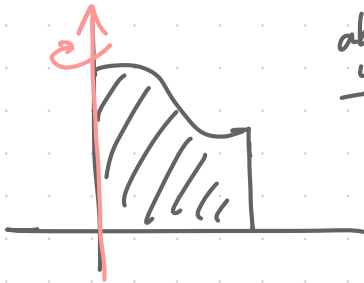
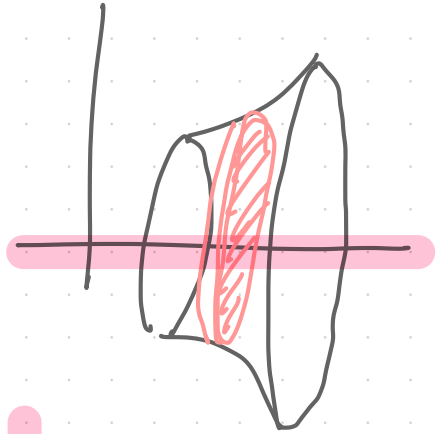
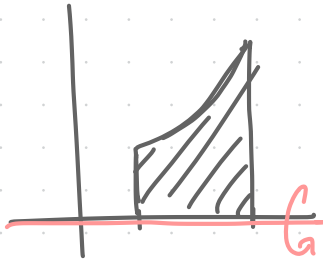


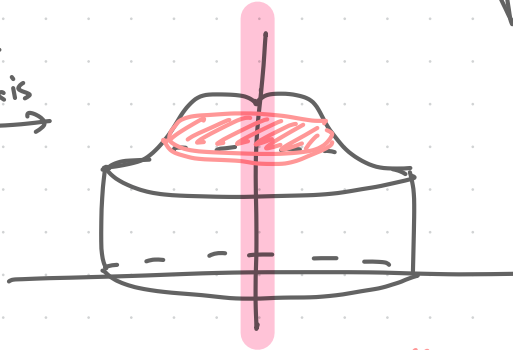
Volumes - Solids of Revolution

Form a 3d-solid by taking a region in the plane and rotating it about an axis called the axis of revolution

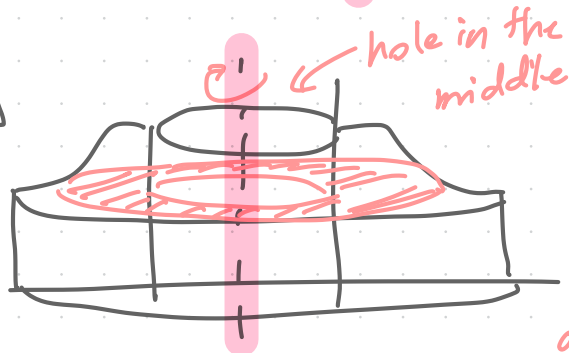
exs:



about
y-axis
→



about
 $x = -1$
↘



cross
sections
are washers

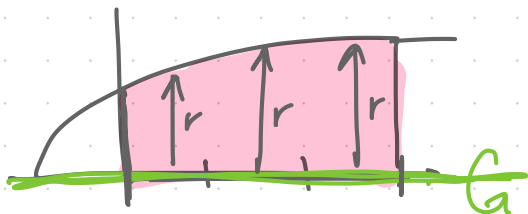
Method of Disks/Washers

for computing volumes of solids of revolution

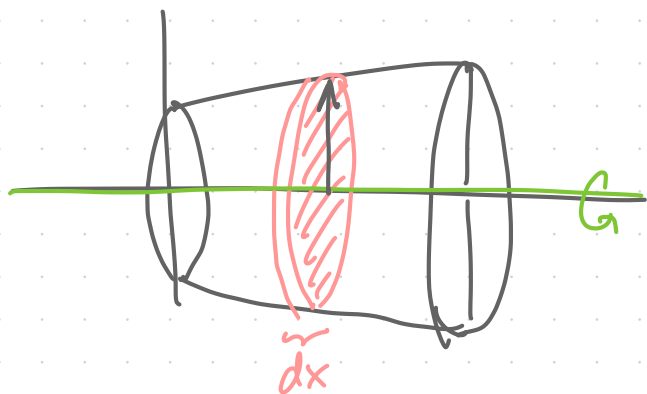
- ① Slice perpendicular to the axis of rev.
(so that your cross sections are disks or washers)
- ② Volume is the integral of cross sectional area & cross sectional area is straightforward (same every time)
- ③ Bounds are determined by the region.

ex: region bounded by $y = \sqrt{x+1}$, $x=0$,
 $x=3$ and $y=0$

rotated about x -axis



radius changes w/ x
and $r = \sqrt{x+1}$

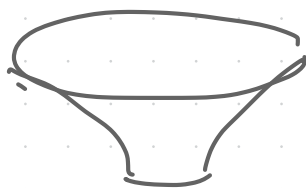
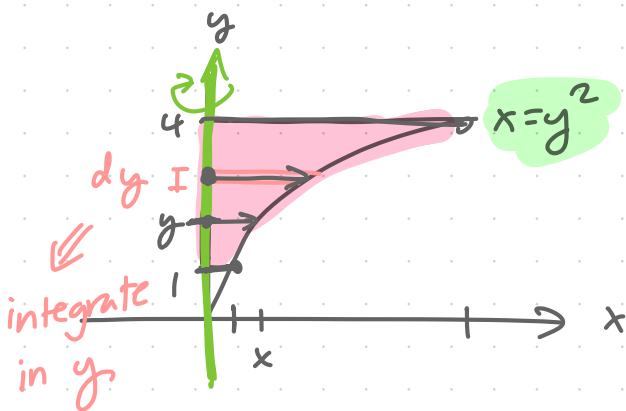


cross sections are disks: $A(x) = \pi r^2$
 $= \pi (\sqrt{x+1})^2$
 $= \pi (x+1)$

$$V = \int_0^3 A(x) dx = \int_0^3 \pi (x+1) dx = \frac{15\pi}{2}$$

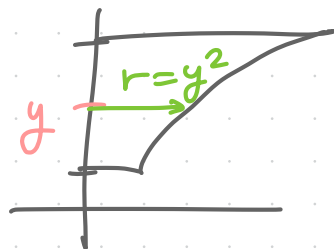
ex: region bounded by $x=y^2$, $y=1$, $y=4$
and $x=0$

rotated about the y -axis



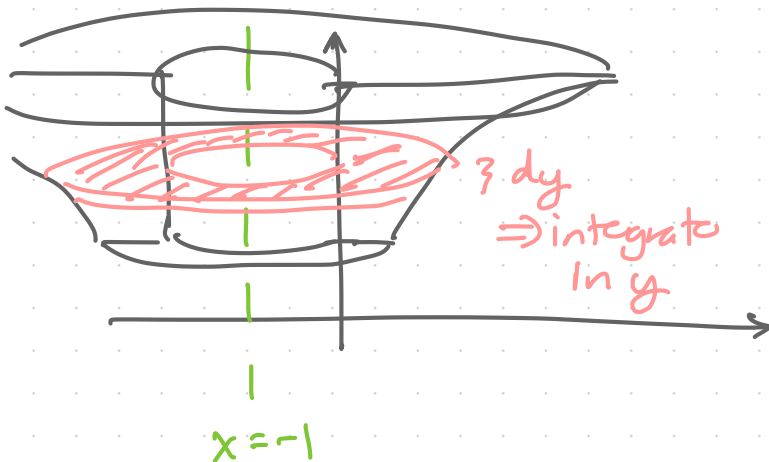
cross sectional area : $A(y) = \pi r^2$
 $= \pi (y^2)^2$
 $= \pi y^4$

* note: radius is always
from axis of revolution
to far side of region



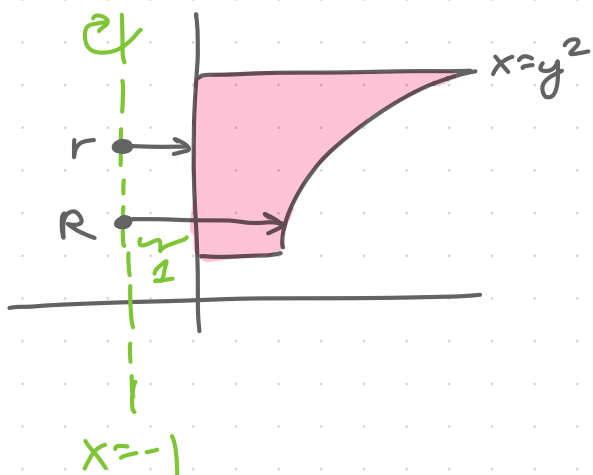
$$\begin{aligned}
 V &= \int_1^4 A(y) dy = \int_1^4 \pi y^4 dy \\
 &= \pi \left[\frac{1}{5} y^5 \right]_1^4 \\
 &= \frac{\pi}{5} (4^5 - 1) \\
 &= \frac{1023\pi}{5}
 \end{aligned}$$

ex: same region as previous ex
rotated about $x = -1$



$$\begin{aligned}
 \text{Area} &= \pi R^2 - \pi r^2 \\
 &= \pi (R^2 - r^2)
 \end{aligned}$$

~~*~~ NOT $(R-r)^2$



inner radius: $r = 1$
 outer radius: $R = y^2 + 1$

$$A(y) = \pi (R^2 - r^2)$$

$$= \pi ((y^2 + 1)^2 - 1)$$

$$V = \int_1^4 A(y) dy$$

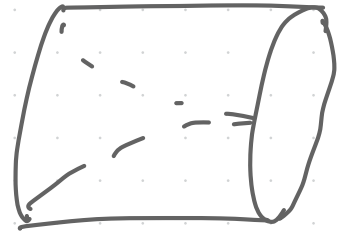
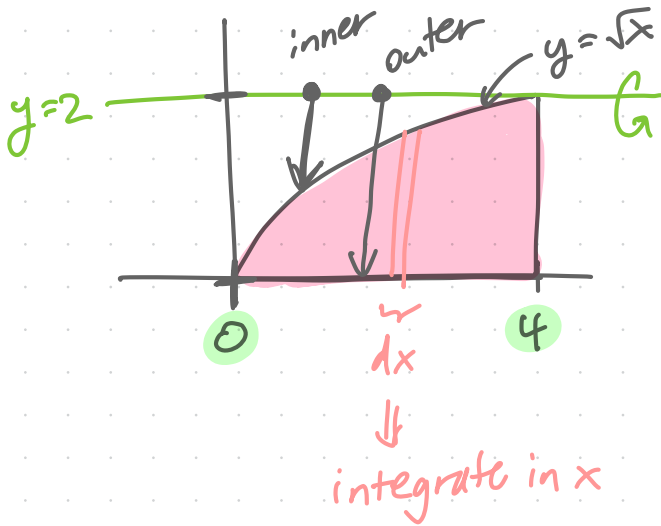
$$= \int_1^4 \pi ((y^2 + 1)^2 - 1) dy$$

$$= \pi \int_1^4 y^4 + 2y^2 + \cancel{1} - \cancel{1} dy$$

$$= \frac{1233\pi}{5}$$

ex: region bounded by $y = \sqrt{x}$, $y = 0$
and $x = 4$

rotated about $y = 2$



washers b/c
hole in the
middle

$$\text{inner: } r = 2 - \sqrt{x}$$

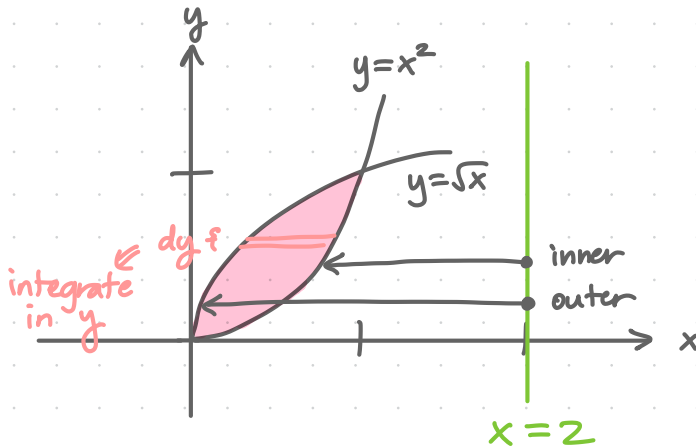
$$\text{outer: } R = 2$$

$$A(x) = \pi (R^2 - r^2) \\ = \pi (4 - (2 - \sqrt{x})^2)$$

inner radius
goes from
axis of rev.
to closest
edge of your
region

$$V = \int_0^4 A(x) dx = \int_0^4 \pi (4 - (4 - 4\sqrt{x} + x)) dx = \frac{40\pi}{3}$$

ex: region bounded by $y=x^2$ and $y=\sqrt{x}$
rotated about $x=2$



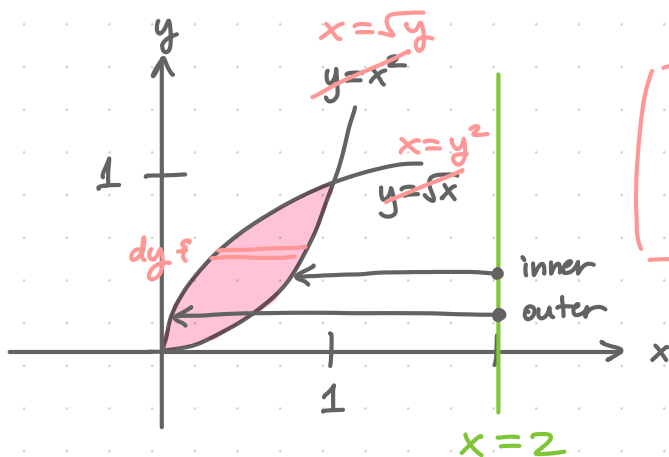
The inner radius stretches from $x=2$ to the curve $y=x^2$.

$$\text{So } r = 2 - x^2 ?$$

BUT we want to integrate in y , so first express the curves as functions of y (i.e. write them as $x = \underline{\hspace{1cm}}$)

$$y = x^2 \Rightarrow x = \sqrt{y}$$

$$y = \sqrt{x} \Rightarrow x = y^2$$



$$S_0 \quad A(y) = \pi(R^2 - r^2) \\ = \pi((2 - y^2)^2 - (2 - \sqrt{y})^2)$$

$$V = \int_0^1 A(y) dy$$

$$= \int_0^1 \pi((2 - y^2)^2 - (2 - \sqrt{y})^2) dy$$

$$= \pi \int_0^1 (\cancel{4} - 4y^2 + y^4) - (\cancel{4} - 4\sqrt{y} + y) dy$$

$$= \pi \int_0^1 y^4 - 4y^2 - y + 4y^{1/2} dy$$

$$= \pi \left[\frac{1}{5} y^5 - \frac{4}{3} y^3 - \frac{1}{2} y^2 + 4 \cdot \frac{2}{3} y^{3/2} \right]'$$

$$= \pi \left(\frac{1}{5} - \frac{4}{3} - \frac{1}{2} + \frac{8}{3} \right) \quad 5 \cdot 2 \cdot 3 = 30$$

$$= \pi \left(\frac{6}{30} - \frac{40}{30} - \frac{15}{30} + \frac{80}{30} \right)$$

$$= \frac{31\pi}{30}$$