enlarged space. We cite it to justify how OGGEP can treat open-system boundary dynamics as fundamental while remaining consistent with microscopic unitarity in the bulk.

Streater, R. F., & Wightman, A. S. (1964). *PCT, Spin and Statistics, and All That.* W. A. Benjamin.
Relevance. This book codifies rigorous axioms for relativistic QFT, including microcausality and locality. We cite it as a foundational reference for AX-1's operator-algebraic encoding of locality and causality.

Susskind, L. (1979). Dynamics of spontaneous symmetry breaking in the Weinberg–Salam theory. *Physical Review D*, *20*(10), 2619–2625.
Relevance. Early naturalness discussions emphasize that symmetry and scale hierarchies are not independent. This supports the motivation for a constitutional axiom (MSMF) aimed at suppressing unexplained parameter sensitivity.

Susskind, L. (1995). The world as a hologram. *Journal of Mathematical Physics*, *36*(11), 6377–6396. https://arxiv.org/abs/hep-th/9409089
Relevance. Susskind's holographic principle paper is a key historical milestone leading to modern bulk–boundary dualities. We cite it to frame OGGEP as a constitution-level insistence that observation is a bulk→boundary export, generalized beyond correlators to full operational semantics.

Susskind, L. (2003). The anthropic landscape of string theory. *Universe or Multiverse?* (conference contribution). https://arxiv.org/abs/hep-th/0302219
Relevance. The landscape framing represents a modern endpoint of unconstrained model freedom: many vacua, many low-energy effective theories, and selection by anthropic arguments. We cite it as a motivation for MSMF, which aims to restore determinism by bounding freedom and demanding symmetry-driven minimality.

Symanzik, K. (1970). Small distance behaviour in field theory and power counting. *Communications in Mathematical Physics, 18*(3), 227–246. DOI: 10.1007/BF01649434.
Relevance. A foundational development of renormalization and scaling in quantum field theory. Together with Callan and Wilson, it underpins the modern RG view used by POS-5 and motivates the identification of a log-scale coordinate with flow in the space of effective theories.

't Hooft, G., & Veltman, M. (1972). Regularization and renormalization of gauge fields. *Nuclear Physics B*, *44*(1), 189–213. https://doi.org/10.1016/0550-3213(72)90279-9
Relevance. The renormalizability of non-Abelian gauge theories is a cornerstone of the SM's consistency. We cite it to illustrate a key lesson: deep consistency conditions (not just phenomenological fits) decide what counts as a viable theory. This motivates postulates that function as consistency filters (e.g. anomaly freedom, RG compatibility).

't Hooft, G. (1980). Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking. In G. 't Hooft et al. (Eds.), *Recent Developments in Gauge Theories* (pp. 135–157). Plenum Press. https://doi.org/10.1007/978-1-4684-7571-5_9
Relevance. Naturalness arguments connect small parameters to enhanced symmetry. We cite this as historical background for MSMF: the GUT-10 constitution pushes the symmetry↔parameter connection to a stronger law, demanding maximal symmetry and forbidding non-protected dials.

't Hooft, G. (1993). Dimensional reduction in quantum gravity. https://arxiv.org/abs/gr-qc/9310026

Relevance. 't Hooft articulated early modern arguments for holography as a fundamental constraint on quantum gravity. We cite it (together with Susskind and Maldacena) to motivate why OGGEP treats the bulk→boundary map as definitional rather than optional.

Touchette, H. (2009). The large deviation approach to statistical mechanics. *Physics Reports* **478**, 1–69. doi:10.1016/j.physrep.2009.05.002.

Relevance. A modern review of large-deviation theory, clarifying how action-like functionals control typical and rare trajectories. It supports POS-UEPby providing a general mathematical framework for extremal principles in open systems.

van Kampen, N. G. (2007). *Stochastic Processes in Physics and Chemistry* (3rd ed.). Elsevier.

Relevance. Develops stochastic-process tools used for mesoscopic physics and chemical kinetics. It provides background for interpreting GDOS as the universal open-system bridge between microscopic laws and coarse-grained dynamics.

von Neumann, J. (1932). *Mathematische Grundlagen der Quantenmechanik*. Springer.

Relevance. von Neumann formalized quantum theory axiomatically but introduced measurement as an additional process distinct from unitary evolution. This motivates GUT's insistence that measurement be defined as a constitutional axiom/postulate rather than left as a dualistic rule.

Ward, J. C. (1950). An identity in quantum electrodynamics. *Physical Review*, *78*, 182. https://doi.org/10.1103/PhysRev.78.182

Relevance. Ward's identity is the prototype relation showing how gauge symmetry constrains quantum amplitudes. We cite it in POS-1 and POS-4 to emphasize that gauge invariance is not only a classical redundancy but an admissibility condition that has nontrivial quantum consequences.

Weinberg, S. (1967). A model of leptons. *Physical Review Letters*, *19*(21), 1264–1266. https://doi.org/10.1103/PhysRevLett.19.1264

Relevance. Weinberg's paper, together with Glashow and Salam, established the electroweak gauge theory. We cite it as part of the historical backbone of gauge invariance and spontaneous symmetry breaking, which GUT-10 aims to generalize to open systems via GDOS.

Weinberg, S. (1972). *Gravitation and Cosmology*. Wiley.

Relevance. Weinberg's text is a standard reference on GR and cosmology, and it exemplifies how a field can be organized around clear principles and operational predictions. We cite it as background for the GR lineage and for the cosmological motivations that feed into GUT-10.

Weinberg, S. (1989). The cosmological constant problem. *Reviews of Modern Physics*, *61*(1), 1–23. doi:10.1103/RevModPhys.61.1.

Relevance. The cosmological-constant problem is a sharp expression of the "too many dials" tension between quantum field theory and gravity. It motivates the GUT emphasis on MSMF/OGN (no hidden knobs) and on seeking geometric constraints that reduce phenomenological freedom in cosmology.

Weinberg, S. (1995). *The Quantum Theory of Fields* (Vol. 1: Foundations). Cambridge University Press.

Relevance. Weinberg's axiomatic and symmetry-driven approach to QFT is a key reference for how locality, causality, and gauge invariance constrain admissible theories. AX-1 is aligned with this tradition but extends it by insisting that the same causal discipline apply to open-system operational evolution and measurement outputs.

Wess, J., & Zumino, B. (1971). Consequences of anomalous Ward identities. *Physics Letters B*, *37*(1), 95–97. https://doi.org/10.1016/0370-2693(71)90582-X

Relevance. Wess and Zumino derived integrability/consistency conditions on gauge anomalies, demonstrating that anomalies have constrained algebraic structure. We cite this work to justify treating anomaly freedom and inflow cancellation as a constitutional filter (POS-3) rather than a model-dependent afterthought.

Weyl, H. (1918). Gravitation und Elektrizität. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 465–480.

Relevance. Weyl's early gauge ideas represent one of the first attempts to relate geometry and electromagnetism. While historically distinct from modern Yang–Mills theory, it anchors the long-standing theme that interactions may be geometric in origin, a theme AX-2 makes constitutional.

Wightman, A. S. (1956). Quantum field theory in terms of vacuum expectation values. *Physical Review, 101*(2), 860–866.

Relevance. An early axiomatic formulation of relativistic quantum field theory in which locality and causality appear as precise algebraic/analytic conditions. It supports the microcausality formulation of AX-1.

Wigner, E. P. (1939). On unitary representations of the inhomogeneous Lorentz group. *Annals of Mathematics*, *40*(1), 149–204. https://doi.org/10.2307/1968551

Relevance. Wigner's classification of particles by symmetry representations established symmetry as the organizing principle of fundamental physics. It motivates AX-4 (MSMF): the search for maximal symmetry is not just aesthetic, but historically productive in determining the space of admissible physical structures.

Wilson, K. G. (1971). Renormalization group and critical phenomena. I. Renormalization group and the Kadanoff scaling picture. *Physical Review B*, *4*(9), 3174–3183.

Relevance. Wilson's RG is the prototype of scale-covariant thinking and motivates POS-RG. Because GUT aspires to unify across scales and domains, it demands that coarse-graining and operational measurement commute in a controlled way.

Wilson, K. G., & Kogut, J. (1974). The renormalization group and the ϵ expansion. *Physics Reports*, *12*, 75–200.

Relevance. A standard reference for the modern RG viewpoint: coarse-graining as a flow on theory space. POS-5 builds on this picture but adds the constitutional demand that RG must commute with the operational dictionary (OGGEP), enabling a geometric RG coordinate in GG-6.

Witten, E. (1998). Anti-de Sitter space and holography. *Advances in Theoretical and Mathematical Physics*, *2*, 253–291. https://arxiv.org/abs/hep-th/9802150
Relevance. Witten's prescription (together with Gubser–Klebanov–Polyakov and Maldacena) provides the canonical bulk→boundary correlator map. We cite it as a concrete historical anchor for OGGEP's demand that a theory includes a bulk→boundary dictionary, and as motivation for extending that dictionary to operational measurement objects.

Yang, C. N., & Mills, R. L. (1954). Conservation of isotopic spin and isotopic gauge invariance. *Physical Review*, *96*(1), 191–195. https://doi.org/10.1103/PhysRev.96.191
Relevance. Yang–Mills theory is the modern origin of non-Abelian gauge symmetry. AX-2 builds on the connection/curvature viewpoint of gauge forces and then strengthens it by asserting that gauge and gravity are parts of one composite geometric structure (GGEP).

Zurek, W. H. (2003). Decoherence, einselection, and the quantum origins of the classical. *Reviews of Modern Physics*, *75*(3), 715–775.
Relevance. Decoherence provides a mechanism for effective classicality in open systems and illustrates why openness and environment-induced selection are essential to operational measurement. This motivates OGGEP and the elevation of open dynamics to the constitutional layer.

K. Arisaka. *GUT-10: Axiom System of a Constitutional Grand Unification*, UCLA-Arisaka/GG-2026-1, preprint (2026).
Relevance. The parent constitutional paper; all four axioms and seven postulates invoked in the present derivation are formally stated and motivated here.

K. Arisaka. *Why Are We Here? A Popular-Level Introduction to the GUT-10/GG-6/GDOS Program*, working manuscript (2026).
Relevance. A popular-level companion explaining the GUT-10/GG-6/GDOS program for a broad audience.

C. Becchi, A. Rouet, and R. Stora. *Renormalization of gauge theories*, *Annals Phys.* **98** (1976) 287.
Relevance. Foundational paper on BRST symmetry; the BRST algebra and its off-shell closure are central to the gauge consistency of the GG-6 bulk QFT.

Y. Kawamura. *Triplet-doublet splitting, proton stability and extra dimensions*, *Prog. Theor. Phys.* **105** (2001) 999.
Relevance. Introduces orbifold GUT breaking to solve the triplet–doublet splitting problem; the boundary-condition symmetry breaking in GG-6 uses the same mechanism.

J. Polchinski. *String Theory, Vol. I: An Introduction to the Bosonic String*, Cambridge University Press, Cambridge (1998).
Relevance. Standard reference for bosonic string theory; GG-6 borrows the Green–Schwarz mechanism from string theory while discarding the full stringy ontology.

J. Polchinski. *String Theory, Vol. II: Superstring Theory and Beyond*, Cambridge University Press, Cambridge (1998).
Relevance. Standard reference for superstring theory; cited for the anomaly-cancellation technology that GG-6 adapts to six dimensions without supersymmetry.

G. 't Hooft and M. Veltman. *Regularization and renormalization of gauge fields*, *Nucl. Phys. B* **44** (1972) 189.
Relevance. Proves renormalizability of non-Abelian gauge theories; the perturbative consistency of the GG-6 zero-mode sector relies on this result.

I. Tyutin. *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Institute preprint 39 (1975), arXiv:0812.0580.
Relevance. Independent discovery of BRST symmetry; together with Becchi–Rouet–Stora, provides the gauge-fixing framework used in the GG-6 BRST algebra.

H. Haber and G. Kane. *The search for supersymmetry: Probing physics beyond the Standard Model*, *Phys. Rept.* **117** (1985) 75.
Relevance. Comprehensive review of supersymmetric phenomenology; cited as a benchmark for the SUSY program that GG-6 explicitly does not require.

S. Nakajima. *On quantum theory of transport phenomena: steady diffusion*, *Prog. Theor. Phys.* **20** (1958) 948.
Relevance. Co-discovers the Nakajima–Zwanzig projection technique; the exact memory-kernel equation is the starting point of the GDOS derivation in the companion paper.

H. Nilles. *Supersymmetry, supergravity and particle physics*, *Phys. Rept.* **110** (1984) 1.
Relevance. Standard review of supergravity and supersymmetric particle physics; cited for the SUSY/SUGRA program whose assumptions GG-6 replaces with the constitutional axiom system.

J. Wess and B. Zumino. *Supergauge transformations in four dimensions*, *Nucl. Phys. B* **70** (1974) 39.
Relevance. Introduces the first four-dimensional supersymmetric field theory; cited as a historical benchmark for the SUSY program that GG-6 does not invoke.

R. Zwanzig. *Ensemble method in the theory of irreversibility*, *J. Chem. Phys.* **33** (1960) 1338.
Relevance. Co-discovers the Nakajima–Zwanzig equation; the projection-operator technique provides the exact non-Markovian backbone of the GDOS derivation.

Jiang, W., Hermolin, J., & Fillingame, R. H. (2001). The preferred stoichiometry of c subunits in the rotary motor sector of *Escherichia coli* ATP synthase is 10. *Proceedings of the National Academy of Sciences of the USA*, *98*, 4966–4971.
Relevance. Confirms the preferred c_{10} stoichiometry of the ATP synthase rotor in *E. coli*, a key experimental integer used in the event-budget verification of the Arisaka Equation.

O'Keefe, J., & Dostrovsky, J. (1971). The hippocampus as a spatial map: Preliminary evidence from unit activity in the freely-moving rat. *Brain Research*, *34*, 171–175.
Relevance. The discovery of hippocampal place cells is a foundational observation for allocentric spatial representation; in the Qi-HAL framework, place-like states instantiate the Inner Hologram and the NHT phase-coding architecture.

Anderson, P. W. (1972). More is different. *Science*, *177*, 393–396.
Relevance. Anderson's argument that each level of complexity exhibits genuinely new organising principles anticipates the pagoda hierarchy: the GDOS framework preserves level-specific emergence while denying level-specific lawlessness.

Metzler, R., & Klafter, J. (2000). The random walk's guide to anomalous diffusion. *Physics Reports*, *339*, 1–77.

Relevance. Foundational paper on anomalous diffusion and Lévy walks; the GDOS derivation of bacterial chemotaxis predicts the biased Lévy walk as a geometric consequence.

Mandelbrot, B. B. (1982). *The Fractal Geometry of Nature*. W. H. Freeman.

Relevance. Mandelbrot established fractal geometry as a mathematical language for scale-invariant natural structures; the Qi-CON pillar derives fractal attractor supports and anomalous dimensions as consequences of the open-system generator rather than as phenomenological observations.

Nakagaki, T., Yamada, H., & Tóth, A. (2000). Maze-solving by an amoeboid organism. *Nature*, *407*, 470.

Relevance. Demonstrates that even a unicellular organism can solve network-optimisation problems, providing a low-floor example of adaptive open-system behaviour that the GDOS framework must accommodate below the neural level.

Fuchs, G. W., Cuppen, H. M., Ioppolo, S., Romanzin, C., Bisschop, S. E., Andersson, S., van Dishoeck, E. F., & Linnartz, H. (2009). Hydrogenation reactions in interstellar CO ice analogues: A combined experimental/theoretical approach. *Astronomy & Astrophysics*, *505*, 629–639.

Relevance. Demonstrates that complex molecular synthesis proceeds under driven nonequilibrium conditions even in interstellar environments, supporting the universality of the GDOS open-system framework beyond terrestrial chemistry.

Haken, H. (1983). *Synergetics: An Introduction* (3rd ed.). Springer.

Relevance. Synergetics develops the slaving principle and order-parameter dynamics for self-organising systems; these ideas are absorbed into the GSSB and GA stages of the master runtime pipeline.

Ballhausen, B., Altendorf, K., & Deckers-Hebestreit, G. (2009). Constant c_{10} ring stoichiometry in the *Escherichia coli* ATP synthase analyzed by cross-linking. *Journal of Bacteriology*, *191*, 2400–2404.

Relevance. Establishes the c_{10} ring stoichiometry of *E. coli* ATP synthase, providing a precise molecular integer that enters the event-budget accounting of the Arisaka Equation at the chemistry scale.

Benham, C. J. (1979). Torsional stress and local denaturation in supercoiled DNA. *Proceedings of the National Academy of Sciences of the USA*, *76*, 3870–3874.

Relevance. Torsional stress and local denaturation in supercoiled DNA illustrate how geometric topology constrains molecular-scale dynamics, motivating the gauge-connection interpretation of structural currents in the chemistry-domain SIP.

Higgs, P. W. (1964). Broken symmetries and the masses of gauge bosons. *Physical Review Letters*, *13*, 508–509.

Relevance. Foundational paper on gauge-boson mass generation. GG-6 reproduces the mass relations from holonomy.

Hopfield, J. J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences of the USA*, *79*, 2554–2558.
Relevance. Hopfield networks provide the canonical model of attractor-based associative memory; in the Qi-HAL framework, the Holographic Attractor Lattice generalises the Hopfield picture to a metastable landscape of selected attractors under open-system dynamics.

Bohr, N. (1913). On the constitution of atoms and molecules. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, *26*, 1–25.
Relevance. Bohr's quantized atomic model establishes the discrete energy levels whose spacing ultimately descends from the Rydberg energy E_{Ryd}; this energy scale reappears in the Arisaka Constant $Q_B = (2/5)\,E_{\mathrm{Ryd}}/K$ as the bridge between physics-natural and chemistry-natural units.

Tononi, G. (2004). An information integration theory of consciousness. *BMC Neuroscience*, *5*, 42.
Relevance. IIT's integrated information measure Φ is compared with GDOS's Qi-HAL universality theorem in the consciousness chapters.

Lane, N. (2015). *The Vital Question: Why Is Life the Way It Is?* W. W. Norton & Company.
Relevance. Lane's emphasis on energy flow in the origin of life aligns closely with the GDOS EEGG ledger approach.

Turner-Evans, D., et al. (2020). The neuroanatomical ultrastructure and function of a biological ring attractor. *Neuron*, *108*, 145–163.
Relevance. Reviews neural ring attractor circuits in *Drosophila*, providing a direct biological realization of the persistent phase-coded memory and evidence-dependent switching described by GA and the DGI–GSSB switch in the Qi-HAL framework.

Wang, J. C. (1979). Helical repeat of DNA in solution. *Proceedings of the National Academy of Sciences of the USA*, *76*, 200–203.
Relevance. Establishes the helical repeat of DNA in solution ($\sim$10.5 bp/turn), a structural integer that illustrates how geometric periodicity constrains the molecular-scale SIP and the topology of the structural-material current J_B.

Simon, H. A. (1962). The architecture of complexity. *Proceedings of the American Philosophical Society*, *106*, 467–482.
Relevance. Simon's "nearly decomposable" hierarchical architecture is a conceptual ancestor of the pagoda hierarchy; the present paper gives it a geometric runtime basis through the story-lift theorem and the DGI shinbashira.

Yoshida, M., Muneyuki, E., & Hisabori, T. (2001). ATP synthase—a marvellous rotary engine of the cell. *Nature Reviews Molecular Cell Biology*, *2*, 669–677.
Relevance. Authoritative review of ATP synthase; the rotary mechanism and energy budget are the empirical data against which the Arisaka Equation is verified.

Watson, J. D., & Crick, F. H. C. (1953). Molecular structure of nucleic acids: A structure for deoxyribose nucleic acid. *Nature*, *171*, 737–738.

Relevance. The double-helical structure of DNA is the foundational molecular template whose geometric and informational properties motivate the chemistry-to-life transition in the GDOS pagoda hierarchy.

Dawkins, R. (1976). *The Selfish Gene*. Oxford University Press.
Relevance. The "selfish gene" perspective anticipates the information-centric view of biology; in the GDOS framework, information is a conserved gauge current (J_{Qi}), not just a metaphor.

Fuller, F. B. (1971). The writhing number of a space curve. *Proceedings of the National Academy of Sciences of the USA*, *68*, 815–819.
Relevance. The writhing number is a topological invariant of closed space curves; it enters the geometric characterisation of DNA supercoiling and provides a concrete example of a curl-type conserved quantity in the chemistry-domain current ledger.

Mallat, S. (2009). *A Wavelet Tour of Signal Processing* (3rd ed.). Academic Press.
Relevance. Wavelet and multiscale decomposition theory provides the mathematical background for the BFFT dyadic protocol, which organises multiscale memory and replay in the Qi-HAL architecture.

Kuhn, T. S. (1962). *The Structure of Scientific Revolutions*. University of Chicago Press, Chicago.
Relevance. Defines "scientific revolution" as used in this book.

Chalmers, D. J. (1995). Facing up to the problem of consciousness. *Journal of Consciousness Studies*, 2(3), 200–219.
Relevance. Defines the "hard problem of consciousness."

Ziegler, H. (1983). *An Introduction to Thermomechanics*. North-Holland, Amsterdam.
Relevance. Maximum entropy production principle.

Kaneko, K. (2006). *Life: An Introduction to Complex Systems Biology*. Springer, Berlin.
Relevance. Spontaneous cell-type differentiation.

England, J. L. (2013). Statistical physics of self-replication. *J. Chem. Phys.*, 139, 121923.
Relevance. Thermodynamic bounds on self-replicator emergence.

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