Given a positive integer n, n factorial (written $n!$) is a shorthand for the product of all positive integers less than or equal to the given integer n.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. (By convention, we say that $0! = 1$.)

1. Evaluate the following.

(a) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

(b) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6(120) = 720$

flipped

(c) $\dfrac{5!}{6!} = \dfrac{120}{720} = \dfrac{1}{6}$

(d) $\dfrac{6!}{5!} = \dfrac{720}{120} = 6$

notice $6! = 6 \cdot 5!$ so

$\dfrac{6!}{5!} = \dfrac{6 \cdot \cancel{5!}}{\cancel{5!}} = 6$ and $\dfrac{5!}{6!} = \dfrac{\cancel{5!}}{6 \cdot \cancel{5!}} = \dfrac{1}{6}$

use this idea for (e).

(e) $\dfrac{102!}{100!} = \dfrac{102 \cdot 101 \cdot \cancel{100!}}{\cancel{100!}} = 102 \cdot 101 = 10302$

2. Simplify each of the following. Assume n is a positive integer.

(a) $\dfrac{(n+\overset{2}{\cancel{5}})!}{n!} = \dfrac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}} = (n+2)(n+1)$

(b) $\dfrac{(n-\overset{3}{\cancel{2}})!}{n!} = \dfrac{(n-3)(n-4)(n-5)!}{n!}$ ← doesn't help...

⇐ expand out denom. b/c its larger, stop once you can cancel w/ num.

what is the difference? the parentheses! really important.

(c) $\dfrac{(2n+2)!}{(2n)!} = \dfrac{(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{(2n)!}}$

← expand until you can cancel w/ n!

(d) $\dfrac{(2n+2)!}{2n!} = \dfrac{(2n+2)(2n+1)(2n)(2n-1)\ldots(n+1)\cancel{n!}}{2 \cdot \cancel{n!}} = \frac{1}{2}(2n+2)(2n+1)\ldots(n+1)$

↑ 2 times n!

(e) $\dfrac{(n!)^2}{((n+1)!)^2} = \dfrac{n!\,n!}{(n+1)!\,(n+1)!} = \dfrac{\cancel{n!} \cdot \cancel{n!}}{(n+1) \cdot \cancel{n!} \cdot (n+1) \cdot \cancel{n!}} = \dfrac{1}{(n+1)^2}$