Instructions: Integrate each of the trig functions and each of the trig functions squared. Four of these are simple antiderivatives, six require a trig identity, and two require a special trick.

1. (a) $\int \sin x\,dx = -\cos x + C$

(b) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$ $u = \cos x$, $du = -\sin x\,dx$

$= \int -\frac{1}{u}\,du$

$= -\ln|\cos x| + C$

(c) $\int \sec x\,dx = \int \sec x\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$

$u = \sec x + \tan x$, $du = \sec x \tan x + \sec^2 x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|\sec x + \tan x| + C$

(d) $\int \cos x\,dx = \sin x + C$

(e) $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$ $u = \sin x$, $du = \cos x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|\sin x| + C$

(f) $\int \csc x\,dx = \int \csc x\left(\frac{\csc x + \cot x}{\csc x + \cot x}\right)dx$

$= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x}\,dx$

$u = \csc x + \cot x$, $du = -\csc x \cot x - \csc^2 x\,dx$

$= \int -\frac{1}{u}\,du$

$= -\ln|u| + C$

$= -\ln|\csc x + \cot x| + C$

2. (a) $\int \sin^2 x\,dx = \int \frac{1}{2}(1 - \cos(2x))\,dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right) + C$

$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

(b) $\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx$

$= \tan x - x + C$

(c) $\int \sec^2 x\,dx = \tan x + C$

(d) $\int \cos^2 x\,dx = \int \frac{1}{2}(1 + \cos(2x))\,dx$

$= \frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right) + C$

$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

(e) $\int \cot^2 x\,dx = \int \csc^2 x - 1\,dx$

$= -\cot x - x + C$

(f) $\int \csc^2 x\,dx = -\cot x + C$