

1B

#1 $\int_0^1 x^2 (x^3 + 2)^5 dx$ $u = x^3 + 2$
 $du = 3x^2 dx$ $x^2 dx = \frac{1}{3} du$
 $x=0 \quad u=2$ $\frac{1}{3} \int_2^3 u^5 du$
 $x=1 \quad u=3$

#2 $\int f(x) dx = e^x \cos x - e^x \sin x - \int f(x) dx$
 Next step is to Add $\int f(x) dx$ to both sides
 $\int f(x) dx + \int f(x) dx = e^x \cos x - e^x \sin x - \cancel{\int f(x) dx} + \cancel{\int f(x) dx}$
 $2 \int f(x) dx = e^x \cos x - e^x \sin x$

#3 $\int_{-1}^4 \frac{x}{(x+7)^3} dx$ $u = x+7$ $x = u-7$
 $du = dx$ $x=-1 \quad u=6$
 $x=4 \quad u=11$

$$\int_6^{11} \frac{u-7}{u^3} du = \int_6^{11} (u^{-2} - 7u^{-3}) du$$

#4 $\int \tan^{10} x \sec^4 x dx$ even secant

$$\int \tan^{10} x \sec^2 x \sec^2 x dx =$$

$$\int \tan^{10} x (\tan^2 x + 1) \sec^2 x dx$$

$$u = \tan x$$

$$du = \sec^2 x dx$$

$$\int u^{10} (u^2 + 1) du = \int (u^{12} + u^{10}) du$$

#5 $\int_{\pi^2/16}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$

u-sub $u = \sqrt{x} = x^{1/2}$

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$2du = \frac{1}{\sqrt{x}} dx$$

$$x = \pi^2/16 \quad u = \pi/4$$

$$x = \pi^2 \quad u = \pi$$

$$2 \int_{\pi/4}^{\pi} \sin(u) du = -2 \cos(u) \Big|_{\pi/4}^{\pi} = -2 \cos(\pi) + 2 \cos(\pi/4) =$$

$$2 + 2 \frac{\sqrt{2}}{2} = 2 + \sqrt{2}$$

#6 $\int \text{Arctan } x \, dx$

by parts

$$u = \text{Arctan } x$$

$$du = \frac{1}{1+x^2} dx$$

$$dv = 1 \, dx$$

$$v = x$$

$$\underbrace{\int \text{Arctan } x \, dx}_{u \, dv} = \underbrace{x \text{Arctan } x}_{uv} - \underbrace{\int \frac{x}{1+x^2} dx}_{v \, du}$$

$$\int \frac{x}{1+x^2} dx \rightarrow \frac{1}{2} \int \frac{1}{w} dw = \frac{1}{2} \ln w = \frac{1}{2} \ln(1+x^2)$$

$$\begin{aligned} w &= 1+x^2 \\ dw &= 2x \, dx \\ \frac{1}{2} dw &= x \, dx \end{aligned}$$

$$\int \text{Arctan } x \, dx = x \text{Arctan } x - \frac{1}{2} \ln(1+x^2) + C$$

#7 $\int e^{-3x} \sin\left(\frac{1}{2}x\right) dx$ by parts

$$u = e^{-3x}$$

$$dv = \sin\left(\frac{1}{2}x\right) dx$$

$$du = -3e^{-3x} dx$$

$$V = -2\cos\left(\frac{1}{2}x\right)$$

$$\int e^{-3x} \sin\left(\frac{1}{2}x\right) dx = \underbrace{-2e^{-3x} \cos\left(\frac{1}{2}x\right)}_{uV} - \int \underbrace{-3e^{-3x} \cdot (-2) \cos\left(\frac{1}{2}x\right)}_{V du} dx$$

$$\int e^{-3x} \sin\left(\frac{1}{2}x\right) dx = -2e^{-3x} \cos\left(\frac{1}{2}x\right) - 6 \int e^{-3x} \cos\left(\frac{1}{2}x\right) dx \quad *$$

$\int e^{-3x} \cos\left(\frac{1}{2}x\right) dx$	$u = e^{-3x}$ $du = -3e^{-3x} dx$	$dV = \cos\left(\frac{1}{2}x\right) dx$ $V = 2\sin\left(\frac{1}{2}x\right)$
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$$\int e^{-3x} \cos\left(\frac{1}{2}x\right) dx = 2e^{-3x} \sin\left(\frac{1}{2}x\right) - \int 2\sin\left(\frac{1}{2}x\right) (-3)e^{-3x} dx$$

$$\int e^{-3x} \cos\left(\frac{1}{2}x\right) dx = 2e^{-3x} \sin\left(\frac{1}{2}x\right) + 6 \int e^{-3x} \sin\left(\frac{1}{2}x\right) dx$$

$$* \int e^{-3x} \sin\left(\frac{1}{2}x\right) dx = -2e^{-3x} \cos\left(\frac{1}{2}x\right) - 6 \int e^{-3x} \cos\left(\frac{1}{2}x\right) dx$$

$$\int e^{-3x} \sin\left(\frac{1}{2}x\right) dx = -2e^{-3x} \cos\left(\frac{1}{2}x\right) - 6 \left[2e^{-3x} \sin\left(\frac{1}{2}x\right) + 6 \int e^{-3x} \sin\left(\frac{1}{2}x\right) dx \right]$$

$$\int e^{-3x} \sin\left(\frac{1}{2}x\right) dx = -2e^{-3x} \cos\left(\frac{1}{2}x\right) - 12e^{-3x} \sin\left(\frac{1}{2}x\right) - 36 \int e^{-3x} \sin\left(\frac{1}{2}x\right) dx$$

Add $36 \int e^{-3x} \sin(\frac{1}{2}x) dx$ to both sides

$$\int e^{-3x} \sin(\frac{1}{2}x) dx + 36 \int e^{-3x} \sin(\frac{1}{2}x) dx = -2e^{-3x} \cos(\frac{1}{2}x) - 12e^{-3x} \sin(\frac{1}{2}x) -$$

$$\cancel{36 \int e^{-3x} \sin(\frac{1}{2}x) dx} + \cancel{36 \int e^{-3x} \sin(\frac{1}{2}x) dx}$$

$$37 \int e^{-3x} \sin(\frac{1}{2}x) dx = -2e^{-3x} \cos(\frac{1}{2}x) - 12e^{-3x} \sin(\frac{1}{2}x)$$

$$\int e^{-3x} \sin(\frac{1}{2}x) dx = \frac{-2}{37} e^{-3x} \cos(\frac{1}{2}x) - \frac{12}{37} e^{-3x} \sin(\frac{1}{2}x) + C$$

#8 $\int \tan^3 x \sec^5 x dx$ odd tangent

$$\int \tan^2 x \sec^4 x \sec x \tan x dx =$$

$$\int (\sec^2 x - 1) \sec^4 x \sec x \tan x dx \quad \begin{matrix} u = \sec x \\ du = \sec x \tan x dx \end{matrix}$$

$$\int (u^2 - 1) u^4 du = \int (u^6 - u^4) du = \frac{1}{7} u^7 - \frac{1}{5} u^5$$

$$\int \tan^3 x \sec^5 x dx = \frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C$$