

Taylor and Maclaurin Series

The Taylor series of a function $f(x)$ centered at a point a is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n.$$

The Maclaurin series of $f(x)$ is the Taylor series based at $a = 0$. In other words, the Maclaurin series is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

<u>Maclaurin Series</u>			<u>Radius of Convergence</u>
$\frac{1}{1-x}$	$= \sum_{n=0}^{\infty} x^n$	$= 1 + x + x^2 + x^3 + \dots$	$R = 1$
e^x	$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$R = \infty$
$\sin x$	$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$R = \infty$
$\cos x$	$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$R = \infty$
$\arctan(x)$	$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$R = 1$
$\ln(1+x)$	$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$	$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$R = 1$

- Using the table of standard Maclaurin series, find the Maclaurin series for $f(x) = x^3 \sin(5x)$ and determine its radius of convergence.

2. Find the Maclaurin series for $f(x) = \frac{e^x - 1}{x}$ and determine its radius of convergence.

3. Find the Maclaurin series for $f(x) = \frac{\ln(1 + 2x^2)}{3x^2}$ and determine its radius of convergence.

4. Differentiate the Maclaurin series for e^x term by term. Re-index the result to start at $n = 0$. What do you notice?

5. Find a Maclaurin series representation for $\int e^{-x^2} dx$.

6. Express $\int_0^1 \cos(x^2) dx$ as a series.