Is There Really a Dictator's Dilemma?
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Online Appendix

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A Repressiveness as cost of challenging the ruler

Setup

In the main text, we formalize the direct effect of repressiveness ω as a cost to the ruler of mobilizing against challenges from the opposition. Here we show that analogous results follow if we instead formalize the direct effect of repressiveness as a cost to the opposition of challenging the ruler.

The ruler (R) and opposition (O) compete for power, where the winner receives a payoff normalized to 1. We assume that the regime has a given level of repressiveness $\omega \in \mathbb{R}_+$. The indirect effect of repressiveness—its effect on the informativeness of the public signal s of the ruler's popularity θ—is unchanged from the main text. We model the direct effect as follows. After observing the public signal s, the ruler chooses whether to mobilize the security apparatus, $\mu \in \{0, 1\}$, against a potential challenge. Mobilization has two effects. First, it may make the ruler *formidable*, in which case challenging the ruler is prohibitively costly. In particular, using $\phi \in \{0, 1\}$ to indicate that the ruler is formidable, we assume

$$\Pr(\phi = 1 \mid \mu) = \mu\pi(\omega),$$

with $\pi(\omega)$ continuous and strictly increasing in ω, $\pi(0) = 0$, and $\lim_{\omega\to\infty} \pi(\omega) = 1$. In essence, a more repressive regime can more effectively mobilize against potential challenges and, as a result, is more likely to become formidable. Second, if the ruler was not already unpopular, mobilization turns (the non-strategic) citizens against the ruler—say, because citizens react negatively to secret-police raids or the presence of troops in the street. Formally, we assume the state variable transitions to $(1 - \mu)\theta$. This latter consequence represents a potential cost of mobilization to the ruler; whether the cost is incurred depends on whether the ruler was popular to begin with.

After observing the public signal s, the mobilization decision μ, and its effect ϕ, the opposition chooses whether to challenge the ruler, $c \in \{0, 1\}$. A challenge is successful, providing a reward of 1, if and only if the ruler is unpopular, $(1 - \mu)\theta = 0$. Potentially

offsetting this (potentially uncertain) benefit, the opposition incurs a cost of challenging the ruler equal to $k \in (0,1)$ if the ruler is not formidable ($\phi = 0$) and $k + f$ if the ruler is formidable ($\phi = 1$). The payoff to the opposition is therefore

$$c \cdot [1 - (1 - \mu)\theta - k - \phi f]. \tag{A1}$$

To rule out trivial cases, we assume $p < 1 - k < f$, which implies that the opposition would challenge the ruler in the absence of additional information or mobilization, but that the opposition would not challenge an (unpopular) ruler who is formidable.

Analysis

Consider first the case in which the public signal $s = 0$, such that it is common knowledge that the ruler is unpopular. In this case, if the ruler does not mobilize, the opposition challenges and defeats the ruler with certainty. In contrast, by mobilizing against a potential challenge, the ruler (who remains unpopular) becomes formidable with probability $\pi(\omega)$. As challenging a formidable ruler is prohibitively costly, the ruler always prefers to mobilize when $s = 0$. The ruler survives with probability $\pi(\omega)$.

Now consider $s = 1$. If the ruler chooses not to mobilize, then the opposition prefers not to challenge if and only if its posterior belief that the ruler is unpopular, given the public signal s, does not justify the cost of challenging, $1 - \tilde{p}(\omega) \leq k$, where $\tilde{p}(\omega)$ is given by Equation 2. As in the main text, this condition holds if

$$\omega \leq \underline{\omega}(p) \equiv q^{-1}\left(\frac{k}{1-k} \cdot \frac{p}{1-p}\right).$$

The ruler therefore prefers not to challenge if $\omega \leq \underline{\omega}(p)$. In contrast, if $\omega > \underline{\omega}(p)$, the ruler has a choice of two lotteries. By not mobilizing, the ruler confronts the opposition (now doubtful of the ruler's popularity, notwithstanding the public signal $s = 1$), in which case the ruler prevails if and only if he is popular, which is true with probability $\tilde{p}(\omega)$. If instead the ruler mobilizes, he becomes unpopular but, with probability $\pi(\omega)$, is also formidable. The ruler therefore prefers not to mobilize if and only if $\tilde{p}(\omega) \geq \pi(\omega)$.

As $\tilde{p}(\omega)$ is defined as in the main text, Lemma 1 holds here. The next proposition immediately follows.

Proposition A1. *Consider the model in which the direct effect of repressiveness is a cost of challenging the ruler. If the public signal $s = 0$, the ruler mobilizes ($\mu = 1$) and the opposition challenges ($c = 1$) if and only if the ruler fails to be formidable ($f = 0$), which occurs with probability $1 - \pi(\omega)$. If $s = 1$:*

1. *If $\omega \leq \underline{\omega}$, the ruler does not mobilize ($\mu = 0$) and the opposition does not challenge ($c = 0$).*
2. *If $\omega > \max\{\underline{\omega}(p), \overline{\omega}(p)\}$, the ruler mobilizes ($\mu = 1$) and the opposition challenges ($c = 1$) if and only if the ruler fails to be formidable ($f = 0$), which occurs with probability $1 - \pi(\omega)$.*
3. *If $p < \overline{p}$ and $\underline{\omega}(p) < \omega \leq \overline{\omega}(p)$, the ruler does not mobilize ($\mu = 0$) and the opposition challenges ($c = 1$).*

In all regions of the parameter space, the ruler's equilibrium strategy in Proposition A1 is identical to that in Proposition 1. The opposition, in contrast, challenges more often. By assumption, mobilization ensures that the ruler is unpopular but not that he is formidable. In particular, with probability

$$\overbrace{(1-p)\left[1-q(\omega)\right]}^{s=0} \cdot \underbrace{\left[1-\pi(\omega)\right]}_{f=0},$$

that is, with greater probability when the regime is minimally repressive, the ruler provokes a challenge that he does not win. But this outcome is also possible when the regime is maximally repressive (case 2 in Proposition A1), as then the ruler is sufficiently confident in his repressive capacity, and sufficiently skeptical of any declarations of support, that he mobilizes when $s = 1$, knowing that this may backfire.

B Private signal of popularity

Setup

In the main text, we assume that the ruler and opposition have symmetric information with respect to the ruler's popularity θ. Here we show that similar qualitative results follow if the ruler has access to private information about his popularity.

The timing is identical to that in the baseline model, with one exception. Following realization of the public signal, but before the ruler chooses whether to mobilize the security apparatus, the ruler observes a private signal $z \in \{0, 1\}$, drawn according to

$$\Pr(z = 1 \mid \theta) = \theta + (1 - \theta)\lambda.$$

The parameter $\lambda \in (0, 1)$ measures the ruler's "intelligence capacity," where smaller values indicate higher capacity. Observe that as λ approaches 1, the extension converges to the baseline model, in which there is no (useful) private information.

With private information, the model is now a signaling game, and the opposition uses the ruler's mobilization decision μ as well as the public signal s to form beliefs about the ruler's popularity. The ruler, in contrast, forms beliefs using both the public signal s and the private signal z. In our analysis, we restrict attention to perfect Bayesian equilibria that satisfy the D1 criterion, which eliminates equilibria supported by implausible beliefs off the equilibrium path.

Analysis

When $s = 0$, such that it is common knowledge that the ruler is unpopular, behavior on and off the equilibrium path is identical to that in the baseline model. The opposition challenges if and only if the ruler does not mobilize. Anticipating this, the ruler mobilizes.

Now consider the case when $s = 1$. This can be understood as a proper subgame, in which the ruler and opposition share the common "prior" $\tilde{p}(\omega, p)$; the ruler chooses $\mu \in \{0, 1\}$, given his private signal z; and the opposition chooses $c \in \{0, 1\}$, given the ruler's decision μ. Let

$\beta_z = \Pr(\mu = 1 \mid z)$ and $\gamma_\mu = \Pr(c = 1 \mid \mu)$ denote the ruler's and opposition's (behavioral) strategy, respectively. Note that, because challenging is costly and always unsuccessful when $\mu = 1$, any equilibrium must have $\gamma_1 = 0$. It remains to characterize β_0, β_1, and γ_0.

First, suppose $\gamma_0 = 1$, so that the opposition is sure to challenge when the ruler does not mobilize. Anticipating the opposition's challenge, the ruler's expected payoff from not mobilizing is

$$\Pr(\theta = 1 \mid z, s = 1) = z \cdot \frac{p}{p + (1-p)\, q(\omega)\, \lambda},$$

versus a payoff from mobilizing of $\pi(\omega)$. Thus, when $z = 0$, the ruler prefers not to mobilize. In contrast, when $z = 1$, the ruler strictly prefers to mobilize if and only if

$$\frac{p}{p + (1-p)\, q(\omega)\, \lambda} < \pi(\omega), \tag{A2}$$

which holds if and only if $\omega > \overline{\omega}_\lambda(p)$, where $\overline{\omega}_\lambda(p)$ is the value of ω for which Condition A2 holds with equality. Clearly, $\overline{\omega}_\lambda(p) > \overline{\omega}(p)$ and $\overline{\omega}_\lambda(p)$ is strictly decreasing in λ. For comparison with the baseline model, observe that $\lim_{\lambda \to 1} \overline{\omega}_\lambda(p) = \overline{\omega}(p)$.

Second, suppose the ruler chooses not to mobilize, while the opposition "knows" (i.e., correctly infers) z. Then, the opposition prefers to challenge when $z = 0$. In contrast, when $z = 1$, the opposition prefers to challenge if and only if

$$\frac{p}{p + (1-p)\, q(\omega)\, \lambda} < 1 - k, \tag{A3}$$

which holds if and only if $\omega > \underline{\omega}_\lambda(p)$, where $\underline{\omega}_\lambda(p)$ is the value of ω for which Condition A3 holds with equality. Clearly, $\underline{\omega}_\lambda(p) > \underline{\omega}(p)$ and $\underline{\omega}_\lambda(p)$ is strictly decreasing in λ. Analogously to the case for $\overline{\omega}_\lambda(p)$, $\lim_{\lambda \to 1} \underline{\omega}_\lambda(p) = \underline{\omega}(p)$.

Proposition A2. *Consider the model in which the ruler receives a private signal before choosing whether to mobilize. If the public signal $s = 0$, the ruler mobilizes ($\mu = 1$) and the opposition does not challenge ($c = 0$). If $s = 1$:*

1. *If $\omega < \underline{\omega}(p)$, the ruler does not mobilize ($\mu = 0$) and the opposition does not challenge ($c = 0$).*

2. *If* $\omega > \max\{\underline{\omega}_\lambda(p), \overline{\omega}_\lambda(p)\}$*, the ruler mobilizes* ($\mu = 1$) *and the opposition does not challenge* ($c = 0$).

3. *If* $\underline{\omega}_\lambda(p) < \omega < \overline{\omega}_\lambda(p)$*, the ruler mobilizes if and only if* $z = 0$ ($\beta_0 = 1, \beta_1 = 0$)*, whereas the opposition challenges if and only if* $\mu = 0$ ($\gamma_0 = 1$).

4. *If* $\underline{\omega}(p) < \omega < \underline{\omega}_\lambda(p)$*, the ruler does not mobilize when* $z = 1$ *and mobilizes with probability* $\beta^* \in (0,1)$ *when* $z = 0$ ($\beta_0 = \beta^*, \beta_1 = 0$)*, whereas the opposition does not challenge when* $\mu = 1$ *and challenges with probability* $\gamma^* \in (0,1)$ *when* $\mu = 0$ ($\gamma_0 = \gamma^*$).

Proposition A2 implies that several important results from the baseline model hold when the ruler has access to private information. When the regime is minimally repressive, so long as the public signal is positive, the ruler does not mobilize the security apparatus regardless of his private information; in the case, the opposition does not challenge. Further, when the regime is maximally repressive, the ruler always mobilizes, which deters the opposition's challenge. In between, when the regime is of intermediate repressiveness, equilibrium strategies are more complicated than the baseline model. If the regime has a "moderately high" level of repressiveness (Case 3 in Proposition A2), the opposition always challenges absent mobilization, whereas the ruler allows his private signal to guide his mobilization decision. In particular, and in direct analogy to the baseline model, when both the public and private signals are good, the ruler is sufficiently confident that he is popular that he does not mobilize the security apparatus, even though he knows that the opposition will challenge. In contrast, if the regime has a "moderately low" level of repressiveness (Case 4 in Proposition A2), equilibrium is in mixed strategies: the opposition challenges with probability strictly between zero and one when the ruler does not mobilize, whereas the ruler mobilizes with probability strictly between zero and one when the private signal is negative. In this case, the ruler takes a "double gamble": both that the opposition does not challenge and that he is popular. Finally, observe that equilibrium behavior in the extension converges to that in the baseline model as λ approaches to 1, given the limits of the thresholds $\underline{\omega}_\lambda(p)$ and $\overline{\omega}_\lambda(p)$.

Proof of Proposition A2. Consider an equilibrium $(\beta_0, \beta_1, \gamma_0)$. In what follows we characterize each quantity case by case. Before doing so, note that the only relevant off-path belief is the opposition's belief when the ruler does not mobilize, $\Pr(\theta = 1 \mid \mu = 0, s = 1)$. Given any γ_0', the ruler receives $\pi(\omega)$ by mobilizing and $1 - \gamma_0' + z \cdot \gamma_0' \cdot \frac{p}{p+(1-p)q(\omega)\lambda}$ by not mobilizing. Because $1 - \gamma_0' + \gamma_0' \cdot \frac{p}{p+(1-p)q(\omega)\lambda} \geq 1 - \gamma_0'$, with the inequality holding strictly when $\gamma_0' > 0$, the D1 criterion requires the opposition's off-path belief to put full probability on the ruler's having received the signal $z = 1$. This implies that, if $\mu = 0$ is off the equilibrium path, then

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \Pr(\theta = 1 \mid z = 1, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, \lambda}.$$

Case 1: $\omega < \underline{\omega}(p)$

Assume $\gamma_0 > 0$. First, $\mu = 0$ must be on the equilibrium path, as otherwise $\omega < \underline{\omega}(p)$ implies that

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, \lambda} > \frac{p}{p + (1-p)\, q(\omega)} > 1 - k,$$

so that it is optimal for the opposition not to challenge following $\mu = 0$, contradicting $\gamma_0 > 0$. Because $\mu = 0$ is on the equilibrium path, $\min\{\beta_0, \beta_1\} < 1$. Suppose $\beta_0 < 1$, which requires

$$1 - \gamma_0 + \gamma_0 \frac{p}{p + (1-p)\, q(\omega)\, \lambda} > 1 - \gamma_0 \geq \pi(\omega)$$

and in turn implies that $\beta_1 = 0$. As a result,

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, (\lambda + (1-\lambda)(1-\beta_0))} \geq \frac{p}{p + (1-p)\, q(\omega)} > 1 - k,$$

which again contradicts $\gamma_0 > 0$. Hence, it must be true that $\beta_0 = 1$ and $\beta_1 < 1$. But this leads to a similar contradiction to $\gamma_0 > 0$:

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, \lambda} > 1 - k.$$

Therefore, it must be true that $\gamma_0 = 0$. Given this, it is optimal for the ruler not to mobilize regardless of z, so that $\beta_0 = \beta_1 = 0$.

Case 2: $\omega > \max\left\{\underline{\omega}_\lambda(p), \overline{\omega}_\lambda(p)\right\}$

In this case, because $\omega > \underline{\omega}_\lambda(p)$,

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) \leq \Pr(\theta = 1 \mid z = 1, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, \lambda} < 1 - k,$$

so that it is optimal for the opposition to challenge following $\mu = 0$. As a result, $\gamma_0 = 1$. Then, when $z = 0$, it is optimal for the ruler to mobilize, leading to $\beta_0 = 1$. Moreover, because $\omega > \overline{\omega}_\lambda(p)$,

$$\frac{p}{p + (1-p)\, q(\omega)\, \lambda} < \pi(\omega),$$

so that it is optimal for the ruler to mobilize when $z = 1$, leading to $\beta_1 = 1$.

Case 3: $\underline{\omega}_\lambda(p) < \omega < \overline{\omega}_\lambda(p)$

Similarly to the previous case, $\omega > \underline{\omega}_\lambda(p)$ implies that $\gamma_0 = 1$ and $\beta_0 = 1$. But here, because $\omega < \overline{\omega}_\lambda(p)$,

$$\frac{p}{p + (1-p)\, q(\omega)\, \lambda} > \pi(\omega),$$

so that it is optimal for the ruler not to mobilize when $z = 1$, leading to $\beta_1 = 0$.

Case 4: $\underline{\omega}(p) < \omega < \underline{\omega}_\lambda(p)$

Assume first that $\gamma_0 = 0$, so that it is optimal for the ruler not to mobilize regardless of z, that is, $\beta_0 = \beta_1 = 0$. But because $\omega > \underline{\omega}(p)$,

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)} > 1 - k,$$

so that the opposition prefers to challenge following $\mu = 0$, contradicting $\gamma_0 = 0$. Hence, it must be true that $\gamma_0 > 0$.

Now assume $\beta_1 > 0$. Because $\gamma_0 > 0$, this requires

$$1 - \gamma_0 < 1 - \gamma_0 + \gamma_0 \frac{p}{p + (1-p)\, q(\omega)\, \lambda} \leq \pi(\omega),$$

which in turn implies that $\beta_0 = 1$. Then, because $\omega < \underline{\omega}_\lambda(p)$,

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, \lambda} > 1 - k,$$

so that it is optimal for the opposition not to challenge following $\mu = 0$, contradicting $\gamma_0 > 0$. Hence, it must be true that $\beta_1 = 0$.

Finally, assume $\gamma_0 = 1$. Then, it is optimal for the ruler to mobilize when $z = 0$, leading to $\beta_0 = 1$. But then, given that $\beta_0 = 1$ and $\beta_1 = 0$,

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, \lambda} > 1 - k,$$

leading to a contradiction to $\gamma_0 = 1$. Hence, it must be true that $\gamma_0 \in (0, 1)$. In turn, because $\gamma_0 \in (0, 1)$, the opposition must be indifferent between challenging and not challenging following $\mu = 0$, which, given that $\beta_1 = 0$, requires

$$\Pr(\theta = 1 \mid \mu = 0, s = 1) = \frac{p}{p + (1-p)\, q(\omega)\, (\lambda + (1-\lambda)(1-\beta_0))} = 1 - k.$$

There is a unique $\beta_0 \in (0, 1)$ that solves this equation, given that the left-hand side is strictly increasing in β_0 and that

$$\frac{p}{p + (1-p)\, q(\omega)} < 1 - k < \frac{p}{p + (1-p)\, q(\omega)\, \lambda}.$$

□