Recall the method of u-substitution for evaluating integrals:

u-Substitution (Change of Variable Method):

If $u = g(x)$ is a differentiable function whose range is and interval I and f is continuous on I, then

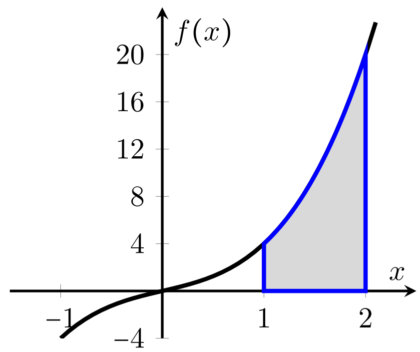
$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

To get a better conceptual understanding of what u-substitution is doing to our integrals, we will examine the graphs of an original integral and its transformed u-substitution integral.

1. We will first examine the integral

$$\int_1^2 2x(1+x^2)\,dx.$$

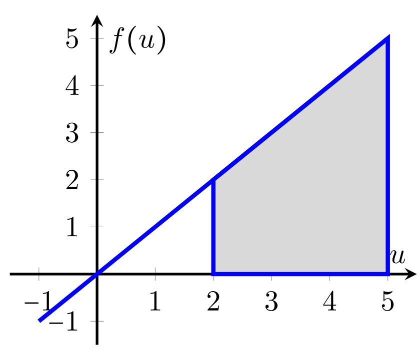
(a) Shade the area represented by the integral on the graph below:



(b) Use technology to calculate the area of this region. $\int_1^2 2x(1+x^2)\,dx$=10.5

(c) Transform the integral using u-substitution. Let $u = 1 + x^2$, then $\int_1^2 2x(1+x^2)\,dx = \int_2^5 u\,du$

(d) Graph the transformed integral on the axis below. *Pay attention to the axis labels!*



(e) Calculate the area of this region. How does it compare to the area for $\int_1^2 2x(1+x^2)\,dx$?

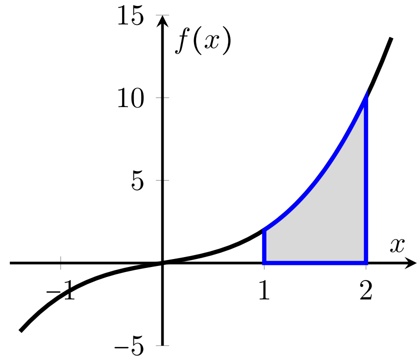
The area under the curve is equal to the integral $\int_2^5 u\,du = \frac{21}{2}$. The u-sub transformed the cubic curve to a line, and the bounds transformed $1 \mapsto 2$ and $2 \mapsto 5$. The areas are equal!

Now we will adjust our integral slightly to see if this impacts anything.

2. Now examine the integral

$$\int_1^2 x(1+x^2)\,dx$$

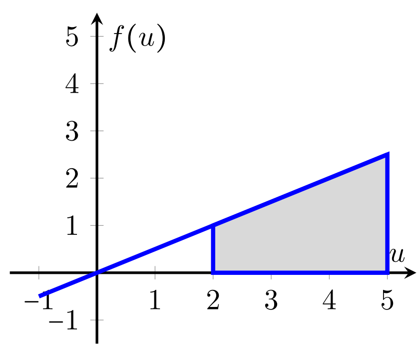
(a) Shade the area represented by the integral on the graph below:



(b) Use technology to calculate the area of this region. $\int_1^2 x(1+x^2)\,dx = 5.25$

(c) Transform the integral using u-substitution. Let $u = 1+x^2$, then $\int_1^2 2x(1+x^2)\,dx = \frac{1}{2}\int_2^5 u\,du$

(d) Graph the transformed integral on the axis below. *Pay attention to the axis labels!*



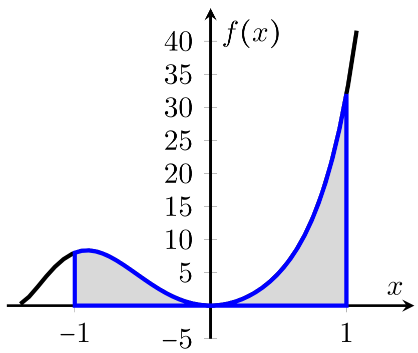
(e) Calculate the area of this region. How does it compare to the area for $\int_1^2 x(1+x^2)\,dx$?

The area under the curve is equal to the integral $\int_2^5 \frac{1}{2}u\,du = \frac{21}{4}$. The u-sub transformed the cubic curve to a line, and the bounds transformed $1 \mapsto 2$ and $2 \mapsto 5$. We can think of this as the areas are equal -OR- we can think the area of $\int_2^5 u\,du$ is double the area of $\int_1^2 x(1+x^2)\,dx$.

3. Using full sentences, write a brief summary of the geometric transformation that occurs to the integral

$$\int_{-1}^{1} 2x^2(3+x^3)^2\,dx$$

when you evaluate it using u-substitution.



If let $u = 3 + x^3$, then we have $du = 3x^2\,dx$. This transforms our integral $\int_{-1}^{1} 2x^2(3+x^3)^2\,dx = \int_{2}^{4} u^2\,du$. So the 8^{th} degree polynomial $2x^2(3+x^3)^2$ is transformed to the quadratic $\frac{2}{3}u^2$. The bounds are transformed $-1 \to 2$ and $1 \to 4$. We can then evaluate the integrals to see the areas are equal.