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Algorithms for Prediction of Smooth Bounded Signals*

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***Abstract*— The paper describes prediction algorithms for continuous time functions that have continuous and bounded derivatives. The synthesis of algorithms are based on Lagrange mean value theorem and its generalization on variable steps. The transient time and error prediction for each algorithm are obtained. Simulation results show the efficiency of the proposed schemes. The proposed prediction algorithms can be applied as disturbance observer in presence of time-delay in measurements.**

I. INTRODUCTION

There are many problems in a control theory related to the prediction of the signals corresponding to the output signal of systems, external disturbances or other processes. This problem is especially relevant for design disturbance observers. Often, such problems need to solve on-line. Most existing results are devoted to the prediction approaches based on the parametric form of signal representations [1-4]. Here it is assumed the knowledge of structure of a disturbance generator. The nonparametric form of signal representations [5-8] are based on measurements of previous values of a signal.

In the class of algorithms based on parametric form of signal representations we note the discrete time algorithms [1] and algorithms with continuous time [2-4]. The huge number of papers are devoted to the identification of parameters of sinusoidal signals [2-4]. Here the dynamical order of these algorithms and prediction accuracy depends on the number of sinusoidal signals.

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In [5, 6] the algorithms based on the nonparametric form of signal representations are used. The design of these algorithms are based on Lagrange mean value theorem. These algorithms used a finite number of moving measurement. For example, in [6] the discrete predictive algorithm is considered for functions which have continuous and bounded derivatives of nth order, where n is the number of the finite differences in the algorithm. In [9] the same class of functions are considered. However, in [9] a maximum range of function is not exceed the value of $1/2h$ (where h is the prediction time). Differently from [6], in [9] the algorithm allows one to reduce the prediction time to $2h$. However, the algorithm from [9] has a limit on the number of measurements which depends on the transient time.

In the present paper we propose the development of results are obtained in [6, 9]. In contrast to [6, 9] we propose the new prediction algorithm with different steps. It is allowed to improve the prediction quality of some signals. Differently from [9] the error prediction is taken into account.

In the paper the scalar function $f(t)$ is considered, where t is time. Assume, that the signal $f(t)$ is measured with delay h, i.e $f(t-l)$, $l \in [0; h)$ is not available for measurement. Additionally assume, that $f(t)$ has a continuous and bounded ith derivative, where the value of i depends on the corresponding approach in the paper. The design of prediction algorithms is based on Lagrange mean value theorem.

II. PREDICTIVE ALGORITHMS

Introduce the first difference of $f(t)$ in the form

$$\Delta^1 f(t) = f(t) - f(t - k_1 h), \tag{1}$$

where $k_1 > 0$ is a design parameter. The second difference is defined as

$$\Delta^2 f(t) = \Delta^1 f(t) - \Delta^1 f(t - k_2 h), \tag{2}$$

where $k_2 > 0$ is a design parameter. Introduce the ith difference in the form

$$\Delta^i f(t) = \Delta^{i-1} f(t) - \Delta^{i-1} f(t - k_i h), \tag{3}$$

where $k_i > 0$ are design parameters, $i = \overline{3, n}$.

Theorem 1. The function $f_a^1(t)$ is the prediction function of $f(t)$, defined by

$$f_a^1(t)=\sum_{i_1=1}^{n}f(t-k_{i_1}h)-\frac{1}{2!}\sum_{\substack{i_1,i_2=1,\\ i_1\neq i_2}}^{n}f(t-(k_{i_1}+k_{i_2})h)+$$
$$+\frac{1}{3!}\sum_{\substack{i_1,i_2,i_3=1,\\ i_1\neq i_2\neq i_3}}^{n}f(t-(k_{i_1}+k_{i_2}+k_{i_3})h)+\ldots \quad (4)$$
$$\ldots+\frac{(-1)^{n+1}}{n!}\sum_{\substack{i_1,i_2,\ldots,i_n=1,\\ i_1\neq i_2\neq\ldots\neq i_n}}^{n}f(t-(k_{i_1}+k_{i_2}+\ldots+k_{i_n})h)$$

where $k_{i_j}\geq 1$, $j = 1, 2, \ldots, n$. The transient time T is defined by

$$T=\sum_{i=1}^{n}k_i h\,. \quad (5)$$

If $f(t)$ has a continuous and bounded first derivative, then the upper bound of the prediction error $e_1(t)$ is given by

$$e_1(t)\leq k_1 h\left(1+\sum_{i=1}^{n-1}\frac{A_{n-1}^{i}}{i!}\right)\sup_{t>T}\left|\dot{f}(t)\right|, \quad (6)$$

where $A_n^l=\dfrac{n!}{(n-l)!}$.

If $f(t)$ has the continuous and bounded derivative of nth order, then the prediction error $e_1(t)$ is given by

$$e_1(t)\leq k_1^n h^n\sup_{t>T}\left|f^{(n)}(t)\right|. \quad (7)$$

In (4) the sum is perceived as the sum of several parameters simultaneously.

Proof of Theorem 1. Rewrite first difference (1) in the form

$$\Delta^1 f(t)=k_1 h\dot{f}\left(t-\theta_1 k_1 h\right), \quad (8)$$

where $\theta_1\in(0;1)$. Taking into account (8), rewrite (2) as follows

$$\Delta^2 f(t)=f(t)-f(t-k_1 h)$$
$$-\left[f(t-k_2 h)-f(t-(k_1+k_2)h)\right] \quad (9)$$

or in the form

$$\Delta^2 f(t)=k_1^2 h^2\ddot{f}(t-\vartheta_1(k_1+k_2)h), \quad (10)$$

where $0 < \vartheta_1 < 1$. Define the third difference as follows

$$\Delta^3 f(t)=f(t)-f(t-k_1 h)$$
$$-\left[f(t-k_2 h)-f(t-(k_1+k_2)h)\right]$$
$$-\left[f(t-k_3 h)-f(t-(k_1+k_3)h)\right] \quad (11)$$
$$+\left[f(t-(k_2+k_3)h)-f(t-(k_1+k_2+k_3)h)\right].$$

Taking into account (7) and (10), rewrite (11) in the form

$$\Delta^3 f(t)=k_1 h\left[\dot{f}\left(t-\theta_1 k_1 h\right)-\dot{f}\left(t-k_2 h-\theta_2 k_1 h\right)\right.$$
$$\left.-\dot{f}\left(t-k_3 h-\theta_3 k_1 h\right)+\dot{f}\left(t-(k_2+k_3)h-\theta_{2,3}k_1 h\right)\right],$$

or as follows

$$\Delta^3 f(t)=k_1^3 h^3\dddot{f}(t-\vartheta_2(k_1+k_2+k_3)h),$$

where $\theta_2,\theta_3,\theta_{2,3},\vartheta_2\in(0;1)$. Analogically, rewrite the nth difference in the form

$$\Delta^n f(t)=f(t)-\sum_{i_1=1}^{n}f(t-k_{i_1}h)$$
$$+\frac{1}{2!}\sum_{\substack{i_1,i_2=1,\\ i_1\neq i_2}}^{n}f(t-(k_{i_1}+k_{i_2})h)$$
$$-\frac{1}{3!}\sum_{\substack{i_1,i_2,i_3=1,\\ i_1\neq i_2\neq i_3}}^{n}f(t-(k_{i_1}+k_{i_2}+k_{i_3})h)$$
$$\ldots+\frac{(-1)^n}{(n-1)!}\sum_{\substack{i_1,i_2,\ldots,i_{n-1}=1,\\ i_1\neq i_2\neq\ldots\neq i_{n-1}}}^{n}f(t-(k_{i_1}+k_{i_2}+\ldots+k_{i_{n-1}})h) \quad (12)$$
$$-\frac{(-1)^{n+1}}{n!}\sum_{\substack{i_1,i_2,\ldots,i_n=1,\\ i_1\neq i_2\neq\ldots\neq i_n}}^{n}f(t-(k_{i_1}+k_{i_2}+\ldots+k_{i_n})h).$$

Applying Lagrange mean theorem to (12), we obtain

$$f(t)=\sum_{i_1=1}^{n}f(t-k_{i_1}h)-\frac{1}{2!}\sum_{\substack{i_1,i_2=1,\\ i_1\neq i_2}}^{n}f(t-(k_{i_1}+k_{i_2})h)$$
$$+\frac{1}{3!}\sum_{\substack{i_1,i_2,i_3=1,\\ i_1\neq i_2\neq i_3}}^{n}f(t-(k_{i_1}+k_{i_2}+k_{i_3})h)+\ldots \quad (13)$$
$$\ldots+\frac{(-1)^{n+1}}{n!}\sum_{\substack{i_1,i_2,\ldots,i_n=1,\\ i_1\neq i_2\neq\ldots\neq i_n}}^{n}f(t-(k_{i_1}+k_{i_2}+\ldots+k_{i_n})h)+R(t),$$

where $R(t)$ is a residue. If $f(t)$ has continuous and bounded first derivative, then, according to Lagrange mean theorem, the residue $R(t)$ is defined as

$$R(t)=k_1 h\left[\dot{f}\left(t-\theta_1 k_1 h\right)-\sum_{i_2=2}^{n}\dot{f}\left(t-k_{i_2}h-\theta_{i_2}k_1 h\right)+\right.$$
$$\left.+\frac{1}{2!}\sum_{\substack{i_2,i_3=2,\\ i_2\neq i_3}}^{n}\dot{f}\left(t-\left(k_{i_2}+k_{i_3}\right)h-\theta_{i_2,i_3}k_1 h\right)\right.$$

$$\left. \times \sum_{\substack{i_2, i_3, ..., i_n=2, \\ i_2 \neq i_3 \neq ... \neq i_n}}^{n} \dot{f}\left(t - \left(k_{i_2} + k_{i_3} + ... + k_{i_n}\right)h - \theta_{i_2, i_3, ..., i_n} k_1 h\right) \right]. \quad (14)$$

Here $\theta_{i_2}, \theta_{i_2,i_3}, ..., \theta_{i_2,i_3,...,i_n}, \in (0;1)$, $i_2, i_3, ..., i_n = 2, 3, ..., n$. Note, that $\theta_{ij} = \theta_{ji}$. For example, $\theta_{1,2} = \theta_{2,1}$, because $f(t-(k_{i_2}+k_{i_3})h) = f(t-(k_{i_3}+k_{i_2})h)$. If $f(t)$ has continuous and bounded nth derivative, then, according to Lagrange mean theorem, the residue $R(t)$ is defined as

$$R(t) = k_1^n h^n f^{(n)}\left(t - \vartheta h \sum_{i=1}^{n} k_i\right). \quad (15)$$

Here $\vartheta \in (0;1)$. Expression (5) follows from (12). Find upper bounds (6) and (7). If $f(t)$ has continuous and bounded first derivative, then prediction error $e_1(t)$ is given by (14). Find the upper bound of the function $e_1(t)$ at the interval $(T; \infty)$ as follows

$$\begin{aligned} e_1(t) &\le k_1 h \sup_{t>T} \left| \dot{f}\left(t - \theta_1 k_1 h\right) \right. \\ &- \sum_{i_2=2}^{n} \dot{f}\left(t - k_{i_2} h - \theta_{i_2} k_1 h\right) + \\ &+ \frac{1}{2!} \sum_{\substack{i_2, i_3=2, \\ i_2 \neq i_3}}^{n} \dot{f}\left(t - \left(k_{i_2} + k_{i_3}\right)h - \theta_{i_2,i_3} k_1 h\right) + ... + \frac{(-1)^{n+1}}{(n-1)!} \\ &\left. \times \sum_{\substack{i_2, i_3, ..., i_n=2, \\ i_2 \neq i_3 \neq ... \neq i_n}}^{n} \dot{f}\left(t - \left(k_{i_2} + k_{i_3} + ... + k_{i_n}\right)h - \theta_{i_2, i_3, ..., i_n} k_1 h\right) \right| \le \\ &\le k_1 h \sup_{t>T} \left[\left| \dot{f}\left(t - \theta_1 k_1 h\right) \right| + \left| \sum_{i_2=2}^{n} \dot{f}\left(t - k_{i_2} h - \theta_{i_2} k_1 h\right) \right| + ... \right. \\ &\left. . + \frac{1}{(n-1)!} \right. \\ &\left. \times \left| \sum_{\substack{i_2, i_3, ..., i_n=2, \\ i_2 \neq i_3 \neq ... \neq i_n}}^{n} \dot{f}\left(t - \left(k_{i_2} + k_{i_3} + ... + k_{i_n}\right)h - \theta_{i_2, i_3, ..., i_n} k_1 h\right) \right| \right] \le \\ &\le k_1 h \sup_{t>T} \left| \dot{f}(t) \right| \left[1 + \frac{A_{n-1}^1}{1!} + ... + \frac{A_{n-1}^{n-1}}{(n-1)!} \right]. \end{aligned} \quad (16)$$

The estimate (6) follows from (16). Estimate (7) immediately follows from (15).

Corollary. Let $k_1 = k_2 = ... = k_n = 1$ in (1)-(3) and $f_a^2(t)$ be a prediction function of $f(t)$. Then, $f_a^2(t)$ is defined by

$$f_a^2(t) = \sum_{i=1}^{n} (-1)^{i-1} C_n^i f(t - ih). \quad (17)$$

The transient time is equal to

$$T = nh. \quad (18)$$

If $f(t)$ has a continuous and bounded the first derivative, then prediction error $e_2(t)$ is given by

$$e_2(t) \le h\left(1 + \sum_{i=1}^{n-1} \frac{A_{n-1}^i}{i!}\right) \sup_{t>T} \left| \dot{f}(t) \right|. \quad (19)$$

If $f(t)$ has a continuous and bounded the nth first derivative, then the prediction error $e_2(t)$ is given by

$$e_2(t) \le h^n \sup_{t>T} \left| f^{(n)}(t) \right|. \quad (20)$$

Proof of Corollary directly follows from the proof of Theorem 1, since for $k_1 = k_2 = ... = k_n = 1$ expression (13) can be rewritten as follows

$$f(t) = \sum_{i=1}^{n} (-1)^{i-1} C_n^i f(t - ih) + R(t). \quad (21)$$

Note that the coefficients $k_i \ge 1$ in Theorem 1 and Corollary. Consider the case where all or part of k_i belong to the interval (0, 1). We will show that in this case we can express the prediction function via the measured variables. Replacing h by h/k, $k > 1$ in (21), rewrite (21) in the form

$$f(t) = \sum_{i=1}^{n} (-1)^{i-1} C_n^i f(t - \frac{i}{k}h) + R(t). \quad (22)$$

Here the residue $R(t)$ is determined according to (14) or (15) by replacing h to h/k.

Consider the number z. Let z be equal to the integer portion of k if k be not an integer, and let z be equal to $k - 1$ if k be an integer. Shifting argument of (22) to the left on the value of h/k consistently z times, we have

$$\begin{aligned} f(t - \frac{1}{k}h) &= \sum_{i=1}^{n} (-1)^{i-1} C_n^i f(t - \frac{i+1}{k}h) + R(t - \frac{1}{k}h), \\ f(t - \frac{2}{k}h) &= \sum_{i=1}^{n} (-1)^{i-1} C_n^i f(t - \frac{i+2}{k}h) + R(t - \frac{2}{k}h), \\ &\vdots \\ f(t - \frac{z}{k}h) &= \sum_{i=1}^{n} (-1)^{i-1} C_n^i f(t - \frac{i+z}{k}h) + R(t - \frac{z}{k}h). \end{aligned} \quad (23)$$

Obviously, the right-hand side of the last equation in (23) contains terms with delay greater than h. Therefore, express $f(t)$

w.r.t. $f(t-\frac{1+z}{k}h)$, …, $f(t-\frac{n+z}{k}h)$. For this end, rewrite system (23) in the form

$$A_1 f(t) + A_2 F_1(t) + A_3 F_2(t) = \widetilde{R}(t), \qquad (24)$$

where

$$A_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ F_1(t) = \begin{bmatrix} f(t-\frac{1}{k}h) \\ f(t-\frac{2}{k}h) \\ \vdots \\ f(t-\frac{z}{k}h) \end{bmatrix},\ F_2(t) = \begin{bmatrix} f(k-\frac{1+z}{k}h) \\ f(t-\frac{2+z}{k}h) \\ \vdots \\ f(t-\frac{n+z}{k}h) \end{bmatrix},$$

$$\widetilde{R}(t) = [R(t), R(t-\frac{1}{k}h), ..., R(t-\frac{z}{k}h)]^T,$$

elements of the matrices A_2 and A_3 correspond to the coefficients of the system (24).

Denote l= [1 0 ... 0] (the length of l is equal to number of equation in (23)) and A_2^L is a left zero divisor of A_2 (i.e. $A_2^L A_2 = \mathrm{O}$, where the O is zero matrix, see [11, 12].

Theorem 2. The prediction function $f_a^3(t)$ for $f(t)$ can be represented as

$$f_a^3(t) = -\left(lA_2^L A_1\right)^{-1} lA_2^L A_3 F_2(t). \qquad (25)$$

The transient time is defined as

$$T = \frac{n+z}{k}h. \qquad (26)$$

If $f(t)$ has continuous and bounded the first derivative, then the upper bound of the prediction error $e_3(t)$ is given by

$$e_3(t) \le \left\| \left(lA_2^L A_1\right)^{-1} lA_2^L \right\| \sqrt{z+1}\,\frac{h}{k}\left(1 + \sum_{i=1}^{n-1} \frac{A_{n-1}^i}{i!}\right) \sup_{t>T} \left|\dot{f}(t)\right|. \qquad (27)$$

If $f(t)$ has continuous and bounded the nth derivative, then the upper bound of the prediction error $e_3(t)$ is given by

$$e_3(t) \le \left\| \left(lA_2^L A_1\right)^{-1} lA_2^L \right\| \sqrt{z+1}\,\frac{h^n}{k^n} \sup_{t>T} \left|f^{(n)}(t)\right|. \qquad (28)$$

Proof of Theorem 2. For the sake of simplicity in the proof of Theorem 2, we will use square matrix A_2^L. However, the number of rows in the matrix A_2^L can be reduced to one, because system (23) has an unique solution $f(t)$ w.r.t. variables $f(t-\frac{1+z}{k}h)$, …, $f(t-\frac{n+z}{k}h)$.

Multiplying the left-hand side of the equation (24) by the matrix A_2^L, we obtain

$$A_2^L A_1 f(t) + A_2^L A_3 F_2(t) = A_2^L \widetilde{R}(t). \qquad (29)$$

Multiplying the left-hand side of (29) by l, we get

$$lA_2^L A_1 f(t) + lA_2^L A_3 F_2(t) = lA_2^L \widetilde{R}(t). \qquad (30)$$

Since $lA_2^L A_1 \neq 0$, then it follows from (30) that

$$f(t) = -\left(lA_2^L A_1\right)^{-1} lA_2^L \left(A_3 F_2(t) - \widetilde{R}(t)\right). \qquad (31)$$

Formula (25) follows from (31). It follows from (31), that the prediction error is given by

$$e_3(t) = \left(lA_2^L A_1\right)^{-1} lA_2^L \widetilde{R}(t). \qquad (32)$$

Since the vector $\widetilde{R}(t)$ in (32) contains the maximum delay $\frac{n+z}{k}h$, therefore, the transient time is given by (26).

Let $f(t)$ has continuous and bounded first derivative. Then the first component of the vector $\widetilde{R}(t)$ is defined as (14), where the value of h is replaced by h/k. Therefore, consider the upper bound of (32) at the interval $(T; \infty)$:

$$\begin{aligned} e_3(t) &\le \left\| \left(lA_2^L A_1\right)^{-1} lA_2^L \right\| \left|\widetilde{R}(t)\right| = \left\| \left(lA_2^L A_1\right)^{-1} lA_2^L \right\| \\ &\times \sqrt{R^2(t) + R^2(t-\frac{1}{k}h) + ... + R^2(t-\frac{z}{k}h)} \le \\ &\le \left\| \left(lA_2^L A_1\right)^{-1} lA_2^L \right\| \sqrt{z+1} \sup_{t>T} |R(t)|. \end{aligned} \qquad (33)$$

Thus, estimates (27) and (28) follow from (33) and (14), (15), where h is replaced by h/k.

Generalize Theorem 2 to the variable step. Let all or part of k_i are less than 1 in (1)-(3). Next, we shift arguments of the function (13) to the left as long as the right-hand side of (13) has delays more than h. As a result, we have a system with an unique solution w.r.t. $f(t)$. Introduce the following notations: $\widetilde{A}_2$ is a matrix with elements corresponding to the coefficients before functions $f(t)$ with time-delays less than h; $\widetilde{A}_2^L$ is a left zero divisor of the matrix $\widetilde{A}_2$; $\widetilde{F}_2(t)$ is a vector consisting of functions $f(t)$ where time-delays do not less than h; $\widetilde{A}_3$ is a matrix whose elements contain the coefficients of the functions with time-delays less than h.

Theorem 3. The prediction function $f_a^4(t)$ for $f(t)$ can be represented as

$$f_a^4(t) = -\left(l\widetilde{A}_2^L A_1\right)^{-1} l\widetilde{A}_2^L \widetilde{A}_3 \widetilde{F}_2(t). \qquad (34)$$

The proof of Theorem 3 is similar to the proof of Theorems 1 and 2.

In a practice the choice of algorithms parameters depend on knowledge of value of residual. Also, these parameters can be chosen from experiment results.

III. EXAMPLE

1) Consider algorithm (4). Let $n = 3$, $k_1 = 1$, $k_2 = 2$ и $k_3 = 3$. Then the algorithm (4) is defined as

$$f_a^1(t) = \sum_{i_1=1}^{3} f(t-k_{i_1}h) - \frac{1}{3}\sum_{\substack{i_1,i_2=1,\\ i_1\neq i_2}}^{3} f(t-(k_{i_1}+k_{i_2})h) + \frac{1}{6}\sum_{\substack{i_1,i_2,i_3=1,\\ i_1\neq i_2\neq i_3}}^{3} f(t-(k_{i_1}+k_{i_2}+k_{i_3})h). \quad (35)$$

The transient time is equal to $T = 6h$.

2) Let $n = 3$ and $k_1 = k_2 = k_3 = 1$. As a result, algorithm (17) is defined as

$$f_a^2(t) = \sum_{i=1}^{3} (-1)^{i-1} C_3^i f(t-ih)\,. \quad (36)$$

The transient time is equal to $T = 3h$.

3) Consider algorithm (25). Let $n = 3$ and $k = 4$. Note that the algorithm from [9] does not implementable for these parameters. The other parameters in algorithm (25) are defined as

$$A_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix},\ A_2 = \begin{bmatrix}-3 & 3 & -1\\ 1 & -3 & 3\\ 0 & 1 & -3\\ 0 & 0 & 1\end{bmatrix},\ A_3 = \begin{bmatrix}0 & 0 & 0\\ -1 & 0 & 0\\ 3 & -1 & 0\\ -3 & 3 & -1\end{bmatrix},$$

$$F_2(t) = [f(t-h),\ f(t-\frac{5h}{4}),\ f(t-\frac{3h}{2})]^{\mathrm{T}}.$$

Define A_2^L in the form

$$A_2^L = \begin{bmatrix}1 & 3 & 6 & -8\\ 1 & 3 & 6 & -8\\ 1 & 3 & 6 & -8\\ 1 & 3 & 6 & -8\end{bmatrix}.$$

As a result, algorithm (25) is defined as

$$f_a^3(t) = 15f(t-h) - 24f(t-\frac{5h}{4}) + 10f(t-\frac{3h}{2})\,. \quad (37)$$

The transient time is equal to $T = 1.5h$.

4) Consider algorithm (34). Let $k_1 = 1/2$, $k_2 = 3/4$ and $k_3 = 1$. The other parameters in (34) are defined as

$$A_1 = \begin{bmatrix}1\\0\\0\end{bmatrix},\ A_2 = \begin{bmatrix}-1 & -1\\ 1 & 0\\ 0 & 1\end{bmatrix},\ F_2(t) = \begin{bmatrix}f(t-h)\\ f(t-\frac{5h}{4})\\ f(t-\frac{3h}{2})\end{bmatrix},$$

$$A_3 = \begin{bmatrix}-1 & 1 & 1 & 1 & 0 & -1 & 0 & 0 & 0\\ -1 & -1 & -1 & 1 & 1 & 1 & -1 & 0 & 0\\ 0 & -1 & -1 & -1 & 1 & 1 & 0 & 1 & -1\end{bmatrix}.$$

Let A_2^L be third order matrix, where all elements are unity. As a result, algorithm (34) is defined as

$$f_a^4(t) = 2f(t-h) + f(t-\frac{5h}{4}) + f(t-\frac{3h}{2}) - f(t-\frac{7h}{4}) - 2f(t-2h) - f(t-\frac{9h}{4}) + f(t-\frac{11h}{4}) - f(t-\frac{5h}{2}) + f(t-3h). \quad (38)$$

The transient time is equal to $T = 3h$.

Let us show the prediction quality of algorithms (35)-(38). Consider the unknown function $f(t)$:

$$f(t) = 0.1t + 1 + \sin(0.05t) + \cos(0.11t) + \sin(0.21t + \pi/3) + \sin(0.27t + \pi/4). \quad (39)$$

Predict function (39) to the time h= 1 (s) on-line. Fig. 1, 2 show the plots of prediction errors $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$, where $e_i(t)$ are defined as $e_i(t) = f_a^i(t) - f(t)$, $i = \overline{1,4}$.

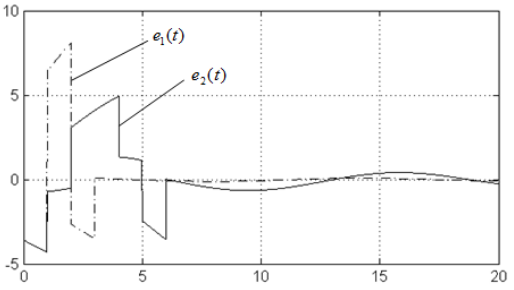


Fig. 1. The plots of prediction errors $e_1(t)$, $e_2(t)$

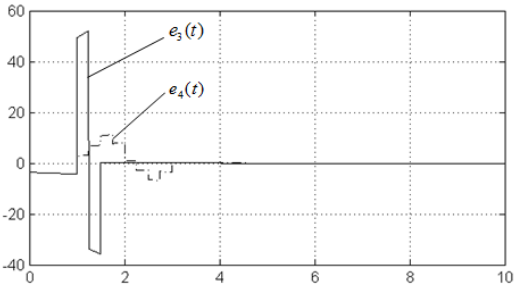


Fig. 2. The plots of prediction errors $e_3(t)$, $e_4(t)$

In Fig. 1, 2 the values of $e_i(t)$, $i = \overline{1,4}$ do not exceed 0.66, 0.12, 0.035 and 0.126 after 6, 3, 1.5, and 3 (s) respectively.

Let us check algorithms (35)-(38) on the random function

$$f(t) = W(p)x(t), \tag{40}$$

where $p = d/dt$, $W(s) = \dfrac{1}{s(20s+1)}$, s is a complex variable, $x(t)$ is a piecewise continuous function with a normal distribution, where the mean, variance and sampling time are equal to 0, 1 and 0.001 (s) respectively.

Predict the function (40) on the value of h = 1 (s) on-line. In Fig. 3 the graph of the random function $f(t)$ is presented. In Fig. 4 and Fig 5 the graphs of $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$ are given.

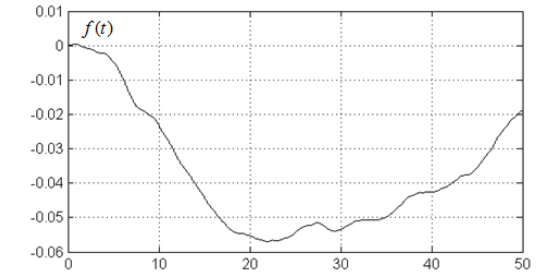


Fig. 3. The plots of random function $f(t)$

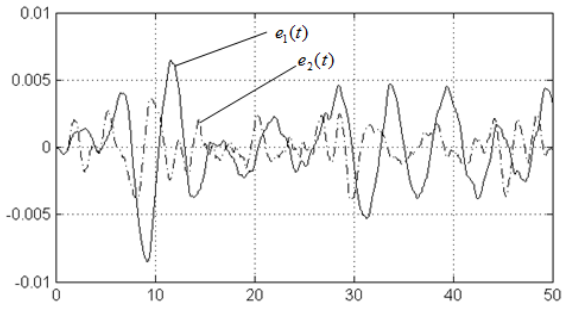


Fig. 4. The plots of prediction errors $e_1(t)$, $e_2(t)$

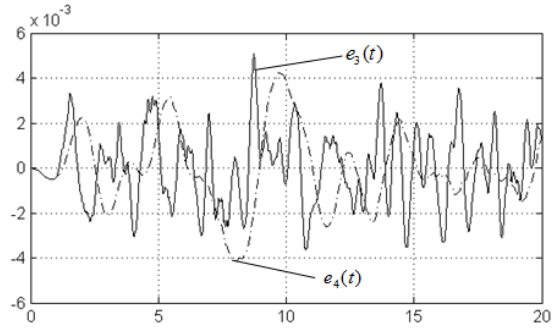


Fig. 5. The plots of prediction errors $e_3(t)$ and $e_4(t)$

In Fig. 4, 5 the values of $e_i(t)$ do not exceed 0.007, 0.004, 0.005 and 0.0045 after 6, 3, 1.5, and 3 (s) respectively. Note that the algorithms [2-4] cannot be applied to the prediction of function (40).

IV. Conclusion

The paper describes the algorithms which allows one to predict the smooth and bounded functions on-line. The synthesis of algorithms is based on Lagrange mean value theorem. The values of prediction time and estimate of the prediction error are proposed. Simulation results show the efficiency of the proposed algorithms.

In a practice the proposed results can be applied for control of plant with input time delay and presence of unknown disturbances [13].Particularly, in [13] the control of electrical motor with unknown torque is considered. Also, the results can be interpreted as disturbance observer in presence of time-delay in measurement.

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