

Definition: For an $n \times n$ matrix A , we say that a nonzero vector $\mathbf{v}$ is an **eigenvector** of A if

$$A\mathbf{v} = \lambda\mathbf{v} \quad \text{for some scalar } \lambda.$$

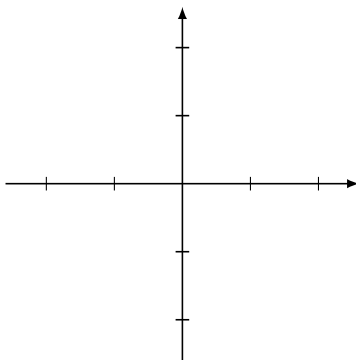
In this case we say that $\mathbf{v}$ is an eigenvector of A with eigenvalue λ .

1. (a) Determine which of the following vectors are eigenvectors of A , where $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$. If the vector is an eigenvector of A , give the corresponding eigenvalue.

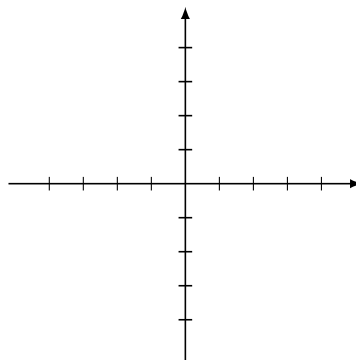
$$\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

- (b) For each vector from part (a), graph the vector and A times the vector on the same axes.

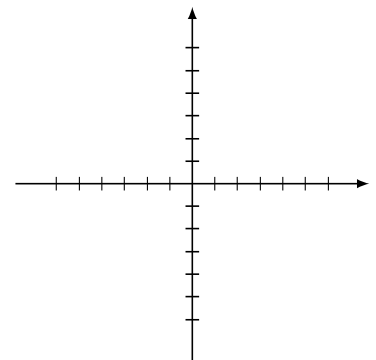
Graph $\mathbf{v}$ and $A\mathbf{v}$



Graph $\mathbf{u}$ and $A\mathbf{u}$



Graph $\mathbf{w}$ and $A\mathbf{w}$



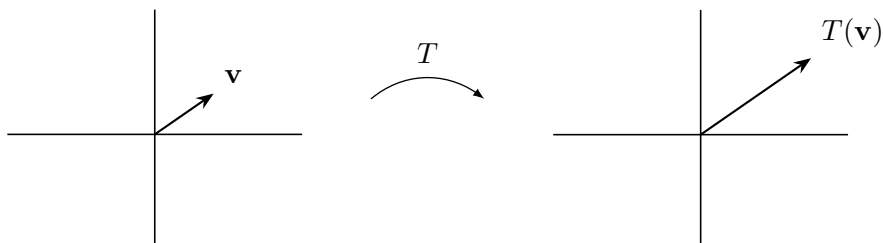
- (c) If $\mathbf{v}$ is an eigenvector of A , what does it mean geometrically?

Connection to Linear Transformations

Consider a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ (which has an associated 2×2 matrix A). If $\mathbf{v}$ is an eigenvector of A with eigenvalue λ , then

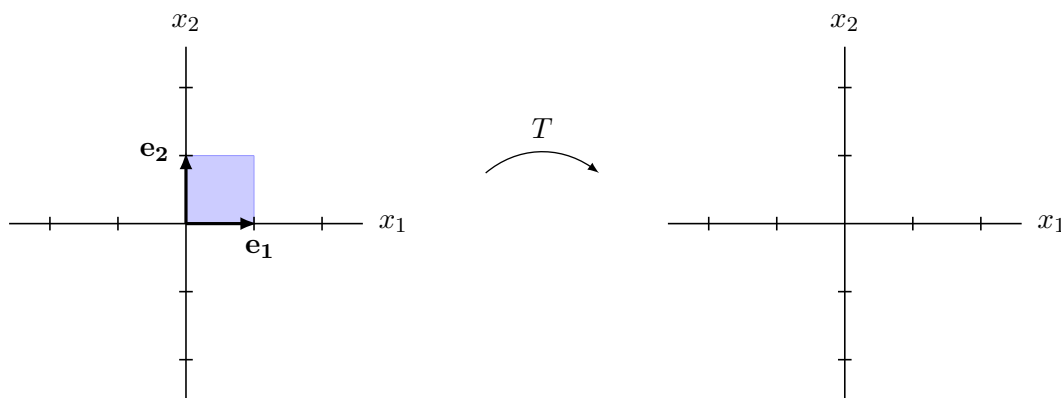
$$T(\mathbf{v}) = \lambda \mathbf{v}.$$

Graphically:



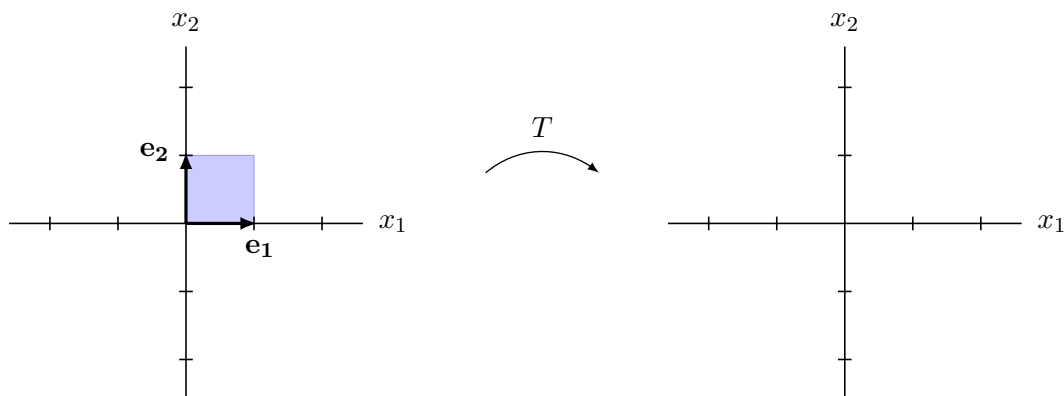
2. Describe in words what the picture above is illustrating in terms of the input $\mathbf{v}$ and the output $T(\mathbf{v})$.
3. (a) Recall that one way to visualize a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ is to look at how it transforms the unit square. Draw the image of the unit square under $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}.$$



- (b) Using your drawing from part (a), find an eigenvector for T . What is its eigenvalue? Can you find any other eigenvectors of T ?

4. (a) Draw the image of the unit square under $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$.



- (b) Using your drawing from part (a), find an eigenvector for T . What is its eigenvalue? Can you find any other eigenvectors of T ?