

LOC 5 SPG 15
①

I)

A)
$$\begin{bmatrix} N_1 & N_2 \\ V_1 & V_2 \\ E_1 & E_2 \end{bmatrix}$$

$$N^0 = N_1 + N_2$$

$$\Rightarrow N_2 = N^0 - N_1 \Rightarrow \frac{dN_2}{dN^0} = -1$$

$$V^0 = V_1 + V_2$$

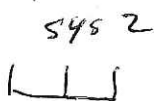
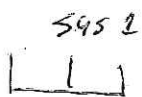
$$E^0 = E_1 + E_2$$

$$E_2 = E^0 - E_1$$

B) $\Omega^0 = \Omega_1 \cdot \Omega_2 = \Omega^0(E_1, V_1)$

II) ASIDE: ON Ω^0 :

EX 1



1 x 4

x

2 x 4

x

3 x 4

x

4 x 4

x

5 x 4

x

6 x 4

x

EX 2
2 systems; 2 ptels
IN sys 1, 1 ptel
sys 2

⇒ MAJOR POINT

$$\Omega^0 = \begin{pmatrix} \text{number} \\ \text{energy} \\ \text{states} \\ \text{sys.} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \text{number} \\ \text{en.} \\ \text{states} \\ \text{sys} \\ 2 \end{pmatrix} = \Omega_1 \cdot \Omega_2$$

4) EX 2 ASIDE 2: SOME (FAIRLY SIMPLE)
COMBINATORICS - USEFUL / ESSENTIAL LATER - WARM-UP

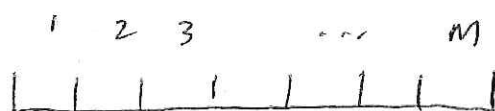
Lec. 5
5/19/25

EX 1 HOW MANY WAYS ARE THERE FOR PLACING N distinguishable objects (particles) in

SAY M boxes (energy levels) w/ LIMIT OF NO MORE THAN 1 ptcl per box.

CASE 1 $M < N$ (unphysical)

SINCE WE CAN'T HAVE MORE PARTICLES THAN PLACES = energy states)



$$\Rightarrow \left(\begin{matrix} N \text{ possibilities} \\ \text{box 1} \end{matrix} \right) \cdot \left(\begin{matrix} N-1 \text{ poss.} \\ \text{box 2} \end{matrix} \right) \cdot \left(\begin{matrix} N-2 \text{ poss} \\ \text{box 3} \end{matrix} \right) \cdots \left(\begin{matrix} N-M+1 \text{ poss} \\ \text{box M} \end{matrix} \right)$$

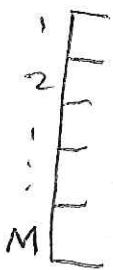
$$\Rightarrow N \cdot (N-1) \cdot (N-2) \cdots (N-M+1) \cdot \frac{[(N-M)(N-M-1) \cdots (N-M)]}{(N-M)!}$$

$$C_1 = \frac{N!}{(N-M)!}$$

FIRST FLYBY COMBINATORICS

LOC 5

EX 2 SAME: MAY OR 1 ^{distinguishable} ptcl. SPQ 15 (3)
per box, but now $M > N$.



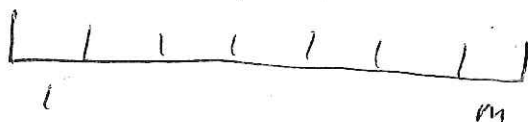
$$C_2^* \Rightarrow \binom{M \text{ poss. boxes ptcl}}{1} \cdot \binom{M-1 \text{ poss. boxes ptcl}}{2} \cdot \binom{M-2}{3} \cdots$$

$$\Rightarrow C_2 = M \cdot (M-1) \cdot (M-2) \cdots (M-N+1) \frac{[(M-N) \cdot (M-N-1) \cdots 1]}{(M-N)!}$$

$$\therefore C_2 = \frac{M!}{(M-N)!}$$

EX 3

HOW MANY WAYS OF PLACING N distinguishable ptcls in M boxes, where there's NO LIMIT ON THE NUMBER OF ptcls per box, and where M can be larger than, or smaller than N .



$$\Rightarrow C_3 = \binom{M \text{ poss locations for ptcl 1}}{1} \cdot \binom{M \text{ poss locations for ptcl 2}}{2} \cdots \binom{M \text{ poss locations for ptcl N}}{N}$$

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$$\text{or } C_3 = M \circ M \circ \dots \circ M$$

$\uparrow \quad \uparrow \quad \quad \uparrow$
 $\text{ptol}_1 \quad \text{ptol}_2 \quad \quad \text{ptol}_N$

$$\boxed{C_3 = M^N}$$

⇒ 1) @ equiv

ROUGH WORK

$$\frac{\partial R}{\partial \sigma_1} \Rightarrow B_1 = B_2 \Leftrightarrow \left. \frac{\partial \ln R_1}{\partial \sigma_1} \right|_{\sigma_1, N} = \left. \frac{\partial \ln R_2}{\partial \sigma_2} \right|_{\sigma_2, N}$$

2) connected to S $\Rightarrow \mu_1 = \mu_2 = \left. \frac{\partial \ln R_1}{\partial \sigma_1} \right|_{\sigma_1, N} = \left. \frac{\partial \ln R_2}{\partial \sigma_2} \right|_{\sigma_2, N}$
 since $S = S(\sigma, N)$

$$\Rightarrow ds = f d\sigma + \frac{f}{T} dT + \mu dN$$

3) ⇒ assuming

$$ds = k \ln R$$

3) AS1 $\Rightarrow S = k \ln R$

$$\Rightarrow \text{then } B_1 = B_2$$

$$\Rightarrow \left. \frac{1}{k} \frac{\partial S}{\partial \sigma_1} \right|_{\sigma_1, N_1} = \left. \frac{1}{k} \frac{\partial S}{\partial \sigma_2} \right|_{\sigma_2, N_2}$$

$$\Rightarrow \boxed{T_1 = T_2} \quad \text{Ans}$$

Ans $\mu_1 = \mu_2$

$$\Rightarrow \left. \frac{1}{k} \frac{\partial S}{\partial \sigma_1} \right|_{\sigma_1, N_1} = \left. \frac{1}{k} \frac{\partial S}{\partial \sigma_2} \right|_{\sigma_2, N_2}$$

$$\Rightarrow \frac{P_1}{kT_1} = \frac{P_2}{kT_2}$$

$$\Rightarrow \boxed{P_1 = P_2}$$

AS120 3

PROPOSITION
 MAC PARTIAL FROM (2.043)

$$R \in \mathcal{C}(\mathcal{V}, N) = f(E) \cdot \mathcal{V} \cdot N$$

$$R = \frac{\partial \Phi}{\partial \sigma} \delta \sigma =$$

$$\frac{\partial}{\partial \sigma} \left(\Phi_1^f \left(\frac{E}{f} \right) \right) \delta \sigma$$

$$= f \Phi_1^{f+1} \left(\frac{E}{f} \right) \frac{\partial \Phi_1}{\partial \sigma} \delta \sigma$$

$$\approx f \left(\frac{E}{f} \right)^{f+1} \frac{\partial \Phi_1}{\partial \sigma} \delta \sigma$$

$$\approx \frac{E^f}{f^f} \delta \sigma$$

$$\therefore \ln R \approx f \ln E - f \ln f + \ln \delta \sigma$$

(I)

(II)

$$= f \ln E + f \ln e$$

Since $f \gg 1$ and $\delta \sigma$

$$\approx f \ln E + \ln \left(\frac{\delta \sigma}{f^f} \right)$$

$$i) \delta \sigma \sim M E$$

$$\sim M(E/f)$$

$$ii) \therefore R \sim \left(\frac{E}{f} \right)^f M \left(\frac{E}{f} \right)$$

$$iii) \ln R \sim f \ln(E/f)$$

$$+ \ln \left(\frac{\delta \sigma}{f^f} \right)$$

$$iv) \therefore \boxed{\ln R \sim f \ln E}$$

$$\sim f \ln \phi_E$$

$$v) \therefore \boxed{\ln R \sim O(f)}$$

rough notes

$$\Phi_1 \sim \left(\frac{E}{f} \right)^f$$

$$\Rightarrow \frac{\partial \Phi_1}{\partial \sigma} \approx \frac{\alpha}{f^f} E^{f-1}$$

$$\Phi_1^{f-1} \approx \left(\frac{E}{f} \right)^{f(f-1)}$$

$$E/f \approx e$$

$$f \sim E/e$$